New Integral Approach to the Specification of STPU-Solutions

Krystian Jobczyk[1,2(✉)], Antoni Ligeza[2], and Krzysztof Kluza[2]

[1] University of Caen, Caen, France
krystian.jobczyk@unicaen.fr
[2] AGH University of Science and Technology, Kraków, Poland
{ligeza,kluza}@agh.edu.pl

Abstract. This paper is aimed at proposing some new formal system of a fuzzy logic – suitable for representation the "before" relation between temporal intervals. This system and an idea of the integral-based approach to the representation of the Allen's relations between temporal intervals is later used for a specification of a class of solutions of the so-called Simple Temporal Problem under Uncertainty and it extends the classical considerations of R. Dechter and L. Khatib in this area.

Keywords: Simple temporal problem under uncertainty · Fuzzy logic · Integral approach · Specification of solutions

1 Introduction

In [2] R. Dechter introduced the so-called *Simple Temporal Problem* as a restriction of the framework of Temporal Constraint Satisfaction Problems, tractable in polynomial time. In order to address the lack of expressiveness in standard STPs, Khatib in [10] proposed some extended version of STP – the so-called *Simple Temporal Problem with Preferences* (STPP). The lack of flexibility in execution of standard STPs was a motivation factor to introduce the so-called *Simple Temporal Problem under Uncertainty* (STPU) in [14]. In order to capture both the possible situations of acting with preferences and under uncertainty, the *Simple Temporal Problem with Preferences under Uncertainty* (STPPU) was described in [13]. Due to – [2] – *The Simple Temporal Problems*(STPs) is a kind of such a Constraints Satisfaction Problem, where a constraint between time-points X_i and X_j is represented in the constraint graph as an edge $X_i \rightarrow X_j$, labeled by a single interval $[a_{ij}, b_{ij}]$ that represents the constraint $a_{ij} \leq X_j - X_i \leq b_{ij}$. Solving an STP means finding an assignment of values to variables such that all temporal constraints are satisfied. Due to [14] – *The Simple Temporal Problem under Uncertainty* extends STP by distinguishing *contingent* events, whose occurrence is controlled by exogenous factors often referred to as "Nature".

Independently of this research path, H-J. Ohlbach proposed in [11] a new integral-based approach to the fuzzy representation of the well-known Allen relations between temporal intervals[1]– initially introduced by J. Allen in [1]. This paper

[1] Such as "before", "after", "during".

L. Rutkowski et al. (Eds.): ICAISC 2016, Part II, LNAI 9693, pp. 317–328, 2016.
DOI: 10.1007/978-3-319-39384-1_27

analysis combines both research paths. In fact, we intend to propose a new-integral-based fuzzy logic system – capable of expressing the chosen relation "before" in terms of Ohlbach's integrals – in this paper. The chosen "before" relation was chosen as some operationally "nice" and paradigmatic example among all Allen's relations, which can be modeled in a similar way. This system is conceived as some extension of the Fuzzy Integral Logic of Pavelka-Hajek from [4] – developed in [8] and –for Allen's relations in [9]. This *manouvre* is dictated by the second main paper purpose: to demonstrate how the integral-based approach to the modelling of Allen's relations allows us to differentiate a potential class of STPU-solutions. Although the specification of a class of the STPU-solutions was made by means of some analytic tools, the introduced formalism supports this analysis, it constitutes its foundation and ensures – thanks the completeness theorem – a coherence between a description of STPU-problems in terms of the proposed formalism and the proposed semantics. An algebraic approach to some unique temporal problems such as scheduling with defects was proposed in [3].

1.1 Paper's Motivation and Formulation of an Initial Problem

The main motivation factor of the current analysis is a lack of an approach to the STPU-solving – capable of elucidating of an "evolution" of solutions. In particular, there is no integral-based approach – in spite of integral-based representation of Allen relations between intervals. In addition, it seems that a theoretic, meta-logical establishing of the STPU[2] has not been discussed yet in a specialist literature. Some possibilities of modelling of preferences in fuzzy temporal contexts were, somehow, demonstrated by authors of this paper in [5–7], but without the explicit referring to STP and its extensions. From the more practical point of view this paper analysis are motivated by the following example of the STPU:

Example: *Consider a satellite which performs a task to observe a volcano Etna in some time-interval [0; 80]. The cloudiness can take place in time interval* $j(x)$*= [20; 50], but it comes out gradually in this time-interval. When to begin the observation task (beginning from the initial time-point) in order to maximize a chance for finishing the satellite observation in a given time-interval [0; 80]?*

We associate this main problem to the following (sub)problems supporting its solution in terms of the features of "before"-relation.

