Mesopotamian Mathematics, Seen "from the Inside" (by Assyriologists) and "from the Outside" (by Historians of Mathematics)

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Abstract

Since the 1950s, "Babylonian mathematics" has often served to open expositions of the general history of mathematics. Since it is written in a language and a script which only specialists understand, it has always been dealt with differently by the "insiders", the Assyriologists who approached the texts where it manifests itself as philologists and historians of Mesopotamian culture, and by "outsiders", historians of mathematics who had to rely on second-hand understanding of the material (actually, of as much of this material as they wanted to take into account), but who saw it as a constituent of the history of mathematics. The article deals with how these different approaches have looked in various periods: pre-decipherment speculations; the early period of deciphering, 1847–1929; the "golden decade", 1929–1938, where workers with double competence (primarily Neugebauer and Thureau-Dangin) attacked the corpus and demonstrated the Babylonians to have possessed unexpectedly sophisticated mathematical knowledge; and the ensuing four decades, where some mopping-up without change of perspective was all that was done by a handful of Assyriologists and Assyriologically competent historians of mathematics, while most Assyriologists lost interest completely, and historians of mathematics believed to possess the definitive truth about the topic in Neugebauer's popularizations.

Keywords

Cuneiform script, decipherment · Mesopotamian mathematics, historiography · Hincks, Edward · Rawlinson, Henry · Oppert, Jules · Thureau-Dangin, François · Neugebauer, Otto

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1 "Through a Glass, Darkly": Historians of Mathematics Before Assyriology

Until 1850, historians of mathematics had no other way to know about pre-classical Near Eastern mathematics[1] than using the information they could draw from classical authors, at best submitted to historical and epistemological common sense—whence the quotations from 1 Corinthians 13:12 in the above headline (which entails no promise like that of St. Paul that in the end we shall see "face to face"). This, for instance, is Jean-Étienne Montucla's (1725–1799) account, in Enlightenment spirit, about "what is told" about the birth of arithmetic (Montucla 1758: I, 46f)[2]:

> The Phoenicians, some say, were the first and the most able merchants of the world; but Arithmetic is nowhere more useful and more necessary than in trade: these people must therefore also have been the first Arithmeticians. Strabon[3] relates this as the accepted opinion of his age; and even if we should believe a historian,[4] Phoenix son of Agenor wrote as first an arithmetic in Phoenician language. On the other hand Egypt boasts of having been the cradle of this art[5]; and since a human intelligence hardly seemed to suffice for so useful an invention, one devised the pious fable that a god was its author, and had communicated it to mankind.[6] At least it was the general opinion, according to Socrates or Plato[7] that *Theut* was the inventor of numbers, calculation and geometry; and it is quite likely that the Greeks took from here the idea to attribute to their Mercury, with whom *Theut* or the Egyptian *Hermes* has a conspicuous connection, the jurisdiction of trade and arithmetic.[8]
>
> But I shall insist no further on these fabulous or risky lines; who wants to discuss the origin of our knowledge somewhat philosophically will see that Arithmetic must have preceded everything else. The first civilized societies could not do without it; it suffices to

[1] A conceptual clarification: The "Near East" encompasses Egypt, the Palestino-Syrian area, Arabia and Mesopotamia—sometimes other neighbouring areas are included as well. Mesopotamia largely coincides with present-day Iraq. Its northern third is Assyria, and the remainder is Babylonia. Chaldea strictly speaking is the southern third (in the third millennium BCE Sumer), but often in the quotations that follow it stands for the whole of Babylonia.

[2] My translation, as everywhere below where nothing else is stated. All translated quotations can be found in original language in the preprint version of the article, on http://rudar.ruc.dk//bitstream/1800/10613/1/Hoyrup_2013_c_Mesopotamian_mathematics_from_the_inside_and_from_the_outside_S.pdf. In cases where the titles of publications have been translated, the genuine titles can be found in the bibliography. The notes to the quotation are due to Montucla, my additions are in square brackets. Similarly below for notes within quotations.

[3] *Geograph*.lib.XVII.

[4] Cedrenus [an 11th-century Byzantine historian].

[5] Diog. Laer. *in proemio*. [Hicks 1925: I, 12].

[6] *In Phædro*. p. 1240 ed. 1602. [274c].

[7] Ibid.

[8] [At this point, the astronomer Joseph-Jérôme Lalande (1732–1807) adds the following in the second edition—the earliest reasoned reference to *Babylonian* mathematics (Montucla 1799: I, 43f): "It is even quite difficult not to affiliate them with the Chaldeans. They, indeed, present us with the first traces of astronomical knowledge, very advanced at that. How would they, without that tool, have been able to discover several astronomical periods, knowledge of which has come down to us!" Apart from that, Montucla's passage is unchanged].

> possess something for being forced to use numbers, and even the first men, if only they had to count days, years, their age, that is enough for saying that they knew Arithmetic. Admittedly, richer or more trading societies may have expanded the limits of this natural Arithmetic by inventing perhaps shortened ways or procedures; and in this sense Strabon has said nothing contrary to reason. As regards *Josephus'* account[9] indicating Abraham as the first of Arithmeticians and making him teach the Egyptians the first elements of Arithmetic, it is easy to see that this historian wanted to adorn the first father of his nation with part of that knowledge which he saw honoured among foreigners. This is one of those pictures that will be favourably received only by some compiler deprived of critical sense and reasoning.

The last line could be directed at Petrus Ramus (1515–1572), in whose *Scholae mathematicae* (1569: 2) this story is taken for a fact (yet with a correct reference to chapter 8 of Josephus).[10]

Abraham Gotthelf Kästner (1719–1800) has no more sources than Montucla and is even more cautious in his *Geschichte der Mathematik* (1796: I, 2):

> To us, the oldest teachers of mathematics were the Greeks. What *they* may have learned from the Orient we only know from their own confessions, and how far their teachers have continued on their own, that they did not consider it necessary to write down. […].

These two quotations, with the addition quoted in note 8, illustrate how much could be known about the mathematics of Mesopotamia and neighbouring areas until the birth of Assyriology.

2 The Beginnings of Assyriology

The earliest dead languages and writing systems to be deciphered were Aramaic dialects—first Palmyrene in 1754, then in 1764 and 1768 Phoenician and Egyptian Aramaic (Daniels 1988, 431); all three scripts were alphabetic, and the basis was provided by bilingual texts containing proper names, which were skilfully exploited by Jean-Jacques Barthélemy (1716–1795).

Much more famous is Jean-François Champollion's (1790–1832) use of the Rosetta Stone in the decipherment of the hieroglyphs and the Demotic script (1824), proving the mixed alphabetic-ideographic character of the former as well as the existence of homophones in the alphabet.

[9] *Ant. Jud.* liv. I c. 9. [Actually chapter 8].

[10] Abraham is at least absent from Giuseppe Biancani's (1566–1624) *Clarorum mathematicorum chronologia* (1615, 39), and also from Gerardus Vossius's (1577–1649) *De universae mathesios natura et constitutione liber* and *Chronologia mathematicorum* (1650), while Polydorus Vergilius (c. 1470–1555) (1546, 59f) has no chapter reference. Since Montucla does not abstain from identifying Ramus by name when chiding him for following “the inclination of the mob toward everything that seems marvellous” (p. 450), the present reference is most likely at least not to be to Ramus alone.

