Mamokgethi Phakeng
Stephen Lerman *Editors*

Mathematics Education in a Context of Inequity, Poverty and Language Diversity

Giving Direction and Advancing the Field

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Foreword

It was with great pleasure that I accepted the invitation by Stephen Lerman and Mamokgethi Phakeng to write a preface for this book which is an homage to Jill Adler, a 'grande dame'—as we say in French—of the field of mathematics education and a model for all of us.

I do not exactly remember the first time I met Jill. I suppose that this was at an international conference, certainly at one of the annual conferences of the International Group on the Psychology of Mathematics Education as, for decades, she has been a very active member of the PME community. In fact, I really came to know her when, in 2002, she was elected vice-president of ICMI or the International Commission on Mathematical Instruction. I had entered the executive committee of ICMI in 1998 and was beginning my second term as vice-president at that time. With Jill Adler, the voice of Africa physically entered the executive committee. As surprising as it may appear, Jill was indeed the first member of an ICMI executive committee to come from an African country! And this was really welcome!

Serving ICMI together during 7 years, we exchanged and collaborated intensively, not as educational researchers usually collaborate, developing joint projects or exchanging on research questions and results of common interest, even if this happened at times. Research was there but often just in the background and also often questioned in its capacity to inform educational actions and policy decisions. This preface is based on my memory of these times, during which I discovered Jill and was deeply impressed by her.

There is no doubt that Jill's two terms in the ICMI executive committee were influential. She played a decisive role in the ICMI move towards putting issues of development, access and equity on the top of its agenda. Her personal experience of serving disadvantaged communities in apartheid and post-apartheid South Africa and the depth and solidity of her analyses based on the rigorous research she had been developing for at least two decades in this African context were invaluable. Her words were strong and convincing.

Thanks to her, a new regional ICMI structure was created for South Africa, and she organised the first associated conference, AFRICME1, in Johannesburg in June

2005, jointly with the annual meeting of the ICMI executive committee. For me, this was a fascinating experience, all the more so as each participating country had been asked to prepare a report on the situation of teacher education in the country. The successive sessions allocated to the plenary presentation and discussion of the different reports were key moments during this conference, with people discovering strong commonalities between their respective situations and some surprising differences also and trying to understand their reasons and then reflecting on how, as a regional community, they could better face their common challenges. This was a source of inspiration for the CANP (Capacity and Networking Project) that ICMI would launch in 2011 in collaboration with UNESCO and with the special support of the International Mathematical Union.

Thanks to her, also, we decided to launch an ICMI study on the teaching and learning of mathematics in multilingual contexts. She convinced us that it was timely to draw the lessons from the number of research studies and experiments which had been developed around these issues and in which researchers from developing countries had played a major role. She convinced us that it was timely to question the common vision that multilingual contexts are only sources of teaching and learning difficulties. Without underestimating the tensions to be addressed, this study whose volume is published this year indeed makes clear that an alternative view is possible and much more productive, considering such contexts in their potential for resourcing mathematics teaching and learning. In fact, this was the collaboration with Jill in the ICMI executive committee that made me discover this area of research, distant from my personal research interests and, I have to confess, not much addressed in my own didactic community despite its evident importance. Through her contributions, but also those of her doctoral students, including Mamokgethi Phakeng who is co-editing this book of course and many others, through my participation in AFRICME conferences, I discovered the importance of this research and the challenges it raises. I am sure that, by reading this book in which these issues are addressed by a diversity of researchers who had the chance of collaborating with Jill, in different parts of the world, not just in Africa, I will learn a lot.

As this book also makes clear, Jill Adler's research is not limited to the linguistic area; her contribution to the theme of teacher education especially is also of major importance. Once again, her deep knowledge of the international state of this domain, so crucial for mathematics education, the interest of the diverse projects she had already piloted at that time, was of inestimable help during her two terms as vice-president in the ICMI executive committee. Reading this book, I am eager to know more about the large-scale and ambitious projects she has led in the recent years, with which I am less familiar, such as the *Wits Maths Connect Secondary Project*, 'aiming to further develop mathematics teaching practice at the secondary level so as to enable more learners from disadvantaged communities to qualify for the study of mathematics-related courses at university', as explained in the citation accompanying the attribution of the 2015 Hans Freudenthal ICMI Medal to Jill.

Jill Adler is a 'grande dame', as I wrote in the first sentence of this preface. I have the deepest admiration for her, for her immense accomplishment as a researcher and

also for the extraordinary way she has been able, without compromise, to make research excellence and educational engagement go hand in hand in her professional life in a diversity of ways that this book beautifully reflects. I hope that this publication will allow all those interested in mathematics education to know more about her research and educational engagement and that it will contribute to make Jill, for many of us, the source of inspiration and the model that she has been for me.

