Modeling and Forecasting of Well-Being Using Fuzzy Cognitive Maps

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Abstract In this paper we address the problem of modeling and forecasting of well-being. First, we apply a graph-based model of a Fuzzy cognitive map to discover cause-and-effect relationships among indicators of well-being. Second, the discovered model is applied to forecast the future state of well-being. The model is constructed using historical multivariate time series containing six consolidated indexes that represent well-being on the considered territory. Experiments with real-world data provided evidence for the usefulness of the proposed approach. Moreover, the interpretation of the obtained FCM graph led to the discovery of unknown dependencies within the data. The analysis of the unknown dependencies requires further research.

1 Introduction

Estimation of well-being is an important problem and a key factor supporting decision-making processes in territory management. The estimation of well-being has raised the active interest of several researchers [3–5, 7, 14, 27, 28]. The estimation of a synthetic well-being index was proposed in [21] where the weighting of the partial indicators was used. The creation of the territory well-being standard includes the following: identification of hierarchy of indicators (i.e., the set of primary indicators and levels of their aggregation), identification of significance coefficients of indicators and identification of normative values of indicators in form of range. This process is performed by experts using historical data based on territory characteristics and specifications [27]. The method of estimation of the territory well-being level is an improvement on the approach to estimation of complex socio-economic

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objects [4]. The theory and estimation of individual and social welfare measures was proposed in [3]. The social context of well-being was investigated in [14]. The constriction of well-being indexes has also been investigated in [5, 7].

The approach for the modeling and forecasting of well-being proposed in this paper is based on the application of the soft-computing model of Fuzzy cognitive maps. A Fuzzy cognitive map is a knowledge representation tool inheriting different aspects of fuzzy sets and neural networks [8, 18]. FCMs model knowledge through fuzzy concepts represented as nodes and relationships between them represented as weighted arcs. The causal relationships among concepts are either determined by experts knowledge or by learning when historical data are available. There is a growing interest in FCMs, especially in the fields of control [30], medicine [29], computer science [20], time series forecasting [10, 11, 15, 23, 24], decision support [31], and machine learning [17]. A review of FCM research is given in [26].

In this paper we address the problem of modeling and forecasting of well-being. The well-being indexes are changing over time and constitute the considered time series. First, we apply a Fuzzy cognitive map to discover cause-and-effect relationships among indicators of well-being. Second, the discovered model is applied to forecast the future course of well-being. The forecasted multivariate time series contain six comprehensive indexes representing well-being in the considered territory. Experiments with real-world data provide evidence for the usefulness of the proposed approach. Moreover, the interpretation of the obtained FCM graph led to the discovery of unknown dependencies within the data.

The rest of this paper is organized as follows. In Sect. 2 the theoretical background on the estimation of well-being is presented. Theoretical background related to Fuzzy cognitive maps is given in Sect. 3. The contribution of this study, i.e., the application of FCM to the modeling and forecasting of well-being, is presented in Sect. 4. Experiments using real-world data are described in Sect. 5. Section 6 concludes the paper.

2 Estimation of Well-Being

Let us assume that $P_k \in \mathfrak{R}$, $k = 1, 2, \ldots, n$ denotes real valued indicators related to well-being (e.g., 'Alcoholism', 'Drug addiction', etc.) where is the number of all considered indicators. The estimation of well-being is based on the values of indicators and consists of the following two steps:

1. Estimation of primary indicators by the calculation of individual well-being indexes i_k. The value of an individual index demonstrates significant improvement if $i_k > 1$. If $i_k < 1$, then it demonstrates significant degradation of the indicator.
2. Estimation of comprehensive indicators by the calculation of consolidated well-being indexes I applying previously calculated individual indexes. The value of

the consolidated index is $I > 1$ or $I < 1$, which demonstrates an improvement in or the degradation of the well-being level, respectively.

Every individual index of the kth indicator is calculated by Formula (1):

$$i_k = 1 + \Delta P_k \cdot S_k, \tag{1}$$

where: ΔP_k is the compliance coefficient of actual values of the kth indicator with standard; $S_k = \pm 1$ is the coefficient which characterizes the 'polarity' of kth indicator, where: $S_k = 1$, when the change of indicator is proportional to the index and $S_k = -1$, when the change of indicator is inversely proportional to the index. The compliance coefficient ΔP_k is calculated using Formula (2):

$$\Delta P_k = \begin{cases} 0, & \text{for: } P_k \in [N_k, Z_k], \\ \frac{P_k - Z_k}{Z_k - N_k}, & \text{for: } P_k > Z_k, \\ \frac{P_k - N_k}{Z_k - N_k}, & \text{for: } P_k < N_k, \end{cases} \tag{2}$$

where: P_k is actual value of the kth indicator; $[N_k, Z_k]$ is the range of normative values of kth indicator, N_k is the lower limit of the range, and Z_k is the upper limit of the range. In cases where the value of the indicator P_k falls within the range of normative values, i.e., $P_k \in [N_k, Z_k]$, the compliance coefficient $\Delta P_k = 0$. In cases where actual value of indicator is above the upper limit of range, the compliance coefficient has a positive value $\Delta P_k > 0$. In cases where the actual value of the indicator is below the lower limit of range, the compliance coefficient has a negative value $\Delta P_k < 0$.