The decipherment of the cuneiform scriptures was a more involved affair—a short description will illustrate how much more involved. It will make it clear why even understanding of cuneiform *mathematics* had to be made in very small and slow steps.

Initially, everything was based on the trilingual inscriptions from Persepolis, which Pietro della Valle (1586–1652) had seen in 1621 to be written from left to right.[11] The development until around 1800 is described by Fossey as on the whole a "period of groping and of hazardous and contradictory hypotheses" (p. 90). Noteworthy positive contributions were, firstly, Carsten Niebuhr's (1733–1813) new and more precise copies of the Persepolis inscriptions—his discovery that three different scripts are involved—and his confirmation of the writing direction (1774: II, 138f, pl. XXIII, XXIV, XXXI); and secondly, at the very close of the period, Friedrich Münter's (1761–1830) dating of the inscriptions to the Achaemenid era (1798, published in Danish in 1800)—his confirmation that three scripts are used—and his arguments that the first of these is alphabetic, the second apparently mixed alphabetic-syllabic and the third perhaps mixed alphabetic-logographic (Münter 1802, 83–86)—his identification of a few signs from the alphabetic script as vowels (ibid., 104–109)—and his identification of its language as Old Iranian (more precisely he suggests Zend). Also of importance was Münter's verification that the Persepolis writing type had also been used in Babylon, and that it had probably originated in Mesopotamia (*ibid*, 129–144).

In 1802, Georg Friedrich Grotefend (1775–1853) presented a memoir to the Göttingen Academy[12] which is habitually taken as the stumbling beginning of decipherment proper. He came to the same conclusions as Münter (whose work only appeared in German during the same year, and which Grotefend may not have known). He went further on three decisive points: showing that all inscriptions were linked to Darius and Xerxes; finding the royal names mentioned in the inscriptions as well as the word for king; and using this to identify a number of letters (he claimed identification of 29 letters of the alphabetic script, 12 of which were later confirmed).

Over the next four decades or so, a number of scholars extended and corrected Grotefend's work, removing false values and adding new ones (not always correctly at first), and identifying the language as an Old Persian dialect distinct from Zend (adding also new inscriptions to the corpus) (Fossey 1904, 112–146). However, all of this concerned the alphabetic script, which was certainly derived from the cuneiform script of Mesopotamia but had a totally different character (and moreover concerned matters without the slightest relation to mathematics).

[11] What follows about work done before 1860 is drawn, when no original sources are referred to, from Charles Fossey's (1869–1946) very detailed exposition of (good and bad) arguments and results (Fossey 1904, 85–220).

[12] Grotefend (1802) was published only in full in (Meyer 1893), for which reason I build on Fossey's account (1904, 102–111) of the arguments that circulated.

Decipherment of the second script (Elamite), using about one hundred signs and being in a language with no known kin, made some but little progress during the same period, and is anyhow irrelevant for the present purpose. Grotefend made some attempts at the third script, which is Akkadian (the language of which Babylonian and Assyrian are dialects). His firm belief that the language had to be an Iranian dialect was one of the reasons he had no success—but until the second half of the 1840s nobody else did much better. In the meantime, excavations had begun, and a much larger, geographically wider and chronologically deeper text corpus was now available.

From 1845 onward, a large number of workers took up discussion and competition about the third script, from which some 300 signs were known: Isidore Löwenstern (1810–1858; 1859; pertinent publications 1845 and onward); Henry Rawlinson (1810–1895; 1846 and onward); Paul-Émile Botta (1802–1870; 1847 and onward); Edward Hincks (1792–1866; 1846 and onward); Félicien de Saulcy (1807–1880; 1847 and onward); Henry de Longpérier (1816–1882; 1847); Charles William Wall (1780–1862; 1848); and Moriz Abraham Stern (1807–1884; 1850)—of whom Rawlinson, Botta and Hincks were by far the most important. Before 1855 it was known that the language of the third script was that of Babylonia and Assyria; that this language (Akkadian) was a Semitic language, and thus a cognate of Arabic and Hebrew; that the same sign might have (mostly several) phonetic and (often several) logographic values, and even function as a semantic determinative (an unexpected function which Champollion had discovered in Hieroglyphics); and that the original shape of the signs had been pictographic. Moreover, Hincks had shown early on that the inventors of the script must have spoken a non-Semitic language. This is all summarized in a letter written by the young Jules Oppert (1825–1905) in 1855 (published as (Oppert 1856)), together with observations and hypotheses of his own. So, from now on large-scale reading of documents could begin—and we may speak of the birth of Assyriology. In (1859), Oppert himself was to stabilize the field—in his obituary of Oppert, Léon Heuzey (1831–1922) was eventually to write as follows (Heuzey 1906, 7):

> After some works on ancient Persian, Oppert concentrated his principal effort on the Assyrian inscriptions. Having been charged by Fresnel with a mission into Babylonian territory, he published at his return, in 1859, a volume, the second tome (in date actually the first) of his *Expédition en Méslopotamie* [sic] in which, using recently discovered sign lists or syllabaries, he established the central rules of decipherment. This volume, Oppert’s *chef d’œuvre*, indicates a turning-point; it put an end to gropings and instituted Assyriology definitively.

3 Assyriologists’ History of Mathematics, 1847–1930

On one account Oppert says nothing in his letter from 1855, even though this was to be one of the things that occupied him during his later brilliant career: mathematics.

However, already in a paper read in 1847 (published as (Hincks 1848)), Hincks had described the "non-scholarly" number system correctly.[13] In comparison, of the 76 syllabic values identified in this early paper only 18 turned out eventually to be correct or almost correct, while 46 had the right consonant but erred in the vowel, and 12 were wholly wrong (Fossey 1904, 185)—which however was already a significant step forward. The discovery of the place-value system followed soon. It was also due to Hincks (1854a, 232), who detected it in a tabulated "estimate of the magnitude of the illuminated portion of the lunar disk on each of the thirty days of the month".[14] A slightly later publication dealing with the numbers associated with the gods (Hincks 1854b, 406f) refers to the "use of the different numbers to express sixty times what they would most naturally do" on the tablet just mentioned; there, 240 is indeed written as iv (Hincks uses Roman numerals for the cuneiform numbers), while "iii.xxviii, iii.xii, ii.lvi, ii.xl, etc." stand for "208, 192, 176, 160, etc.".

Rawlinson also contributed to the topic in (1855) (already communicated to Hincks when the second paper of the latter was in print, in December 1854). A five-page footnote (pp. 217–221) within an article on "The Early History of Babylonia" points out that the values ascribed by Berossos (Cory 1832, 32) to σάρος (*šār*), νήρος (*nēru*) and σώσσος (*šūši*), respectively 3600, 600 and 60 years, are "abundantly proved by the monuments" (p. 217). As further confirmation Rawlinson presents an extract of "a table of squares, which extends in due order from 1 to 60" (pp. 218–219), in which the place-value character of the notation is obvious but only claimed indirectly by Rawlinson. The note goes on as follows:

> while I am now discussing the notation of the Babylonians, I may as well give the phonetic reading of the numbers, as they are found in the Assyrian vocabularies.

