

Chapter 2

Current Research on Prospective Secondary Mathematics Teachers' Knowledge

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1 Introduction

Teachers' knowledge has been a major focus in the preparation of prospective teachers for a long time. Teachers need to know about the subject that they teach, they need to know how to teach it, and they need to know how to act and behave as teachers. Teacher education institutions organize teacher education programs around three strands, that Winslow and Durand-Guerrier (2007) named as content knowledge, pedagogical knowledge, and didactical knowledge, a distinction based on Shulman's (1986) seminal work that stands as the theoretical basis of a large number of studies in mathematics education. In this chapter, we address research on prospective teacher knowledge in mathematics and didactics of mathematics or knowledge of mathematics teaching.

Ponte and Chapman (2008, 2016) conducted systematic reviews of the research literature from 1998 until 2013 and concluded that some of the important developments in our field are: recognition that mathematical and didactical knowledge required for teaching is of special type; development of ways in teacher education where prospective teachers revisit familiar content in unfamiliar ways to develop the underlying meanings of the mathematics; and understanding the difficulty of prospective teachers to develop knowledge of mathematics teaching and designing tools to promote this knowledge. Although most studies have focused on prospective primary school teachers, there is a recognition that prospective secondary school teachers' (PSMTs)' knowledge of mathematics and mathematics teaching in sec-

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ondary schools is of a different nature, and new theoretical and methodological frameworks are needed to study it (Speer et al. 2015). In this chapter, we report findings from our survey on studies related to PSMTs' knowledge.

2 Methodology of the Survey

We searched each journal by using the following keywords: “prospective teachers”, “future teachers”, “teacher candidates”, “pre-service teachers”, “knowledge”, and “secondary”. We identified fifty-nine relevant papers, by reading the abstract and the methodology section. In addition, given the importance of large-scale studies on prospective mathematics teachers' knowledge, we identified other relevant papers and reports on these studies. Next, we reviewed and coded the papers and reports according to the following dimensions: (i) focus of the study and its research questions, (ii) main theoretical ideas underpinning it, (iii) methodological elements (setting, participants, instruments/tasks, data and process of data analysis), (iv) main findings, and (v) contribution of the study. Finally, we constructed a table with short descriptions for each paper related to the five dimensions. We first classified the papers in terms of their focus in three main thematic areas, as addressing: (a) the exploration of PSMT knowledge, (b) the impact of teacher education practices on PSMT knowledge, and (c) the process of PMST knowledge development in the context of teacher education programs. Initially, we provide some factual information about the mathematical content areas that the papers address, the dimensions of teacher knowledge, and the theoretical and methodological perspectives used. Then, we discuss the papers grouped in each of the three thematic areas in more detail presenting their main findings and contribution.

3 Basic Information About Research on PSMT Knowledge

3.1 *Mathematical Content*

Prospective teachers' knowledge of mathematical content has been studied from quantitative and qualitative perspectives. The large-scale TEDS-M international study (Tatto et al. 2012) addressed content knowledge in four content subdomains (number and operations, algebra and functions, geometry and measurement, and data and chance) and in three cognitive dimensions (knowing, applying and reasoning) (Döhrmann et al. 2012; Li 2012). The German study COACTIV (Krauss et al. 2008) is another example of quantitative large-scale study that addresses content knowledge. The papers reviewed, which referred to these studies mostly reported findings regarding content knowledge as a single construct although they

Table 1 The mathematical areas addressed in research studies

Mathematical areas	No. of papers
Specific mathematical content	22
Algebra/numbers	10
Geometry	5
Calculus	5
Statistics	2
Mathematical processes	15
Problem solving and modeling	11
Reasoning and proof	4
Not specifically defined	22

Bold indicates mathematical content (22) includes algebra/numbers, geometry, calculus and statistics (which values 10+5+5+2 add up to 22). The same for the mathematical processes (15), that include problem solving and modeling and reasoning and proof (adding also 11+4 = 15)

differentiated between mathematical subjects and between countries as it was difficult to report reliable scores for various mathematical subjects. On the other hand, qualitative studies usually focus on a specific mathematical content or process with emphasis on algebra, problem solving, and modeling, and tend to address this mainly in terms of structure and understanding (Table 1).