After the calculation of all individual indexes i_k, the consolidated well-being index I is calculated by Formula (3):

$$I = \sum_{k=1}^{n} u_k \cdot i_k, \tag{3}$$

where: $u_k > 0$ is a significance coefficient of kth indicator, where it is assumed that $\sum u_k = 1$, and n is the number of all individual indicators.

To illustrate the described calculation procedure, we provide a numerical example. Table 1 presents individual indicators used to calculate the comprehensive indicator of 'psycho-emotional tension'.

Table 1 Individual indexes for the consolidated index 'Psycho-emotional tension'

k	Indicator	P_k	N_k	Z_k	ΔP_k	i_k	u_k	S_k
1	Children's drug addiction	7.16	4.20	7.20	0	1.00	0.21	−1
2	Teenage drug addiction	180.56	120.00	150.00	1.02	−0.02	0.20	−1
3	Drug addiction	265.38	200.00	250.00	0.31	0.69	0.15	−1
4	Alcoholism	1333.63	1100.00	1300.00	0.17	0.83	0.15	−1

First, according to Formula (2), for all primary indicators P_k, we calculate compliance coefficients ΔP_k. For example, as can be noted in Table 1, the actual value of 'Children's drug addiction' falls within the normative range, therefore the coefficient of compliance is identified as: $\Delta P = 0$. The actual values of other indicators fall above the upper limit of the range; therefore, the compliance coefficient is calculated according to the second condition in Formula (2).

Second, we calculate the individual well-being indexes i_k according to Formula (1). Taking into account the negative polarity S_k for 'Teenage drug addiction', the corresponding individual index is calculated as: $i_2 = 1 + 1.02 \cdot (-1) = -0.02$.

In the last column of Table 1 we placed significance indexes corresponding to the individual indexes. Using the values of individual indexes and applying Formula 3, we calculate the consolidated well-being index. (I) for 'Psycho-emotional tension' as $I = 0.21 \cdot 1.00 + 0.20 \cdot (-0.02) + 0.17 + 0.15 \cdot 0.69 + 0.15 \cdot 0.83 = 0.23426$.

This method provides the estimation of well-being by assessing the changes in the indicators values relative to their normative values.

3 Introduction to Fuzzy Cognitive Maps

Let us assume we observe real-valued variables $v_1, v_2, \ldots, v_n \in V$, where: V is the set of the considered well-being indicators. Let C denotes a set of fuzzy sets, where every set $c \in C$ is a node of the FCM. At time step $t \in [0, 1, \ldots, t_e], t_e \in \aleph$, the value of $v_i(t)$ is mapped by the fuzzification function to the state of the corresponding concept $c_i(t) = \mu_i(v_i(t))$. The value of $c_i(t)$ is the degree in which $v_i(t)$ belongs to the fuzzy set c_i. The fuzzification is usually simplified as a normalization: $c_i(t) = \frac{v_i(t)-\min(v_i)}{\max(v_i)-\min(v_i)}$.

The FCM is defined as an ordered pair $< C, W >$, where C is the set of concepts and W is the connection matrix that stores the weights $w_{ij} \in [-1, 1]$ assigned to the pairs of concepts. The value $w_{ij} = 1$ expresses full positive and $w_{ij} = -1$ full negative impact of the ith causal concept on the jth effect concept respectively. The intermediate values of weights refer to partial causality [8].

The FCM model can be exploited for the prediction of a concept's states $c_i'(t)$ and, after their defuzzification, the corresponding values of varibles $v_i'(t)$. The prediction is carried out using Eq. (4):

$$c_j'(t) = f\left(\sum_{i=1, i\neq j}^{n} w_{ij} c_i(t-1)\right), \tag{4}$$

where $n = card(C)$ is the cardinality of set $C, f(x)$ is the transformation function. The transformation function restricts the weighted sum of concepts states into the interval $[0, 1]$. For the purpose of this study, we use the logistic transformation: $f(x) = \frac{1}{1+e^{-gx}}$, where $g > 0$ is the parameter that determines the gain of the transformation.

