Bond Graphs in System Modelling

R. Ibănescu

Abstract The paper focuses on the presentation of a graphical modeling method named a bond graph method that is applicable to various dynamic systems such as mechanical, electrical, hydraulic, thermal, chemical and magnetic or a combination of these named hybrid systems or multidisciplinary systems. The method is based on an analysis of the power flow in a system from the power sources to its working elements. A brief presentation of the method illustrates its basic concepts and intends to outline the ease and the benefits of using it and its huge potential for multidisciplinary systems modelling, as well.

Keywords Bond graph · Mathematical model · Block diagram model · Hybrid systems

1 Introduction

A frequently used procedure in systems design and analysis consists in constructing a mathematical model. There are different methods to obtain such a model. Unfortunately, there is no rule to choose a method or another technique, for this reason, it is the engineer's decision, according to her/his experience and abilities. All these methods have in common that they lead directly and immediately to a system of ordinary differential equations (ODE) or, worse, to a system of algebraic-differential system of equations (DAE). Some methods, such as Lagrange's equations method, are a sort of 'black box' method, which gives little insight into the physical phenomena that lies behind the functioning of a particular system. It becomes more and more clear that the power exchanges between the parts of a device play the most important role in system analysis and modeling. This aspect urged scientists to find a new approach to system modeling, in order to exploit power transfer. In the middle of the last century, Paynter completed a new

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modeling method, based on a power exchange, called a Bond Graph [1]. The birthday of this method is considered to be April 24, 1959, as the author himself declared. This new modeling method has developed very quickly, due to its great advantages and it became a target of scientific work for many researchers [2–18]. What can be more attractive than a graphical method, suitable for various energetic domains (mechanical, electrical, hydraulic, thermal etc. or a combination of them, forming hybrid systems [18–23]), based on a thorough analysis of a power exchange between system elements and providing directly a simulation scheme, suitable for appropriate software, without writing any equations? These advantages determined a quite important part of the scientific world to focus on the bond graph method and to improve its capabilities. Nowadays, the bond graph methodology is extended to many fields, such as nuclear reactors [24, 25], optimal control [26], bioengineering [27–29], mechanisms and machines [30–33], environment engineering [34], chemical industry [35], switching systems [36], uncertain models [37], robotics [38], population interactions [39], renewable energy [40–45], genetic programming [46], estimation and control [47], acoustics [48], magnetic systems [49], identifiability [50], finite element [19], fault diagnosis [51], statics of systems [52, 53], theory of sonics [54] etc. On the other hand, a lot of dedicated software has been created, in order to directly implement the bond graph model in computers. The present work is organized as follows: Sect. 2—About Power and Energy Variables; Sect. 3—The Bond Graph Elements; Sect. 4—Causality in Bond Graphs; Sect. 5—Construction of the Bond Graph; Sect. 6—Exploitation of the Bond Graph as a Meta Model; Sect. 7—An Illustrative Exemplary Use of the Bond Graph Method for a Hydro-Mechanical System; Sect. 8—Conclusions.

2 About Power and Energy Variables

It is well known that every moving system has a power source. The idea is to consider the power as a single physical quantity that causes the system to function and then, to analyze the power as the result of the simultaneous presence of the two physical quantities, whose product is power. Let's consider, for example, a socket as a power source. The power is given as the product of voltage and current. The voltage is constant, but the current varies, depending on the necessary power. For example, a bulb will need a smaller current than a heating device. This point of view lets one consider the voltage as a *cause* physical quantity and the current as an *effect* physical quantity. More than that, the socket can be considered as an *ideal source of power*, as long as the maximum power it can provide is not exceeded. The socket is more specifically named an ideal source of voltage, because the voltage is the cause. Of course, there are also ideal sources of current. There are ideal sources of power in any other usual energetic field and one of the physical quantities of power will always be the cause, while the other one will represent the effect.

In the bond graph methodology, the two components of power (denoted $P(t)$) are generally named *effort* (denoted e) and *flow* (denoted f), one being always the cause

Table 1 The classical correspondence of effort and flow in the main energetic fields

Energetic field	Effort e	Flow f
Mechanical translation field	Force F [N]	Velocity v [m/s]
Mechanical rotation field	Torque M [Nm]	Angular velocity ω [s^{-1}]
Electrical field	Voltage u [V]	Current i [A]
Hydraulic field	Pressure P [Pa]	Volume flow rate Q [m^3/s]
Thermal field	Temperature T [K]	Entropy flow rate $\dot{S}$ [J/Ks]

Table 2 The classical correspondence of generalized momentum and generalized displacement in the main energetic fields

Energetic field	Generalized momentum p	Generalized displacement q
Mechanical translation field	Momentum p [Ns]	Linear displacement x [m]
Mechanical rotation field	Angular momentum L [Nms]	Angular displacement θ [rad]
Electrical field	Magnetic flux Φ [Wb]	Electrical charge q [C]
Hydraulic field	Pressure momentum p_P [Ns/m^2]	Volume V [m^3]
Thermal field	–	Entropy S [J/K]

and the other being the effect ($P(t) = f(t)e(t)$). They are called *power variables*. The correspondence of the effort and the flow in mechanical, electrical, hydraulic and thermal fields, from the classical point of view, are shown in Table 1.

Another important physical quantity is the energy stored in some elements of a system. The stored energy can be expressed by using two kinds of variables, named *energy variables*. They are also the state variables in the bond graph methodology. These variables are the *generalized momentum* (denoted by p) and *the generalized displacement* (denoted by q).

The correspondence of the generalized momentum and the generalized displacement in mechanical, electrical, hydraulic and thermal fields, from the classical point of view, is shown in Table 2.

The following relations exist between the power variables and energy variables:

$$\dot{q}(t) = f(t) \quad \dot{p}(t) = e(t)$$

$$q(t) = \int_0^t f(\tau)d\tau + q(t_0) \quad p(t) = \int_0^t e(\tau)d\tau + p(t_0). \tag{1}$$

For an easier and more intuitive use, the *mass flow* ($\dot{m}$ [kg/s]), the *heat flow* ($\dot{Q}$ [J/s]) or the *enthalpy flow* ($\dot{H}$ [J/s]) may be used, instead of *volume flow* or *entropy flow*. In this case, the bond graph model will be called a *pseudo bond graph*, because the product $e(t) \cdot f(t)$ is no longer a power. The pseudo bond graphs are mainly used in process engineering systems modeling. The systematic construction and the use of a bond graph model remain applicable, but a pseudo bond graph cannot be connected to a real bond graph model.