All three of "cuneiform's 'holy' triad", as Rawlinson, Hincks and Oppert were called by Samuel Noah Kramer (1897–1990) (1963, 15), were indeed quite aware that numbers and what had to do with them was important for understanding Mesopotamian history and culture.[15]

The reason that this was so is reflected further on in Heuzey's obituary:

> Oppert's scientific activity followed many directions: historical texts and religious texts, bilingual (Sumero-Assyrian) texts and purely Sumerian texts, juridical texts and divination texts, Persian texts and neo-Susian texts, there is almost no branch of the vast literature of

[13]This system is sexagesimal but not positional until 100, after which it is combined with word-signs for 100 and 1000.

[14]Archibald Henry Sayce (1845–1933), when returning to the text in (1875, 490; cf. Sayce 1887, 337–340), reinterprets the topic as a table of lunar longitudes. Geometrically, the two interpretations are equivalent, but the final verb of the lines (DU, "to go") suggested this new understanding.

[15]In contrast, the just published Blackwell *Encyclopedia of Ancient History* planned the same number of pages for Mesopotamian mathematics and Mesopotamian hairstyles. It should be added that those who planned the volume had little idea about Mesopotamia (nor were they very interested in receiving advice, however).

the cuneiform inscriptions he has not explored. The most special questions, juridical, metrological, chronological, attracted his curiosity […].

Evidently, administrative, economical and historiographic documents could—and can—only be understood if numeration and metrology were/are understood. Reversely, such documents, in particular administrative and economical records, are and were important sources for understanding numeration and metrology. Oppert's observations on "the notation for capacity measures in cuneiform juridical documents" from (1886) offer an example.

For a long time, however, they were far from being the only sources for knowledge and assumptions. Already the decipherment of the scripts had drawn much on sources from classical Antiquity (how else could the names of the Achaemenid kings have been known?) and on comparison with Zend, Hebrew and Arabic. Similarly, known or supposedly known metrologies and numerical writings from classical Antiquity were drawn upon—sometimes with success (Rawlinson's use of Berossos is an example), sometimes with exaggerated confidence in the stability and uniformity of metrologies. Didactical materials such as bilingual lexical lists and tables also played their role (as they had done in the decipherment); so did astronomical texts (as already for Hincks and Rawlinson in 1854–55).

Three illustrative examples are (Norris 1856, Smith 1872, Oppert 1872). Edwin Norris (1795–1872) not only draws much on Biblical material in his dubious article but also reads the cuneiform signs on Mesopotamian weight standards as Hebrew characters ("I thought the first word looked like מנה"—p. 215). George Smith (1840–1876) makes use of metrological lists in order to establish the sequence of length units and their mutual relations, and of "lion" and "duck weights" (that is, stone sculptures of these animals on which their weight is inscribed) and of written documents in order to reach a similar understanding of the weight system (which turns out to be contradictory).

Oppert makes use of similar material. But he also believes in a shared stable "ancient" metrology,[16] and draws in particular on Hebrew parallels (and on Hebrew measures which he assumes *must* have had a parallel[17]). Friberg (1982, 2) justly characterizes the outcome as "somewhat premature", even in comparison with other publications from the period.

The use of the place-value principle not only for integers but also for fractions was established in Johann Strassmaier's (1846–1920), Josef Epping's (1835–1894) and Franz Xaver Kugler's (1862–1929) analysis of the Late Babylonian astronomical texts, beginning with (Strassmaier and Epping 1881); so far, however, it

[16]We may see this belief as the last scholarly and pseudo-scholarly survivor of the Renaissance faith in ancient *prisca sapientia*. Paradoxically, Oppert had pointed out already in (1886, 90) that there were "in Assyria and Chaldea, as everywhere else, ceaseless variations in the measures", which should have warned him against the dangers inherent in the comparative method.

[17]See for example p. 427f on the postulated unit "hair", which leads him to rather far-fetched hypotheses (presented "with all reserve", it is true).

was understood only in analogy with the use of sexagesimal fractions in Ptolemaic and modern astronomy.[18]

In a way, Hermann Hilprecht's (1859–1925) *Mathematical, Metrological and Chronological Tablets from the Temple Library of Nippur* (Hilprecht 1906) constitutes a decisive step. As we have seen, tables of squares and metrological lists had already been used in the early period by Rawlinson (1855) and Smith (1872). Hilprecht, however, put at the disposal of Assyriologists a large number of arithmetical and metrological tables. Unfortunately, his failing understanding of the floating-point character of the place-value system; the still strong conviction that the classical authors could provide interpretations of Mesopotamian texts; and a belief that everything Babylonian had to be read in a mystico-religious key[19] caused him not only to read very large numbers into the texts but also to understand a division of 1;10 (or 70) by 1 as 195,955,200,000,000 ÷ 216,000 (p. 27), where the denominator was then explained from a (dubious) interpretation of the passage about the "nuptial number" in Plato's *Republic* VIII, 546B–D (pp. 29–34) and coupled to postulated cosmological speculations:

> How can this number influence or determine the birth and future of a child? The correct solution of the problem is closely connected with the Babylonian conception of the world, which stands in the centre of the Babylonian religion. The Universe and everything within, whether great or small, are created and sustained by the same fundamental laws. The same powers and principles, therefore, which rule in the world at large, the macrocosm, are valid in the life of man, the microcosm.

So, while Hilprecht's publication represented a material step forward, his approach remained that of the nineteenth century.

Franz Heinrich Weißbach's (1865–1944) article "about the Babylonian, Assyrian and Old Persian weights" (Weißbach 1907), on the other hand, inaugurated a new trend. As formulated by Powell (1971, 188), "the study of Mesopotamian weight norms can be divided into two eras: the pre-Weissbach and the post-Weissbach eras". Weißbach discarded the comparative method, concentrating (like George Smith) on what could be derived from Mesopotamian sources and artefacts. He did not convince those who were committed to the "comparativist paradigm"; instead, the process confirms the observation made by Max Planck (1950, 33) (concerning Ludwig Boltzmann) and famously quoted by Thomas Kuhn (1970, 151), namely that

[18]Basing himself on indirect evidence and on Greek writings, Johannes Brandis (1830–1873) had already claimed that the unending sexagesimal fraction system of the Greek astronomers had to be of Chaldean origin "even if we never find direct or indirect testimony ascribing it to them" (Brandis 1866, 18).

[19]Hilprecht quotes this passage from Carl Bezold's (1859–1922) "concise survey of the Babylonian-Assyrian literature" (Bezold 1886, 225): "As far as we know by now, Babylonian-Assyrian mathematics primarily served astronomy, and this on its part a pseudo-science, astrology, which probably arose in Mesopotamia, propagated from there into the Gnostic writings, and was inherited by the Middle Ages, although we are not yet able to reconstruct this whole chain, the links of which are often broken from each other".