After performing the prediction of the concepts state, to obtain the predicted values of variables $v_j'(t)$, denormalization is performed by using the formula: $v_j'(t) = c_j'(t)(\max(v_j) - \min(v_j)) + \min(v_j)$.

For the purpose of this paper we decided to apply Mean Absolute Error (MAE), the simplest approach to the calculation of forecasting errors, which is given by the following formula:

$$e = \frac{1}{n \cdot card(T)} \sum_{t}^{card(T)} \sum_{j=1}^{n} |v_j'(t) - v_j(t)|, \tag{5}$$

where T denotes the considered (learning or testing) period in which the errors were accumulated, $card(T)$ is the length of the considered period of time calculated in time steps, and n is the number of variables. $v_j'(t)$, $v_j(t)$ denotes the predicted and actual values of the time series, respectively.

The set of concepts C is provided by an expert, and only the matrix W is learned, using historical data. There are two known approaches to learning FCMs: adaptive and population-based. Adaptive algorithms are based on the idea of Hebbian learning borrowed from the theory of artificial neural networks. The adaptive learning methods involve: DHL [19], BDA [16], AHL [22] and other algorithms. The population-based approaches for learning FCMs are: RCGA (real coded genetic algorithm) [32], PSO-based algorithm (applies particle swarm optimization method) [25], simulated annealing optimization-based algorithm [12], and differential evolution-based algorithm [17].

As reported in the literature [9], the RCGA is one of the most competitive among the population-based. For that reason, it has been selected to be used in this study.

4 An FCM-Based Model of Well-Being

The first goal of this study is to create an FCM model representing the dependencies among consolidated indexes of well-being. The resulting FCM model is applied as a decision support tool for policy makers but also as a predictive model allowing the forecasting of well-being. To accomplish the aforementioned objective we map the considered consolidated indexes to the concepts of Fuzzy cognitive maps. The mapping is shown in Table 2.

Table 2 Mapping between consolidated indexes and the concepts of FCM

Index	Concept
Population structure	c_1
Labour market	c_2
Housing facilities	c_3
Standard of living	c_4
Psycho-emotional tension	c_5
Medical provision	c_6

As stated in Sect. 2, all consolidated indexes assume values in the range [0, 1]; therefore, the normalization of the original and denormalization of the forecasted time series are not required.

For the purpose of this paper, the evolutionary approach based on the RCGA is applied. The RCGA creates the population of genotypes; each of them is a vector of weights of a candidate FCM. The goal of the evolutionary algorithm is to optimize the matrix W with respect to the predictive capability of the FCM. The applied RCGA algorithm relies on the template of a genetic algorithm (Algorithm 1).

Algorithm 1: Genetic Learning of FCM.

Input: Multivariate time series $\{V(1), V(2), \dots, V(t_e)\}$.
Output: Optimized matrix: W.

Initialize randomly the first population $P_k, k = 1$ of genotypes;
While (stopping-criterion is not satisfied) {
  $Evaluation(P_k)$;
  $P_{k+1} \leftarrow Selection(P_k)$;
  $Mutation(P_{k+1})$;
  $Crossover(P_{k+1})$;
  $k \leftarrow k + 1$;
}
return $p_{best} \in P_k$ - the genotype with the highest fitness value;

Index k denotes the number of generations. The constituents of the algorithm are the following:

- **Genotype**. Every genotype $p \in P$ includes the vector of numbers coming from the matrix W of the candidate FCM. Subsequent rows of W are placed linearly one after the other into the vector of genotype. The elements on the diagonal of the matrix W are omitted, as they do not take part in reasoning.
- **Evaluation of Genotypes**. To use the RCGA for learning FCMs, we defined the fitness function as $fitness(FCM) = -e$, where e is the accumulated forecasting error calculated for the learning period.
- **Selection**. During the selection process, a new population P_{k+1} of genotypes is produced. The newly created population is later supplemented using the operators of mutation and crossover. For the purpose of this paper the elite selection is applied [13].
- **Mutation and crossover**. To supplement the population, the offspring of the elite genotypes are produced using standard probabilistic mutation and one-point crossover. The probabilities of mutation and crossover are the parameters of the evolution.
- **Stopping-criteria**. The algorithm stops when at least one of the following conditions holds:

1. no improvement in the best fitness value has been recognized after k_{run} consecutive generations, k_{run} is the parameter,
2. the maximum number of generations k_{max} has been reached, k_{max} is the parameter.

5 Experiments

For the validation of the proposed approach, we use real-world data gathered for an industrial city in Siberia (Novokuznetsk, Russia). The data, given in Table 3, are publicly available [1, 2, 6]. For the purpose of our experiments the data has been divided into learning and testing parts. The learning part contained data from five years (2005–2010). Testing was performed for three years (2011–2013).