3 The Bond Graph Elements

Similar behavior, from an energy processing point of view, leads to some *generic elements*. Each of them is governed by a *generic constitutive law* and represents a group of real physical elements. For example, a generic element, called a *capacitor*, has the following corresponding real elements: a linear spring in the mechanical translation field, a torsion spring in the mechanical rotation field, a capacitor in the electrical energy field, a storage tank in the hydraulic energy field and a heated enclosure in the thermal energy field. No matter the system type, the bond graph modeling method uses only nine generic elements for constructing its model.

3.1 Sources of Power

The elements providing power are named sources and are denoted by S. A source imposes one of the power variables (the *cause* physical quantity) and the other will result from the system necessities (the *effec*t physical quantity). If the source imposes the effort, then it is called an *effort source*, and is denoted by S_e. If the source imposes the flow, then it is called a *flow source*, and is denoted by S_f. When the cause of the physical quantity must be modified, according to the system behavior via a feedback control, the source is called a *modulated source* and capital M is added to the notation (MS_e or MS_f). Power consumption for doing a work can be modeled by a source and, in these circumstances, the source is called a *sink* and is considered a *negative source*. An intuitive example is a weight raised by a lifting system.

3.2 Storage Elements

The elements that store energy in an *inductive* or *inertial* manner are denoted by I and are called *inductances* or *inertial* elements. The generic constitutive equation is $p - k_I f = 0$, where k_I is the parameter of I element. The elements that store energy in a *capacitive* manner are denoted by C and are called *capacitors* or *capacitive* elements. The generic constitutive equation is $k_C e - q = 0$, where k_C is the parameter of C element. In Table 3, we present the I and C elements in the principal energy fields.

Table 3 I, C and R elements in the main energy fields

Energetic field	I element parameter constitutive equation	C element parameter constitutive equation	R element parameter constitutive equation
Mechanical translation field	Body in translation motion $k_I = m$ [kg]-mass $p - mv = 0$	Linear spring $k_C = 1/k_e$, k_e [N/m]-spring stiffness $F - k_e\, x = 0$	Linear damper $k_R = \gamma$ [Ns/m]-viscous friction coefficient $F - \gamma v = 0$
Mechanical rotation field	Body in rotation motion $k_I = J$ [kgm^2]-moment of inertia $L - J\omega = 0$	Torsion spring $k_C = 1/k_t$, k_t [Nm]-torsion spring stiffness $M - k_t\, \omega = 0$	Torsion damper $k_R = \gamma_t$ [Nms]-torsion viscous friction coefficient $M - \gamma_t\, \omega = 0$
Electrical field	Coil $k_I = L$ [H]-inductance $\Phi - Li = 0$	Capacitor $k_C = C_e$ [C]-capacity $C_e u - q = 0$	Resistance $k_R = R_e$[Ω]-electric resistance $u - R_e i = 0$
Hydraulic field	Fluid in laminar flow $k_I = L_f$ [kg/m^4]-fluid inductance $p_P - L_f\, Q = 0$	Tank $k_C = C_f$ [m^{4}s^2/kg]-fluid capacity $V - C_f P = 0$	Pipe of constant area $k_R = R_f$ [kg/s/m^4]-fluid resistance $P - R_f\, Q = 0$
Thermal field	–	Heated substance mass $k_C = C_t$ [J/kg]-thermal capacity $Q - C_t\, T = 0$	Wall crossed by a heat flux $k_R = R_t$ [K/W]-thermal resistance $T - R_t \dot{Q} = 0$

3.3 Dissipative Elements

There are elements that irreversibly transform energy into heat and dissipate heat in the environment. These elements are called *resistors*, *dissipators* or *resistive elements* and are denoted by R. The generic constitutive equation is $e - k_R f = 0$, where k_R is the parameter of the R element, whose elements in the principal energy fields are presented in Table 3.

3.4 Transformers and Gyrators

There are two conserving power bond graph elements, which transform the power components, according to the following equation:

$$e_{in} f_{in} = e_{out} f_{out}. \tag{2}$$

The first element is called a transformer and is denoted by TF. It has a transformation ratio k_{TF}, which permits us to write two equations that connect efforts with efforts and flows with flows, as shown in Fig. 9. Some examples are: the gear, the pulley, the lever and the electrical transformer.

The second element is called a gyrator and is denoted by *GY*. It has a transformation ratio k_{GY}, which permits us to write two equations that connect efforts with flows, as shown in Fig. 9.

3.5 Connection Elements

The constitutive elements of any system are connected to each other and the power is transmitted between them. The connection between the bond graph elements is made by two kinds of elements, named *one junction*, denoted by *J1* or simply 1 and *zero junction*, denoted *J0* or simply 0. Both elements are of power conservative type. On April 24, 1959, when Henry Painter invented them, he understood that the bond graph methodology is "complete and constituted a formal discipline".

The one junction (1) connects elements having the same flow variable and the zero junction (0) connects elements having the same effort variable. The powers getting in the junction have a plus sign and the powers getting out the junction have a minus sign. The principles associated with these junctions may be regarded as generalizations of the well-known Kirchhoff's laws.

When n elements are connected at a one junction, the power conserving equation has the form:

$$\sum_{j=1}^{n} \alpha_j P_j = \sum_{j=1}^{n} \alpha_j e_j f_j = 0, \quad \alpha_j \in \{-1, +1\} \quad j = \overline{1, n}. \tag{3}$$

Taking into account that the flows are equal, the one junction constitutive relationships are:

$$f_1 = f_2 = \cdots = f_n \text{ and } \sum_{j=1}^{n} \alpha_j e_j = 0, \quad \alpha_j \in \{-1, +1\} \quad j = \overline{1, n}. \tag{4}$$

For a zero junction, Eq. 3 stands, but in this case, the efforts being equal, the constitutive equations become:

$$e_1 = e_2 = \cdots = e_n \text{ and } \sum_{j=1}^{n} \alpha_j f_j = 0, \quad \alpha_j \in \{-1, +1\} \quad j = \overline{1, n}. \tag{5}$$

Two systems, composed of three elements, namely: an inertia element (the mass m), an effort source (the force F_1) and a capacitor element (the linear spring of stiffness k_e) are represented in Fig. 1. The power of force F_1 is positive and the powers of the force of inertia F_2 and of the elastic force F_3 are negative.