Paris, France Michele Artigue

Contents

Introduction

This volume is dedicated to the remarkable career of Jill Adler and the role she has played in growing mathematics education research in South Africa, Africa and beyond. Her work epitomises what is referred to as the 'engaged scholar', i.e. doing rigorous and theoretically rich research at the cutting edge of international work in the field which at the same time contributes to critical areas of local and regional need in education. Included in the latter is her enormous contribution to research capacity development—her work with research teams and graduate students has made a substantial contribution particularly in South Africa during a critical time of political transition.

Jill is one of the world's leading experts in mathematics education research, and her exemplary career is a continuous source of inspiration for generations of researchers and students. She started her career as a secondary teacher and materials developer for adults and alienated youth excluded from school mathematics learning in apartheid South Africa for 15 years. She built a trajectory of research since entering academia in 1989 rooted in this professional work. It took forward her concern with mathematics teaching quality in general and equitable access and achievement in post-apartheid South Africa in particular. Her research insights have had a substantial impact on mathematics education, particularly in the area of teacher education as well as teaching and learning mathematics in multilingual classrooms. A prolific author and sought-after keynote speaker, her seminal book *Teaching Mathematics in Multilingual Classrooms* was instrumental in leading research internationally in the field. Her work on mathematics teacher education has been published in French and Portuguese.

In 2009, based on a proposal that conceptualised a research-informed, grounded intervention into the deepening crisis in math teaching and learning in SA secondary schools, Jill became the first recipient of a prestigious South African Chair in Mathematics Education jointly funded by government and business. Her work in this chair project focused on developing teachers' mathematics knowledge for teaching and impacting on learner access and achievement in selected schools.

It emerges from problems of practice, particularly in South Africa, and has been shaped by findings arising from her earlier research.

In 2006, she was appointed to a Chair in Mathematics Education at King's College, London (KCL). She established the UK arm of research on mathematics for teaching, an indication of the wider relevance of her work. This project embedded research capacity building where she works with a group of novice researchers who are building their research trajectories related to their professional work as teacher educators. Jill has played a key role of scholarly leadership role in mathematics education nationally, regionally and internationally.

Jill's accomplishments are legendary among her students. Multiple Ph.D. and masters students have graduated under her supervision, but most importantly most have published from their studies. Several of these students are highly respected scholars themselves, and others hold leadership positions in research; adding to the human research capacity, she has been instrumental in developing South Africa.

Jill has played an important role in the International Commission on Mathematical Instruction (ICMI) putting South Africa and Africa on the map. Between 2003 and 2009, she was vice-president of the International Commission on Mathematical Instruction and during this period launched the African ICMI regional congress (AFRICME), contributing significantly to participation from African countries in ICMI and enabling development of mathematics education research in the region. Jill has provided South Africans and Africans in mathematics education with an incredible role model of leadership and research excellence. Her message to all of her students has always been that they do not only have a role to play in participating in the international arena of mathematics education but that they also have a role to play in leading research internationally.

This volume is built out of contributions continuing in many ways the line of thoughts promoted/developed by Jill, giving a glimpse of the diversity of her research. This volume also reveals some insights of her remarkable personal life. All those who have had the privilege to meet and work with her have been engaged by her passionate and lively personality and inspired by her example of staying focused and being faithful to her pursuit of access to quality mathematics education for all.

The chapters in this Festschrift are authored by Jill's former Ph.D. students from Wits University and a few select colleagues from different parts of the world that she collaborated with as well as leading scholars who she worked with in PME, ICMI and her many international assignments. In essence, this Festschrift celebrates Jill's contribution not only to mathematics education but also to our contributions as her friends and colleagues. For many of us, our professional lives would not have been the same without the lessons and the approaches to scholarship that we learned from Jill.

Both of us have known Jill as a professor, colleague and friend and can never hope to describe how much she has meant to us in each role. We could certainly never dare to give voice to the gratitude of others, but we hope this volume will serve to remind her of our esteem for her and her work.

The chapters in this collection all acknowledge the effect Jill's work has had on them and on Jill as a friend and inspiration, whilst also discussing their own work, thus pushing the field forward, again a tribute to Jill. Naturally there are overlapping themes addressed in all of the chapters but they fall into a number of sections.

Four chapters are written by her former students. The chapter by Renuka Vithal and Busisiwa Goba, Renuka's former doctoral student (called in the field a 'grandchild'), focuses on Jill's supervision set against the background of an academic community emerging post-apartheid and the development of the research community across South Africa. The chapter by Mamokgethi Phakeng and Anthony Essien looks specifically at Jill's contribution to research on language. Mellony Graven's chapter draws on reflections on being supervised by Jill by three former students, Mamokgethi, Thabiso Nyabanyaba and Mellony herself, set within a framework of communities of practice. A key feature of Jill's supervision of all her very many graduates has been the creation of cohorts working together. Mellony provides a theoretical rationale and elaboration of those experiences. The chapter by Kate le Roux, Willy Mwakapenda, Nancy Chitera, Vasen Pillay, Craig Pournara and Bruce Tobias carries forward themes in a chapter that Jill wrote with Stephen in 2003 on ethics in mathematics education research, called *Getting the description right and making it count: Ethical practice in mathematics education research*. They describe Jill's work that has come to count in the community and follow this with descriptions of their own research histories in response to the challenges set in that chapter and in Jill's work more generally.