3.2 Aspects of PSMT Knowledge

A categorization of the papers according to the aspects of knowledge they address is presented in Table 2. Most studies that focused on PSMT knowledge of mathematics in mathematical contexts used interviews based on mathematical tasks (e.g., Tsamir et al. 2006), mathematical items in survey instruments (e.g., Döhrmann et al. 2012; Huang and Kulm 2012), or interactions in teacher education settings where the solution of a mathematical problem was a main task (e.g., Shriki 2010). Those studies that explored PSMT mathematical knowledge in teaching contexts mostly included settings as the analysis of students’ work (e.g., Magiera et al. 2013) or the comparison of different textbooks (e.g., Davis 2009). Shulman’s (1986) constructs of content

Table 2 Aspects of PSMT knowledge addressed in research studies

Aspects of PSMT knowledge	No. of papers
Knowledge of mathematics	41
Studied in mathematical contexts	27
Studied in teaching context	14
Knowledge of mathematics teaching	11
Relationship of knowledge of mathematics and mathematics teaching	12

knowledge (CK) and pedagogical content knowledge (PCK) are central to most studies, but some papers draw also on theoretical notions such as the distinction between common content knowledge (CCK) and specialized content knowledge (SCK) of Ball et al. (2008), on the notion of “deep mathematical knowledge” (Hossin et al. 2013) and “teacher knowledge on what else is needed beyond specific content knowledge” (Clark 2012). Despite the fact that many papers strived to address the specific features of PSMT knowledge of mathematics, still a number of them treat PSMTs as students who showed a rather deficient knowledge of mathematics.

Knowledge of mathematics teaching was less central in the research papers reviewed. This refers mostly to teaching of different mathematics topics, and it was often related to mathematics knowledge. The TEDS-M large-scale study (Blömeke et al. 2014) focused on the interrelationships between CK, PCK and general pedagogical knowledge in three participating countries, while the study conducted by Aguirre et al. (2012) included cultural and social elements in teacher knowledge.

3.3 Theoretical and Methodological Perspectives of PSMT Knowledge

The theoretical perspectives adopted by most of the papers belong to the cognitive/constructivist tradition with a few papers using socio-cultural and sociological lenses. The frameworks from Shulman and Ball and her collaborators are major theoretical references while often complemented with other theoretical perspectives. For example, Ticknor (2012) used a situated perspective, “person-in-practice-in-person” of Lerman (2000) to study the mathematical content knowledge developed in an abstract algebra course focusing on how prospective teachers impacted a community of practice, and how practicing in that community impacted the prospective teachers’ mathematical identities. Adler and Davis (2006) used Bernstein’s (1996) educational code theory and Ball and Bass’ (2000) notion of “unpacking” in the mathematical work of teaching to study the mathematical knowledge promoted in mathematics courses for teachers in South Africa. In terms of the methodological frameworks, most studies followed the interpretive paradigm with qualitative small-scale approaches (39/59) while the others adopted quantitative (14/59) or mixed methods (6/59).

4 Exploration of PSMT Knowledge

4.1 Large-Scale Projects

Several important large-scale research projects addressed issues of prospective mathematics teacher knowledge in relation to program features. One of these

studies is the *Mathematics Teaching in the 21st Century* (MT21) international study that addressed the preparation of middle school mathematics teachers with participation of six countries: South Korea, Taiwan, Bulgaria, Germany, US and Mexico (Schmidt et al. 2011). Regarding mathematics preparation (CK), coursework in linear algebra and calculus and on more advanced mathematics corresponded to higher individual scores (in the two Asian countries) whereas coursework in advanced school mathematics did not. These scores were very much in line with the opportunities to learn (OTL) provided to prospective teachers. For mathematics pedagogy (PCK), only Bulgaria and Mexico had low scores, and the relationship with the OTL was still significant but much lower than regarding CK.