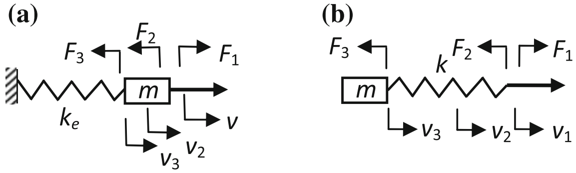


Fig. 1 Example of three elements connected to: **a** one junction **b** zero junction

In Fig. 1a, the three elements have the same velocity and they are connected to a one junction. The constitutive equations are:

$$v_1 = v_2 = v_3 \text{ and } F_1 - F_2 - F_3 = 0. \tag{6}$$

In Fig. 1b, the elements are acted upon by the same force and they are connected to a zero junction. The constitutive equations are:

$$F_1 = F_2 = F_3 \text{ and } v_1 - v_2 - v_3 = 0. \tag{7}$$

4 Causality in Bond Graphs

The causality issue is essential in bond graph modeling and its elements are shown below.

4.1 Bonds and Ports

The graphical symbol of the power exchange between the bond graph elements must have the possibility to emphasize, on one hand, a simultaneous transmission of the two components of the power and, on the other hand, the direction of the power exchange. A half of an arrow, named a *bond*, is chosen to do this, as it is shown in Fig. 2.

The power is provided by the generic bond graph element A and is received by the generic bond graph element B. The flow is usually written on the half arrow side.

Another important problem is to graphically emphasize which of the power signals is *cause* (*input*) and which is *effect* (*output*) for the elements A and B. We can have two situations for the bond presented in Fig. 2, as shown in Fig. 3.

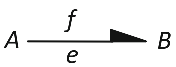


Fig. 2 The graphical representation of the power transmission between two generic bond graph elements A and B by a bond

Fig. 3 The two possible causal aspects for the bond presented in Fig. 2

Fig. 4 The two causal augmented bond for the situations presented in Fig. 3

In Fig. 3a, the element A has the flow as cause and the effort as effect. The element B has the effort as cause and the flow as effect. In Fig. 3b, the situation is reversed.

In order to graphically express these situations, a causal stroke perpendicular to the bond is drawn at the end where the element having the effort as cause is located. The causal bond for the situation depicted in Fig. 3a is represented in Fig. 4a, while for the situation depicted in Fig. 3b, the causal bond is represented in Fig. 4b. A bond provided with a causal stroke is called a *causal augmented bond*. There is no connection between the direction of the half arrow and the position of the causal stroke.

A point where a bond graph element exchanges power with another is named a *port*. There are elements with *one port* (sources, dissipative elements, inertia elements and capacitive elements), elements with *two ports* (transformers and gyrators) and *multiport* or *n-port* elements (junctions).

4.2 Causality Assignment

The causality assignment to the bonds is related to the types of the bond graph elements which are connected and this assignment should respect some rules. The parameter of the element is written near the symbol of the element and is separated by a colon.

The sources have a unique possibility for causal assignment, as illustrated in Fig. 5. The case of negative sources is also presented.

I and C elements can have two kinds of causality. When the input must be integrated in order to obtain the output, the element has *integral causality*, as is shown in Figs. 6a and 7b.

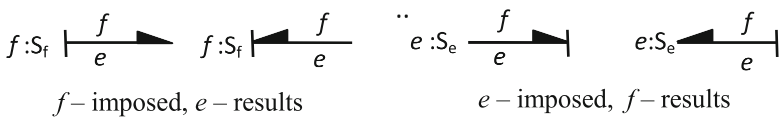


Fig. 5 Causality of the sources

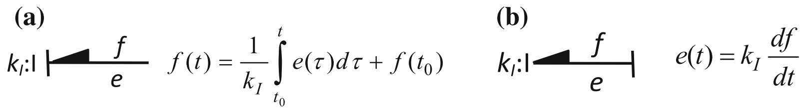


Fig. 6 Causality of I elements: **a** integral causality **b** derivative causality

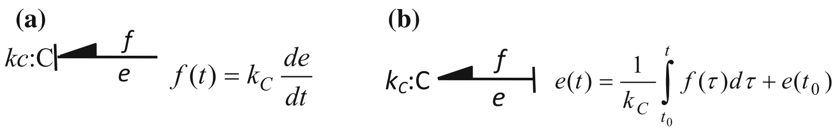


Fig. 7 Causality of the C elements: **a** derivative causality **b** integral causality

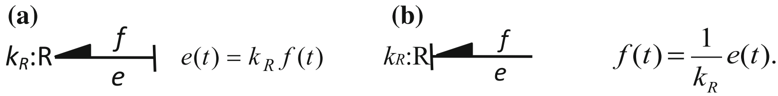


Fig. 8 Causality of R elements: **a** resistance causality **b** conductance causality

When the input must be differentiated in order to obtain the output, the elements I and C have *derivative causality*, as is shown in Figs. 6b and 7a. The half arrows point always to I and C elements, because they accumulate energy. The integral causality is always preferred to derivative causality.

R elements can have *resistance causality* if the input is the flow and *conductance causality* if the input is the effort, as can be seen in Fig. 8a, b, respectively. The half arrows point always to a R element.

Each of TF and GY elements can have two kinds of causality, as shown in Fig. 9, where the constitutive equations are written on the right-hand side. The half arrows must be oriented in the same direction, imposed by the power flow.

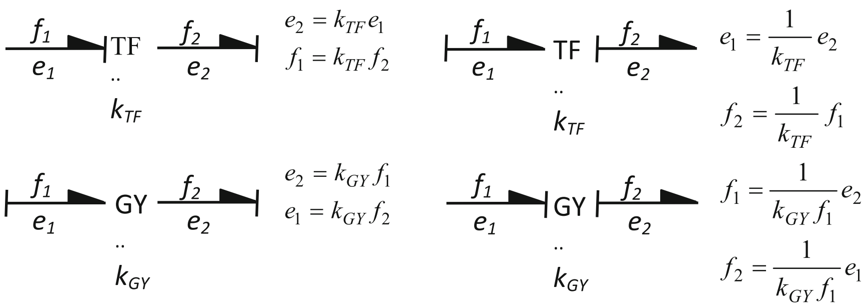
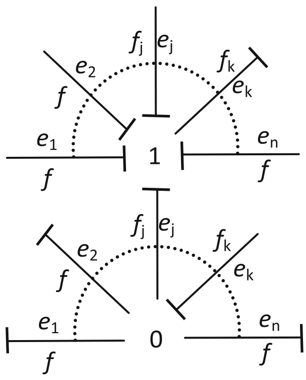


Fig. 9 Causality of TF and GY elements



$$f_j = f_k \quad j = \overline{1,n} \quad j \neq k$$

$$e_k = -\frac{1}{\alpha_k}\sum_{\substack{j=1\\ j\neq k}}^{n} \alpha_j e_j \quad \alpha_j, \alpha_k \in \{-1,+1\}$$

$$e_j = e_k \quad j = \overline{1,n} \quad j \neq k$$

$$f_k = -\frac{1}{\alpha_k}\sum_{\substack{j=1\\ j\neq k}}^{n} \alpha_j f_j \quad \alpha_j, \alpha_k \in \{-1,+1\}.$$

Fig. 10 Causality of J1 and J0 elements

The causality of J1 and J0 elements is imposed by one bond, as shown in Fig. 10. The half arrows are not represented, because they have no connection with causality assignment.