> a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it.

The following generations of Assyriologists, indeed, less trained in Hebrew and classical scholarship but familiar with the results of a mature discipline, followed the model set by Weißbach and by the immensely influential François Thureau-Dangin (1872–1944). The earliest work on metrology and mathematical techniques of the latter had been published in 1897 (a sophisticated interpretation of the intricate calculations on a field plan from the outgoing third millennium BCE); he was going to publish on metrological questions for decades to come.[20]

So far, Assyriologists had been concerned with *mathematics in use*, namely in use in non-mathematical documents (including astronomy and texts serving in elementary mathematical training). The first to come to grips with what became known among historians of mathematics as "Babylonian mathematics" from the 1930s onward—namely mathematics which was complicated enough to be counted as mathematics by those same historians in the twentieth century—was the 25-years old Ernst Weidner (1891–1976) in (1916) (he had already published a volume on Babylonian astronomy in 1911 (Jaritz 1993, 15)). Weidner's article begins with the observation that

> As regards the knowledge of the Akkadians in the area of mathematics we are still quite badly informed.[21] Beyond a few tables containing square and cube numbers and rather many multiplication tables we have nothing real except building- and field-plans, which already in quite early time makes us presuppose an Akkadian ability also to accomplish rather difficult calculations.

which summarizes the situation perfectly. Two difficult texts had been published in 1900, Weidner says[22] (only in cuneiform, with neither transliteration nor translation, which presupposes no understanding and conveys none); but these texts are then characterized as "probably the most difficult transmitted in cuneiform", which explains that nothing had been done on them. In the Berlin Museum he had now seen other texts of the same type, and he analyses two problems from one of them (VAT 6598): two different approximate calculations of the diagonal of a rectangle (a first and a corrupt second approximation, see (Høyrup 2002, 268–272)).

Compared to the analysis of the same text offered by Otto Neugebauer (1899–1990) in (1935), Weidner's interpretation contains some important mistakes, for which reason it can certainly be characterized as premature. However, Weidner's short paper, together with the commentaries of Heinrich Zimmern (1861–1931)

[20]Outside Assyriology, in particular among natural scientists taking interest in Antiquity and its mysteries, the comparativist trend is still alive and kicking—see (Berriman 1953; Rottländer 2006; Lelgemann 2004).

[21][A footnote refers to Moritz Cantor's *Vorlesungen* I, on which below.].

[22]Now known as BM 85194 and BM 85210.

(1916) and Arthur Ungnad (1879–1945) (1916, 1918) provided the first understanding of Babylonian mathematical *terminology*.[23]

In (1922), Cyril John Gadd (1893–1969) published a text dealing with subdivided squares, and added some further important terms (not least those for square, triangle and circle). Since the text in question contains no calculations, only terms for mathematical objects occur, none for operations. In (1928), finally, Carl Frank (1881–1945) published the collection of *Straßburger Keilschrifttexte*—a spelling that adequately reflects his working situation: he had made the copies before the War, when Strasbourg was Strassburg, and only received his own material in 1925, with no possibility of collating. None the less, Frank's book added another batch of terms. Because Frank's texts are even more difficult than the short ones dealt with by Weidner, Zimmern and Ungnad, and because Frank translated all sexagesimal place value numbers into modern numbers (repeatedly choosing a wrong order of magnitude), his understanding of the texts was rather defective.

This is how far Assyriologists went in the exploration of cuneiform mathematics until 1930—when Assyriology was half of its present age.

4 Historians of Mathematics Until c. 1930

On the whole, historians of mathematics depended during the same period not only on the material put a their disposal by Assyriologists but also on their interpretations.

In the posthumous (Hankel 1874), Hermann Hankel (1839–1873) dealt with "the Babylonians" (once, p. 65, accompanied by the Assyrians) on scattered pages of his discussion of the "pre-scientific period". Given the difficulties of Assyriologist with not only absolute but also relative chronologies until (Hommel 1885), it is no wonder that Hankel's observations are messy on this account. Substantially, he speaks about the sexagesimal divisions of metrologies (pp. 48f; not mentioning that not all subdivisions are sexagesimal, which was known at least since (Smith 1872)); a hunch of sexagesimal fractions (pp. 63, 65; but only to one place, and understood as written with a denominator which is "usually omitted"); the existence of tables of squares and astronomical tables (the two texts used by Hincks and Rawlinson in 1854–55), from which the hypothesis is derived that the Babylonians were interested in arithmetical series (p. 67); and a low level of geometry, concluded on the basis of the "building art deprived of style" (p. 73). Iamblichos's claim that Pythagoras had his knowledge of the harmonic proportion from the Babylonians is mentioned but explicitly not endorsed (p. 105).

[23]Only the terms for (what can approximately be translated as) square and cube roots were known since Moritz Cantor's use of Hilprecht's material in (1908). Quite a few of Weidner's readings later turned out to be philologically wrong while their technical interpretation was adequate. What was correct, however, was important later on, and some of the philological errors were still taken over in Neugebauer's early interpretations without great damage.

In the first edition of volume I of his *Vorlesungen* from (1880), Moritz Cantor (1829–1920) dedicates separate chapters to the Egyptians and the Babylonians—the latter on pp. 67–94. He is much better informed than Hankel—in part, it must be admitted, from publications that had appeared too late to be taken into account by Hankel, such as (Oppert 1872).[24] He offers an orderly exposition of the numerals and the "natural fractions" 1/6, 1/3, 1/2, 2/3 and 5/6. Further, he describes the tables of squares and cubes (which in (1908) he was going to see as tables of the corresponding roots), and he discusses the sexagesimal place value principle in connection with astronomy. Geometry is dealt with on the basis of geometric decorations, Herodotos and other Greek authors, and the Old Testament. Babylonian numerology is also discussed, in particular the ascription of numbers to the gods.

In the third edition from (1907), Cantor deals with the Babylonians before the Egyptians (pp. 19–51). The five extra pages allow him to tell the historiography of the field, but apart from a suggestion of that reinterpretation of Rawlinson's tables of squares as tables of square roots which he was to publish in full in (1908), nothing substantial is changed in the account of Babylonian mathematics. There are, however, some remarks about the material published by Hilprecht in (1906), with faithful adoption of his immense numbers (pp. 28f).

Hieronymus Georg Zeuthen's (1839–1920) *Geschichte der Mathematik im Altertum und im Mittelalter* (1896) dedicates a chapter (pp. 8–13) to what the Egyptians and the Babylonians knew in mathematics at the moment they came into touch with the Greeks, and which the Greeks might possibly have taken over from them (thus pp. 8f). Of the six pages, 26 *lines* deal with the Babylonians. 21 of these lines refer to astronomy and the division of the circle into 360°, and 5 to the possibility that Greek numerology was in debt to Babylonians and Chaldeans.