Another project is TEDS-M (Tatto et al. 2010) which surveyed 17 country-regions into the approaches, structures, and characteristics of such programs. The theoretical framework draws on the CK and PCK notions of Shulman (1986). This study is the first international study on mathematics teacher knowledge and offered us important theoretical and methodological perspectives that take into account contextual characteristics of mathematics teacher education in the participating countries (Tatto et al. 2012). Prospective primary and lower secondary teachers' knowledge was assessed through questionnaires including items for testing CK and PCK and general pedagogical knowledge (GPK). There were two different groups of lower secondary teachers, one being prepared to teach up to grade 10 (PG5-Program Group 5) and another being prepared to teach up to grade 11 and above (PG6-Program Group 6).

The TEDS-M main report (Tatto et al. 2012) provides results about several variables, including CK and PCK. Regarding CK, the score of participants from PG6 varied widely, with more than 200 points of difference between the highest and the lowest mean score. Prospective teachers from Taiwan, Russia, Singapore, Germany, and Poland outperformed the participants from the other countries, with a mean score above 559 points (Anchor Point 2). Prospective teachers of PG5 had less variation in their scores with the top performing countries being Singapore, Switzerland, and Poland, with a score above the 500 points (the international mean). Regarding PCK, PG5 participants from Switzerland, Singapore, Poland and Germany had scores above the international mean whereas PG6 participants from Taiwan, Germany, Russia, Singapore, USA, and Poland had scores above 509 (the single Anchor Point).

Blömeke et al. (2013) identified subgroups of countries with specific weaknesses and strengths related to content domains, cognitive demands and item formats. For example, prospective teachers from countries of the East Asia tradition (Taiwan and Singapore) performed better in mathematics content items and in constructed-response items, of the Western tradition (USA, Germany and Norway) did particularly well on data handling and items related to mathematics teaching, and of the Eastern European tradition (Russia and Poland) were strong on non-standard mathematical operations. Blömeke and Delaney (2012) also conducted a literature review of comparative studies in the context of the TEDS-M study discussing its

conceptual framework, methodology and main findings. TEDS-M's conceptual and methodological framework measured teacher competences by distinguishing several aspects of teacher knowledge, linking them to beliefs including cognitive and affective dimensions and stressing its situative and applied nature. In addition to the ranking of countries in terms of aspects of prospective teacher knowledge that have already been mentioned above, predictors of teacher knowledge were included. Gender effects (males performed better than females in CK), language effects (teachers whose first language matched the official language of instruction in teacher education performed better both in CK and PCK), prior knowledge (high-school achievement and the number of mathematics classes at school had a positive impact on both CK and PCK) and motivation (subject related motives were positively related both to CK and PCK) were some individual predictors. Institutional predictors, which had a strong influence both on PSMTs' CK and PCK, included opportunities to learn mathematics in teacher education and the quality of teaching method experiences.

Another study that addressed prospective mathematics teachers' knowledge is COACTIV (Krauss et al. 2008). This study sought to establish construct validity for the notions of CK and PCK. The main sample was practicing mathematics teachers ($n = 198$) while prospective secondary school mathematics teachers ($n = 90$) and school students in advanced grade 13 mathematics courses ($n = 30$) were two contrast groups used to validate the instruments. The study concluded that PCK is deeply interrelated with CK and that CK is a prerequisite for PCK. The PCK measure had three subscales, Tasks, Students, and Instruction, with Tasks having the lower correlation to CK. Supporting Shulman's (1986) notion of PCK as an amalgam of CK and GPK, the findings suggest that there are two possible routes to develop PCK, one based on very strong mathematical competence and another based on pedagogical knowledge common to teaching other subjects. It also concluded that prospective mathematics teachers have statistically significant lower CK and PCK regarding gymnasium practicing teachers, albeit not much strong in absolute terms (8.5 vs. 6.6 in CK and 21.0 vs. 18.2 in PCK). It also showed that prospective mathematics teachers significantly outperformed school students in both kinds of knowledge (18.2 vs. 9.7 in PCK and 6.6 vs. 2.6 in CK). This may suggest that both PCK and CK are acquired at university in teacher education programs while their development during the teacher career is not very significant. This reinforces the importance of university and teacher education studies in the development of prospective teachers' knowledge. The COACTIV study was successful in establishing construct validity for CK and PCK as separate notions and suggested that PCK is the most important factor that explains secondary school students' learning (Baumert et al. 2010). However, as Krauss et al. (2008) indicate, its measurement instruments still have room for further improvement, for example, striving to construct PCK items that are not influenced by CK and providing a more suitable representation of geometry items.