Problem 1: *Does the Allen relation "before" take a one or many values in the integral-based depiction? If many, show which values from [0,1]-interval can be taken by this relation in their integral-based depiction for linear functions.*

Problem 2: *If the "before"-relation can be evaluated by values from [0,1], decide for which real parameters* $C > 0$ *this relation takes values no smaller than 0,7?*

[2] Establishing as completeness of system describing the STPU w.r.t its models.

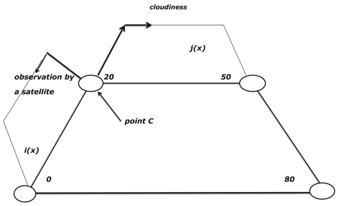

Fig. 1. STPU for observation task of the satellite

2 Terminological Background

The proper analysis will be prefaced by introducing a terminological background regarding concepts of the fuzzy intervals, operations on them and the Ohlbach's representation of Allen's interval relations.

Definition 1 *(Fuzzy Interval). Assume that* $f : \mathcal{R} \mapsto [0.1]$ *is a total integrable function (not necessary continuous). Than the fuzzy interval* i_f *(corresponding to a function f) is defined as follows:* $i_f = \{(x, y), \subseteq \mathcal{R} \times [0.1] | y \leq f(x)\}$.

A fuzzy set (in a comparison with a crisp one) is illustrated on the picture (Fig. 2):

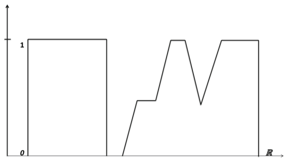

Fig. 2. A crisp and a fuzzy interval

Operations of an intersection and a union of two fuzzy intervals are defined with a use of the appropriate t-norms. Classically: $(i \cap j)(x) =^{def} min\{i(x), j(x)\}$ and $(i \cup j)(x) =^{def} max\{i(x), j(x)\}$.

Some Basic Transformations on Fuzzy Sets. We can associate some additional transformation with fuzzy intervals – presented in details in [11,12]. We restrict their list to the following, especially useful:

$$identity(i) =^{def} \quad i,$$
$$integrate^{+}(x) =^{def} \int_{\neg\infty}^{x} i(y)dy/|i|,$$
$$integrate^{-}(x) =^{def} \int_{x}^{+\infty} i(y)dy/|i|,$$
$$cut_{x_1,x_2}(x) = 0, \text{ if } x < x_1 \text{ or } x_2 \leq x; i(x) - \text{ otherwise.}$$

1. Before. In order to define this relation let us assume that some point-interval relation:'p before j' is given and let us denote it by $B(j)$. In order to extend $B(j)$ to the interval-interval relation (for j and some interval i), we should average this point-interval *before*-relation over the interval i. Since fuzzy intervals form subsets of R^2, all these points satisfying this new relation $before(i,j)$ are given by the appropriate integral, namely: $\int i(x)B(j)dx/|i|$. ($|i|$ normalizes this integral to be smaller than 1.)

Infinite Intervals: This general methods should be somehow modified w.r.t the situation when either i or j or both intervals are infinite. If i is $[a,\infty)$-type, than nothing can be after i, thus $before(i,j)$ must yield 0. For a contrast, if j is $(-\infty,a]$-type, than nothing can be before j, what leads to the same value 0.

It remains the case, when i is $(-\infty,a]$-type, but j is finite or of $[a,\infty)$-type. In this case we should find some alternative, because $\int i(x)B(j)dx$ will be infinite. Therefore we take an intersection $i \cap_{min} j$ instead of the whole infinite i. Since j is not of a $(-\infty,a]$-type, the intersection $i \cap_{min} j$ must be finite and the $before(i,j)$ is given by:

$$before(i,j) =^{def} \int (i(x) \cap_{min} B(j))dx/|i(x) \cap_{min} j(x)|.$$

In results, for some point-interval relation $B(j)$ the new interval-interval relation $before(i,j)$ should be represented as below:

$$before(i,j) = \begin{cases} 0 & if\ i = \emptyset\ or\ i = [a,\infty)\ or\ j = \emptyset \\ 1 & if\ \ i = (\infty,a]\ and\ i \cap j = \emptyset \\ \int i(x) \cap_{min} B(j)/|i(x) \cap_{min} j(x)| & if\ i = (\infty,a] - type \\ \int i(x)B(j)/|i(x)| & otherwise \end{cases}$$