Johannes Tropfke (1866–1939) follows Hankel's pattern in the first volume of his *Geschichte der Elementarmathematik* (1902), mentioning the Babylonians now and then but not treating Babylonian mathematics per se—the obvious choice, given his full title "history of elementary mathematics presented systematically". But he only speaks about the sexagesimal system (mentioning Rawlinson's "square table" but without describing it). Only on two (quite dubious) points does he go beyond Hankel: he considers Iamblichos a certain source, and he claims (p. 304) that the Babylonians knew the solution 3–4–5 to the "Pythagorean equation"; he gives no source, and would have been unable to, since no pertinent text was known at the time. Most likely, he misremembers Cantor's idea (1880, 56) ("admittedly, for the moment without any foundation", thus Cantor) that *the Egyptians* might have used 3–4–5 triangles on ropes to construct right angles.

[24]Already Cantor's "mathematical contributions to the cultural life of the nations" had contained a chapter on the Babylonians (Cantor 1863, 22–38). At the time, however, he had only been able to speak about the decipherment; about "Oriental" culture in general; and about the writing system, about integer numerals and about the possible use of some kind of abacus (a hypothesis which he repeats in the *Vorlesungen*).

In general, historians of mathematics were not interested in Babylonian matters during the period. Inspection of 21 of the first 26 volumes of the series *Abhandlungen zur Geschichte der Mathematik*[25] (1877–1907) reveals no single article on the subject. For good reasons, as revealed by what Hankel and Cantor had been able to say about it—what Assyriologists had succeeded in finding out was still so tentative and so incoherent that it invited more to speculation than to solid work. The other possible explanation—that the authors should have been interested only in the higher level of mathematics—can be safely disregarded for the period before 1914, witness the many articles on elementary topics published in the same series.

5 The Long 1930s—Neugebauer, Struve, Thureau-Dangin, and Others

Beginning in 1929, the distinction between Assyriologists and historians of mathematics becomes irrelevant (for a while). This is the period when the advanced level of Old Babylonian and Seleucid mathematics was deciphered for good, after the modest but decisive beginnings made by Weidner, Zimmern, Ungnad, Gadd and Frank.

Neugebauer, it is true, is normally counted as a historian of mathematics. If anything, historian of astronomy would be the correct denomination—as we shall see, mathematics only occupied a rather short stretch of his life. But he had also been trained in Assyriology by none less than Anton Deimel (1865–1954), as he tells with gratitude in (1927, 5). Vasilij Vasil'evič Struve (1889–1965) was an Egyptologist but had also been trained in Assyriology, which was soon to become his main field. Thureau-Dangin was one of the most eminent Assyriologists of his (and all) times, but the contrast between his works from the 1920s (and before) and those from the 1930s demonstrates how much the discussions (and competition) with Neugebauer and the perspective of the history of mathematics had changed his approach. Hans-Siegfried Schuster (1910–2002), who made an important contribution in c. 1929, became an Assyriologist but participated in Neugebauer's seminar in Göttingen (Kurt Vogel, personal information; (Neugebauer 1929, 80)); Heinz Waschow (1914–?) studied not only Oriental philology (including Assyriology) from 1930 until 1934 but also applied mathematics (Waschow 1936, unpaginated CV). Albert Schott (1901–1945), the last of Neugebauer's contacts, had a strong interest in astronomy (but the numerous references to his assistance in (Neugebauer 1935–1937) all refer to strictly philological matters). Kurt Vogel (1888–1985), who sometimes took part in the discussion, was a mathematician and historian of mathematics but also trained in Egyptian (as well as Greek, and later also medieval Italian and German) philology—not to speak of his war experience as a military engineer.

[25]The exceptions are vols 2, 16, 19–20 and 25, to which I have no access; there is no reason to believe they should change the general picture.

Since Neugebauer's person was all-important for what happened in the 1930s, some words about his background may be fitting. His *Doktorarbeit* from (1926) had dealt with the Egyptian fraction system, but already while working at it he had become interested in the mathematics of the Sumerian cultural orbit as a parallel that might throw light on Egyptian thought, and been convinced (with due reference to Thureau-Dangin) that metrology was all-important for the development of early mathematics (Neugebauer 1927, 5).

In 1929, he launched *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik* together with Julius Stenzel (1883–1935) and Otto Toeplitz (1881–1940). Since the co-editors were 17 respectively 19 years older than Neugebauer, there can be little doubt that the initiative was his. He may not have written the *Geleitwort* ("accompanying observations"), but if not he must at least have agreed with it (Neugebauer et al. 1929, 1–2):[26]

> […] Through the title "Sources and Studies" we want to express that we see in the constant reference to original sources the necessary condition for all historical research that is to be taken seriously. It will therefore be our first aim to makes *sources* accessible, that is, in as far as possible to present them in a form that can meet the requests of modern philology but also, through translation and commentary, enable the non-philologist to always convince himself of the precise words of the original. To really meet the legitimate wishes of *both* groups, philologists and mathematicians, will only be possible if a genuine *collaboration* between them is brought about. To prepare that will be one of the most important tasks of our enterprise.
>
> The technical realization of this programme we intend to achieve through the publication of two irregular publication series. One, A, will accommodate the major proper editions, containing the text in its original language, philological apparatus and commentary and a translation that is as literal as possible, and which will make the contents of the text as easily accessible also to the non-philologist as can reasonably be done. Each issue of these "Sources" is to be a self-contained piece. The issues of Section B, "Studies", are to contain each a collection of articles having a closer or more distant relation to the material coming from the Sources.
>
> The "Sources and Studies" are to provide contributions to the *history* of mathematics. But they do not address specialists of the history of science alone. They certainly intend to present their material in such a form that it can *also* be useful for specialists. But they also address all those who think that mathematics and mathematical thinking is not only the concern of a particular science but also deeply connected with our total culture and its historical development, and that the attention to the *historical* genesis of mathematical thinking can provide a bridge between the so-called "sciences of the spirit" [*Geisteswissenschaften*] and the seemingly so a-historical "exact sciences". Our final aim is to participate in the building of such a bridge. […]

So, a common endeavour between philologists and historians of mathematics was aimed at, for the benefit of both groups as well as a broader educated public. That those publications about Babylonian mathematics that appeared in the journal did not cast much light on the role of mathematics in general culture was not a result of failing will; as Neugebauer had to point out in (1934, 204), one should "not

[26]Some stylistic features do point to Neugebauer as the writer. As revealed by the quotation in note 41, at least Toeplitz also had an attitude in conflict with the present text.

forget that we still know practically nothing about the whole setting of Babylonian mathematics within the context of general culture”.

In the first issue, Neugebauer and Struve (1929) published an article “about the geometry of the circle in Babylonia” (actually also about other geometrical objects). Among the results is the identification of a technical term for the height of plane or solid geometric figures. The explanation is philologically mistaken, but as in the case of Weidner’s similar errors this is not decisive, as pointed out by Thureau-Dangin (1932a, 80) in the note where he gives the correction.