The chapter by Richard Barwell and David Pimm looks at language and mathematics, the topic of Jill's Ph.D. research and early publications. They pick up on her focus on the political issues around multilingual classrooms, drawing on tensions and dilemmas, always concerns of Jill's. We have already mentioned the chapter by Mamokgethi and Anthony Essien that also examines language.

Jill has made substantial contributions to research on mathematics teaching and teacher education. Dan Chazan and David Pimm focus in particular on dilemmas, an issue that was central to Jill's conceptualisation of her doctoral research field. Their chapter is built around an edited conversation, a framing that offers a rich dialogue on their and Jill's ideas. On teacher education, Paula Ensor examines the development of a knowledge base for mathematics teachers, pre-service from her own research and in-service from the project led by Jill and Zain Davis. The chapter by Ruhama Even, Michal Ayalon and Shai Olsher reflects on Jill's contribution to the problems of the field as a background and context for their study on the way that collaboration between teachers, mathematicians and textbook writers can transform connections and participation in mathematics classrooms.

Anna Sfard and Hamsa Venkat discuss the dilemma of an activist researcher engaging in issues that matter hugely to the participants in the research, students and teachers in post-apartheid schooling whilst also setting up the separation that is needed for good research.

Finally, two chapters perform overviews of Jill's life and work as we have all known her. Avraham Arcavi and Ronnie Karsenty offer a critical reflection on key concerns that have run throughout her work. They pose a series of key questions

faced by the field and follow with their thoughts regarding these questions, inspired by Jill's work.

The chapter by Stephen is biographical. There is a great deal that is not generally known about Jill and Stephen, and with some 'insider' help, it reveals some insights that will engage all readers.

We hope and expect that you will enjoy and learn from this book. We have benefited enormously in putting it together and reading the contributions of Jill's colleagues.

Kgethi Phakeng and Stephen Lerman

Chapter 1
Adler's Contribution to Research on Mathematics Education and Language Diversity

Mamokgethi Phakeng and Anthony A. Essien

Introduction

Research on the relationship between language and mathematics learning is recent—it only took off after the 1974 UNESCO Symposium that was held in Nairobi focusing on the "Interactions between linguistics and mathematical education". This area of study has seen growth since then and Jill Adler's work has contributed much. This chapter explores Adler's journey and its contribution to research on mathematics and language diversity. We begin the chapter with a description of the genesis of the journey and highlight two of her main contributions to research in this area of study. From these discussions we draw our main argument that the contributions were shaped by three interrelated factors: Jill's consistent focus on problems of practice in multilingual mathematics classrooms, the theoretical frameworks and methodologies she used, as well as the timing and context of her research.

The Beginning of the Journey

Adler's research on mathematics education and language diversity began in 1989, and was the subject of her Ph.D. dissertation in which she studied mathematics teachers' knowledge of their practice in multilingual classrooms. In this study Jill worked with six secondary school mathematics teachers, and Mamokgethi

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(first author) was one of them. Her main question was, "how and why [teachers] do what they do. . . in multilingual settings" (Adler, 2001, p. 35). The goal was thus to describe teachers' specific professional "knowledgeability" in the domain of teaching mathematics in multilingual classrooms in post-apartheid South Africa. There was no attempt to judge or evaluate if their practices were effective in achieving certain educational objectives and ideals.

This Ph.D. research began at a critical time in South Africa, which was then an apartheid state with structural racism; segregation; two official languages; inequality; poverty and a strong civil society. Data collection was conducted during the years 1992 and 1993—a time of radical changes in South African schools, with the classes becoming multilingual and multiracial as South Africa was busy negotiating its way out of its apartheid past. Asking questions about teaching and learning mathematics in multilingual classrooms was therefore timely, not only for South Africa but also for the international mathematics education community which was at the time also grappling with what was fast becoming a commonsense notion that mathematics learning is enhanced by conversation. This was a critical time in mathematics education, which saw the publication of the Cockroft report in 1982 and the first edition of the NCTM Curriculum and Evaluation Standards for School Mathematics in 1989. Both these documents foregrounded learning to communicate mathematically as a central aspect of what it means to learn school mathematics. So the question was, if communication is important for mathematics learning, then in which language should that communication be if the learners are not fluent in the Language of Learning and Teaching (LoLT). Adler explored what teachers know and do to enable access to mathematics as they teach in multilingual settings. This she did through analysis of empirical data (Interviews, lesson observations, workshops) from three language environments: the first two were where English was an additional language and the third was where English was a foreign language.

