Chapter 6
When Mathematics Meets Real Objects: How Does Creativity Interact with Expertise in Problem Solving and Posing?

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Abstract The paper analyzes the results of activities undertaken by Mathematics students enrolled in a pre-service teacher-training program. Students were given the task to describe the way of building a figure from which one could get a box, to identify the geometric properties that allow producing the box, and to propose new models from which new boxes can be obtained. For creativity analysis, a cognitive flexibility framework has been used, within which students' cognitive variety, cognitive novelty, and their capacity to make changes in cognitive framing are analyzed. The analysis of some specific cases led to the conclusion that creativity manifestation is conditioned by a certain level of expertise. In the process of building a solution for a nonstandard problem, expertise and creativity support and mutually develop each other, enabling bridges to the unknown. This interaction leads also to an increase in expertise. Moreover, in order to get individual relevant data, the identification of creativity should take place based on tasks situated in the proximal range of the person's expertise but exceeding his/her actual level of expertise at a time.

Keywords Mathematical creativity • Modelling • Cognitive flexibility • Expertise

6.1 Introduction

What is the relationship between expertise and creativity? This is a question that has generated lots of controversy in literature over time. Some authors (eg Diezmann and Watters 2000) argue that expertise is a precondition for creativity. Other authors (eg Craft 2005), accepting the existence of "small *c* creativity", say the contrary, arguing that because creativity can occur in any person, we must accept a spectrum of knowledge – therefore of expertise, in connection with creativity (Craft 2005).

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We started our project with the intention to answer the question: *How does students' mathematical creativity manifest in a context in which technology and modelling interact with theoretical mathematics?* Our study was initiated by the fact that we noticed very different behaviors in terms of creativity when placing a group of students - prospective mathematics teachers in a context of problem solving and problem posing that involves modelling. Thus, while progressing with our analysis, another question became dominant, which actually includes the previous one: *How does creativity interact with expertise in problem solving and posing?* As a result, we intend to study the link between creativity and expertise in a complex situation, which occurs in a context combining problem solving, problem posing, and modelling.

6.2 Framework

6.2.1 Problem Solving and Problem Posing

In his well-known book *How to Solve It*, Pólya (1945) identified four steps in solving a problem: (i) understanding the problem; (ii) developing a plan; (iii) carrying out the plan; and (iv) looking back. Subsequently, lots of frameworks have been developed for studying the problem solving process (eg Schoenfeld 1992).

We have found a variety of approaches for studying problem posing in the literature, as well (eg Brown and Walter 2005; Jay and Perkins 1997; Singer et al. 2015). In this paper, we accept Silver's position, stating that problem posing refers to the generation of (completely) new problems, and also to the re-formulation/modification of given problems (Silver 1994). We specifically address here the context of problem modification.

A conceptual cognitive framework for problem solving, with various applications in problem posing was developed by Singer and Voica (2013). This framework highlights four operational categories: decoding, representing, processing, and implementing (Singer and Voica 2013).

6.2.2 Mathematics Modelling

Mathematical modelling can be seen as a process of translating between the real world and mathematics in both directions (Borromeo Ferri 2006). In recent years, the following description of an ideal modelling process (according to Blum and Leiss 2007) is frequently discussed: starting from *a real world situation*, this is simplified and/or structured: one thus arrives to *a real model of the situation*. This is transposed in a mathematical language, thus generating *a mathematical model*. The processing of the mathematical model leads to some results, which are then interpreted and validated into the real situation.

In the present study, students had to describe mathematically a real object – therefore to build a mathematical model of the real object, and then to extend the model so that to design new more complex objects. Using Kaiser and Sriraman's (2006) terminology, this type of task is framed into realistic or applied modelling (solving real world problems, understanding of the real world, promotion of modelling competencies).

6.2.3 Creativity

Creativity had long been viewed as a domain-general phenomenon. However, recently, new evidence show that creativity is not only domain-specific, but it even seems to be task specific within content areas (eg Baer 2012).

There is no consensus concerning the definition of creativity and its framework of study; there is no consensus in studying mathematical creativity either. There is however certain consensus regarding the difference between (advanced) research mathematicians creativity –considered as "extraordinary" or "absolute" creativity, and creativity in school mathematics – part of "everyday" or "relative" creativity (eg Craft 2003; Lev and Leikin 2013; Sriraman 2005). In addition, "big "C" creativity" and "small "c" creativity" are largely discuss (eg Bateson 1999; Gardner 2008).

Usually, creativity is studied starting from Torrance's tests, which is based on four related components: fluency, flexibility, novelty, and elaboration. Starting from here, various frameworks for studying creativity have been generated, usually adapted to specific types of tasks.

For problem solving context, Leikin (2013) uses multiple-solution tasks as a lens to observe creativity. The interplay between individual and expert solution space is an expression of creativity and the dimensions of her model are originality, fluency and flexibility, which are aggregated into creativity score by a research-based and, subsequently refined, scoring technique.

The construct of *spaces of discovered* properties is at the core of a new framework (Leikin and Elgrabli 2015), advanced to explore the complex relationship between creativity and knowledge in the context of an investigation task set in a dynamic geometry environment. The discovered properties were assessed from the point of view of their novelty, complexity of auxiliary constructions, and the complexity of their proofs.

For problem posing context, Kontorovich and Koichu suggested a framework based on four "facets": resources, heuristics, aptness, and social context in which problem posing occurs (Kontorovich and Koichu 2009). A more recent refinement of this framework has integrated task organization, knowledge base, problem posing heuristics and schemes, group dynamics and interactions, and individual considerations of aptness as parameters in analyzing creativity in problem posing situation (Kontorovich et al. 2012).

A different approach to creativity, one based on organizational theory, has been taken by Voica and Singer (2011, 2013). Their framework relies on the concept of cognitive flexibility. Cognitive flexibility is described by: cognitive variety, cognitive novelty, and changes in cognitive framing. Cognitive variety manifests in the formulation of different new problems/properties from an input stimulus; cognitive novelty captures the innovative aspect in the posed problem – its distance from the starting element; while changes in the participant's mental frame refer to shifts in the "on-focus" elements during the problem posing. Thus, cognitive flexibility arises as a complex, non-linear interplay between these dimensions. Consequently, the construct of cognitive flexibility opens up the possibility to capture different ways of being creative, namely through the differing loads on the three dimensions.

In the present study, we use the cognitive-flexibility framework in analyzing data. We consider that this framework better corresponds to our case, in which communication tasks related to problem solving, problem posing and modeling of problem situations occur. By using this framework, we can capture, beyond mathematical creativity, implications related to communication and social interactions reflected in problem posers' cognitive approach.

6.2.4 Experts Versus Novices

Expertise implies the existence and use of two types of knowledge: explicit knowledge of facts, principia, formulae pertaining to the domain, and implicit knowledge of how to operate with them (Sternberg 1998).

Glaser (1999) argued that, because self-monitoring – the ability to observe and, if necessary, reshape one's performance – is a hallmark of expertise, this skill should be emphasized in instruction. How to arrive at doing these in the real classroom? Although a very tempting concept from the point of view of artificial intelligence, the idea of expertise was not very much explored in psychology in relation to education. The criteria developed by Glaser (1988) for comparing experts and novices are still valid. Glaser characterizes expertise through six features ("generalizations" in Glaser's terminology: knowledge organization and structure, depth of problem representation, theory and schema change, proceduralized and goal oriented knowledge, automaticity, and metacognitive self-regulatory skills; because we use these features further, we detail them below.

