Graph Theory in Product Development Planning

I. Kutschenreiter-Praszkiewicz

Abstract Development of process planning methods is an important research area. Product development planning takes into consideration lead-times of tasks and activities, bill of materials, quality plans, and risk analysis. Production process planning can be supported by graph theory. Methods such as CPM, PERT, GERT are useful in product development planning. In this article, a methodology of risk identification and assessment is combined with the GERT method.

Keywords Product development planning · GERT · Risk assessment

1 Introduction

Product development has to meet tight budgets and deadlines [1]. The aim of this article is to join planning methods based on graph theory with the risk management method.

Uncertainties and risk connected with product development projects comes from different sources and they have to be modelled respectively [2].

Deciding about product development, several criteria should be taken into consideration simultaneously. The most important of them are: financial criteria (project should consume limited costs), time criteria (project should be finished in given time) and results criteria (project should be focused on particular results).

In order to be able to integrate multi-dimensional criteria of project realization and evaluation, it is necessary to aggregate evaluation criteria and combine them with a stochastic method of project planning.

I. Kutschenreiter-Praszkiewicz (✉)
University of Bielsko-Biała, Bielsko-Biała, Poland
e-mail: ipraszkiewicz@ath.bielsko.pl

S. Zawiślak and J. Rysiński (eds.), *Graph-Based Modelling in Engineering*,
Mechanisms and Machine Science 42, DOI 10.1007/978-3-319-39020-8_12

Stochastic planning methods help to create schedules that take into consideration all possible solutions before the final schedule is identified. The most common stochastic planning methods are PERT and GERT [3].

GERT is a graph based project planning method that can handle the successive probability of possible project outputs.

2 GERT

Graphical Evaluation and Review Technique, commonly known as GERT, is a network analysis technique used in project management that allows probabilistic treatment of both network logic and estimation of activity duration [4–7]. The technique was first described in 1966 by Pritsker [8]. GERT allows loops between tasks and application of three types of nodes in the graph:

- node “and” marked in the graph as a circle indicates that all tasks have to be done,
- node “or” marked in the graph as a 45° rotate square indicates that at least one of the previous tasks has to be done,
- node “either” marked in the graph as a 45° rotated square with the line perpendicular to the square diagonal means that exactly one from the previous tasks has to be done.

A GERT graph is built with the one start node and some end nodes, which means different possibilities of project ending are possible.

In a GERT graph, edges indicate tasks for which project resources, such as time or costs and probability are allocated.

To create a GERT graph, it is necessary to follow the steps below:

- first step: transform linguistic discretion of the project into a stochastic graph. Indicate project tasks and relations between them,
- second step: determine data regarding project tasks. Fix tasks duration or costs and their probability,
- third step: determine substitute transmutation—which gives information regarding probability of project success and expecting total project time or costs. Graph has to be reduced according to principles given in Table 1,
- fourth step: interpret the results.

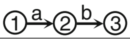
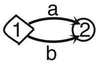
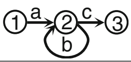
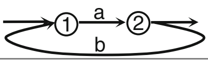
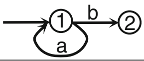
The expected time of a reduced graph can be calculated according to formula (1) [3]:

$$\mu_{1(1,n_k)} = \frac{\sigma}{\sigma_s} W_{1,n_k}(s)|_{s=0} \tag{1}$$

where:

$W_{1,nk}(s)$ graph transmittance between node 1 and n_k.

Table 1 Graph algebra

Sub graph	Sub graph probability calculation
	$p_e = p_a\, p_b$
	$p_e = p_a + p_b$
	$p_e = \frac{p_a \cdot p_c}{1 - p_b}$
	$p_e = \frac{p_a}{1 - p_a p_b}$
	$p_e = \frac{p_b}{1 - p_a}$

The presented approach used is a moment generating function that helps finding the mean and the variance of a random variable. Variance can be calculated according to formula (2) [3]:

$$\sigma^2_{1,n_k} = \mathrm{Var}(X) = \mu_{2(1,n_k)} - [\mu_{1(1,n_k)}]^2 \tag{2}$$

where:

$$\mu_{2(1,n_k)} = \frac{\sigma^2}{\sigma_s^2} W_{1,n_k}(s)|_{s=0}. \tag{3}$$

Probability related to tasks can be assessed with the use of a risk management method.