5 Construction of the Bond Graph Model

There is a specific methodology for constructing the bond graph model of mechanical systems and another one for non-mechanical systems.

5.1 Construction of Acausal Bond Graph Models for Mechanical Systems

The construction of an acausal bond graph model for mechanical systems involves the following steps:

1. Consider a J1 element for each well individualized absolute velocity in the system, that is for each I element.
2. Add a J0 element for each velocity resulting from an algebraic sum of two velocities coming from two J1 elements that is, for each relative velocity.
3. Attach the S_e, S_f, I, C and R elements to the corresponding J1 and J0 elements.
4. Introduce TF and GY elements between the corresponding J1 elements.
5. Assign the bonds with half arrows according to the flux of power between the elements.
6. Simplify the bond graph, if it is needed, and then, put numbers on the bonds.

5.2 Construction of Acausal Bond Graph Models for Non-mechanical Systems

The construction of an acausal bond graph model for non-mechanical systems involves the following steps:

1. Consider a J0 element for each point characterized by a well—individualized potential, pressure or temperature.
2. Consider a J1 element for each S_e, S_f, I, C and R element. J1 element is introduced between two J0 elements that represent the potential, the pressure or the temperature, elements S_e, S_f, I, C and R are connected to.
3. Introduce TF and GY elements.
4. Assign the bonds with half arrows, according to the power flow direction between the elements.
5. Identify the elements J0, characterized by a reference (usually null) potential, pressure or temperature, for example the mass of an electric circuit, the atmospheric pressure or the reference temperature of the system (e.g. 0 °C, 20 °C or 0 K). Such a J0 element is eliminated with all its adjacent bonds.
6. Simplify the bond graph, if it is needed, and then, put numbers on the bonds.

5.3 Construction of the Causal Bond Graph

The bond graph model is finished after the causal assignment of bonds. This operation is fulfilled by following the next three steps:

Step 1

1.1. A source is chosen and is causally assigned, getting the imposed causality of the element.
1.2. A unique causality assignment results for some adjacent bonds, because of the specific rules for elements J0, J1, TF and GY. In this way, the causality propagates at all the bonds that can get an unambiguous causality.
1.3. Steps 1.1 and 1.2 are then applied to all sources.

Step 2

2.1. An I or C element is chosen and is causally assigned with integral causality.
2.2. Analogue operations to those specified in 1.2 are applied.
2.3. Steps 2.1 and 2.2 are then applied to all I and C elements.

Step 3

3.1. An unassigned R element is conveniently causally assigned.
3.2. Analogue operations to those specified in 1.2 are applied.
3.3. Steps 3.1 and 3.2 are then applied to all R elements.

In some cases, it is not possible to have integral causality for all I and C elements, without violating the causality assignment rules for one or more J1, J0, TF and GY elements. In these cases one or more energy storage elements will be in derivative causality.

6 Exploitation of Bond Graph as a Meta-Model

The bond graph model gives rise to three ways for using it. The first way is to deduce a *differential* or *algebraic-differential system of equations*, based on the bond graph model. The second way is to obtain another graphical model, named a *block diagram model*, which can be very easy implemented in appropriate software, such as MATLAB SIMULINK. The third way is to draw the bond graph model, as it was obtained, directly into *dedicated software*, such as 20sim or many others. The last two ways do not require solving any equation.

6.1 Deriving State-Space Models

The mathematical model is a state-space model in energy variables, when the bond graph model *does not contain* storage elements in *derivative causality*.

When the constitutive equations of the elements are linear, the next steps should be followed:

1. Identify the variables $e(t)$ of the effort sources and $f(t)$ of the flow sources.
2. Identify the energy variables p_{I} and q_{C} and the power variables f_{I} and e_{C} of I and C elements.
3. Write the equations $\dot{p}_{\mathrm{I}}(t) = e_{\mathrm{I}}(t)$ and $\dot{q}_{\mathrm{C}}(t) = f_{\mathrm{C}}(t)$ for each I and C element.
4. Make explicit each $e_{\mathrm{I}}(t)$ and $f_{\mathrm{C}}(t)$ by using the constitutive equations of R, J0, J1, TF and GY elements. The state-space system of equations in terms of variables p_{I} and q_{C} is finally obtained, based on the constitutive equations of I and C elements: $f_{\mathrm{I}} = \frac{1}{k_I} p_I$ and $e_{\mathrm{C}} = \frac{1}{k_C} q_C$. The variables of sources will occur, as well. The number of equations equals the number of storage elements. The system can be easily expressed in terms of power variables f_{I} and e_{C}.

The system of equations will contain algebraic equations if the bond graph model contains storage elements in derivative causality, because the energy variables of the storage elements in derivative causality are not independent. The mathematical model is a differential-algebraic system in this case.