The preceding article in the same issue is by Neugebauer alone (1929). It offers a new analysis of some of Frank’s texts, and manages to elucidate much which had remained in the dark for Frank. Neugebauer’s main tool is of astonishing simplicity: he retains the sexagesimal shape of numbers, while Frank, in order to get something more familiar to a modern mathematical eye, had translated them into decimal numbers (and often translated them into a wrong order of magnitude, as observed above). Beyond that, Neugebauer offers a number of improved readings.[27] Some of the problems, it turns out, contain questions of the second degree. Neugebauer concludes (pp. 79f) in these words:

> One may legitimately say that the present text presents us with a piece of Babylonian mathematics that enriches our all too meagre knowledge of this field with essential features. Even if we forget about the use of formulas for triangle and trapezium, we see that complex linear equation systems were drawn up and solved, and that the Babylonians drew up systematically problems of *quadratic* character and certainly also knew to solve them – all of it with a computational technique that is wholly equivalent with ours. If this was the situation already in Old Babylonian times, hereafter even the later development will have to be looked at with different eyes.

A note added after the proofs had been finished then reveals that a text has been found which *solves* mixed second-degree problems, referring to the essential role played Schuster for understanding this, while an article written by Schuster (1930) and appearing in the second issue analyses the solution of four such problems in a Seleucid text.

The conclusion just quoted announces the approach which was to be that of the 1930s. Since the meanings of terms for mathematical operations were derived from the numbers that resulted from their use, the operations were almost by necessity understood as arithmetical operations; as a rather natural consequence, problems were understood as (arithmetical) equations and equation systems. And of course Neugebauer, as everybody else, expressed amazement that complicated matters such as second-degree equations were dealt with correctly.

Neugebauer knew very well that Old Babylonian (1800–1600 BCE, according to the “middle chronology”) and Seleucid (third-second century BCE) mathematics were formulated in different terminologies. But he believed that the difference was one of terminology and implicitly supposed, as we see, that there must have been steady progress of knowledge from the early to the late period.

[27]“Moreover, even the readings themselves can be considerably improved, once the substantial contents has been elucidated” (p. 67).

A number of publications from Neugebauer's hand (and three from that of Waschow (1932a, b, c)) followed in *Quellen und Studien* B until 1936 (in vol. 4, from 1937–38, Neugebauer has turned completely to astronomy). In 1935–37, Neugebauer also published in *Quellen und Studien* A the monumental *Mathematische Keilschrifttexte* (MKT) (Neugebauer 1935–1937). They can be said to bring to completion the interpretation of his (1929)-paper; but they also make clear that Neugebauer had not left behind his interest in metrology and other simple matters—he was not looking merely after matters that might be seen as analogous to modern equation algebra. The conclusion of volume III (Neugebauer 1935–1937, III, 79f) gives two warnings to the reader. Firstly, that MKT is a *source edition*—"It does not belong among the tasks that I have proposed for myself in this edition to develop the consequences which can be drawn from this text material". Secondly,

> Since our knowledge of these things is of relatively recent date, and current datings had to be pushed considerably, there is an obvious danger to overestimate the mathematics of the Babylonians. In order to somehow gloss over the lack of a basis in sources, many familiar books change elementary mathematical things into "propositions" and "discoveries" that must be ascribed to great men. It seems to me that we should not stamp the Babylonians as such discoverers. What is often overlooked and cannot be sufficiently emphasized is the terrible difficulty and slowness of the development of the very simplest fundamental mathematical concepts, first of all of a genuine computational technique. This, however, is not the achievement of a single person; it can only be understood within a historical process, inextricably attached to the emergence of a whole culture. Once this stage has been reached, then there is nothing in Babylonian mathematics that must be seen as an unexpected brilliant performance.

The last sentence refers to Neugebauer's hypothesis (which he considers an established fact) (Neugebauer 1935–1937, III, 79),

> that Babylonian mathematics first grew out of the numerical methods of sexagesimal calculation, the practical advantage of which was fully understood, and then, decisively sustained by the possibilities offered by ideographic writing, soon reached a strongly "algebraic"[28] treatment of purely mathematical problems that were of or could be reduced to linear or quadratic character.

Thureau-Dangin, as we have seen, had been interested in metrology and mathematical techniques since (1897). He started dialogue with Neugebauer in (1931) (making a philological correction that also concerns Frank, whom he does not mention). His weighty *Esquisse d'une histoire du système sexagésimal* (1932c), however, is rather a crown on his work from the 1920s, describing both the sexagesimal place-value system and the non-positional system and non-sexagesimal fractions, together with their uses.[29] But very soon, Thureau-Dangin moved from

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[28][The quotes around the word *algebraic* indicate that Neugebauer refuses to make hypotheses about which kind of algebraic thought is involved in the texts. The many algebraic formulas in his commentary are not meant to map the thinking of the authors of the texts; they show why the calculations are pertinent (or, rarely, why they are not)].

[29]This booklet had no strong impact—it drowned in the fury surrounding the new discoveries of the time. However, a revised English translation (including much about the Babylonian "algebra") appeared in *Osiris* in (1939) on George Sarton's initiative (p. 99).

purely philological emendations and addenda to the publication of new mathematical texts and to considerations of their mathematical substance—for example in (1932b, 1934, 1936)—and to a synthesis about "the method of false position and the origin of algebra" (1938a) along with the source edition *Textes mathématiques babyloniens* (1938b).[30] In several of these works Thureau-Dangin can be seen to be much less wary than Neugebauer when speaking of the algebraic thinking of the Babylonians. He also shows himself familiar with a very wide range of later mathematical sources, from Diophantos, Ptolemy and al-Khwārizmī to Stevin and Wallis.

Then, in 1937–38, this "heroic period" ended abruptly. In 1945, it is true, Neugebauer and Abraham Sachs published *Mathematical Cuneiform Texts* (Neugebauer and Sachs 1945),[31] an edition of texts from American collections that had not been included in MKT, and Neugebauer's popularization *The Exact Sciences in Antiquity* from (1951) (revised in 1957) contains a chapter on the topic; but apart from that Neugebauer only published two or three small items on Babylonian mathematics after 1937, dedicating instead himself wholly to the history of astronomy (and to the launching of the *Mathematical Reviews*, after the National Socialists had seized power over his earlier creation *Zentralblatt für Mathematik*). Schuster published nothing in the area after 1930 (he is better known as a Hittitologist), while Waschow entered the army in 1934, writing at the same time a dissertation on Kassite letters (1936).[32] In 1938 he published a book (*4000 Jahre Kampf um die Mauer*) about siege techniques since Old Babylonian times, after which I have been unable to find information about his fate (I would guess that as

[30]This is what von Soden (1939, 144) tells about the purpose of this parallel edition: "This new work is not meant to replace Neugebauer's MKT; indeed, the phototypes and autographs are not repeated, nor are all texts treated anew. Th.-D.s aim was instead, leaving the arithmetical tables completely aside (only the introduction speaks briefly about them) to make those problem texts that are sufficiently well preserved to allow at least a generally satisfying understanding available to as many researchers as possible in a cheaper edition, since the exorbitant price of the MKT unfortunately hampers its wider diffusion." But further: "While thus the specialist researcher will also in future not be able to give up Neugebauer's MKT as the complete source collection, with the just mentioned exception [two small texts from Susa with area calculations published by Vincent Scheil in 1938], then precisely he will also not be able to pass over Th.-D.'s new edition, as nobody will be able to digest in brief the large number of corrected readings and the immensely weighty lexical, grammatical and substantial observations, masterly concise though they are." In Thureau-Dangin's own words (TMB, xl): "The present volume contains no text which has not been published elsewhere in its original form [that is, without a translation of ideograms into syllabic Akkadian]. The main task I have set myself while preparing it has been to make documents accessible to the historians of mathematical thought.".