In order to solve this problem we will consider two fuzzy intervals $i(x)$ and $j(x)$. For simplicity (but without losing of generality) we can take into account a single Allen relation $before(i,j)(x)$ between them localized w.r.t the y-axis as given on the picture (Fig. 3):

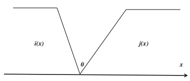

Fig. 3. Fuzzy intervals $i(x)$ and $j(x)$

3 Some Extension of the Fuzzy Integral Logic of Hajek for the Fuzzy Allen Relation "before"

3.1 Requirements of the Construction

We will extend the *Fuzzy Integral Logic* of Hajek from [4] in order to express the interval-interval relation "before". In order to render it in a language of our system we need introduce a new relation symbol, say $B(i,j)$ for atomic terms i, j (denoted by fuzzy intervals). In accordance with the Ohlbach's definition of this relation, one also need introduce the following: a) a symbol, say $B(i)(x)$ to represent the atomic interval-point relation $before(i,x)$ (an interval i is'before' a point x) and b) a constant for normalization factor N. The point-interval relation $B(i)(x)$ etc. will be denoted by a symbol: $\hat{B}^i_x$. Because of the need of a clear distinction between the FLI -syntax and its semantics with Allen's relations – the fuzzy intervals $i(x), j(x)$ will be represented in the FLI -syntax by formulas ϕ^i_x and ϕ^j_x (resp.). In results, we will write: $\int \psi^i_t \hat{B}^j_t dt$ instead of the Ohlbach's formula: $\int i(x)B(j)(x)dx$ etc.

3.2 Syntax and Semantics

Language. For these purposes we introduce our FLI in an appropriate language L of Lukasiewicz Propositional Logic (LukPL) with the following connectives and constants: $\rightarrow, \neg, \iff, \wedge$ (weak conjunction), $\otimes$ (strong conjunction), $\vee$ (weak disjunction), $\oplus$ (strong disjunction) and propositional constants 0 and 1. We extend by new constants: $\hat{r}_1, \hat{r}_2, \hat{r}_3 \ldots$, representing in the language $\mathcal{L}(FLI)$ the rational numbers: $\hat{r}_1, \hat{r}_2, \ldots, s_1, s_2 \ldots$ etc. We enrich this language by $\exists$– and $\forall$-quantifiers to the full language of Rational Pavelka Predicate Logic RPL$\forall$.
The alphabet of $\mathcal{L}$(FLI) consists of[3]:

- propositional variables: $\phi, \chi, \psi, \ldots, a_i, b_i, \ldots x, y, t \ldots$
- functional symbols: $\phi_t, \phi_{x-t}, \chi_t, \chi_{x-t}, \cdots$
- predicates (of point-interval relations): $\hat{B}^i_t$, $\hat{D}^i_t$, $\hat{M}^i_t$, $\hat{S}^i_t$, $\hat{F}^i_t \ldots$..
- rational constant names: $\widehat{r_1}, \widehat{r_2}, \ldots \widehat{0}, \widehat{1}$, scalar constants: $\widehat{N}, \widehat{M} \ldots$ etc.
- quantifiers: $\forall, \exists| \int ()dx, \int\int ()dxdy, \int_0^\infty ()dt, \int_{t_0}^{t_1} ()dt \ldots$
- operations: $\rightarrow, \neg, \vee, \wedge, \bullet, \oplus, \otimes, =$.