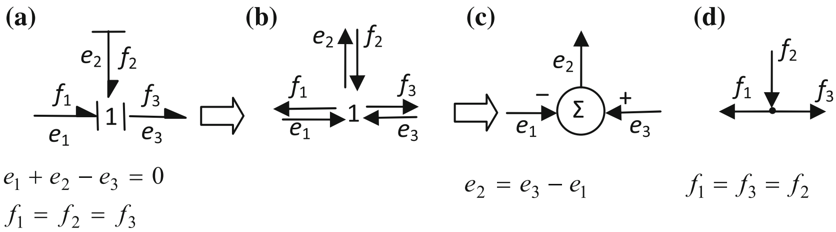


Fig. 11 Transformation of a J1 element from bond graph to block diagram model. **a** J1 element. **b** Graph of signal processing. **c** Summation. **d** Multiple output node

6.2 Deriving Block Diagram Models

The block diagram model is very easily obtained from the bond graph model, by simply transforming the bonds into two signals. One signal is for the effort and one signal is for the flow. The direction of the signals is given by the causality. The signal associated to effort is directed toward the element located near the causal stroke and the signal associated to flow is in the opposite direction. A graph called a *graph of signals processing* is obtained after this operation. This graph is then transformed into a block diagram model. Each J0 and J1 element is split in a *multiple output node-summation* pair. In the case of a J1 element, the summation is assigned to the efforts and the multiple output node is assigned to the flows. In the summation, all the efforts are inputs and only one effort is output. The output effort corresponds to the bond that imposes the velocity on each J1 element. Each effort gets a sign resulting from the equilibrium equation of the efforts. In the multiple output node, all the flows are output, except one, which is input. The input flow corresponds to the bond that imposes the velocity on J1 element. An example for three bonds connected to a J1 is given in Fig. 11.

The element J0 behaves similar to element J1, but the role of flow and effort are reversed.

The elements I and C in integral causality become *integrator blocks*, while I and C elements in derivative causality become *derivative blocks*. The elements R become *gain blocks*. The TF element is replaced by two gain blocks connecting effort to effort and flow to flow, respectively. The element GY is replaced by two gain blocks connecting efforts to flows. The sources are replaced by *source blocks*. A block diagram, processing only signals, finally results.

6.3 Software Environment for the Bond Graph Method

Dedicated software has been especially created for bond graph models. The bond graph model is directly drawn on the computer and, after introducing the numerical values of all required parameters, the simulation may run on its own. It is the most amazing modality of exploiting the bond graph model. The usual software is presented in [2].

7 An Illustrative Example—Use of Bond Graph Method for a Hydro-Mechanical System

The bond graph modeling method is illustrated by an example consisting in a hydro-mechanical system, depicted in Fig. 12. The hydraulic part is composed of a constant pressure pump of magnitude P = 1,000,000 N/m^2, which supplies oil to a hydraulic cylinder, through a pipe. The piston radius is r = 0.06 m. The length of the pipe is l = 1 m and the area is A = 0.002827 m^2. The dynamic viscosity of the hydraulic oil is μ = 0.1 Ns/m^2 and the density is ρ = 900 kg/m^3. The hydraulic inertia of the oil in the pipe $L_h = \rho l/A$ = 318,309.88 kg/m^4 and the resistance of the pipe R_h = 128 µl/A = 314,380.13 kg/(sm^4), for a presumed laminar flow.

The mechanical part consists in a spring located on the other part of the piston, having stiffness k_e = 50,000 N/m. The piston rod pushes an external weight of mass m = 100 kg, including the mass of the piston and of its rod. The mass is subjected to the action of a damper with the proportionality coefficient γ = 4000 Ns/m. The mass must overcome a constant force F = 1000 N, as well.

The bond graph of the hydraulic part of the system is depicted in Fig. 13. There are three J0's for three absolute pressures. The first J0 is considered for the reference pressure p_0, that is the atmospheric pressure, the second J0 is considered for the pressure p_1 of the source and the third J0 is considered for the pressure p_2 in the hydraulic piston. There are three J1 elements between them. The source of pressure is connected to the left one. The elements I and R, corresponding to the inertia of the hydraulic oil in the pipe and to the resistance of the pipe to oil flow, are connected to the middle one. A negative source of pressure (a sink), for the pressure exerted by the fluid on the piston, is connected to the right one. The final bond graph, obtained after removing the reference pressure together with its bonds, is represented in the right-hand side of Fig. 13.

The bond graph of the system mechanical part is depicted in Fig. 14. The piston is subjected to a force F_p, due to oil pressure that has been modeled by an effort source, and to an elastic force F_e, exerted by the spring, which has been modeled by a C element. The viscous friction force is modeled by an R element and the inertia

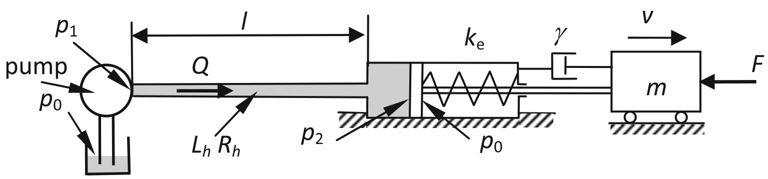


Fig. 12 The hydro-mechanical system

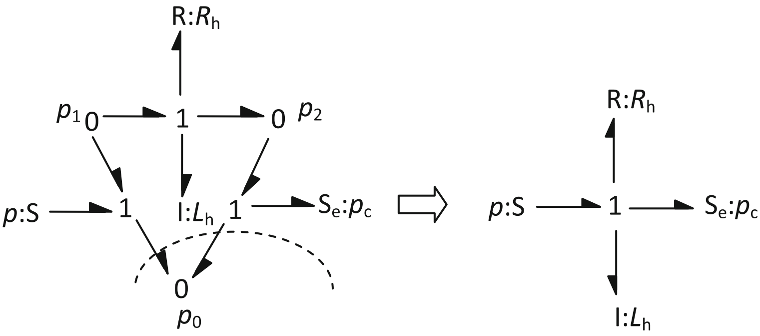


Fig. 13 The acausal bond graph model of the hydraulic part

of the mass m by an I element. The force F that must be overtaken is modeled by a negative source of effort. All these elements are connected to a J1 element corresponding to the common velocity of the mass, of the rod and of the piston.

The two bond graphs are connected by a TF element, based on the constitutive equations $F_p = p_c A$ and $Q = vA$, where A is the piston area, v is the piston velocity and Q is the volume flow.

The final bond graph is depicted in Fig. 15. The causality is assigned beginning with the two sources of effort. Then, the element I, corresponding to the mass m, is assigned in integral causality and all the bonds will receive a causal stroke. The notations for the elements and variables in connection with a bond will have as subscripts quite the bond number, for example: I_9, e_1 or f_4.