- In terms of knowledge structure and organization, the expert has structured information items that are integrated into previous knowledge organizations so that they are rapidly selected from memory in large units, while novices possess punctual knowledge, consisting of isolated elements that display a superficial understanding of domain-specific key concepts and terms. (A)
- Regarding the complexity of problem-solving representation, the novice solves a task starting from its surface features, while the expert makes interferences and identifies principles underlying the surface structures. (B)
- In changing thinking schemes, the expert amends his/her own knowledge theories, and develops schemes that facilitate more advanced thinking, while novice manifests rigidity in changing a thinking scheme. (C)

- In terms of goal-oriented procedural knowledge, the expert displays functional knowledge, while novice possesses information without clearly understanding the applicability conditions. (D)
- In terms of automation that reduces the concentration of attention, an expert can focus attention while alternates between basic capacity and higher levels of strategy and understanding, using automate thinking to achieve good performance, while novices have difficulty in sharing attention, they frequently get lost in details and are unable to concentrate on essential facts. (E)
- Regarding metacognitive capacities of self-regulation, the expert check rapidly and intuitively the solution to a problem, proves accuracy in judging its difficulty, in assessing own knowledge and understanding, can ask questions, predict the outcome of the work, and use time effectively, while the novice tackles a linear approach, without looking ahead, and without controlling timing and work outcomes. (F)

6.2.5 Expertise and Creativity

During this study, we have started by exploring students' creativity and we came up by analyzing the students' level of expertise. We therefore ask ourselves: what is the relationship between these concepts? There are conflicting views about it, depending on how creativity is defined, but also depending on the domain being surveyed. We will further refer to creativity in school mathematics. For Diezmann and Watters, for example, for a student to be creative, he/she needs some intellectual autonomy and expertise (Diezmann and Watters 2000). Expertise is therefore seen as a necessary precondition for the manifestation of creativity. In his studies, Baer nuanced this relationship: he admits as obvious that some degree of expertise is important for the expression of creativity, but the question is what kind of expertise is required in a particular domain (Baer 1998, 2010).

On the other hand, Craft (2005) admits that every student is capable of creative manifestations; the consequence would be that expertise is not absolutely necessary for the manifestation of creativity or, at least, that we should accept a spectrum of knowledge at different levels.

6.3 Method

6.3.1 Sample and Task

The data comes from students in mathematics – prospective teachers who have received the same task during a Mathematics Education course. The task (listed in Annex) had two parts. In the first part, students approached a task of communication ("telephoned" description of a geometric configuration – Fig. 6.1.) consisting of producing a list of instructions based on which an interlocutor who did not have access to seeing the configuration have to reproduce it. After finishing this activity,

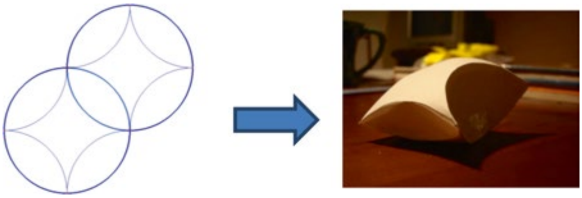


Fig. 6.1 The images initially showed to students (the box was presented as physical object, in the classroom)

students had to interact with the person "at the other end (of the phone)", and to improve the instruction list taking into account the received feedback and, eventually, to validate the new list of instructions with another partner (the validation consisted of that the partner was able to make an object that meets certain geometric properties).

In the second part of the task, students explored geometric properties of the given configuration, and tried to develop generalizations.

For the first part, students could work in groups of two, while for the second they had to work individually. There were students who preferred to work alone for the entire task. To solve the task, the students had a period of three weeks. In total, 26 students responded to this task: they constitute our sample for research.

6.3.2 A Modeling Context

The task proposed to students involves a modelling process. This is because, in a first phase of the task, properties related to the technological process for obtaining the box are to be interpreted in mathematical terms; thus one builds the mathematical model of the real object. In the second phase, this model was faced up with the possibility of extension, which allows obtaining new objects of the same category. The validation of the new mathematical proposals was made by obtaining geometric configurations and the actual construction of new boxes.

In achieving the mathematical model, students were exposed to a context of communication and social interaction, which led to the description of the model in an implementable technological manner (the students listed the steps of a technological process). This is another argument for interpreting the task as being a modelling one.

6.4 Results

6.4.1 What Elements of the Geometrical Configuration Were Relevant for Students?

Students' instruction lists and their recommendations for constructions show that they focused on the decomposition of the given figure in certain components. We briefly present the elements that students highlight in formulating instructions

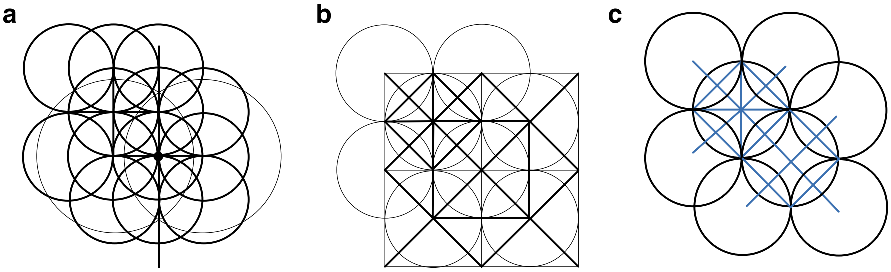


Fig. 6.2 Configurations that students perceived within the initial figure: (**a**) network circles; (**b**) squares and inscribed circles; (**c**) squares and circumscribed circles

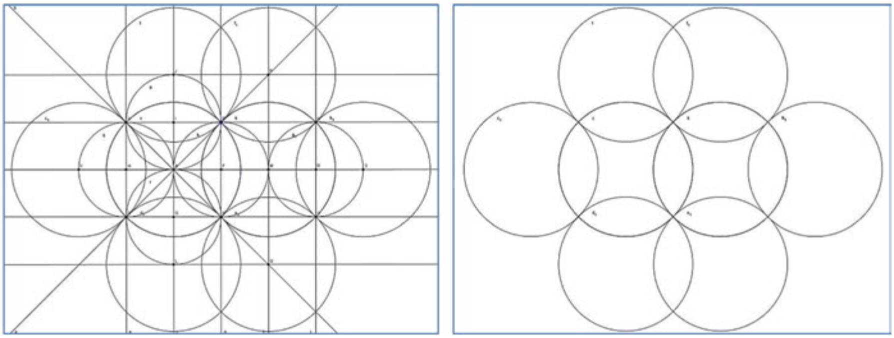


Fig. 6.3 Drawings made by Miron and Ana using GeoGebra

for (telephoned) reproduction of the given figure and for obtaining the box. (A broader discussion on the results presented in this paragraph is found in Pelczer et al. 2015). We have seen a variety of starting approaches. Below, there are a few selected. In the issued instruction lists, students frequently showed some networks/tessellations of the plan, which guided the construction achievement. Most often, there is about a network of circles or plane coverage with squares and circles inscribed or circumscribed to them. Figure 6.2 shows three configurations that students perceived within the initial figure, namely: A) network of circles; B) squares and inscribed circles; C) squares and circumscribed circles.

A particular situation occured in the response given by one of the teams who used GeoGebra (although this software is not recommended by the curriculum). The team Miron & Ana included Fig. 6.3 in their solving. Here, the first figure shows a plan coverage with squares and circles inscribed and circumscribed to them, while the second figure ("clean and ready to cut") only highlights a pattern of circles.

6.4.2 What Geometric Properties Do Students Identify?

For identifying geometrical properties it is not enough for the students to observe the initial configuration because the task statement does not contain data about the figure; they need to translate facts related to the technological process into a mathematical language. Therefore, in describing the configuration, students had to identify dominant perceptual elements of the mathematical model.