Set of Formulas FOR: The class of well-formed formulas FOR of $\mathcal{L}$(FLI) form *propositional variables* and *rational constants* as *atomic* formulas. The next - formulas obtained from given $\phi, \chi \in FOR$ by operations $\neg, \vee, \wedge, \rightarrow$, $\oplus, \otimes, \forall, \exists, \int ()dx$ and the formulas obtained from $\phi_i, \chi_i \in FOR$ by

[3] This system is a (slightly modified) system introduced in [8].

operations $\neg, \vee, \wedge, \rightarrow, \oplus, \otimes, \bullet, \forall, \exists, \int_0^\infty ()dt, \int_0^{t_0} ()dt, \int_{t_0}^{t_1} ()dt$. Finally, formulas obtained from $\phi_i \in FOR$ and rational numbers by operations $\neg, \vee, \wedge, \rightarrow, \oplus, \otimes, \bullet$ belong to FOR as well. These classes of formulas exhaust the list of FOR of $\mathcal{L}$(FLI).

Example: $\int_0^\infty \phi_t \bullet \chi_{x-t} dt \rightarrow \widehat{r} \in FOR$, but $\int \phi dx \rightarrow \int_0^\infty \chi_t dt$ does not.

The mentioned system FLI arises in $\mathcal{L}$(FLI) by assuming the following

Axioms: – partially considered by Hajek in [4]:

$$\int (\neg\phi) dx = \neg \int \phi dx, \int (\phi \rightarrow \chi) dx \rightarrow (\int \phi dx \rightarrow \int \chi dx)$$

$$\int (\phi \otimes \chi) dx = ((\int \phi dx \rightarrow \int (\phi \wedge \chi) dx) \rightarrow \int \chi dx))$$

$$\int \int \phi dx dy = \int \int \phi dy dx^{4} \text{ (Fubini theorem):}$$

and new axioms defining the algebraic properties of convolutions[5]:

$$\int_0^\infty \phi_t \bullet \chi_{x-t} dt = \int_0^\infty \phi_{x-t} \bullet \chi_t dt$$

$$\widehat{r} \int_0^\infty \phi_t \bullet \chi_{x-t} dt = \int_0^\infty (\widehat{r}\phi_t) \bullet \chi_{x-t} dt, (\text{r}-constant) \text{ (associativity)}^{6}$$

$$\int_0^\infty \phi_t \bullet (\chi_{x-t} \oplus \psi_t) dt = \int_0^\infty \phi_t \bullet \chi_{x-t} dt \oplus \int_0^\infty \phi_t \bullet \psi_{x-t} dt \text{ (distributivity)}$$

As inference rules we assume *Modus Ponens,* generalization rule for $\int -symbol$ and two new specific rules: $\frac{\phi}{\int \phi dx}, \frac{\phi \rightarrow \chi}{\int \phi dx \rightarrow \int \chi dx}$ and the same rules for indexed formulas and convolution integrals.

Semantics. Our intention is to semantically represent $\int$-formulas of $\mathcal{L}$(FLI) by 'semantic' integrals. Because all of the considered point-interval relations $D(p, j), M(p, j)$ etc. are functions for the fixed j, than such "semantic" integrals can be defined on a class of the appropriate functions.

More precisely, we will understand such integrals I as a mapping $I : f \in Alg \mapsto If(x) \in [0, 1]$ (where Alg is an algebra of functions from $M \neq \emptyset$ to [0.1] containing each rational function $r \in$ [0.1] and closed on $\Rightarrow$ (see: [4], p. 240))

[4] If both sides are defined.

[5] We only present the axioms for convolution in the infinite domain. The axioms in other cases are introduced in the same way.

[6] That is wrt the scalar multiplication.

satisfying the conditions corresponding to the presented axioms of $\mathcal{L}$(FLI). For example, it holds:

$$I(1-f)dx = 1 - Ifdx,\ I(f \Rightarrow g)dx \leq (Ifdx \Rightarrow Igdx) \tag{1}$$

$$I(Ifdx)dy = I(Ifdy)dx \tag{2}$$

$$I(f(t)g(x-t)dt = Ig(t)f(x-t)dt \tag{3}$$

$$rIf(t)g(x-t)dt = I(rf(t))g(x-t)dt \tag{4}$$

We omit the whole presentation of these corresponding conditions. They can be found in [8] and partially in [4].