For the learning FCM we used the RCGA algorithm with the following parameters: cardinality of the population = 100, maximal number of iterations k_{max} = 500, number of iterations without the change of fitness k_{run} = 10, probability of mutation = 0.1, and probability of crossover = 0.8. After numerous trials we set up the gain of the transformation function as $g = 0.5$, and the cardinality of the elite population to 20 %. In Table 4 we present the obtained values of FCM weights.

As can be noted in Table 4, high bidirectional dependency has been found for the concepts $c_1 - c_2, c_1 - c_3, c_1 - c_4$. This means that the population structure is highly positively related to the labor market, housing facilities and standard of living. On the other hand, fairly high negative weights have been recognized for the dependencies related to the concept of psycho-emotional tension. Increased standard of living may lead to decreased psycho-emotional tension. w_{45}: 'Standard of living' (c_4) $->$ 'Psycho-emotional tension' (c_5). Moreover, decreased psycho-emotional tension improve the indicator related to the population structure. w_{51}: 'Psycho-emotional tension' (c_5) $->$ 'Population structure' (c_1).

Further analysis of the obtained dependecies is a challenge for future research and should be made by the domain specialists. To present the obtained FCM to the domain experts we illustrate it in Fig. 1. We show only the weights $w_{ij} \geq 0.8$ and $w_{ij} < -0.2$

Table 3 Well-being indexes of the comprehensive indicators (Novokuznetsk, 2005–2013)

Indicator	2005	2006	2007	2008	2009	2010	2011	2012	2013
Population structure	0.58	0.78	0.84	0.95	0.85	0.75	0.85	0.8	0.82
Labour market	0.8	0.84	0.82	0.8	0.73	0.25	0.6	0.74	0.76
Housing facilities	0.75	0.81	0.81	0.75	0.74	0.7	0.85	0.87	0.8
Standard of living	0.67	0.71	0.74	0.82	0.76	0.68	0.74	0.75	0.73
Psycho-emotional tension	0.6	0.54	0.32	0.56	0.46	0.46	0.64	0.74	0.76
Medical provision	0.67	0.8	0.87	0.95	0.91	0.7	0.85	0.87	0.86

Table 4 Causal dependencies among comprehensive indicators

Concept	c_1	c_2	c_3	c_4	c_5	c_6
Population structure c_1	–	0.90	0.96	0.82	0.57	0.87
Labour market c_2	0.84	–	0.66	0.29	0.58	−0.03
Housing facilities c_3	0.87	0.67	–	0.59	−0.03	−0.06
Standard of living c_4	0.80	0.87	0.15	–	−0.27	0.30
Psycho-emotional tension c_5	−0.28	0.06	0.03	0.07	–	−0.05
Medical provision c_6	−0.07	0.14	−0.04	0.9	0.04	–

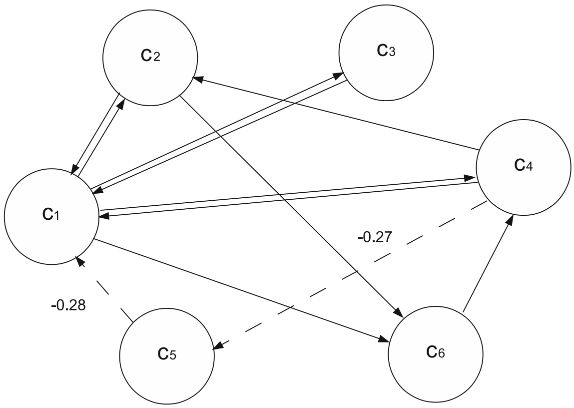

Fig. 1 FCM model of well-being

Table 5 Mean absolute errors

2011	2012	2013
0.0389	0.0510	0.0382

The obtained model has been applied to forecasting future state of well-being. The forecasting accuracies calculated as MAPE for every year within the testing period are given in Table 5.

Taking into account that the obtained errors are accumulated over six variables of the multivariate time series, they can be evaluated as very low. They are also satisfactory from the considered domain of application. This provides evidence that the applied FCM model can be effectively used for modeling and forecasting of well-being.

6 Conclusions

In this study we proposed a new FCM-based model of well-being. The model has been constructed using real-world data. The discovered model graphically illustrates the dependencies between the consolidated well-being indexes and as such can be a valuable decision support tool for policy makers. Moreover, the obtained model has been applied to the forecasting of the future course of well-being. The obtained results are very encouraging and thus motivate our further research in the considered domain. The limitation of the demonstrated approach is a small amount of available data. In spite of that, the paper proposes a general approach that can be easily scaled up in the future.

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