3 Risk in Product Development

Risk can be described as the chance of an event happening which may have an impact on project objectives [9].

Risk can arise from different sources and have different consequences. The source of risk may be outside of the organization (external risk), as well as inside organizations (internal risk). External risk can be caused by natural environment (earthquake, flood, etc.), sociocultural factors (strike, disturbances, etc.), technological factors (new materials, new technology, etc.), economic factors (financial crisis, business partners instability, …), political/legal factors (e.g. new regulations can cause opportunity or threat for an enterprise).

Risk could cause additional project cost, extended project time, influence project results quality, etc.

Additional project activity needed in case of a risk event happening can be managed with the use of a chosen risk management standard.

The procedure of risk management includes the following steps [10]:

- establishing risk context—identifying field of risk analysis (external context, internal context, risk criteria,…),
- risk identification—defining what can happen and why, when, where, how,
- risk analysis—estimating level of risk, determining the probability and consequences of risk future occurrence,
- risk evaluation—risk classification into given groups (e.g. low, medium, and high). For that purpose a risk matrix could be created,
- risk treatment (risk can be: accepted, transferred or reduced)

Risk management processes can be combined with the GERT method (Fig. 1). Results of risk analysis which are possible disturbances in project realization can be integrated with a project planning method. Among different project planning methods, like e.g. Critical Path Method (CPM), Program Evaluation and Review Technique (PERT) or the Gant chart, only GERT allows for back coupling modelling, taking into account their probability and consequences. GERT also allows for variants modelling of a project task. CPM is a deterministic planning method

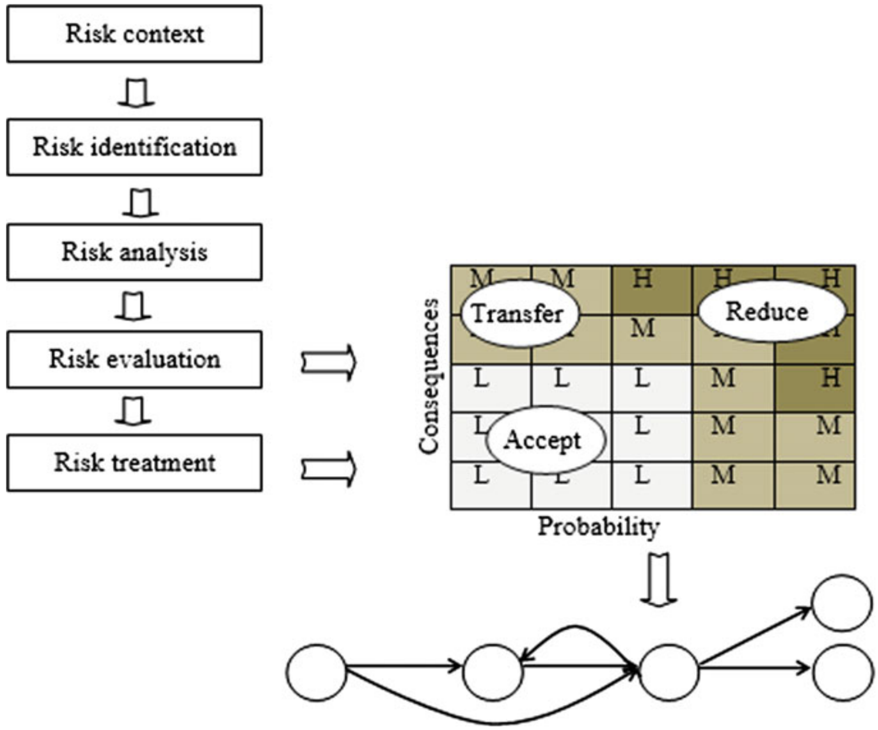


Fig. 1 Risk in product development planning

that needs precise data related to a project task. PERT takes into consideration uncertainty related to project task duration but it is not possible to model different variants of a project task.