For example, students have noticed that to obtain the box, some parts of the figure should coincide when overlapped. "Perfect" overlap was expressed in some cases through congruence. There are also cases where students retain from overlapping parts of the figure only the equality of their areas: the mathematical property identified in this case is "weak" because it does not translate, in mathematical terms, the complexities of the real object. In other words, in this case the properties suggested by students do not allow a unique characterization of the given configuration, but have degrees of freedom that lead to a broader class of configurations. Consequently, two categories of properties that students remark within the given configuration occur: strong properties and weak properties.

More precisely

Strong property: is part of a mathematical model that uniquely characterizes the initial figure from which the box is obtained. In other words, it is a property belonging to a minimal set of necessary and sufficient conditions that ensure identical reproduction of the object.

Weak property: expresses mathematical features necessary but not sufficient, of the given figure. In other words, it provides a class of possible configurations of the given basic elements in which the initial configuration is found, but one can find there other configurations as well.

Table 6.1 shows the geometrical properties identified by the students from our sample through the model specifications that allow building the box. For the clarity of presentation, we organized the students identified properties into 5 categories of content. We have also selected some significant comments of students for the characterization of the respective property. They reveal types of constraints identified in the mathematical model, which conditioned the making of the box.

Most students identify, in the given configuration, equal circles and regular polygons. Out of these, some remain in the straightedge-and-compass constructability, ie they focus on polygons that can be built in this way. Table 6.1 shows separately content categories inscribed/circumscribed and regular polygons. Although this seems to be a redundancy, because any regular polygon is an inscriptible one, we distinguished among these categories because while some students consistently use circles to build polygons, others operate with regular polygons without needing the support of a circle.

Table 6.1 Dominant features of the mathematical model used for building the box

Dominants identified by students	Weak properties	Strong properties
Highlighting congruence	The lateral faces ("lenses") have equal surfaces.	Lateral faces of the box are congruent figures.
	"*When pasting the figure it should perfectly overlap*" – Paul	"*The figures formed by the intersections of circles are all congruent each other – so we can put perfectly on each other and form the sides of the box*" – Andreea
	The number of "convex lens" is even.	Interior arches are congruent with each other and are congruent with large arcs on the circles.
	"*Because they overlap two by two*" – Paul	"*Interior arches are equal to itself – otherwise, bonding would not be possible*" – Catalin
Emphasis on geometric transformations	The plane figure has "stability" in rotations towards the centers of the circles – Cristina	Figure axis of symmetry is the common chord "*If you fold on the dotted line, figures overlap*" – Anca
	The figure has as a symmetry line the centers line. – Dana	"*It helps to assemble the box*" – Rodica
	The faces of the box have symmetry axes. – Madalina	The second circle is a translation of the first circle – Adriana
Emphasizing tessellations	The squares used have sides equal to the diameter of the initial circle – ie one can use the circle inscribed in the square.	
Highlighting inscribed/ circumscribed polygons	Some polygons are cyclic.	The property of inscriptibility essentially intervenes in the square.
	"*In a circle a polygon can be inscribed*" (Georgeta)	"*The Square fits" perfectly "in a circle*" – Andreea
Emphasis on regular polygons	Square	Regular polygons can be constructed with compass and straightedge.
	"*We can see equidistant points corresponding to a square*" – Catalin	"*In fact, the essential property in the construction of this figure is breaking the circle into four equal arcs, namely the opportunity to build the angle* $\frac{\pi}{2}$. *Reformulated, it is constructible regular polygons*" – Miron. (Miron states this without being asked a generalization at this stage.)
	"*Square is a regular polygon*" – Gabriela	

6.4.3 What Changes Do Students Propose to the Initial Figure to Get Other Boxes?

The second task proposed to students required from them to alter the initial figure to get two new boxes of different shapes. To meet this requirement, students had to consider the mathematical model (reached by identifying the properties of the original figure), to extend/modify the model, and to validate the new model by effectively obtaining new boxes. Table 6.2 shows the dominants of the mathematical models used by the students in our sample to develop other types of boxes, different from the original. The dominants are given in terms of geometric properties that students perceived as essential in guiding the transfer from the initial object to new constructions.

Table 6.2 Dominants of the mathematical model used by students in the generation of new boxes

Dominant used by students	Solution – the modified box	Nr of stud arriving to the solution	Comment
Focus on the net of a solid	icosahedron	**1**	Concentration on the final product, they just keep the idea of container.
	dodecahedron	**1**	
	parallelepiped	**1**	
	(regular) octahedron	**2**	
	right-regular pyramid with congruent edges	**1**	
	cylinder	**1**	
Plan coverage with regular polygons	triangular box	**1**	Students use tessellations with squares or equilateral triangles.
	"spectacle case" box	**3**	
	hexagonal box	**1**	
	"heart-shaped box"	**1**	
Focus on inscriptibile/ circumscriptibile polygons	regular octagon	**1**	Metric aspects are ignored; for the first three cases, the common chord is a diagonal in polygons, not a side.
	regular dodecagon	**1**	
	regular 16-gon	**1**	
	equilateral triangle	**1**	
	regular hexagon	**1**	
Emphasis on the use of a regular polygon	regular pentagon	**2**	To achieve the figure, students use practical tools (ruler to scale, protractor, square ruler) or technology (GeoGebra)
	regular hexagon	**2**	
	regular octagon	**1**	
Focus on constructability with compass and straightedge	regular pentagon *and* hexagon	**1**	Students presented effective (ideal) constructions, using onlycompass and straightedge
	regular hexagon *and* octagon	**8**	

We have identified three types of approaches used by students in modifying the initial given figure.

(a) **A Theoretical Approach**

This approach is characterized by "perfect" figures: the students that adopted this approach propose changes related to the idea of regular polygon that can be constructed with compass and straightedge. Typically, students who have this approach minimally change the initial context, they just change the number of sides. In general, these students did not pay attention to the practical purpose of the task, focusing on the rigor of the mathematical constructions.

(b) **A Technological Approach**

Students who adopt this approach are not interested in the rigorous construction of the figure because they have alternative instruments (ruler, protractor, or square ruler; graphic computer programs), and the focus is on obtaining the final product. For these students, the practical verification (even if there are flaws in combining the elements to obtain the product) replaces proof and argumentation.

(c) **Focus on plane figures**, **with no analogical 3D transfer**

Some students retain from the task only that we want to form "a container". These students went back to their basic knowledge (such as the classical net pattern of a cylinder or octahedron), actually neglecting the task constraints.

6.5 Discussion

6.5.1 Some General Comments

We will comment on the geometric properties identified by students (Table 6.1) and on their perceptual clues in generating new boxes (Table 6.2) from the view of modeling. We note that geometric transformations have not been used to generate new configurations: they just remained at the level of the language used by students to describe the mathematical model. The properties that highlight the congruence of elements of the original figure was obtained by the mathematical translation of a technological process (the effective realization of the box), while the students who relate to an unfolded net of a solid as a way of generating new "products" seem to retain only this aspect – ie that the connection plane-space goes through unfolding and make a transfer conditioned by this stereotype.

Most of the students' proposed changes (18 new proposals) are based on regular polygons, constructible with compass and straightedge. In fact, starting from square (seen as a regular constructible polygon), students undergo a process of generalization and propose in 8 out of the 9 cases, boxes that use regular hexagons and regular octagons. For these students, we found a certain automatism: they use an algorithm corresponding to a general property (constructability of polygons with compass and straightedge).

Plane tessellation with regular polygons as a dominant feature of the mathematical model represents a creative potential, yet untrained in the Romanian school. Some students retain that, following the instructions indicated by them, a square coverage of the plane appears as background. Subsequently, they do coverage plane with squares or equilateral triangles and, starting from this background, they propose new geometric configurations that can lead to obtain boxes.