Main Contributions

Adler's thesis entitled *Secondary School Teachers' Knowledge of the Dynamics of Teaching and Learning Mathematics in Multilingual Classrooms* was the springboard from which her first publications on the dynamics of multilingualism and the teaching and learning of mathematics in multilingual classrooms (Adler, 1995, 1997, 1998) emerged. Prior to her work on teaching and learning mathematics in multilingual classrooms, research on communicating mathematically "normalised" the classroom and presented an assumed unilingual homogeneity of the mathematics classroom. Pimm's seminal work explored how the interrelationship between ordinary language (English) and mathematical language can pose difficulty for mathematics teaching and learning (Pimm, 1981, 1987). This work of Pimm emphasised the assumed important role of language and communication in mathematics learning. It was Sfard, Nesher, Streefland, Cobb, and Mason (1998) who

interrogated the assumption and concluded that the question is not "whether to teach through conversation but rather how", and that "communication skills cannot be taken for granted [. . .] if conversation is to be effective and conducive to learning" (p. 50). They (Sfard et al.) further argued that communication is an art, and as such, needs to be taught. It is important to note that Sfard et al.'s interrogation did not consider the specific case of mathematics classrooms in which learners are multilingual and/or not fluent in the LoLT. It was Adler who asked questions about the role of language and communication in the teaching and learning of mathematics in multilingual classrooms. We elaborate further on this later.

The focus on the specificity of classrooms where children learn mathematics in a language that is not their home language came into focus in the paper by Austin and Howson (1979), which was a follow up on the Nairobi symposium. The paper concluded that the challenge of language and mathematics learning and teaching is not just an issue for developing countries but for the whole world. In developing countries the challenge is that of learners learning mathematics in a language that is not their mother tongue; in developed countries such as Wales, the USA, Belgium and Canada, there are communities of immigrants with well-established "minority" languages and in some countries, there are instances where problems arise because of the non-standard nature of the local vernacular (e.g. Jamaica, England and the USA). Austin and Howson acknowledged the fact that language diversity is a political matter and thus change in society may lead to policy change. This matter of the political role of language in mathematics education was not taken up by research till the publication by Setati (2005).

Various mathematics education research post the work of Austin and Howson (e.g. Clarkson, 1991, 1992; Cummins, 1979; Pimm, 1981, 1987; Pimm & Keynes, 1994) focused on the interplay between language and mathematics and pointed to the intricate link between language competence and mathematical aptitude. Prompted by the need to provide research evidence that could inform the debate according to which the use of the learners' home languages impeded or enhanced mathematical understanding, Dawe (1983) and Clarkson (1992) investigated the effect of bilingualism on their capacity for learning in school. Dawe worked with bilingual Punjabi, Mirpuri, Italian and Jamaican students growing up in England, while Clarkson focused on Papua New Guinea where students learned mathematics in English, which is not their home language. Both studies found that bilingual students with proficiency in both mother tongue and English outperformed students who were proficient in only one of either mother tongue or English, and bilingual students with low competence in both languages performed very poorly.

In all these studies, multilingualism and the intricacies of teaching and learning in multilingual classrooms were not explicitly in focus. Even when multilingual learners were involved in the study (e.g. the study by Adetula (1990) on word problems in home language and English), multilingualism and its implications/impact on teaching and learning did not form part of the discussions or findings resulting from these studies. This gap was subsequently bridged by Jill's first publication on the subject (Adler, 1995) which investigated "the dynamics of multilingualism and the teaching and learning of mathematics in junior secondary

classrooms in South Africa". While earlier research focused on what second language learners can or cannot do mathematically, our view is that Jill's work foregrounded a number of issues pertinent to the teaching and learning of mathematics that led to fundamental theoretical shifts in this area of study.

First, it acknowledged that learning and teaching mathematics in multilingual classrooms is complex and that teachers have to grapple with dealing with this complexity. Adler (1995, p. 265) expresses this complexity in these words:

> . . .the dynamics of teaching and learning mathematics in multilingual classrooms is not simply about proficiency in the language of learning; nor is it only about access to the (English) mathematics register; nor should it be reduced to social diversity and social relations in the classrooms. The three, while analytically separable, are in constant interplay in the cultural processes that constitute school mathematics learning

Jill Adler's book (Adler, 2001) entitled "Teaching and Learning Mathematics in Multilingual Classrooms" was the first in mathematics education in which the word "multilingual" has been featured in the title. In the book, Jill uses the language of dilemmas to describe and explain the complexities involved in teaching and learning in multilingual classrooms. Her focus on the three main dilemmas (transparency, mediation and code-switching) opened up avenues for exploring tensions in multilingual classrooms as both explanatory tools and sources of praxis in the teaching and learning process. What is important to note is that there was an admission in her work that these teaching dilemmas did not necessarily have to be resolved but managed by the teacher. This position positions the teacher as a key player in dealing with dilemmas in multilingual classrooms. As she argues in the book "teachers manage their dilemmas" (p. 15), and that "Every day, teachers in South Africa (and in many other countries) manage their mathematics teaching in multilingual settings" (p. 35). Given this position, Adler (2001) further contends that the "language of dilemmas can assist teachers to identify, recognise, talk about and act on the tensions in their practice" (Adler, 1998, p. 32).