A project focused on product development can be modelled with the use of a GERT graph, which includes main project tasks as well as risk that can occur during project realization. Project tasks could be performed in concurrent ways, but some of the project tasks could be completed with the use of different methods or tools. In the beginning of a project, the objectives are defined but the way to achieve them can be different, so the alternatives should be modeled in an applied planning method. Also, the residual risk and activity needed to reduce medium and high risk should be modelled using a product development planning method.

4 Risk Management and the GERT Method: A Joint Example

The example is focused on project planning with joint risk management and the GERT method. Any enterprise has to distinguish tasks in order to develop product. Each task must end with given results. The following tasks were specified in the example of a toothed gear production process planning of a customized product:

– brief foredesign,
– engineering documentation,
– production documentation,
– processing,
– tests.

In the presented approach, risks were identified and assessed. The following risks were identified:

(a) foredesign failure,
(b) engineering documentation failure,
(c) production documentation failure,
(d) product failure,
(e) project abandonment.

Failure probability was assessed as 0.01. Risk consequences in the case of forced foredesign failure (a) are brief foredesign repeating and correction which takes approximately 2 h. Engineering documentation failure (b) brings about 12 h, product documentation failure (c) brings about 2 h, product failure (d) brings about 8 h. The identified risks were presented in a risk matrix (Fig. 2).

The identified risks were accepted and the following tasks were added to the list of project tasks:

Consequences	≤40	M	M	H	H	H
	≤16;40>	M	M	M	M	H
	≤8;16>	L (b) (d)	L	L	M	H
	≤1;8>	L (a) (c)	L	L	M	M
	<1	L (e)	L	L	M	M
		<0,05	≤0,05;0,1>	≤0,1;0,2>	≤0,2;0,5>	≤0,5
		Probability				

Fig. 2 A risk matrix example

- foredesign correction,
- engineering documentation correction,
- production documentation correction,
- product correction.

Each task had a determined probability of occurrence and moment-generated functions of task times. For each task transmittance was determined (Table 2). Numerical values were based on a set of examples.

Table 2 Product development tasks

Project task	Task description	Task dependency	p_i	M_i	$W_i = p_i*M_i$
A	Brief fore design	Start	1	e^{12s}	e^{12s}
B	Engineering documentation	After A	1	e^{20s}	e^{20s}
C	Production documentation	After B	0.99	e^{10s}	$0.99e^{10s}$
D	Processing	After C	0.9	$e^{74.46s}$	$0.9e^{74.46s}$
E	Tests	After D end task	1	e^{8s}	e^{8s}
F	Fore design correction	Parallel A, opposite direction	0.01	e^{2s}	$0.01e^{2s}$
G	Engineering documentation correction	Lap after B	0.01	e^{12s}	$0.01e^{12s}$
H	Production documentation correction	Lap after C	0.01	e^{2s}	$0.01e^{2s}$
I	Product correction	Lap after D	0.01	e^{8s}	$0.01e^{8s}$
J	Project abandonment	End after A	0.01	e^{s}	$0.01e^{s}$
K	Project abandonment	End after B	0.01	e^{s}	$0.01e^{s}$
L	Project abandonment	End after C	0.01	e^{s}	$0.01e^{s}$

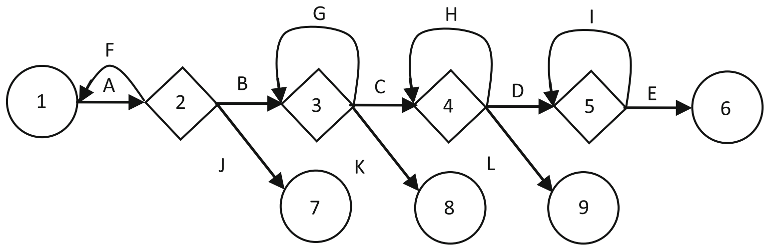


Fig. 3 GERT graph of product development project

The GERT graph of the product development tasks described in Table 2 was presented in Fig. 3. Edges in the presented graph indicate project tasks, which were denoted by capital letters. Letters from A to E indicate main project tasks and letters from F to L indicate tasks which come from a risk management method.