The weak properties identified by students appear in an incomplete mathematical modelling. However, they allow more degrees of freedom, because they can lead to a wider class of geometrical configurations: therefore, they have the potential to facilitate a more creative approach. Strong properties usually lead to a mathematical model very well-articulated. The existence of this model seems to be sufficiently rigid to direct the solution and push students towards a theoretical approach. We note that, the more theoretically advanced is the mathematical model (as in the case of the theorem of characterization of regular constructible polygons), the stronger it controls generalizations. As a result, although in this case many potential solutions appear, they follow the same pattern, they are in the same equivalence class, so once the student has demonstrated mastery of this instrument, his/her results cannot be recorded as cognitive variety.

6.5.2 A Few Case Studies

The sample of students used for this study is relatively small. Therefore, a quantitative analysis would not be relevant. On the other hand, we try to understand the relationship between creativity and expertise. Both features can be better captured by analyzing individual student responses. Therefore, we further include case studies in which students discuss how they have responded to the task, from two perspectives: proven expertise in the formulation of solutions, and their degree of creativity. We will try every time to identify, in student's cognitive behavior, evidence for the criteria that distinguish between novice and expert, detailed in Sect. 6.2.4, and the cognitive flexibility components, within the framework used to identify creativity. In some cases it was possible to make, for a student, clear distinctions novice – expert or creative – uncreative. There are also situations where, based on available data, we could not make such distinctions.

6.5.2.1 Case 1 (Emilian)

Emilian identifies the following geometric properties of the given figure: the points on the two circles are equidistant, forming two squares; the interior arches are congruent with the "proper" arches; inner arcs do not intersect (except their ends). These conditions define the initial geometric configuration, which shows that Emilian is able to infer the necessary and sufficient conditions underpinning this configuration.

In modifying the configuration, Emilian first tries to customize to the triangular case – this shows specific behavior in problem solving. He realizes that, in this case, one of the identified conditions regarding the arcs (condition which is automatically checked in the given configuration!) cannot be met. He has a moment of doubt (he writes: “I think the box cannot be made, at least not in this way”), then he returns and delete/cut out some of these comments. He has a few paragraphs originally written, on which he returned and cut out. This behavior, on the type “step back” of observing own solving process, is also obvious in the analysis of the list of instructions. Initially, it contained 13 items; subsequently, based on observations made on the person who followed these instructions, the list was reformulated. Even if the task did not require (explicitly) this experiment being rebuilt (ie the new list of instructions to be proposed to another person), based on the new observations made, the list of instructions was changed again. This behavior shows, from a cognitive view, that Emilian has the ability to change his thinking schemes. The changes proposed by Emilian – boxes using regular hexagon and regular octagon denote abstract mathematical thinking. Even if Emilian do not explain why “skip” over the case of polygons with 5 or 7 sides, the fact that he began his analysis with the case of triangle proves understanding of the restrictions imposed by the constructability with compass and straightedge. The avoidance of certain numbers shows that Emilian possesses structured information (results about constructability with compass and straightedge), which he activates in this case. All these bring evidence for the existence of a certain way of structured organization of knowledge. Emilian obtains the figures through constructions made using only compass and straightedge, and claims that in the hexagonal box type, he checked his conjecture by building the box; thus proving purposely oriented procedural knowledge.

Once the checking made for one of the boxes, Emilian seems convinced that the other box fulfills the requirements without any supplementary checking. He thus expands the observed properties to the octagonal box, proving metacognitive capacities of self-regulation.

Previous comments show that Emilian proves theoretical expertise: he shows abstract thinking, he explicitly identify necessary and sufficient geometrical conditions allowing the construction of the object. In other words, his expertise compels him to assume a rigorous mathematical modeling of the object. To what extent does he show creativity in solving the task? The fundamental element to which Emilian refers is a regular polygon constructible with compass and straightedge. Once this frame built (mentally) – shaped by defining geometric properties (ie necessary and sufficient), he manages to identify and further modify essential elements (in this case – the number of sides of the polygon) and generate new valid configurations; this is about the capacity of changing an initial mental frame within the persistence of his assumed mathematical model. Emilian includes proof of the impossibility of building a triangular box in his response, by the same process. Subsequently, he generates boxes of different number of sides (6 and 8). This approach, of inductive type (starts with the minimum possible number of sides continues by varying the number) suggests that Emilian knows that the generalization process can be continued. We interpret this behavior as specific to cognitive variety. This shows that Emilian approaches creatively the given task.

6.5.2.2 Case 2 (Andreea)

The solving proposed by Andreea focuses on the technological process. Her instructions state, first, the materials necessary to achieve the box construction and are very detailed ("make notches using the cutter – but do not cut (!) the arches inside, so we can easily bend them"). Unlike Emilian, her constructions use ruler to scale and square ruler. This shows that Andreea is not interested in the mathematical "theoretical/abstract" aspect of the task, but of the pragmatic ones. She intuitively identifies the geometric properties that allow obtaining the box, which she formulate using a common language (such as "so we can put perfectly one on each other and form the lateral sides of the box", "the square perfectly falls in a circle", etc.). Andreea proves a type of goal oriented procedural knowledge.

Andreea identifies a defining property of the initial configuration – namely, that the marked points on the two circles determine a square in each. She claims that the square "fits perfectly in a circle" (meaning that it is a cyclic polygon), and that this applies to any regular polygon; as a result, we can use any regular polygon instead of the square, the only changes being that the number of lateral sides of the box increases and the box shape changes. In other words, Andreea identifies principles underlying the original structure – ie the property that the used polygons should be regular.

Andreea proves effectiveness in solving the task. She does not question constructability – as Emilian, but construction: for this, she neglects the details of the figure, focusing on the property she found as dominant, and generate (for example) a non-rigorous drawing, yet very clear in respect to information transmitted (Fig. 6.4). This shows that Andreea can develop her thinking schemes by synthesizing information.

As evidence of her technological orientation skills, Andreea uses GeoGebra to get the figures she suggested. The existence of this universal tool – GeoGebra ensures Andreea that the construction can be made for an arbitrary number of sides of regular polygons. Once generated the plane configuration, Andreea seems convinced that the effective realization of the box doesn't bring any difficulty – it is made similarly with the original case. This shows metacognitive capacities of self-regulation – it is no needed to recheck something that works analogically!

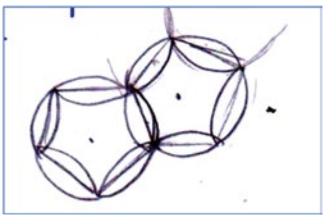


Fig. 6.4 Representation made by Andreea to explain how to obtain a pentagonal box

Is Andreea creative? We will further analyze this issue, to show that the answer is affirmative. Andreea succeeds to understand the properties of the original figure, even if sometimes her language is too approximate from a strictly mathematical view. For example, she notices that the arches that appear in the initial figure are equal, and the common side of the squares determines “congruent arcs on both sides” – ie, she notices the symmetry of the figure. Starting from the fact that the square is a regular polygon, Andreea says that we can use for the requested construction any regular polygon, but it is difficult to identify the centers of the circle describing the inner arcs. This looks as she evolves within a well-defined frame, but she does not pay attention to metric details because she can use a tool (GeoGebra) for every conceivable situation. Once the frame built, the variations she proposes (changing the number of sides of polygons consistently) show capacity of frame change.

In her response, Andreea includes only one box – namely, a pentagonal box. However, we concluded that she displays, in fact, cognitive variety: once a property with potential for generalization (ie the regularity of the polygons) determined, Andreea knows that she can unrestrictedly change the number of points of division – ie she can get many (new) models of boxes! She is not “restricted” in the construction of these new boxes, as the used instrument (GeoGebra) allows unrestricted freedom to vary a parameter of the geometric configuration (ie number of sides).