4.2 *Content-Specific Character of the Research*

Studies on PSMT conceptions on specific mathematical concepts in algebra, geometry and statistics, or problem solving/modeling and reasoning/proof indicate that many prospective mathematics teachers for lower and upper secondary education have not developed a deep mathematical knowledge that can inform their teaching towards developing understandings of mathematical concepts and reasoning.

Concerning mathematical concepts, we provide examples of some of these studies addressing the different mathematical areas presented in Table 1. In algebra/numbers, Sirotic and Zazkis (2007) investigated PSMT knowledge of irrational numbers through an analysis of their characteristics (intuitive, algorithmic, formal) and showed inconsistencies between PSMT intuitions and formal and algorithmic knowledge. Caglayan (2013) studied PSMT sense making of polynomial multiplication and factorization modeled with algebra tiles and found three different levels of understanding: additive, one way multiplicative and bidirectional multiplicative. Huang and Kulm (2012) measured PSMT knowledge of algebra for teaching and especially focused on the function concept through a survey instrument aiming to identify PSMT understanding of school and advanced mathematics as well as their views on the teaching of algebra identifying certain limitations in all areas. Alajmi (2015) focused on the algebraic generalization strategies used by PSMTs in linear, exponential and quadratic equations showing that they had difficulties in generalizing algebraic rules especially with exponents, in line with a similar finding from TEDS-M (Tatto et al. 2012).

Yanik (2011) explored PSMTs' knowledge of geometric translations and concluded that PSMT conceived translations mainly as physical motions based on their previous experiences. In calculus, the study of Tsamir et al. (2006) on PSMT images of the concept of derivative and absolute function showed that PSMT gave correct definitions but could not use them appropriately in solving a given task. However, their engagement in evaluating their own responses brought some changes in their initial solutions. Hannigan et al. (2013) focused on conceptual understanding of statistics and the relationship with attitudes towards statistics and found that PSMTs had low conceptual understanding of statistics and positive attitudes, with a low correlation between conceptual understanding and attitudes.

Several studies focused on mathematical processes, problem solving strategies, and modeling. Demircioglu et al. (2010) studied PSMT metacognitive behavior and showed that this behavior was not related to their achievement and type of problems. Regarding modeling, all the papers focused on the construction of mathematical models by PSMT. Daher and Shahbari (2015) showed different ways of how technology was integrated in the modeling process. Delice and Kertil (2015) also looked for PSMT connections of the modeling process to different forms of representations, and indicated difficulties of PSMTs in making such connections. Carrejo and Marshall (2007) investigated the modeling process in the context of a

physics course and showed that PSMTs began to question the nature of mathematics in their attempt to make connections to the real world. The study of Eli et al. (2011) also focused on the mathematical connections that PSMTs made while engaged in card-sorting activities and found that most of the PSMTs' connections were procedural and categorical.