Fig. 14 The acausal bond graph model of the mechanical part

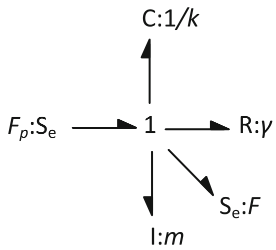
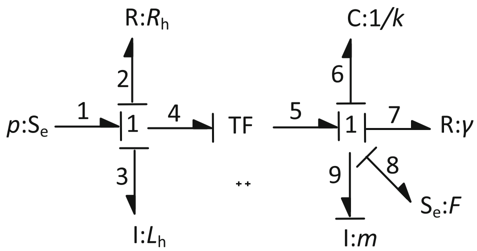


Fig. 15 The bond graph model of the whole system



Because the element I_3 is in derivative causality, the mathematical model consists of two differential equations and an algebraic one:

$$\dot{p}_8 = -\left(\frac{R_h A^2}{m} + \frac{\gamma}{m}\right) p_8 - \dot{p}_3 A - k_e q_6 - pA - F \tag{8}$$

$$\dot{q}_6 = \frac{1}{m} p_8 \tag{9}$$

$$p_3 = \frac{L_h A}{m} p_8. \tag{10}$$

This is not an unusual situation and it can be solved in two ways. The first one is to differentiate Eq. 10 and to consider a three differential equations system whose unknowns are: p_8, q_6 and p_3. The second one is to differentiate Eq. 10, to replace $\dot{p}_3$ in Eq. 8 and to finally obtain a system of two differential equations. In both situations, the systems must be expressed in an explicit form in order to perform a numerical simulation.

The bond graph shown in Fig. 15 generates the block diagram model pictured in Fig. 16. This is the most useful feature of bond graph modeling method, because any signal in the system can be seen and saved by simply attaching a scope. The derivative block du/dt must be replaced by a transfer function block $s/(as+1)$,

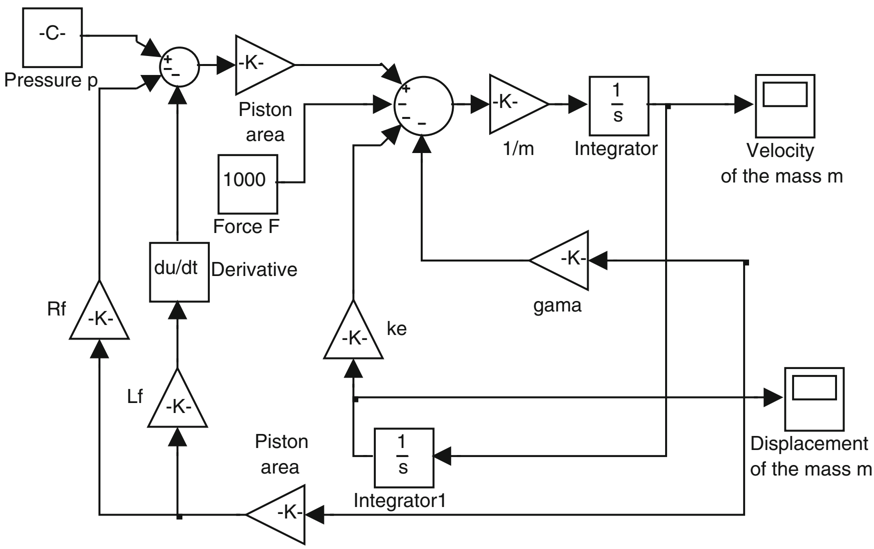


Fig. 16 The system block diagram model obtained from the bond graph model

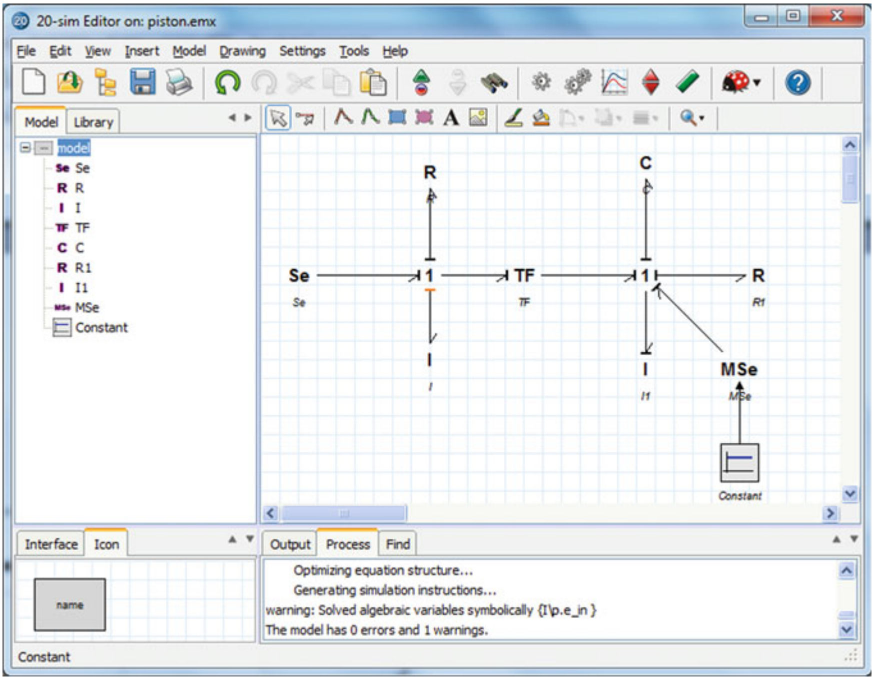


Fig. 17 The bond graph model in 20sim

which approximates the operation of differentiation [55]. This procedure prevents the occurrence of an error, for some sets of system parameter values. Possible errors are produced by the derivative block during the numerical simulation. The value of constant a, from the transfer function, must be estimated according to the frequency domain associated with the regular system operation, that is:

$$\frac{1}{a} \geq 10 \cdot (\max|\lambda_j|), \; j = 1, 2, \ldots, n \tag{11}$$

where: λ_j (j = 1, 2,…, n) are the eigenvalues of the system matrix. In this case, a very good value for a is 0.02.

The construction of the bond graph model in dedicated software is also a good option. In Fig. 17, the system bond graph created in 20sim is shown. This is an object oriented modeling method.

The diagram pictured in Fig. 18 shows the displacement of mass m. A complete analysis of system behavior, based on Eqs. 8, 9 and 10 is performed in [56].