Interpretation. Let assume that $Int = (\triangle, \|\phi\|)$ with $\triangle \neq \emptyset$ and a (classical fuzzy) truth-value interpretation–function: $\|\|$ of formulas of $\mathcal{L}$(FLI). The propositions of Łukasiewicz logic are interpreted in the sense of $\|\|$ as follows: $\|\neg(\psi)\| = 1 - x$, $\| \rightarrow (\phi, \psi)\| = min\{1, 1 - x + y\}$, $\| \wedge (\phi, \psi)\| = min\{x, y\}$, $\| \wedge (\phi, \psi)\| = max\{x, y\}$, $\| \otimes (\phi, \psi)\| = max\{0, x + y - 1\}$, and $\| \oplus (\phi, \psi)\| = min\{1, x + y\}$ for any $x, y \in$ MV-algebra A[7].

We inductively expand now this interpretation for new elements of the grammar $\mathcal{L}$(FLI) as below. **Definition of the Model:** We define a *model* M as a

syntax ($\phi \in \mathcal{L}(FLI)$)	**fuzzy semantics** ($\|\phi\|_{FLI}$)
a_i, b_i	objects A_i, B_i for $i \in \{1, \ldots, k\}$
ϕ_i	functions $f(i)$ for $i \in \{x, t, x-t, t-x\}$
$\int \phi dx, (\int_0^\infty \phi dx)$	$Ifdx, (I_0^\infty fdx)$
$\phi_i \bullet \chi_i$	$\|\phi_i\| \star \|\chi_i\|$ (i like above)
$\phi \otimes \chi$	$min\{1, \|\phi\| + \|\chi\|\}$
$(\|\phi_i \otimes \chi_i\|)$	$min\{1, \|\phi_i\| + \|\chi_i\|\}$
$\int_0^\infty \phi_t \bullet \chi_{x-t} dt$	$I_0^\infty g(t) \star f(x-t))dt$
$\widehat{r} \int_0^\infty dt$	$\|\widehat{r}\| \star \|I_0^\infty f\|dt = rI_0^\infty fdt$
$\frac{\int \phi_t^i \bullet \widehat{B}_{x-t}^j dt}{\widehat{N}}$	$I\frac{i(t)B(j)(x-t)\star f(x-t))dt}{N}$

n-tuple of the form: $M = \langle |M|, \{r_0, r_1, ..\}, f_i, If_idx, I_0^\infty f_ig_jdt\rangle$ where $|M|$ is a countable (or finite) set $\{r_0, r_1, \ldots\}, f_i$ are respectively: a set of rational numbers belonging to $|M|$, and atomic integrable functions. If_i are integrals on the algebra Alg of subsets of $|M|$ and $I_0^\infty f_ig_jdt$ are convolutions of f_i and g_j.

7 We omit a detailed definition of MV-algebra as a structure that algebraically interprets a language of a fuzzy logic, it can be easily found in [4].

FLI turns out to be complete w.r.t such a model and undecidable. If a model M is given, we write $\|\phi\|_{M,v}$ for a denotation of the *truth value under evaluation* v for each formula of $\mathcal{L}(FLI)$ as a function: $\mathcal{L}(FLI) \rightarrow [0,1]$. If M is a model and v is a valuation, than: $\|\widehat{r}\|_{M,v} = r, \|x\|_{M,v} = a \in [0,1]$ for a variable x and for a predicate $Pred(t_1,\ldots,t_k)$ it holds: $\|Pred(t_1,\ldots t_k)\|_{M,v} = Rel(\|t_1\|_{M,v},\ldots \|t_k\|_{M,v})$ for Rel interpreting $Pred$ in a model M.

Example 1: Consider two intervals $i(x)$ and $j(x)$ such that $i,j \subseteq [a,b]$ and two actions A and B associated with $i(x)$ and $j(x)$ (resp.) Let denote this by $i(x)^A$ and $j(x)^B$ The fact that action A is parallel to B can be represented by an integral-based $during(i(x)^A, j(x)^B)$-relation and interpreted by a model $\mathcal{M} = \langle [a,b], i^A, j^B, \int_a^b i^A D(i(x)^A, j(x)^B)dx/|i^A|\rangle$.

Completeness Theorem for *FLI*: *For each theory T over predicate* $\mathcal{L}$ *(FLI) and for each formula* $\phi \in \mathcal{L}(FLI)$ *it holds:* $|\phi|_{FLI} = \|\phi\|_{FLI}$.

Proof is very similar to the completeness proof for the Hajek's integral logic from [4], so it will be omitted here.