Reasoning and proof was also the focus of some papers. Yenem-Karpuzcu et al. (2015) studied PSMT covariational reasoning abilities showing different levels for low and high achievers. Zazkis and Zazkis (2015) used PSMT scripted dialogues between teacher and students related to the proof of Pythagoras' theorem to address how they comprehend students' understanding of proof showing that PSMTs mostly considered errors on algebraic manipulations and did not assess proof comprehension in a holistic way. Stylianides et al. (2007) studied PSMT knowledge on mathematical induction, identifying certain difficulties on the essence of the base step of the induction method, the meaning of the inductive step, and the possibility of the truth set of a statement proved by mathematical induction to include values outside its domain of discourse. Corleis et al. (2008) examined PSMTs' CK and PCK about argumentation and proof in Germany and Hong Kong and indicated that PSMTs from Hong Kong performed better in their CK about proof and argumentation than those from Germany, while there was no difference in PCK.

4.3 Relation of PSMT Knowledge to Teaching

A number of studies focused on the relationship between PSMTs' CK and PCK showing rather positive relations. Van den Kieboom et al. (2014) reported that PSMTs' algebraic proficiency was related to the questions that they asked while interviewing students. Positive relations also were reported in the studies of Karp (2010), Charalambus (2015) and Mamolo and Pali (2014). Whereas the study of Karp (2010) showed that lack of PCK creates difficulties in PSMT field experiences, the study of Morris et al. (2009) focused on how PSMT unpack learning goals into subconcepts and found that although PSMTs identified such subconcepts they could not use them in the context of teaching. Similarly, Johnson et al. (2014) found that the PSMT's use of definitions and examples while doing mathematics did not seem to influence their teaching. Magiera et al. (2013) also reported that PSMTs' algebraic thinking and its relation to the analysis of tasks and students' algebraic thinking were not smoothly related. The study of Capraro et al. (2012) on problem solving also showed that mathematical competence does not translate to pedagogical effectiveness. Finally, the study of Subramaniam (2014) examined PSMT PCK for teaching the estimation of length measurement by examining their personal benchmarks and showed that holding mathematical knowledge does not guarantee knowledge for teaching.

4.4 *Epistemological and Theoretical Issues*

Three papers focus on epistemological and theoretical issues related to PSMT knowledge. Moreira and David (2008) addressed the differences between school and academic mathematics knowledge related to number systems pointing out that mathematics teacher educators need to be aware of these differences. Speer, King and Howell (2015) discussed the relevance of frameworks of studying mathematics teacher knowledge at the primary level up to the secondary and college level. They argued that frameworks for primary teachers have to be extended, as there are differences in the nature of knowledge required for secondary and college mathematics teachers. Koirala et al. (2008) developed an assessment performance task and rubric to measure PCK based on the analysis of students' needs and on the design of lesson plans.

5 Impact of Teacher Education Practices on PSMT Knowledge

The impact of teacher education programs on PSMT knowledge has been studied both in the large-scale studies TEDS-M and COACTIV and in small-scale studies. TEDS-M (in Li 2012) shows that there is difficulty on making direct connections between teachers' performance and their program of studies even within an education system (e.g. in Singapore). However, in the case of the US, it appears that selecting more mathematically able students in teacher education and providing key mathematics and mathematics pedagogy opportunities to learn in the courses, has a positive impact on the development of PSMT knowledge. The study of Wang and Tang (2013) uses the data from TEDS-M and analyses the opportunities to learn (OTL) offered in the context of teacher education programs for prospective secondary mathematics teachers in fifteen countries. The results show that three profiles of OTL appear at tertiary-level mathematics, school-level mathematics, mathematics education and general education. Tertiary-level mathematics demand extensive and intensive coverage of topics, Mathematics education courses focus more on students' cognitive understandings and abilities while general education emphasizes the relation to school practice and the comprehensive coverage of topics. In the case of COACTIV study and in particular in the context of COACTIV-R study that focused on professional competences of prospective teacher, it appears that offering formal learning opportunities at the teacher education level promotes PSMT knowledge (Kunter et al. 2013). Through the small-scale studies, different teacher education practices seem to promote PSMT knowledge. One of them is PSMT engagement in tasks with certain characteristics. Zbiek and Conner (2006) argued that PSMT engagement in modeling tasks indicates changes in PSMT motivation and understanding of the modeling process. The study of Stankey and Sundstrom (2007) showed how a high school task can be extended to

teacher education while the study of Levenson (2013) focused on the process of selecting and analyzing tasks related to mathematical creativity showing that PSMT take into account not only features and cognitive demands in their analysis and choices but also affective factors. Steele et al. (2013) suggested that the connection between CCK and SCK can be developed through PSMT engagement with rich tasks first as learners, sharing solutions, and then analyzing the tasks as teachers.