The second contribution, and related to the first, comes from the fact that Jill's work is informed by sociocultural theory of mind where consciousness is constituted in and constitutive of activity in social, cultural and historical contexts. She also used a participatory-inquiry approach to understand teaching and learning school mathematics in multilingual classrooms (see, for example, Adler, 1997). This theoretical lens allowed her to move from seeing learners' home language as a problem to seeing it as a potential resource that the teachers can draw on to facilitate mathematics learning. Thus Jill's work on multilingual mathematics education moved away from deficit theories of multilingualism to address issues on how the linguistic resources that multilingual learners bring to class (the home languages of learners in particular) can be adequately harnessed to enable their mathematics learning.

Final Remarks

While Adler's work brought to the fore the importance of describing classrooms as multilingual, research in this area of study has tended to treat bilingualism as a form of multilingualism and use the two words interchangeably, which in our view is a weakness. As Setati Phakeng (2016) argues, there are significant differences between the politics of bilingualism and the politics of multilingualism.

> While multilingualism is about inclusion and recognition of all languages, bilingualism is about competition between two languages to the exclusion of others. In all the contexts that are labelled as bilingual there is an existence of other languages that are wittingly or unwittingly silenced. (Setati Phakeng, 2016, p. 11)

Contexts that are regarded or described as bilingual are in fact multilingual but foreground two dominant traditions that are often in competition. For example, Canada is regarded as a bilingual country, with English and French as official languages, despite the fact that there are indigenous people who speak a variety of languages that are never counted. A bilingual language policy is often used as an apparatus of politics to appease two competing language traditions. These politics inevitably shape language choices, and how language(s) are used in mathematics classrooms. It is recognition of the specificity of multilingual classrooms that highlighted the dilemma of code-switching as experienced by Thandi (the first author), one of the six teachers in Adler's work (Adler, 2001). Thandi's classroom was multilingual and her learners were not fluent in English, the LoLT. Thandi explained this dilemma in one of her writings as follows:

> This is a dilemma because as a mathematics teacher I would like to have my students understand the mathematical concepts and at the same time to have them master English as a language, especially that they learn mathematics in English. Grasping the concepts might mean allowing the students to use the language they understand better; in which case they will be free to communicate in their groups although their use of English will not improve. On the other hand if they are forced to have their discussions in English they may either not do as required or they may withdraw and not communicate enough in their groups. (Setati, 1993, p. 2)

Switching from one language to another is no trivial matter in a multilingual classroom because learners have different home languages and the teacher has to ensure that in addition to learning mathematics, learners should also improve their fluency in the LoLT.

In uncovering the dilemmas teachers in multilingual mathematics classrooms have to deal with, Jill Adler also revealed an additional burden that the apartheid education system imposed on black African teachers and learners—one that the country is still dealing with 22 years after democracy. It is Jill Adler's work that made code-switching the new normal in multilingual classrooms, particularly in South Africa, ensuring that the learner's home languages are not regarded as a problem but as a resource for teaching and learning. This work remains a key reference for anyone doing research on mathematics and language diversity and an inspiration for all who do research in contexts of poverty and inequality.

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Chapter 2
Bakhtin and Some Dilemmas of Mathematics–Language

Richard Barwell and David Pimm

One of Adler's early academic articles, published in 1988 in *Educational Studies in Mathematics*, concerned mathematical learning by adults by means of curricular newspaper inserts. This work was rooted in the previous decade of massively missed and disrupted schooling in Soweto and elsewhere, due to school boycotts and protests against the apartheid government in South Africa. Her work was an evaluative attempt to explore the issue of successful engagement with the mathematical under-education of a significant part of the South African population: the use of a mass, distance-education medium to convey a sense of 'democratisation of knowledge' (p. 61). Even then, Adler was attuned to linguistic issues with respect to the language of learning and teaching (LoLT) in relation to the intended audience for such non-standard mathematics materials, not least when she refers to English as being 'a second language' (p. 66) for the audience.