4 Solving of the Problem

In this section we intend to solve the main problem – defined and depicted on a picture in the introductory section with two problems associated to it. Anyhow, we preface this solution by considerations focused on analytic features of $before(i,j)$-relation in terms of integrals. In particular, we present a graph of the function representing this relation provided that the atomic point-interval relation is linear. We decide on this linearity assumption because of a simplicity of the further analysis.

4.1 A Formal Depiction of the Problem and Some Introductory Assumptions

We begin with the formal depiction of the presented problems. For that reason, let us note that the interval-interval definition of Allen's relation $before(i,j)(x)$ is of the type:

$$before(i,j)(x) = \frac{\int i(x)j(x)dx}{max_a \int i(x-a)j(x)dx} \tag{5}$$

According to the above requirement let us consider their unique form for $i(x)$ and $j(x)$ given by linear functions, i.e. $\begin{cases} i(x) = & Ax, A > 0, B < 0, \\ j(x) = & Bx, A < 0, B > 0 \end{cases}$ (see: Fig. 1)

Than:

$$(1) = \frac{AB\int x^2dx}{AB\int (x-a)xdx} = \frac{\int x^2dx}{\int (x^2-ax)dx} = \frac{x^3}{3[\frac{x^3}{3}-\frac{ax^2}{2}]} = \frac{x^3}{3[\frac{2x^3-3ax^2}{6}]} = \frac{2x^3}{2x^3-3ax^2} \tag{6}$$

for some done $a \in R$.

It remains now to investigate the function $f(x) = \frac{2x^3}{2x^3-3ax^2}$ in order to find its values in the fuzzy interval [0,1].

4.2 Investigation of Properties of the Considered Allen's Before-Relation in the Integrals-Based Representation

In this subsection we check the analytic properties of the function $\frac{2x^3}{2x^3-3ax^2}$ representing the considered Allen's relation $before(i,j)(x)$ for fuzzy intervals $i(x)$ and $j(x)$ given by linear functions.

(a) ***Domain of*** $f(x)$.
$2x^3 - 3ax^2 \neq 0 \iff x \neq 0$ or $x \neq 3/2a$, so $x \in R/\{0, 3/2a\}$.

(b) ***Limits:***
$\lim_{x\to-\infty} \frac{3x^3}{2x^3-3ax^2} = [\frac{-\infty}{-\infty}] = 1$, $\lim_{x\to\infty} \frac{3x^3}{2x^3-3ax^2} = [\frac{\infty}{\infty}] = 1$,
$\lim_{x\to(\frac{3}{2}a)^-} \frac{2x^3}{2x^3-3ax^2} = [\frac{2(\frac{3a}{2})^3}{0^-}] = -\infty$, $\lim_{x\to(\frac{3}{2}a)^+} \frac{2x^3}{2x^3-3ax^2} = [\frac{2(\frac{3a}{2})^3}{0^+}] = \infty$.
$\lim_{x\to(\frac{0}{0})^-} \frac{2x^3}{2x^3-3ax^2} = [\frac{0}{0^-}] = 0$, $\lim_{x\to(\frac{0}{0})^+} \frac{2x^3}{2x^3-3ax^2} = [\frac{0}{0+}] = 0$.

It allows us to visualize the graph of the function as follows: Therefore, $0 \leq \frac{2x^3}{2x^3-3ax^2} \leq 1$ for $x \in (-\infty, 0)$ (Fig. 4).

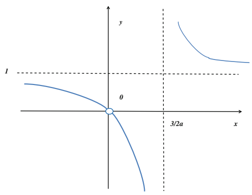

Fig. 4. An outline of the function $\frac{2x^3}{2x^3-3ax^2}$

4.3 Some Modification of the Initial Assumptions

Let us note that the above solution holds by assumption that intervals $i(x)$ and $j(x)$ meets in a point $x = 0$ as on the picture. One needs therefore a function $F(x-B) = \frac{2(x-B)^3}{2(x-B)^3-3a(x-B)^2}$. Its graph stems from the earlier graph of $\frac{2x^3}{2x^3-3ax^2}$ *via* translation by a vector (0, B). It looks like this: We are now interested in the

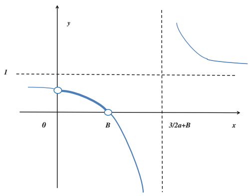

Fig. 5. A diagram of a function $f(x)$ in the required vector translation.