There are studies referring to instructional sequences in teacher education that appear to be effective in developing PSMT CK. Bleiler et al. (2014) proposed an instructional sequence to improve PSMT proof validation of students' arguments in number theory. The sequence consists of five activities including marking students' responses to proof tasks, analyzing a video extract of a teacher facing a student's inductive argument, discussing in groups, and validating proof arguments provided by students in other research studies, grading individually students' proof arguments, justifying the score, and providing feedback to the students. However, their findings do not show any change in PSMT proof validation before and after the teacher education course. Similarly, Moon et al. (2013) referred to a three-week teaching unit designed to overcome PSMTs' difficulties in understanding the big ideas related to connections among representations in the context of conic curves showing that PSMTs had difficulties relating to the variation, the Cartesian connection, and graphs as locus of points. Prediger (2010) suggested a number of teacher education strategies that support the development of PSMT diagnostic competence in algebra (e.g., evaluating students' and their peers' responses).

Adler and Davis (2006), Hossin et al. (2013), and Adler et al. (2014) refer to a mathematics enhancement program. Adler and Davis (2006) reported findings from a survey of teacher education programs in South Africa that aimed to develop PSMT CK and show that the mathematics taught was compressed without promoting mathematical ideas and reasoning. Hossin et al. (2013) studied the impact of this course on the development of PSMT mathematical and teaching identity and identified several issues with the course regarding the process of developing mathematics teachers. Adler et al. (2014) indicated that PSMT conceived "understanding mathematics in depth" because of their participation in this course showing that their conceptions were influenced by the way that mathematics was considered in the course.

Some studies focus mainly on teacher education strategies that support PSMT to develop PCK or pedagogical knowledge. Viseu and Ponte (2012) showed the impact of a course that integrates the use of ICT tools (emails and forum) on the development of a PSMT planning and teaching. The PSMT started to use tasks that are more open and initiated more productive classroom communication. Jenkins (2010) showed that PSMT advanced their PCK by being engaged in preparing task-based interviews, doing and analyzing the interviews, preparing a report linking their findings to the research discussed in the course, and sharing this with their peers. Sanchez-Matamoros et al. (2014) described a teaching module aimed to promote PSMT noticing of students' thinking of the derivative of a function through a number of different tasks such as analyzing students' work and solving problems themselves. The module focuses on the learning trajectory of the

derivative concept and the findings show that it had a positive impact on PSMT noticing of students' thinking. Aguirre et al. (2012) focused on the process of supporting PSMT to develop PCK by taking into account cultural and social issues. The designed course and the assignment given to PSMT asked them to analyze their own teaching by using categories that also address culturally responsive characteristics. Although PSMT were receptive to these approaches, they did not develop the pedagogical ways of addressing them into their teaching.

A number of studies investigated ways of developing both CK and PCK. Groth and Bergner (2013) focused on the development of CK and PCK in statistics and in particular in analyzing categorical data. The activities in which they engaged PSMT were analyzing themselves data and reading papers about learning and teaching categorical analysis. They showed that various types of knowledge structures developed through the analysis of PSMT writing prompts from their readings and the analysis of students' errors. Clark (2012) showed a positive impact of a history of mathematics course designed to show the development of mathematics, the cultural and historical influences and the integration of history in teaching on PSMT development of mathematical and pedagogical awareness. Tsamir (2005) introduced PSMT to the theory of intuitive rules and showed development of PSMT CK and PCK. The PSMT were asked to construct intuitive and counter intuitive tasks about "same A–same B" and report episodes that they identified in their practicum analyzing them by using this theory. Finally, Davis (2009) showed that reading and planning of PSMT from two different textbooks had a positive impact on PSMT CK and PCK of exponential function.