To calculate probability of project success it is necessary to determine substitute transmittance. Equation (4) presents the dependency between node 1 and 6 related to the graph presented in Fig. 3.

$$W_{1-6} = \frac{W_A \cdot W_B \cdot W_C \cdot W_D \cdot W_E}{(1 - W_A \cdot W_F) \cdot (1 - W_G) \cdot (1 - W_H) \cdot (1 - W_I)} \tag{4}$$

$$W_i = p_i \cdot M_i \tag{5}$$

where:
W_i transmittance of i task,
p_i probability of i task,
M_i moment—generated function of i task time

Based on the data presented in Table 2 probability of project success was determined:

$$W_{1-6} = \frac{e^{12s} \cdot e^{20s} \cdot 0.99 \cdot e^{10s} \cdot 0.9 \cdot e^{74{,}46s} \cdot e^{8s}}{(1 - 0.01 \cdot e^{2s} \cdot e^{12s}) \cdot (1 - 0.01 \cdot e^{12s}) \cdot (1 - 0.01 \cdot e^{2s}) \cdot (1 - 0.01 \cdot e^{8s})}$$

$$W_{1-6} = 0.928.$$

Expected total time of the project was calculated according to formula (6) where the transmittance is differentiating at zero:

$$\mu_{1(1,6)} = \frac{\sigma}{\sigma_s} W_{1,6}(s)|_{s=0} \tag{6}$$

$$\mu_{1(1,6)} = 116.281h.$$

Total project time variance was calculated according to formulas (7) and (8) with the use of a second derivative:

$$\sigma^2_{(1,6)} = \mu_{2(1,6)} - [\mu_{1(1,6)}]^2 \tag{7}$$

where:

$$\mu_{2(1,6)} = \frac{\sigma^2}{\sigma_s^2} W_{1,6}(s)|_{s=0} \tag{8}$$

$$\sigma_{(1,6)} = \sqrt{\mu_{2(1,6)} - [\mu_{1(1,6)}]^2} = 32.542.$$

5 Conclusions

The presented approach can be used in an enterprise for product development planning. Those projects are characterized by lack of knowledge regarding project tasks. It is possible that a project will be completed before achieving its project goal. Commonly used project planning methods, such as CPM and PERT, are not sufficient.

Enterprises implement risk management procedures, therefore the GERT method which helps integration of product development planning with risk management is promising.

Lack of knowledge regarding project tasks can be modelled with the use of probability; also risk is modelled with the use of its consequences and probability.

In project development planning and risk analysis, probability can be assessed by frequency analysis or set up by experts. The weak side of the proposed approach can be lack of statistics which help in probability assessment or lack of experts who will be able to assess probability.

References

1. Eckert, C., Clarkson, J.: Planning development processes for complex products. Res. Eng. Design **21**, 153–171 (2010)
2. Werners, B., Weber, R.: Decision and planning in research and development. In: Zimmermann, H. (ed.) Practical Applications of Fuzzy Technologies. Springer, US (1999)
3. Kukuła, K.: Badania operacyjne w przykładach i zadaniach. PWN, Warszawa (2004)
4. Kurihara, K., Nishiuchi, N.: Efficient Monte Carlo simulation method of GERT-type network for project management. Comput. Ind. Eng. **42**, 521–231 (2002)
5. Kosztyn, Z., Kiss, J.: Stochastic network planning method. In: Elleithy, K. (ed.) Advances Techniques in Computing Sciences and Software Engineering. Springer B.V. (2010)

6. Kutschenriter-Praszkiewicz, I.: Systemy bazujące na wiedzy w technicznym przygotowaniu produkcji części maszyn. Wydawnictwo ATH, Bielsko-Biała (2012)
7. Wang, ChN, Yang, G.K., Hung, KCh., Chang, K.H., Chu, P.: Evaluating the manufacturing capability of a lithographic area by using a novel vague GERT. Expert Syst. Appl. **38**, 923–932 (2011)
8. Pritsker, A.: GERT: graphical evaluation and review technique. National Aeronautics and Space Administration. Memorandum RM-4973 NASA. The RAND Corporation Santa Monica, California (1966)
9. Lu, J., Jain, L.C., Zhang, G.: Handbook on Decision Making, Vol 2: Risk Management in Decision Making. Springer, Berlin (2012)
10. Mathew, J., et al.: Engineering asset management and infrastructure sustainability. Springer, London (2011)