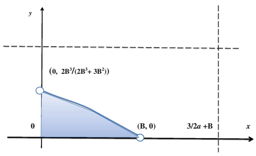

Fig. 6. The fragment of a graph of a function $F(x - B)$, which we are interested in – as a visual representation of our problem solution.

part of this graph between $x_0 = 0$ and $x_1 =$ B. Immediately from the graph one can see that for $x = B$ the function $F(x - B)$ is not defined, but $\lim_{B \to 0} F(x - B) = 0$. On can easily compute that $F(0 - B) = \frac{-2B^3}{-2B^3 - 3B^2 a} = \frac{2B^3}{-B^3 + 3B^2 a} < 1$. It can be visualized as follows: Therefore, the investigated function takes the values from the interval $I = (0; \frac{2B^3}{2B^3 + 3B^2 a})$ (Figs. 5 and 6).

Example: For B = 1 we obtain an interval $I_1 = (0, \frac{2}{2+3a})$ for done $a \in R$.

4.4 Further Properties of This Integral-Based Representation of Allen's Before-Relation

At the end we intend to show that our function $\frac{2x^3}{2x^3 - 3ax^2}$ is uniformly continuous. It means that the change the fuzzy values (one for another) is "lazy" and non-radical in the whole interval (0, B) (for arbitrary pairs of x_1 and x_2 from this interval if only $|x_1 - x_2| < \rho$ for some arbitrary $\rho > 0$.)

For this purpose let us consider its module of continuity:

$$|\frac{2x_1^3}{2x_1^3 - 3ax_1^2} - \frac{2x_2^3}{2x_2^3 - 3ax_2^2}| = |\frac{2x_1^3(2x_2^3 - 3ax_2^2) - 2x_2^3(2x_1^3 - 3ax_1^2)}{(2x_1^3 - 3ax_1^2)(2x_2^3 - 3ax_2^2)}| = \quad (7)$$

$$|\frac{-6ax_1^3x_2^2 + 6ax_2^3x_1^3}{(2x_1^3 - 3ax_1^2)(2x_2^3 - 3ax_2^2)}| = |\frac{6ax_1^2x_2^2(x_2 - x_1)}{(2x_1^3 - 3ax_1^2)(2x_2^3 - 3ax_2^2)}| \leq \frac{6a|x_1 - x_2|}{M} = \frac{6a\rho}{M}, \quad (8)$$

where M is the appropriate lower bound of the last denominator. Hence $\forall \epsilon > 0 : |\frac{2x_1^3}{2x_1^3 - 3ax_1^2} - \frac{2x_2^3}{2x_2^3 - 3ax_2^2}| < \epsilon$ if only assume $\rho \leq \frac{M\epsilon}{6a}$, what justifies a desired uniform continuity of our function.

4.5 Solving of the Main Problem

The arrangements, presented above, allow us to solve the main problem with the observation task of a satellite and the problems associated to it in the introductory part. In order to make it let us recall that:

$$before(i,j) = \frac{\int i(x)Bef(j)(x)dx}{max_a \int i(x-a)Bef(j)(x)dx}, \tag{9}$$

for some point-interval relation $Bef(j)(x)$. Meanwhile, we have just shown that for linear functions defining the fuzzy intervals this general definition can be given by:

$$before(i,j) = \frac{2x^3}{2x^3 - 3ax^2} \tag{10}$$

and $0 \leq before(i,j)(x) \leq 1$ holds for $x \in (0; \frac{2C^3}{2C^3+3C^2a})$. Nevertheless, by our assumption $C = 20$ (min) we obtain that: $x \in (0; \frac{2\bullet 20^3}{2\bullet 20^3+3\bullet 20^2a}) = (0; \frac{2\bullet 8000}{2\bullet 8000+1200a})$. Assuming for simplicity $a = 1$ we can get $x \in (0; \frac{16000}{17200}) = (0; 0,9302)$.

Therefore, our function takes values from $(0; 0,9302)$.