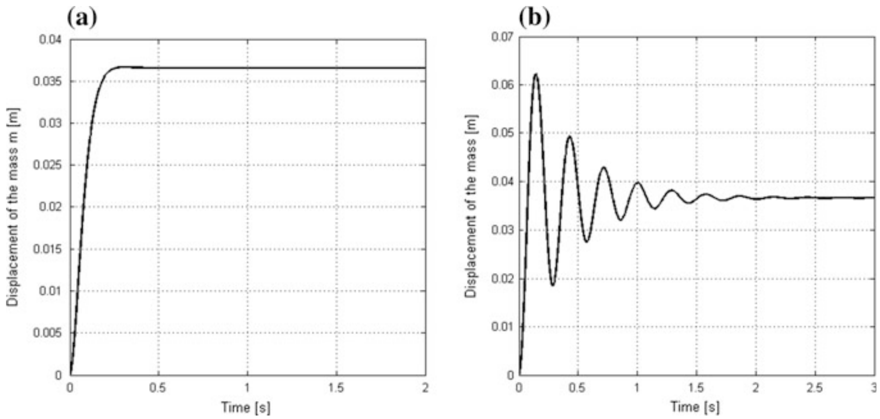


Fig. 18 The displacement of the mass m: **a** for γ = 4000 Ns/m **b** for γ = 200 Ns/m

8 Conclusions

The bond graph modeling method is based only on a deep and thorough analysis of the power flow in the system, from the source to the working elements. Any physical phenomenon misinterpretations are revealed by the violation of bond graph rules. So, the modeler has full control over the accurate interpretation of physical phenomena developed during system functioning.

There are three possibilities to exploit the bond graph model:

1. The first possibility consists in deducing a system of equations, usually a differential algebraic one. It has to be transformed in a system of differential equations after some algebraic manipulations. The system must be finally put in explicit form, in order to be numerically solved. These mathematical operations are encountered in any other modeling method.
2. The second possibility consists in getting another graphical model, named a block diagram. This model can be implemented in dedicated software, the most common being Matlab-Simulink, with all the benefits of such a model. The most important advantage consists in the possibility of visualizing any signal in the system, by simply attaching a scope to the desired signal. There is no need to solve any differential equation.
3. The third possibility consists in drawing the bond graph directly in specialized software, such as 20sim. The simulation can be performed after introducing the required parameters. There is no need to solve any differential equation, as well.

It is generally very useful in practice to apply two different modeling methods in case of very complex systems, in order to check the accuracy of results. The present paper strongly recommends the bond graph method to be one of these two.

The above issues confirm the main advantages of the bond graph method in system modelling.

References

1. Paynter, H.M.: Analysis and Design of Engineering Systems. M.I.T. Press (1961)
2. Borutzky, W.: Bond Graph Methodology. Springer, Development and Analysis of Multidisciplinary Dynamic System Models (2010)
3. Breedveld, P.-C., van Amerongen, J.: Dynamische systemen: modelvorming en simulatie met bondgrafen. Open Universiteit, Heerlen (1994)
4. Brown, F.T.: Engineering System Dynamics: A Unified Graph-Centered Approach, 2nd edn. CRC Press, New York (2006)
5. Dauphin-Tanguy, G.: Les Bond Graphs. Hermes Science Europe Ltd., Paris (2000)
6. Gawthrop, P.J., Smith, L.: Metamodelling: Bond Graphs and Dynamic Systems. Prentice Hall (1996)
7. Karnopp, D.C., Margolis, D.L., Rosenberg, R.-C.: System dynamics. Modeling and simulation of mechatronic systems, 4 edn. Willey (2006)
8. Kypuros, J.A.: System Dynamics and Control with Bond Graph Modeling. CRC Press, New York (2013)
9. Mukherjee, A., Karmakar, R.: Modelling and Symulation of Engineering Systems through Bondgraphs. Narosa Publishing House, New Delhi (2000)
10. Mukhaerjee, A., Karmakar, R., Samantaray, A.K.: Bond Graph in Modeling, Simulation and Fault Identification. I.K. International Publishing House Pvt. Ltd., New Delhi (2006)
11. Păstrăvanu, O., Ibănescu, R.: Bond-Graph Language in Modeling and Simulation of Physical-Technical Systems. "Gheorghe Asachi" Publishing House, Iași (2001)
12. Samantaray, A.K., Ould Bouamama, B.: Model-based Process Supervision. A Bond Graph Approach. Springer, London (2008)
13. Singh, M.K., Singh, B.R., Faruqi, M.A.: Modelling and Simulation of Dynamic Half Car Using Bond Graph. LAP Lambert Academic Publishing, Saarbrücken, Germany (2014)
14. Taghouti, H., Mami, A., Jmal, S.: Nouvelle Technique de Modélisation et Simulation par Bond Graph. Applications aux Circuits Hauts Fréquences et Antennes Patch. Edition Universitaires Europeennes, France (2014)
15. Thoma, J.U.: Introduction to Bond Graphs and Their Applications. Pergamon Press, Oxford (1975)
16. Thoma, J.U.: Simulations by Bond Graphs-Introduction to a Graphical Method. Springer (1990)
17. Thoma, J.U., Ould Bouamama, B.: Modeling and Simulation in Thermal and Chemical Engineering (A Bond Graph Approach). Springer (2000)
18. Wang, D., Yu, M., Low, C.B., Arogeti, S.: Model–Based Health Monitoring of Hybrid System. Springer, New-York (2013)
19. Damić, V.: Modelling flexible body systems: a bond graph component model approach. Math. Comput. Model. Dyn. Syst. **12**(2–3), 175–187 (2006)
20. Das, S.: Mechatronic Modeling and Simulation Using Bond Graphs. CRC Press, New York (2009)
21. Muvengei, O., Kihiu, J.: Using Bond Graphs in Simulating Hydro-Mechanical Systems: Case Study of an Excavator. VDM Verlag, Saarbrücken (2011)
22. Roddeck, W.: Grundprinzipien der Mechatronik. Modellbildung und Simulation mit Bondgraphen. Springer, Wiesbaden, Germany (2013)
23. Vijay, P., Samantaray, A.K., Mukherjee, A.: A bond graph model-based evaluation scheme to improve the dynamic performance of a solid oxide fuel cell. Mechatronics **19**, 489–502 (2009)
24. Sosnovsky, E., Forget, B.: Bond graph for spatial kinetics analysis of nuclear reactors. Ann. Nucl. Energy **56**, 208–226 (2013)
25. Sosnovsky, E., Forget, B.: Bond graph representation of nuclear reactor point kinetics and nearly incompressible thermal hydraulics. Ann. Nucl. Energy **68**, 15–29 (2014)
26. Mouhib, O., Jardin, A., Marquis-Favre, W., Bideaux, E., Thomasset, D.: Optimal control in bond graph formalism. Simul. Model. Pract. Theory **17**, 250–256 (2009)