Adler's abiding interest lies in the *political* implications of language in practice in mathematics classrooms, not solely because of the cultural importance ascribed to success in mathematics but also, we feel, because of there being some specific interactions of significance to be found within mathematics–language, an equally weighted hybrid term coined to signal their unseparateness. In the closing chapter of her 2001 book (based on her doctoral dissertation that studied multilingual mathematics classrooms in South Africa), she offers a number of questions, questions that 15 years later are still pertinent and viable:

> Herein lies a profound challenge for mathematics education research and practice. If the costs of obtaining meaningful mathematical communication are so high, can they possibly be made widely available? Or does meaningful mathematics conversation as a route to

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> mathematical learning become, however unintentionally, the preserve of the privileged few? Expressed in more political terms: in whose interests is the dominant construction of *mathematically rich and meaningful communication*? (p. 142; italics in original)

One of our aims in this chapter is to explore these questions. In order to do so, we first need to review some key ideas in Adler's study. In particular, we focus on two: dilemmas and resources. Our review of these ideas is informed by and elaborated through a Bakhtinian theoretical perspective.

Dilemmas and Resources

Teaching Mathematics in Multilingual Classrooms reports Adler's study of the challenges faced by mathematics teachers in post-apartheid South Africa's diverse multilingual classrooms. (In what follows, any page reference in isolation refers to this book.) In this study, she worked with six secondary school mathematics teachers in Greater Johannesburg schools, who were engaged in what Adler characterises as three different contexts. Two taught in township schools, in which students and teachers spoke a variety of different African languages, of which Setswana was the most common of students' main languages. Two teachers were from what had previously been white suburban schools, in which English remained the dominant language and specifically the language used for teaching mathematics. At the time of the study, the students in these 'well-resourced' schools came from a variety of language backgrounds, but are described by Adler in this context as learning English as an additional language (p. 38). Finally, two of the teachers taught in urban private schools. Again, these schools were relatively 'well resourced'. These two participating teachers were white English-speakers, while the students were racially and linguistically diverse.

While Adler selected these three contexts in order to look at different versions of multilingual mathematics teaching, there are important socio-economic differences that intersect with the language profile of each school:

> What of course is obscured in the discussion of these three multilingual contexts is that support for English as LoLT—the English language infrastructure—does not operate in isolation from other social dimensions of schooling. While the historically white schools might provide the strongest support for English as LoLT, as black students entered these schools, they found themselves in a culturally alienating environment. Cultural alienation must thus intersect with possibilities for learners to benefit from their immersion in an English language environment. (p. 145)

Language, then, cannot be separated from other social, cultural and political dimensions of education.

Adler set out to work with these six teachers in order to understand the challenges they faced in teaching mathematics, each in their own version of a multilingual classroom. Through a process of video-recording their teaching and then reviewing the videos during interviews with the teachers, she developed two important ideas that have influenced much subsequent research on the learning

and teaching of mathematics in multilingual classrooms around the world. The first and more explicitly developed idea is the notion of *teaching dilemma*, which Adler used to characterise some of the challenges expressed by teachers during the research. The second idea, the notion of language as a *resource*, is present but not foregrounded throughout the book. Adler (2000) developed this idea, however, within a broader discussion of resources in an article entitled *Conceptualising Resources as a Theme for Teacher Education*.

The notion of dilemma became the focus of Adler's analysis of her conversations with and observations of the six teachers, as well as a way to explain language-related phenomena that arose in the teachers' mathematics teaching. For Adler, these dilemmas arise as tensions 'inherent' in teaching; for us, as we argue below, these tensions are inherent in language.

The dilemma of code-switching arises when students and teachers share a main language, such as Setswana, in a context in which English is the language of learning and teaching. In South Africa, at the time of the study (as now), English is seen as important for participation in higher education or more desirable employment. For many students, schooling provides a significant opportunity for learning English—or, at least, for learning forms of English that are more valued in South African society. On the other hand, in mathematics classes where many of the students might be considered to be learning English, teaching in English makes the task of learning mathematics more difficult.

Adler, then, identified a dilemma that arose for some of the mathematics teachers in her study: use English, because English is seen as valuable, but risk students not understanding mathematics; or 'switch' to a shared home language, such as Setswana, so that students can understand the mathematics better, but have fewer opportunities to learn English. Adler notes that for some teachers, this dilemma is continual (p. 2). In subsequent work, Setati (e.g. 2008) has argued that this dilemma is political in nature; that is, the dilemma arises from a broader sociopolitical context, one in which some languages (e.g. English) are considered more valuable than others, and in which acquiring greater fluency in these languages at whatever cost is seen as more important than learning mathematics.

The dilemma of mediation arises in many classrooms but, argues Adler, is particularly acute in multilingual classrooms. In classrooms influenced by 'reform' approaches to mathematics teaching, the inclusion of students' voices, mathematical meanings and interpretations is seen as important, based on the idea that students should learn to think through mathematics for themselves. She also stresses the importance of this kind of approach in the development of democratic values in the then newly free South Africa. Of course, this process is mediated by the teacher.