6.5.2.3 Case 3 (Paul)

Paul is prolific in identifying geometrical properties of the given configuration. He sets out 10 geometric properties, some of which are “dependent” (can be deduced from the properties listed above) – and, consequently, could be missing. We interpret his desire to formulate more geometric properties than necessary as an argument for the fact that Paul can change his thinking schemes and to focus in turn on some other aspects of the given geometric configuration.

Paul expresses the properties of the given configuration in two language registers. On the one hand, Paul connects the geometric context of the initial figure with a strong mathematical result, such as the theorem of characterization of regular polygons constructible with straightedge and compass (ie a regular polygon with n sides is built if and only if $n = 2^k \Pi p$, where the product contains only prime distinct Fermat numbers). The correlation of these properties with the context (in which the constructability with compass and straightedge was not explicitly stated) suggests that Paul has a knowledge organization of expert type, because he can quickly select, from memory, that specific information which is necessary and useful in solving the current task.

On the other hand, Paul seems that he does not only want to identify and convey properties, but he also wants to explain them suggestively. In this respect, he uses intuitive descriptions or names, such as “biconvex lens”, “the box resembles to a cuboid covered with two blankets bond in the corners”. For Paul, the link between

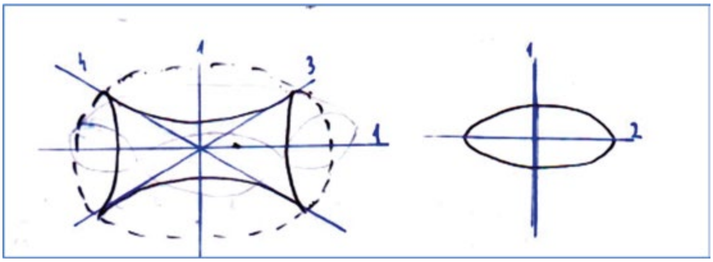


Fig. 6.5 Paul's drawing, which suggests that he may use an elliptical figure for getting a box

theory and practice is much stronger than for Emilian or Andreea. This is reflected in the plasticity of language and in the fact that, unlike other colleagues, he moves from the general result (ie the theorem of constructability of a regular polygon) to the concrete situation in which the theorem is applied. Paul's expertise doubles and supports his creativity. He is prolific in identifying properties of the given figure, which indicates cognitive variety. Moreover, although he does not explicitly state, he seems convinced that one can build a box by distorting the initial figure such as circles become ellipses (see Fig. 6.5). If this was indeed his intention, Paul shows reframing, therefore a high level of creativity.

6.5.2.4 Case 4 (Dana)

Dana has generated a list of instructions containing eight items. Her instruction list starts from two secant circles and from the symmetrical points of the centers of the circles to the intersection points. Subsequently, she builds arcs of circles with centers in these points. Dana's instructions and comments do not specify whether the initial circles are equal, or if quadrilaterals obtained are squares. Dana implicitly assumes, however, that these conditions are met. In fact, if we follow her instruction list (with the supplementary hypothesis of congruence of the initial circles), we get a box in which the base is a rectangle (see Fig. 6.6) Dana is however not aware of this fact that could lead her to an immediate generalization; she is focused only on the figure and she believes that in this way, she gets squares, regardless of the distance between the centers of the two circles.

For the initial figure, Dana notes that "the intersection of the two squares is another square having as side the radius of the two circles". She breaks down the initial figure into "small" squares (as in Fig. 6.7), and then she generates new figures, made of triangles, which keep the "zigzag" pattern.

It seems that Dana retains only surface features of the task (ie a specific pattern of squares that cover, in her perception, the initial figure) and uses this pattern for another geometric figure – ie equilateral triangle. Not coincidentally, the figures

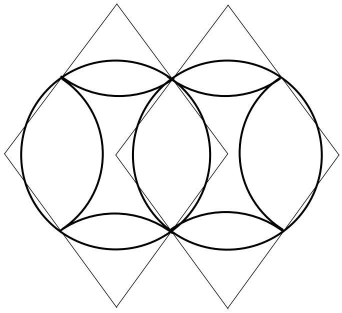


Fig. 6.6 The figure generated following Dana's instructions

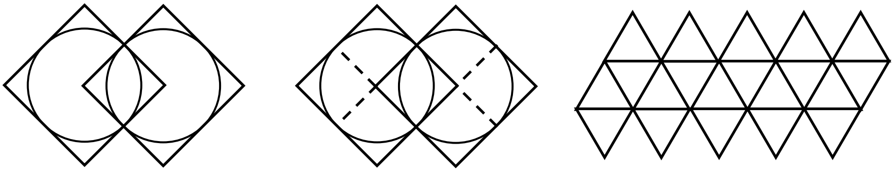


Fig. 6.7 Dana's patterns identified in the initial figure and applied to the figure she generated

generated by Dana (as alternatives to the given figure) no longer contain circles or circle arcs: Dana identifies only superficially the geometric figure baseline (the 2 circles have equal radii; we built two symmetrical squares; their intersection is also a square; the figure has two symmetry axes), and none of them is about the built arches. All this shows that Dana is rather novice in exploring the task.

Dana retains only one aspect – namely, that in the end, we obtain a container. The background she identified, consisting of matching squares arranged diagonally, suggests the use of figures previously known, representing the unfolded net of some regular polyhedron (octahedron and icosahedron).

We can say that Dana denotes cognitive novelty because her chosen changes are significantly far from the initial context. However, she thus slides out of the problem frame due to insufficient understanding of the geometric properties of the given figure (her generated construction leads to circumscribed rhombuses and inscribed rectangles, missing the condition of equal circles). At a careful analysis, we note that, in fact, she exploits a simple regular easily identifiable pattern. This is a relevant case for the situation that creativity does not advance too much because expertise is missing (in the Glaser's sense). Apparently, this is in contradiction with the fact that Dana is a student with high academic results. Perhaps her learning is often a surface one, based on memorization and not on depth analysis of mathematical contexts – but we do not have more data to advance this hypothesis.

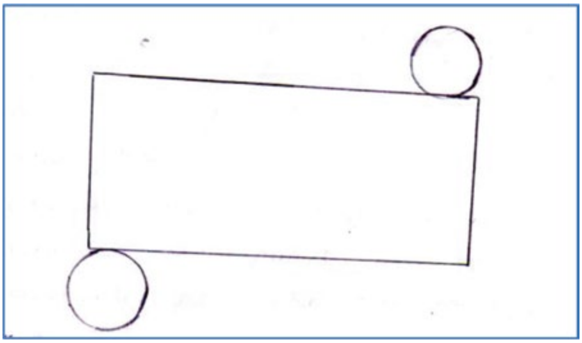


Fig. 6.8 The change proposed by Georgeta

6.5.2.5 Case 5 (Georgeta)

The structure generated by Georgeta differs from those of all his colleagues. All the other students in our sample have generated a list of instructions specifying (and numbering) the steps. Georgeta has designed its instructions as a descriptive prose. Many of her instructions are non-essential and unclear. For example, there are indications of the colors to be used for certain details of figure and comments like "common part of the two circles must be quite large, but smaller than the radius". By this, she proves superficial understanding of key concepts and terms. Georgeta believes that the defining geometrical property of the given configuration is that "in a circle can be inscribed any geometric figure, more exactly, polygons". These statements have shown us that she is novice. As a change from the original, Georgeta proposes the drawing of Fig. 6.8, in which an unfolded cylinder appears. She insists that it causes a box, while it has no other geometric properties compared to the initial context.

With the proposed change, Georgeta depart significantly from the given pattern. Is this evidence of cognitive novelty? We incline to think it is not.