[31]Curiously enough, (Neugebauer and Sachs 1945) is much less afraid of ascribing modern mathematical concepts to the Babylonians than Neugebauer had been in the 1930s—such as logarithms, p. 35, cf. (Neugebauer (1935–1937), I, 363–365). Whether this is due to Sachs's influence or Neugebauer himself had been convinced by what others had read into (Neugebauer 1935–1937) I am unable to say.

[32]The edition of one long Seleucid text (BM 34568) in (Neugebauer 1935–1937, III, 14–22) is also, according to Neugebauer, "apart from a few trifles due to Herrn Dr. Waschow". This work must be dated between 1935 and 1937.

an officer he lost his life during the war). Albert Schott concentrated on astronomy, while Kurt Vogel's *Habilitationsschrift* (1936) dealt with Greek logistics. Thureau-Dangin returned to other Assyriological questions.

In (1961), Evert Bruins (1909–1990) and Marguerite Rutten (1898–1984) published a volume with mathematical texts from Susa. They had started work around 1938, and Bruins was very proud of having been trained by Thureau-Dangin.[33] No wonder that the volume is wholly in the style of the 1930s—yet on a much lower philological level than what had been published during this epoch, and full of groundless speculations and misreadings (with interspersed good ideas, it should be added).

6 Assyriologists, 1940–1980

After 1940, Assyriologists would usually put aside any tablet containing too many numbers in place-value notation as "a matter for Neugebauer" (thus Hans Nissen, at one of the Berlin workshops on "Concept Formation in Mesopotamian Mathematics" in the 1980s). In consequence, very few new texts (apart from the batch from Susa) were published during the following four decades.

There is one important exception to this generalization (and a few other less important ones). Between 1950 and 1962 the Iraqi Assyriologist Taha Baqir (1912–1984) published four papers in the journal *Sumer* with new texts excavated between 1945 and 1962 (Baqir 1950a, b, 1951, 1962). These were highly important for several reasons: They came from a region from which until then no mathematical texts were known; like the Susa texts their provenience was known, since they were regularly excavated; but unlike what had happened to the Susa texts, the excavations were carefully made, for which reason the texts can also be dated.[34] von Soden (1952) suggested a number of improved readings with implications for the interpretation,[35] and Bruins (1953) tried (as usually) to show that everything von Soden had said was absurd; but the impact of Baqir's papers on historians of mathematics was almost imperceptible—one joint article by the mathematician

[33]He returns to this link time and again in the numerous angry letters I have from his hand. I suppose he can be believed on this account, his general unreliability notwithstanding. According to the preface (TMS, xi), Rutten made the hand copies and collaborated with Bruins on the translation. However, already the translation of word signs into Akkadian contains so many blunders of a kind no competent Assyriologist would commit that Bruins can be clearly seen to have had the upper hand concerning everything apart from the hand copies.

[34]A further text covering three tablets was found on the ground, apparently left behind by illegal diggers as too damaged. It was published by Albrecht Goetze (1897–1971) in (1951).

[35]Until then, von Soden had never worked directly on mathematical questions himself; but he had always been interested in the topic, as can be seen from his careful and extensive reviews of Neugebauer 1935–1937 (1937) and TMB (1939). He also made a review of TMS in (1964), an indispensable companion piece to the edition itself.

Karl-Bernhard Gundlach (*1926) and Wolfram von Soden (1908–1995) from (1963) deals with one of Baqir's texts and a text from Susa.

Already in 1945, Goetze had contributed a chapter "The Akkadian Dialects of the Old–Babylonian Mathematical Texts" to (Neugebauer and Sachs 1945, 146–151). In contrast to the volume as a whole, this chapter falls outside what had been done in the 1930s.[36] In these pages, Goetze makes a careful classification of all Old Babylonian mathematical texts known by then that contained enough syllabic writing to allow orthographic analysis.

Occasionally, some Assyriological publication would touch at numero-metrological questions, but not very often.[37] We have to wait until the early 1970s before an Assyriologist took up systematically the kind of work which Thureau-Dangin and others had pursued in the 1920s. In (1971), Marvin Powell submitted his doctoral dissertation on *Sumerian Numeration and Metrology*, soon followed by a major paper on "Sumerian Area Measures and the Alleged Decimal Substratum" (1972a). Also in 1972 a short paper from his hand (1972b) on "The Origin of the Sexagesimal System: the Interaction of Language and Writing", followed, and in (1976) a longer one on "The Antecedents of Old Babylonian Place Notation and the Early History of Babylonian Mathematics", published in *Historia Mathematica*. The latter two articles took up topics which both Thureau-Dangin and Neugebauer had tried their teeth on around 1930, yet for lack of adequate sources from the third millennium without reaching solid results. In (1978) and (1979), Jöran Friberg, paradoxically a mathematician of merit and no Assyriologist but using approaches and methods that had been characteristic of the Assyriological tradition, made a break-through on the numerical and metrological notations of the fourth millennium; with minor corrections, his results were later confirmed by the Berlin Uruk project (Damerow and Englund 1987).

7 Historians of Mathematics

During the same decades, little original work on Mesopotamian mathematics was made by scholars who would primarily be classified as historians of mathematics. They can be seen to have regarded the analysis in (Neugebauer 1935–1937) and (Neugebauer and Sachs 1945) as exhaustive—as it actually was on most accounts,

[36]The outcome can be seen as an extension of a division of the corpus into a "northern" and a "southern" group which Neugebauer had suggested in (1932, 6f); but Neugebauer's arguments had been of a wholly different nature.

[37]In 1978–79, Carlo Zaccagnini thus published at least four papers on the metrologies of peripheral areas.I disregard publications in Russian, most noteworthy of which is (Vajman 1961)—my reading of Russian, which reached the level of "rudimentary" 25 years ago, has vanished completely since then for lack of practice. An exhaustive survey, often with discussion, of all at least minimally pertinent publications (also those in Russian) for the period 1945–1980 will be found in (Friberg 1982, 67–130).

as long as Neugebauer's and Sachs's approach *as understood by historians of mathematics* was taken for granted.

There are again a few exceptions. The most substantial of these is a sequence of proposed interpretations of the famous text Plimpton 322, originally published in (Neugebauer and Sachs 1945, 38–41) and considered there as an early instance of number theory. Most noteworthy during the early period is (Bruins 1957), where a derivation of its Pythagorean triples from pairs of reciprocals is proposed (an interpretation which has been confirmed with modifications and extra arguments since then by Friberg (1981) and Robson (2001)). It may be considered a manifestation of the new "modernizing" orientation of (Neugebauer and Sachs 1945) that this possibility had been overlooked, given that Neugebauer had believed in the 1930s that the whole second-degree "algebra" came from the place-value system (above, text around note 28).