The dilemma here for teachers in Adler's study was that it was difficult for them to judge when to intervene and to what extent. In the multilingual classrooms in the study, the teachers were aware that students would sometimes 'lose track' of their thinking. By intervening, the teachers could support the students to focus on more conventional mathematical meanings; at the same time, such intervention may divert students from developing their own capacity for mathematical thinking.

Finally, *the dilemma of transparency* concerns the role of explicit attention to mathematical language in English. This dilemma also arises in relation to moments when teachers might or might not intervene in students' mathematical expression. In this case, however, it concerns the mathematically appropriate nature of students' expressions of their ideas. Students may produce valuable mathematical ideas but not express them in conventionally acceptable ways. They 'do it right but say it wrong' (p. 4), as one teacher in the study said. This teacher, however, questioned her practice of including lengthily explicit attention to how things *should* be said in mathematics, to the point where, she felt, the explicit mathematical focus became lost (p. 115). This dilemma therefore concerns when to direct attention to mathematical language itself, and to what extent it should be highlighted. Drawing on Lave and Wenger's (1991) work, Adler adopted the term 'transparency' to capture the idea that language should at times be 'invisible', i.e. present but not attended to, while retaining the possibility of being made visible, not least through explicit and public teacher attention.

These three dilemmas can also be seen in several other studies looking at learning and teaching mathematics in multilingual or bilingual contexts (e.g. Khisty, 1995; Moschkovich, 2009; Setati, 2005). Adler sums up their nature as 'at once personal and contextual, a function of the teachers themselves, their mathematical and pedagogical goals, and the diverse multilingual contexts in which they work' (p. 5). Intriguingly, Adler, in passing, characterises these dilemmas using an alternative formulation: as dilemmas of access, voice and meaning (p. 5).

The word 'resource' is quite widely used in Adler's book. Sometimes in the context of 'teaching and learning resources' (e.g. p. 9), sometimes the dilemmas themselves are offered as teacher education resources (e.g. p. 15). Sometimes it is the students' primary languages that are the explicit resource (e.g. p. 34) and sometimes a practice (e.g. code-switching, p. 84). It occurs in the title of Chap. 6 and often in the construction 'X is a resource for Y' or 'language as a resource rather than a problem' (p. 84). But the largest number of uses comes with a discussion of Lave and Wenger's notion of 'transparent resource' in Chap. 8.

A now almost invisible metaphor at work is that of language as resource ('linguistic resources' seems an almost unexceptional expression). What does it mean to declare language a resource and what are some consequences? And what, as with all metaphors, are some of the costs of so doing? Is language to be seen as a natural resource, like say a mineral? Is it an artificial one? Is it material? Is it simply lying around, present-at-hand to use Heidegger's term. Or is it perhaps ready-to-hand, to be 'picked up' and used, without awareness? What might a parallel be to Heidegger's example of a hammer breaking?

And why might this metaphor matter? One reason is to do with its growing ubiquity: for instance, its widespread use by sociolinguists of multilingualism (e.g. the various chapters in Blackledge and Creese, 2014). In mathematics education, there is some work which makes use of Ruiz's separating out of language-as-resource from language-as-problem and language-as-right, in his 1984 paper commenting on American language planning. Instances include Planas and Civil (2013) and Planas and Setati-Phakeng (2014); the latter write of language as a

pedagogical resource in mathematics classrooms (in interaction with governmental language policy decisions) and assert that language-as-resource will help in 'reducing some of the unequal conditions of learning mathematics in multilingual classrooms' (p. 884). Ruiz offers a number of reasons for preferring language-as-[economic] resource: enhancing minority language status, easing tensions between minority languages and the majority language, and 'offering a more consistent way of viewing the role of non-English languages in the US' (1984, p. 25).

A partial parallel can be seen with the contemporary educational use of the term 'collaboration' in relation to classrooms, whose strong connotations of Vichy France during the Second World War as the antithesis of resistance seem completely absent, not least in respect of imposed power and collective punishment. Houssart (2001) writes engagingly about 'The whisperers', a small group of boys who at times supported one another and communicated with one another to *resist* the public discourse of the teacher. Too often, classroom accounts seem to ignore issues of power and political struggle at work in mathematics classrooms, in particular ignoring that in some sense one can only *collaborate* with an enemy.

Without overplaying the etymological card, the word 'resource' has some interesting dictionary connotations: according to the *Oxford English Dictionary*, a *resource* is something intended to be of help or aid, an assistance that may be resorted to in a difficulty or emergency, something that supplies a deficiency or need, that serves as an expedient to help overcome or to mitigate adverse circumstances. In other words, resources are assets to be drawn on when necessary: but it is people in difficulty who need resources. So, once again, language seen as a resource indirectly marks the circumstances and the resource-users as being in difficulty or as suffering from a deficiency: moreover, the resource they need is, apparently, external to them.