6.5.2.6 Case 6 (Cristina)

Cristina's instruction list starts from the description of three special "preliminary" constructions with compass and straightedge: the midpoint of a segment, the perpendicular from a point on a line, the circle inscribed in a square. Her instructions contain 11 items: most of them are synthetically formulated. Cristina gives in her instruction list "milestones" – brief indications to verify the construction accuracy. This ability to synthesize the information transmitted, but also to keep a protective attitude towards the reader, proves the goal oriented procedural knowledge – which is guiding the solver.

Cristina equally proves synthetic when she identifies geometrical properties of the given configuration: they refer to invariance through symmetries and rotations. These properties are seen in relation to the final object (the box); for example, the symmetry to the common chord is the condition that "causes the box to have

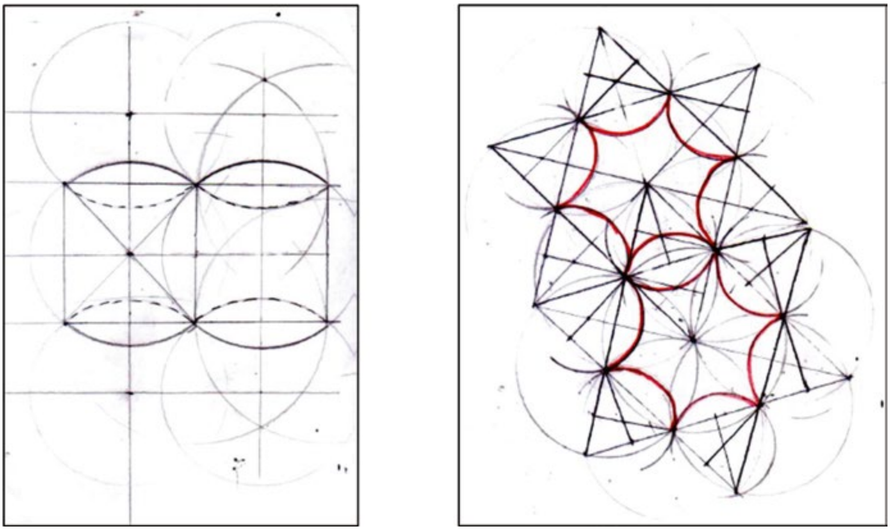


Fig. 6.9 Cristina's drawings for getting new boxes

walls – when bending the box, the walls have to overlap". The element through which Cristina seems to modify the initial configuration is the coverage of the plane with figures of the same shape. For the given figure, the background she perceives is a tessellation with congruent squares. Cristina keeps this tessellation as a way of generating a new box (Fig. 6.9a) or use a tessellation with equilateral triangles (Fig. 6.9b).

Cristina keeps a method similar to that of her instruction list for drawing the inside arcs. More precisely, these are arcs of the circles circumscribed to squares or equilateral triangles from the tessellation. Cristina works with a weaker condition: in the second proposal, the arcs are no longer symmetrical towards the common chord, and this is why the sides of the box do not perfectly match. In principle, this weakening of a condition could allow a bigger number of possible solutions (at the expense of object's "perfection"). Could this be an evidence for creativity?

The weakening of conditions is actually a gap in her response, to the extent that she is not aware of the consequences: she actually did not realize the implications, even if she made the box and so checked that it can be built. Specifically, Cristina is unaware that in the new construction, the sides of the box do no longer "perfectly" overlap, as happens in the initial model.

6.5.2.7 Case 7 (Adelina)

Adelina preferred to solve alone the whole task (not in a team of two, as most of her colleagues did). Her list of instructions contains 10 items; the language used is not mathematically rigorous, but instructions can be easily followed. Her instructions are focused on obtaining the figure, not on getting the box: once the figure drawn,

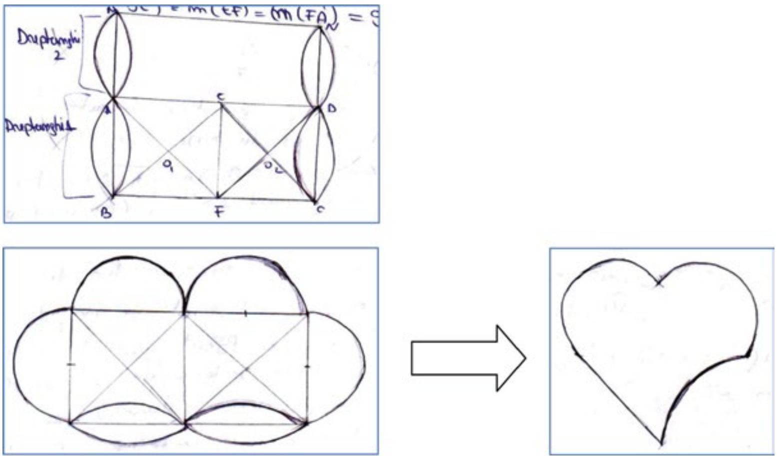


Fig. 6.10 Adelina's proposals for new boxes

Adelina believes that her mission was accomplished. Adelina identifies only two geometric properties of the initial configuration, namely: 1. quadrilateral determined by the points of intersection of the two circles of the figure and the centers of the circles is a square; 2. the circles have been divided into four equal parts.

Even if the mathematical model described by Adelina is incomplete (it does not say anything about the inner circle arcs), her instruction list shows that she internalized the context and can give directions to complete its reproduction. This shows us that Adelina displays functional knowledge.

To change the initial figure, Adelina proposes the models shown in Fig. 6.10.

Adelina achieved a first product resulting from a correct mathematical modeling, whose shape is found even among usual items around us (a spectacle case), although she does not mentioned this as such. The utility of the obtained product indicates transfer capacities (Gardner 1993). The second product obtained – also by a correct mathematical modeling, has, in addition, aesthetic value. The fact that these new objects have practical and aesthetic values is another argument for her procedural functional knowledge. Compared to its peers, Adelina proposes very different models. So we can say that she denotes cognitive novelty. For her both new models, she keeps the same background (easily to identify congruent squares) and the same way of building arches (parts of the circumscribed circles to such squares). Adelina evolves within a well-defined framework and manages to make substantial changes to it, while keeping it consistent.

6.5.2.8 Case 8 (Anca)

Anca has generated a list of six initial instructions. In her instructions, she implicitly assumes that the person to follow the list knows some mathematical concepts, at least at a basic level (eg perpendicular lines, reflective symmetry of a point, square circumscribed to a circle, etc.). At the end of this list, Anca includes a commentary

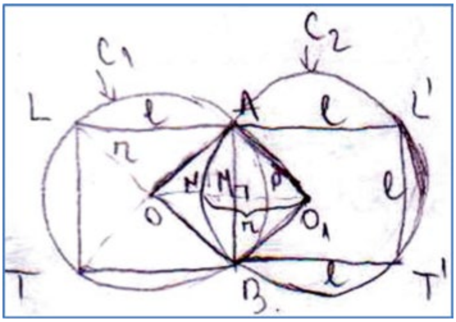


Fig. 6.11 Notations made by Anca for the metric description of the initial configuration

under the title "philosophy of the instructions," in which she claims the construction accuracy. She also includes extensive comments on the difficulties faced by people to whom he proposed making the box: some of these difficulties arise from misunderstandings on mathematical concepts. The fact that Anca redesigned not just the lists of instructions, but the entire solution to the task as a whole (she asked to resubmit a new version of solving the whole task, because she believed that she can better explain how to solve it) shows, on the one hand, her capacity of changing thinking schemes, and, on the other hand, she proves metacognitive capacities of self-regulation. Anca prefers to describe metrically the geometric properties of the given configuration: she expresses the lengths of the various segments as function of the radius of the initial circles (Fig. 6.11).