6 The Process of PSMT Knowledge Development in the Context of Teacher Education Programs

Few studies focus on the actual process of PSMT development in the actual teacher education program analyzing interactions in order to trace teacher knowledge at mathematical and pedagogical level. Ticknor (2012) investigated whether PSMTs who participate in an abstract algebra course made links with high school algebra by relating individual's mathematical history to the community of the classroom of the course and vice versa and concluded that such links are not easy. Assuming mathematical creativity as a component to teacher knowledge, Shriki (2010) addressed how it can be developed in a context of a methods course. The PSMT initially focused on the creative product considering mathematics as a closed domain while later in the course they focused on the creative process viewing mathematics as an open domain. Tsamir (2007) analyzed a lesson in a teacher education course focusing on psychological aspects of mathematics education and in particular on the role of intuitive rules in learning. Her main finding is that intuitive rules acted as a tool for supporting PSMT reflection on their own methods and intuitive solutions. Ryve et al. (2012) addressed how mathematics teacher

educators establish “mathematics for teaching” in teacher education programs by using variation theory to analyze classroom interaction in a teacher education course. Parker and Adler (2014) studied knowledge and practice in mathematics teacher education focusing on both knowledge of mathematics and knowledge of mathematics teaching and their co-constitution. They recognize shifts between mathematics and mathematics teaching but claim that the recognition and realization rules for the privileged text (using Bernstein’s theory of pedagogic discourse) with respect to mathematics teaching were available.

7 Final Remarks

Research continues to show that the PSMT knowledge of mathematical content and processes is plagued with difficulties and low conceptual understanding of many concepts. However, the studies do not stem from a common framework regarding what must be required from prospective secondary teachers, and requirements established by researchers seems to vary in nature and depth. In this area, an important step forward would be the establishment of such frameworks (as suggested by Speer et al. 2015) and a common understanding of important steps in the development in PSMT mathematical knowledge (learning frameworks). Concerning knowledge of mathematics teaching, didactical knowledge or PCK, we seem to have an even more precarious situation, given the scarce number of studies in the field and the fuzziness that still accompanies this notion. As the work of Kaarstein (2015) showed, PCK is an elusive notion, and its distinction of mathematics knowledge is often problematic. The large-scale national and international studies on teacher knowledge also point towards a very complex relation between PCK and CK.

The studies on the impact of teacher education practices and the processes of how PSMT knowledge develops in teacher education programs suggest that the active engagement of participants in doing mathematics and discussing strategies and results has a positive influence in their mathematics learning. In addition, PSMT active engagement in preparing tasks, analyzing students’ work, giving feedback to students, and discussing with colleagues and teacher educators are also positive influences on their knowledge about mathematics teaching. Looking closely at students’ thinking is a major trend in the research carried out in the last ten years and may have an important impact on PSMT learning. When they are in fieldwork placements, ICT may be a useful means for communication and interaction. For dealing with specific topics, we will probably need local theories that indicate what kinds of tasks, materials and environments promote a stronger development. Moreover, taking into account the complexity of mathematics teaching, we need to extend our teacher education practices into directions so that this complexity becomes transparent to PSMT. Addressing complexity in teacher education challenges researchers and educators to consider PSMT knowledge and its development under new more participatory theoretical perspectives.

Besides the focus on knowledge, we also need to strengthen the focus on how PSMT develop knowledge (Cochran-Smith and Villegas 2015). There are already a good number of studies giving hints on how this may occur in specific courses within university contexts. However, we also need to know how PSMT PCK is fostered through their practicum or in other kinds of fieldwork, since field placements are dubbed as powerful settings for the development of PSMT knowledge in all of its dimensions.

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