Other exceptions are a publication of some merit by Gandz (1948), sent to the journal *Osiris* around 1938 but then delayed by the war; and a republication of one of Gandz's results in (1955) by Peter Huber, who had not noticed Gandz's work.

However, while little new research was done on Mesopotamian mathematics by historians of mathematics, "Babylonian mathematics" was close to becoming the standard introduction to histories of mathematics.

The way it was dealt with is well illustrated by Asger Aaboe's (1922–2007) *Episodes from the Early History of mathematics* (1964). Aaboe starts by observing that a modern schoolboy transposed to Babylonia or ancient Greece would find the "physics" of classical Antiquity utterly unfamiliar (p. 1). Mathematics, however, would

> look familiar to our schoolboy: he could solve quadratic equations with his Babylonian fellows and perform geometrical constructions with the Greeks. This is not to say that he would see no differences, but they would be in form only, and not in content; the Babylonian number system was not the same as ours, but the Babylonian formula for solving quadratic equations is still in use.

That is, firstly: mathematics is a topic outside history, changing "in form" only. Secondly, the "contents" of mathematics consists in "formulae". Aaboe himself may have believed to continue Neugebauer's approach, but in reality the programme of *Quellen und Studien* has been betrayed. The "seemingly so a-historical 'exact sciences' " have become, precisely, *a-historical*. The lack of information about the social context of Babylonian mathematics is no longer a deplorable fact, as for Neugebauer in (1934)—the absence of information about its creators is just taken note of, while institutional setting etc. constitute non-questions.[38]

[38]"Of the creators of Babylonian mathematics we know nothing whatsoever except the result of their work" (p. 6). That the texts are school texts is intimated by photos of presumed schoolrooms from Mari (which are actually store-rooms) and occasional references to a "schoolboy"—but schooling seems to be just as timeless as mathematics. In 1964, we may observe, more was known about the Old Babylonian scribe school than in 1934—cf. (Kramer 1949; Falkenstein 1953; Gadd 1956).

Turning to the contents, we see that the reader learns that the sexagesimal place-value system is *the* Babylonian number system. Aaboe ignores that it was used only for intermediate calculations; in school; and in (late Babylonian) mathematical astronomy. He is unaware that a different system was used in "real-life" juridical and economical documents[39]—he only knows about inconsistency and failing rationality.[40]

When going beyond place-value computation Aaboe deals with three more advanced topics. The first is treated through two "algebraic" problems about square areas and appurtenant sides from BM 13901, quoted in Neugebauer's translation but then immediately transformed into modern algebraic symbols; the second is YBC 7289, a tablet showing a square with diagonals and three inscribed numbers corresponding to the side, the diagonal and their approximate ratio, which allows immediate discussion in terms of $\sqrt{2}$; the third the calculation of a height in an isosceles trapezium. It is mentioned (p. 23) that the first two are Old Babylonian and the third Seleucid, but it is claimed (as does *not* correspond to the information that could be extracted from (Neugebauer 1935–1937), and as is in any case quite wrong) that all three could have been written in any period. The conclusion discusses "algebra" once again, and states that

> Quadratic equations are often given in the equivalent form of two equations with two unknowns, such as
>
> $$x + y = a, \quad xy = b,$$
>
> whence one finds immediately that x and y are the solutions of
>
> $$z^2 - az + b = 0$$

without mentioning that such problems deal with rectangular areas and sides, nor that the "one" who "finds immediately" is Aaboe himself or some other modern calculator, and that no corresponding step can be found in the original texts.

Aaboe's book was intended as supplementary high-school reading, and can thus be understood according to Toeplitz' "genetic method" (1927), the introduction of modern concepts through pedagogically motivating idealized quasi- or

[39]However, all of this is described in (Thureau-Dangin 1939), who distinguishes the "abstract" (namely place-value) system "intended only to serve as an instrument of calculation" (p. 117) from the ordinary sexagesimal but non-positional system.

[40]"It should be added that an entirely consistent use of the sexagesimal system is to be found only in the mathematical and astronomical texts, and even in astronomical texts one can find year numbers written as, e.g., 1–*me* 15 (meaning 1 hundred 15) instead of 155. In practical life the Babylonians showed the same profound disregard for rationality in their use of units for weight and measure as does the modern English-speaking world" (p. 20). The year number in question is written in precisely that number system which Hincks had deciphered in 1847, cf. note 13—the very first contribution to the study of Assyro-Babylonian mathematics.

pseudo-history.[41] However, the typical general histories of mathematics published during the period share the basic character of Aaboe's presentation—see my anatomies of (Hofmann 1953), (Boyer 1968) and (Kline 1972) in (Høyrup 2010). Only another book written for the high-school level (but here the German *Gymnasium*), Vogel's *Die Mathematik der Babylonier* from (1959) stands out—with its awareness that the place-value system was a scholarly system; because of its interest in metrologies and in computations dealing with everyday life; and with its discussions of ways of thought.[42] Vogel, indeed, had worked on the material himself already in the 1930s, and he had always been interested in ways of thought and in the mathematics of practical life, while Aaboe had only worked on Seleucid astronomical texts, and Joseph Ehrenfried Hofmann (1900–1973), Carl Boyer (1906–1976) and Morris Kline (1908–1992) at best on Neugebauer's translations—but apparently more often on his popularizations and his explanatory commentaries without distinguishing the latter from what was done in the sources.

8 After 1980

After c. 1945, the historiography of Mesopotamian mathematics had thus been an almost dead topic, little considered by Assyriologists and treated under the point of view of "historical mathematics" by those who otherwise wrote about the history of mathematics.[43]

Beginning with the above-mentioned works of Powell and Friberg this situation was going to change once again. But this is where my own work in the field started, first on the connection between mathematics, general socio-cultural context and educational situation, from 1982 onward on the concepts and operations of Old Babylonian mathematics, so here I shall stop—adding only that in recent years a number of younger scholars trained in mathematics as well as Assyriology have entered the field, adding new approaches and returning to the Assyriological questions of the earlier twentieth century with the luggage of a century of extra textual and archaeological discoveries, thus being able to integrate the mathematical

[41]"Nothing is farther from me than to teach a history of the infinitesimal calculus; I myself as a student ran away from a lecture of that kind. History is not at stake, but the genesis of problems, acts and demonstrations, and the decisive turning points in this genesis" (Toeplitz 1927, 94).

[42]Dirk Struik's (1894–2000) *Concise History of Mathematics* from (1948) deals with Mesopotamian mathematics too briefly to allow description in similar depth (pp. 23–32). Struik's layout, however, is similar: The analysis is embedded in general social history; non-positional as well as place-value system are described; but like Vogel, Struik has no possibility to go beyond Neugebauer.

[43]Boyer had written about the "*concepts* of the calculus" (1949), and Kline's title refers to "mathematical *thought*". Hofmann had written among other things about Ramon Lull's squaring of the circle in (1942), and had tried there to penetrate the thinking and motives of Lull (without which he would indeed have been unable to conclude anything of interest).

dimension with studies of social, political and economic history. The field remains alive—but mathematicians may not find it very interesting for their purpose.

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