Typically, the quantitative metric approach of a configuration is a barrier to generalization/transfer because quantitative information limits the chance of identified generic properties. Anca proposes three modifications to the initial configuration: she replaces squares with regular octagons, with regular dodecagon, respectively with regular 16-gons. For the new situations, she explains how regular polygons can be built with straightedge and compass (mainly building bisectors of angles, but she does not perform the constructions, including only schematic representations of them). Anca possesses goal oriented procedural knowledge.

We note that Anca manages to overcome the "barrier" of metric results and identifies a property with potential for generalization – ie "square is a regular polygon." Perhaps, she sees regular polygons in quantitative context (lengths of sides and measures of angles), not in a qualitative one (invariance over symmetries and rotations). The focus on a particular property of the initial configuration, which allows generalization shows that Anca may overcome interferences and identify principles underlying the surface structures. We may ask where her "jump over hexagon" comes from – ie why Anca, unlike the majority of students who have generalized based on the idea of a regular polygon did not consider the case of hexagon. A possible answer is suggested by the way she imagine the new boxes (Fig. 6.12). Anca keeps as invariant the configuration of two equal circles that intersect over arches of 90°. She then divides each of these circles in a same number of congruent arcs, such as the intersection points of circles to be the dividing points. Therefore, her self-imposed restriction (the relative position of the two circles) requires dividing the number of points to be multiple of 4.

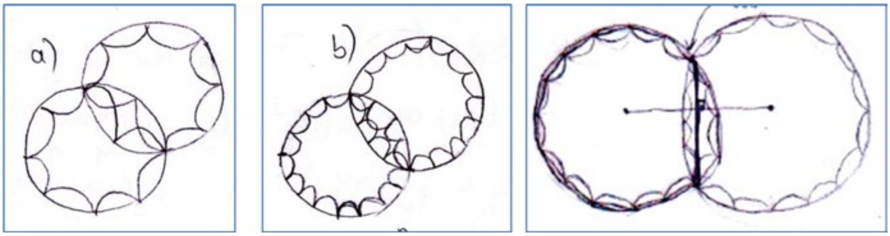
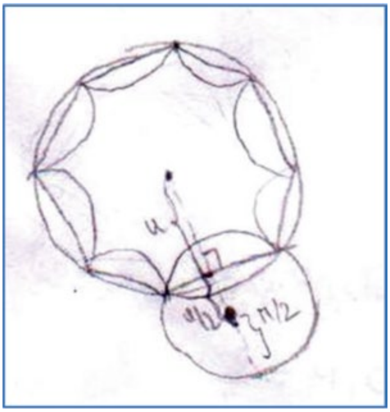


Fig. 6.12 Anca's imagined configurations for her new boxes

Fig. 6.13 The construction pattern of the inner circle arcs indicated by Anca



Anca is thoughtless in tracing the arcs of the circle. She proposes a construction described in metric terms that he believes is generally applicable (Fig. 6.13), but which cannot be applied in all the described cases. Because of this, the "lenses" Anca obtained shows no symmetry and the boxes imagined do not close "perfectly" (as in Cristina's case).

Anca evolves in a well-defined cognitive frame and makes changes in this frame, varying the number of sides of polygons. She also proves cognitive variety – by her new generated models. Anca identifies a general process of obtaining new configurations – namely, for a given configuration, doubling the number of points of division by building bisectors of angles. In this way, the idea that implicitly appears is that the number of sides may vary indefinitely – which is another argument for cognitive variety.

6.5.2.9 Case 9 (Miron)

Miron's instruction list begins with mentioning a list of the necessary materials and continues in some detail (eg: he mentions the fact that two distinct points determine a line, and lists basic compass and straightedge constructions, such as drawing a segment determined by two points). The proper list of instructions contains 14 items. The instructions contain milestones – indications on how the solver can verify his/her construction. Miron proves very synthetic in identifying the essential elements of the given figure:

> *In fact, the essential property in the construction of this figure is the possibility of breaking the circle into four equal arcs, namely the opportunity to build the angle π/2. Reformulated, it is about constructible regular polygons.*

He thus proves that he can mobilize thinking schemes, easily moving from the original context to a generalized representation of it. He recalls the theorem about regular polygons constructible with straightedge and compass ("A regular polygon with n sides is constructive $\Leftrightarrow n = 2^k p_1 p_2 \ldots p_r$, where p_i are distinct prime Fermat numbers- ie $2^{2^m} + 1$"), proving that he can rapidly select from memory items of structured information when needed. Miron notes that the square obviously satisfies the theorem conditions, but the instruction list for the initial figure are specific to this case and are not useful in the generalizations that follow. Miron's proposed new cases are those of a regular hexagon and regular octagon. He presents the constructions steps in a highly synthetic and generalized formulation:

- *We choose n equally-spaced points on the circle (how exactly to do that depends on n, but it is always possible).*
- *For any two consecutive, we build another circle that contains them and has the same radius as the initial (a compass and straightedge elementary construction in at most four steps).*
- *We now have a "star" with n corners inside the initial circle. We choose any of the other circles and repeat the procedure (of the construction of another "star" inside it).*
- *We reached the desired figure that can be cut.*

When putting the construction into act, he uses GeoGebra to make the "classical" compass and straightedge construction (to specify the division of a circle into n equal arcs). The technology in this case is just a good instrument (it has accuracy and shortens time) that replaces physical objects such as paper, straightedge and compass, keeping all valences of the ideal construction.

He alternates schemes and procedures which he combines in a manner that focuses on optimization and getting results simultaneously. Miron proves metacognitive capacity of self-regulation, high transfer capacity and, in general, the type of expertise of a mathematician.

Comparing to how another student (Andreea) used GeoGebra, we can see that Miron – with mathematics expertise, used the software as only a support to enhance and concentrate the force of the theory, while Andreea – with a rather pragmatic expertise uses the facilities of the software in actual construction without questioning the geometric accuracy. Figure 6.14 shows the images used by Miron to construct the regular octagon-based box.

He does no need to identify the initial figure geometric properties that allow the construction of the box (of the type: symmetry, congruency) because he internalized a general scheme available for construction. This scheme – the constructability theorem – offers the individual cases to perform the initial construction and the pattern that allows generalization. In these circumstances, we can ask how creative is a solution induced by the in-depth knowledge of a strong theorem. Perhaps the given context is not enough challenging for him to provoke creativity.

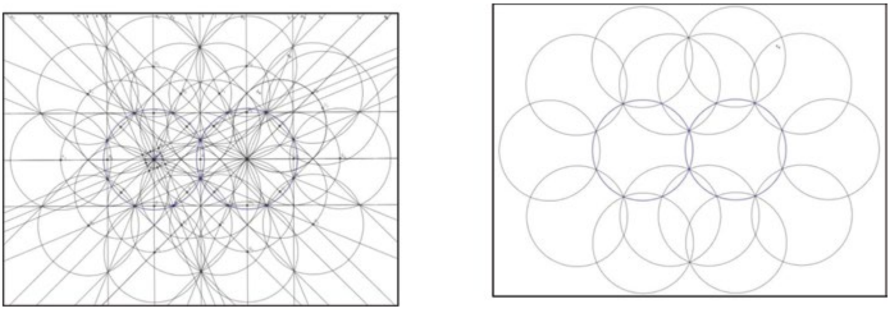


Fig. 6.14 Miron's images obtained using GeoGebra

6.5.3 Comparative Remarks

We will look in more detail into the cases presented above.

Emilian, Andrei and Paul proved validated task-related expertise because their cognitive behavior allowed checking the assumed criteria that confront expert-novice abilities. However, their type of expertise is manifested in different ways: Emilian proves metacognitive capacity for self-regulation and general use of problem-solving tactics; Andreea is a practitioner expert type focused on a technological approach, showing high procedural and goal-oriented knowledge; Paul proved stronger transfer skill for making connections between theory and practice than Emilian and Andreea, and has developed a meta-cognitive capacity to explain the identified properties using suggestive expressions.