27. Diaz-Zuccarini, V., LeFevre, J.: An energetically coherent lumped parameter model of the left ventricle specially developed for educational purposes. Comput. Biol. Med. **37**, 774–784 (2007)
28. Diaz-Zuccarini, V., Rafirou, D., LeFevre, J., Hose, D.-R., Lawfort, P.V.: Systemic modeling and computational physiology: the application of Bond Graph boundary conditions for 3D cardiovascular models. Simul. Model. Pract. Theory **17**, 125–136 (2009)
29. Le-Rolle, V., Hernandez, A., Richard, P., Buisson, J., Carrault, G.: A model of the cardioavascular system using Bond Graphs. Innovation et Technologie en Biologie et Médicine-une Revue de Technologie Biomédicale ITBM-RBM **26**, 243–246 (2005)
30. Narwal, A.-K., Vaz, A., Gupta, K.-D.: Study of dynamics of soft contact rolling using multiband graph approach. Mech. Mach. Theory **75**, 79–96 (2014)
31. Romero, G., Felez, J., Maroto, J., Mera, J.M.: Efficient simulation of mechanism kinematics using bond graphs. Simul. Model. Pract. Theory **17**, 293–308 (2009)
32. Xiaotian, L., Anlin, W.: Definitions of causality in bond graph model for efficient simulation mechanism. Mech. Mach. Theory **80**, 112–124 (2014)
33. Yutao, L., Di, T.: Dynamics modeling of planetary gear set considering meshing stiffness based bond graph. Procedia Eng. **24**, 850–855 (2011)
34. Selişteanu, D., Roman, M., Şendrescu, D.: Pseudo Bond Graph modeling and on-line estimation of unknown kinetics for a wastewater biodegradation process. Simul. Model. Pract. Theory **18**, 1297–1313 (2010)
35. Couenne, F., Jallut, C., Maschke, B., Breedveld, P.C., Tayakout, M.: Bond graph modeling for chemical reactors. Math. Comput. Model. Dyn. Syst. **12**(2–3), 159–174 (2006)
36. Nacusse, M.A., Junco, S.J.: Switchable structured bond: A bond graph device for modeling power coupling/decoupling of physical systems. J. Comput. Sci. **5**, 450–462 (2014)
37. Djeziri, M.A., Ould Bouamama, B., Merzouki, R.: Modelling and robust FDI of steam generator using uncertain bond graph model. J. Process Control **18**, 149–162 (2009)
38. Ragusila, V., Emami, M.-R.: Modelling of a robotic leg using bond graphs. Simul. Model. Pract. Theory **40**, 132–143 (2014)
39. Assimacopoulos, D.: Population interactions modeled by bond graphs. Apply Math. Model. **10**, 234–240 (1986)
40. Abbes, M., Farhat, A., Mami, A., Dauphin-Tanguy, G.: Pseudo bond graph model of coupled heat and mass transfer in a plastic tunnel greenhouse. Simul. Model. Pract. Theory **18**, 1327–1341 (2010)
41. Bakka, T., Karimi, H.R.: A bond graph approach to modeling and simulation of nonlinear wind turbine system. In: Nonlinear Dynamics Science and Engineering, vol. 3. Springer, Berlin, pp. 41–61 (2013)
42. Badoud, A.E., Khemliche, M.: Bond Graph Modeling and Diagnosis in Wind Energy Conversion System. LAP Lambert Academic Publishing, Saarbrücken (2014)
43. Kurniawan, A., Pedersen, E., Moan, T.: Bond graph modeling of wave energy conversion system with hydraulic power take-off. Renew. Energy **38**, 234–244 (2012)
44. Roman, M., Bobaşu, E., Selişteanu, D.Ş.: Modelling of biomass combustion process. Energy Procedia **6**, 432–440 (2011)
45. Sanchez, R., Medina, A.: Wind turbine model simulation: A bond graph approach. Model. Pract. Theory **41**, 28–45 (2014)
46. Seo, K., Fan, Z., Hu, J., Goodman, E.-D.: Toward a unified and automated design methodology for multi-dynamic domain systems using bond graphs and genetic programming. Mechatronics **13**, 851–885 (2003)
47. Gawthrop, P.J., Ronco, E.: Estimation and control of mechatronic systems using sensitivity bond graphs. Control Eng. Pract. **8**, 1237–1248 (2000)
48. Kime, N.M., Ryan, M.J., Wilson, P.S.: A bond graph approach to modeling anuran vocal production system. J. Acoust. Soc. Am. **133**(6), 4133–4144 (2013)
49. Roy, S., Umanand, L.: Magnetic arm-switch-based three-phase series-shunt compensated quality AC power supply. IET Electr. Power Appl. **6**(2), 91–100 (2012)

50. Delgado, M., Profos, G.: Identifiability of Dynamic Systems Represented by Bond Graphs. Math. Comput. Model. Dyn. Syst. **5**(2), 89–112 (1999)
51. Chatty, N., Ould Bouamama, B., Gehin, A.L., Merzouki, R.: Signed bond graph for multiple faults diagnosis. Eng. Appl. Artif. Intell. **36**, 134–147 (2014)
52. Ibănescu, R.: Statics of Frames By Bond-Graphs. Buletinul Institutului Politehnic Iaşi, Section: Mathematics. Theoret. Mech. Phys. L (LIV), **3–4**, 93–104 (2004)
53. Ibănescu, R., Ungureanu, C.: Approach of a Particle Statics Problem by Using the Bond-Graph Modeling Method. Appl. Mech. Mater. **657**, 599–603 (2014)
54. Ibănescu, R., Ibănescu, I., Melnichi, R., Irimiciuc, N.: Similarity between sonic systems and electrical circuits emphasized by The Bond-Graph Method. Buletinul Institutului Politehnic Iaşi, Section: Machine Construction, XLVII (LI), **3–4**, 387–393 (2001)
55. Ibănescu, R., Păstrăvanu, O.: Numerical problems in Block-Diagram Simulation of Bond-graph Models with Derivative Causality—A MATLAB-SIMULINK-based Case Study. Buletinul Institutului Politehnic Iaşi, Section: Mathematics. Theoret. Mech. Phys. XLVIII (LII), **1–2**, 75–86 (2002)
56. Ibănescu, R.: The bond-graph model containing derivative causality. In: Proceedings of 15th International Conference Modern Technologies, Quality and Innovation, Chișinău, Republic of Moldova http://www.modtech.ro/2011/papers.php. pp. 493–496 (2011)