Problem 2: *For which parameters $C > 0$ the $before(i,j)(x)$ -relation takes values no smaller than 0,7?*

Solution: $0,7 \leq \frac{2C^3}{2C^3+3C^2}$. It is equivalent to $0 \leq \frac{2C^3}{2C^3+3C^2} - \frac{0,7\bullet(2C^3+2C^2)}{2C^3+3C^2} = \frac{1,3(C^3-2,1C^2)}{2C^3+3C^2}$. Let's consider the equation $\frac{1,3(C^3-2,1C^2)}{2C^3+3C^2} = 0$ (for $C \neq 0$). It leads to the equation $1,3(C^3 - 2,1C^2 = 0 \iff C^2(1,3C - 2,1) = 0 \iff 1,3C = 2,1 \iff C = \frac{2,1}{1.3}$. Therefore, our unequality holds for $C \in (-\infty; 0) \bigcup (\frac{21}{13}; \infty)$. Because we are only interested in $C > 0$, so the only solution is given by an interval $(\frac{21}{13}; \infty)$.

5 Concluding Remarks

It has emerged that the integral-based approach to the Allen temporal relations allows us to specify the class of STPU-solutions. It also appears that the intuitively graspable point-solutions are preserved as the appropriate ones – as a board case solution in considered situations. Finally, the construction of a fuzzy logic system and its completeness ensures that models of STPU (in terms of before-relation) really refer to the formal descriptions of STPU in the appropriate languages. It seems that the similar procedures can be repeated for other Allen relations in the integral-based Ohlbach's depiction.

Anyhow, the considered STPU-problem belongs to the class of relatively elementary problems. It seems that many similar problems, with a higher complication degree – such as STPPU-problems – could be investigated in the similar way. In this perspective, the analysis of the current paper seems to be open.

Acknowledgement. Authors of this paper are grateful to Katarzyna Grobler for some useful remarks and comments.

References

1. Allen, J.: Maintaining knowledge about temporal intervals. Commun. ACM **26**(11), 832–843 (1983)
2. Dechter, R.: Temporal causal networks. Artif. Intell. **49**, 61–95 (1991)
3. Grobler-Debska, A., Kucharska, E., Dudek-Dyduch, E.: Idea of switching algebraic-logical models in flow-shop scheduling problem with defects. In: Methods and Models in Automaton and Robotics (MMAR), pp. 532–537 (2013)
4. Hajek, P.: Metamathematics of Fuzzy Logic. Kluwer Academic Publishers, Dordrecht (1998)
5. Jobczyk, K., Ligeza, A.: Temporal planning in terms of a fuzzy integral logic (fli) versus temporal planning in pddl. In: Proceedings of INISTA, pp. 1–8 (2015)
6. Jobczyk, K., Ligeza, A., Bouzid, M., Karczmarczuk, J.: Comparative approach to the multi-valued logic construction for preferences. In: Rutkowski, L., Korytkowski, M., Scherer, R., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.) Artificial Intelligence and Soft Computing. LNCS, vol. 9119, pp. 172–183. Springer, Heidelberg (2015)
7. Jobczyk, K., Ligeza, A., Karczmarczuk, J.: Fuzzy temporal approach to the handling of temporal interval relations and preferences. In: Proceedings of INISTA (2015)
8. Jobczyk, K., Bouzid, M., Ligeza, A., Karczmarczuk, J.: Fuzzy integral logic expressible by convolutions. In: Proceeding of ECAI 2014, pp. 1042–1043 (2014)
9. Jobczyk, K., Bouzid, M., Ligeza, A., Karczmarczuk, J.: Fuzzy logic for representation of temporal verbs and adverbs 'often' and 'many times'. In: Proceeding of LENSL 2011 Tokyo (2014)
10. Khatib, L., Morris, P., Morris, R., Rossi, F.: Temporal reasoning about preferences. In: Proceedings of IJCAI-01, pp. 322–327 (2001)
11. Ohlbach, H.: Relations between time intervals. In: 11th Internal Symposium on Temporal Representation and Reasoning, vol. 7, pp. 47–50 (2004)
12. Ohlbach, H.-J.: Fuzzy time intervals and relations-the futire library. Research Report PMS-04/04, Inst. f. Informatik, LMU Munich (2004)
13. Rossi, F., Yorke-Smith, N., Venable, K.: Temporal reasoning with preferences and uncertainty. Proc. AAAI **8**, 1385–1386 (2003)
14. Vidal, T., Fargier, H.: Handling contingency in temporal constraints networks: from consistency to controllabilities. J. Exp. Tech. Artif. Intell. **11**(1), 23–45 (1999)