We note that in all these three cases, students manifested a creative behavior:

Emilian firstly investigates the case of a box with a triangular base, he identifies arguments by showing the impossibility of construction, and then he generalizes. Andreea includes a single new model box (pentagonal). She however indicates a construction with potential of generalization, performed with a "universal" instrument – GeoGebra: she is confident that this process will work for any number of sides, and therefore she does not need to include other cases. Paul is prolific in identifying properties, showing cognitive variety. Through drawings, he suggests a substantial change frame, because he finally replaces circles with ellipses. Paul recalls a general result, regarding the polygon constructability with straightedge and compass; once identified the theoretical background, he particularizes the theorem and provides two new constructions.

Dana and Georgeta behave as novices. The properties they identified with respect to the initial configuration are weak properties. Dana identifies a pattern and appears to extend this pattern to generate new boxes. Georgeta relates to unfolding a solid and proposes as a new model an unfolded net of a cylinder. At a first view, Dana and Georgeta seem more creative because their proposals are significantly far from the initial model. They yet focus on superficial aspects, such as a simple pattern of squares distribution and/or the idea of container, and this lack of consistency shows that, in fact, they do not behave mathematically creative.

It would be expected that weak properties, allowing more degrees of freedom, have the potential to facilitate a more creative approach. However, we found that they actually lead to an insufficiently consolidated frame (probably caused by an insufficient level of expertise), instead of leading to spectacular generalizations.

The above comments suggest the conclusion that expertise seems to be a precondition for creativity. We will show that this statement should be at least nuanced.

The data available for Cristina, Adelina and Anca did not allowed us to consider them experts in every sense of the word. Rather, they have a moderate level of expertise, having characteristics of expert behavior, but also novice features. For example, Cristina is not aware of the consequences of weakening some requirements; Adelina shows a superficial understanding of some of the concepts used; and Anca based her constructions on metric inputs, but which may not apply in certain situations. We classified the proposals of the three students as creative. Cristina proposes two new boxes, totally different proving cognitive variety. Adelina's proposals have functional and aesthetic valences, and are very different from all the other proposals. Anca suggests a construction of a generalized manner that allows many more new products, thus showing cognitive variety.

Therefore Cristina, Anca and Adelina behave creatively. It seems that a rather moderate level of expertise allows expression of their creativity.

To verify this hypothesis, we consider the case of Miron. Obviously, Miron is the expert par excellence. He summarizes, in his solving, the problem nature, he quickly selects items he needs from memory, and "closes" the problem by applying a general result that solves a whole class of problems of the same type. Moreover, he "hijacks" a tool like GeoGebra using it for a compass and straightedge construction, and including it in his theoretical approach.

In his case, his high level of expertise as related to the task practically cancels the problem. In this case, it becomes legitimate to ask if does make sense to put the question of a creative answer in Miron's case Why did this question arise? Because Miron, by mastering powerful mathematical tools, manages to reduce a problem that for others is complex to schemes automatically activated. For this reason, because the solution is based on results already known to him, his creative contribution is at most in appropriately correlating concepts and procedures, ie in small changes well controlled within a cognitive frame clearly emphasized from the beginning. Meanwhile, cognitive novelty, and cognitive variety are practically undetectable. As a result, we believe that, in the Miron's case, we cannot detect creativity on this task. We make the assumption that facing more complex tasks that would require a higher level of expertise, Miron could be highly creative. This hypothesis was confirmed by the information later obtained about him, beyond this task. We learned that Miron is already included in a mathematics research program and that he has already published original results.

Therefore, the determination of creativity should happen at a level that exceeds the person's expertise at that moment. It appears as a corollary that *creativity is not an absolute parameter*. The manifestation of creativity depends on the context, as confirmed by other studies.

6.6 Conclusions

In this paper, we have studied how students' creativity manifests in a complex context that involves modeling, problem solving and problem posing. A first conclusion refers to how students use the defining elements perceived in an initial given figure. We have seen that these elements are further used for generalization and transfer. So, the way in which students perceive the initial figure is fundamental for solving the task and for posing new coherent modifications.

A second conclusion refers to the relationship creativity – expertise. The students that seemed more creative at a first sight, proving that they are novices in the domain, produced either non-functional or inappropriate objects. Conversely, students who showed a high degree of expertise utilized strong mathematical results (such as constructability with compass and straightedge) and made incremental changes by varying a simple parameter (in our example, the number of sides of regular polygons).

The analysis of some specific cases led to the conclusion that creativity manifestation is conditioned by a certain level of expertise. In the process of building a solution for a nonstandard problem, expertise and creativity support each other and enable bridges to the unknown, mutually developing each other. This interaction leads also to an increase in expertise.

We have seen that, because contextualization, it is practically not possible to find tasks that would allow discerning creativity for a broad range of skills. If the task is at a cognitive level accessible to a majority, a person with high level of expertise will make appeal to tools that automatize the response; if the task is challenging for a person with a high level of expertise, then it is not cognitively accessible to a larger sample, in order to make comparisons.

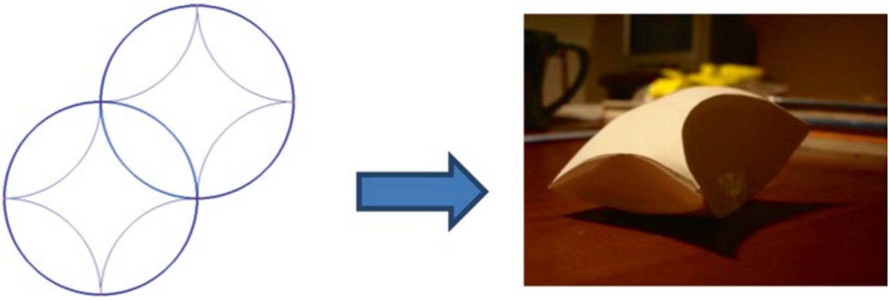
We unravel from here that a possible method of training excelling students is through practicing tasks appropriate to their level of mathematical abilities, but containing nonstandard challenging components, in order to train metacognitive self-regulation capabilities through creative leaps.

Therefore, to create the context in which a student can advance, it is necessary to determine the type of task for which he/she manifests expertise and to integrate this task in a challenging context. Our study shows that this approach seems to work for advanced students. Further research will focus on a methodology to check if it may work for students of any level.

Annex

The Given Task

From the figure below one can get a "fantasy box" [a.n. the box was presented "physically" by the teacher].



I. *The first two questions constitute a group task (2 people). For this part, the group members will receive the same score.*

1. Write specific instructions for constructing this figure. The instructions will contain only words, no drawings, diagrams or pictures.
2. Give these instructions to another person who does not know what you want to achieve. Ask that person to follow instructions. Do not interact with that person, do not give indications, or help. Note (or record) what happens. If the person has difficulty in representing the figure, or something unforeseen happens, it's OK: this only shows that your instructions are not enough precise and should be reviewed. You will not be penalized if the first set of instructions is not quite accurate.

   (a) Write a report as detailed as possible (but no longer than 3 pages!) about what happened;
   (b) Write a revised instruction list and possibly repeat the experiment with another person.

II. *Answer the following 3 questions individually.*

3. What geometric properties are used in the construction of this box? Explain your answer.
4. The fantasy-box has a "squared" shape ☺. How could you modify the original drawing to get boxes of other shapes? Build two new figures and make sure you can get boxes starting from the figures you indicated.
5. Do the proposed figures above use other geometric properties than the ones of the original box? Explain your answer, and if it is yes, please specify which are these properties.

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