There are two related effects of speaking (and thinking) of language as a resource: the first has to do with language being 'thingified', objectified, while the second arises from this consequent externalisation, namely a separation from self (a related observation concerns how the term 'human resources' serves to commodify people). Yet language does not exist without us.

Further, we live in an age where an awareness of the finitude of resources is growing appreciably, as well as the fact that resources get used up. How does this view fit with language? What of unequal distribution and a consequent need for redistribution, its connection to Bourdieu's notion of 'cultural capital' (and 'capital' is an appreciably more nuanced and theoretically examined notion than 'resource')? And what of the folding in of 'resources' to a market metaphor: their accumulation, exploitation, management, development, a source of competition? Lastly, 'language', *tout court*, seems so undifferentiated, so unspecified. Which aspect is of use and for what purpose or goal? And why is it of use?

To understand more about the source of both dilemmas and resources, we draw on Bakhtin's theory of language tensions.

Resourcing Dilemmas

Bakhtin (1981) developed a rich and complex theory of language based on his analysis of language in novels. This theory conceptualises language as fundamentally dialogic as well as situated in time and space. By 'dialogic', Bakhtin is arguing that meaning in language arises from the relations between aspects of language, rather than from these separate aspects themselves, such as words, utterances, genres, styles, registers and languages. For example, in a mathematics classroom discussion about shapes, the meaning of 'convex' is relative to the meaning of 'non-convex' (Barwell, 2015a). The two words are in dialogue with one another.

In seeing language as situated in time and space, Bakhtin is highlighting how any utterance is in dialogic relation to preceding utterances, both from the immediate interactional context, as well as reaching back both through the history and the geography of language. Ways of using language always have a history and meaning derived from this history of use. Moreover, this history is not simply of abstract words; it is the history of people using words—of their voices. In Bakhtin's account, voice refers to a crucial dimension of meaning. He suggests that when we use words, they are 'half someone else's' (p. 293): they carry the echoes of previous voices using those same words. When students first use a word like *hypotenuse*, for example, their use carries the voice of their teacher as much as their own. Indeed, Bakhtin argues that learning to expropriate language and to shape it to our own intentions is difficult. This difficulty is part of the challenge facing students in multilingual mathematics classrooms, who must draw on the utterances of others, often in other languages, to make mathematical meaning for themselves.

Bakhtin understood that language in use always reflects two poles simultaneously. One pole is what Bakhtin's translators have termed *heteroglossia*, which refers to the diversity inherent in language at all levels (e.g. phonemes, words, accents, genres, languages). Without variety in language use, it would not be possible to make meaning. Busch (2014), based on a careful reading of Bakhtin's original Russian texts, argues for different forms of heteroglossia, proposing three aspects: multidiscursivity, multivoicedness and language diversity. Multidiscursivity refers to the various forms of language that can recognisably be associated with some activity, group of people, time, etc. This includes the languages of mathematics, of schools, of particular classrooms, and so on. Multivoicedness refers to the presence of the multiple individual voices that can be heard in any utterance, such as when students echo their teacher's use of language, or through mimicry, parody or repetition. Finally, language diversity refers to the presence of multiple languages, such as the mix of languages in the different classrooms in Adler's research.

Bakhtin terms the second, opposing pole as *unitary language*, which refers to the idea of language or language forms as abstract, singular systems. This view is particularly apparent in dictionaries, grammars, or guides to writing or speaking in specific genres. It is also apparent in the popular pastime of publicly correcting

common ‘mistakes’ in punctuation, grammar, etc. The point here is that these ideas are abstract *ideals* about how language *should* be. Language in use is actually much more diverse as it shifts and changes in different contexts and circumstances and for different purposes.

Having defined these two poles, Bakhtin argues that there are ‘forces’ at work within language both towards heteroglossia and towards unitary language. He uses the metaphor of centrifugal force to refer to the tendency towards change, diversity and difference, and centripetal force to refer to the ideological drive for uniformity, standardisation and correctness. In mathematics, for example, there is a constant process of diversification (there are many ways to talk and write about mathematics in different contexts) as well as the ever-present idea of Mathematical English (or other language). The term ‘unitary language’, however, suggests a type of language rather than a set of forces at work: to speak of ‘homoglossa’ (literally, ‘same tongue’, a word formed in parallel with heteroglossia—‘different tongues’) might at least bring out the oppositional sense conveyed by the physical metaphor of ‘centripetal’ and ‘centrifugal’ forces. (See Pimm, 2014, for more exploration of some consequences of this physics metaphor.) Similarly, in Lave and Wenger’s (1991) discussion of transparency, they insist that visible/invisible is ‘not a simple dichotomous distinction, since these two crucial characteristics are in a complex interplay’ (p. 102). To this extent, they echo the same quality of Bakhtin’s centripetal and centrifugal forces, namely that both are always present in every utterance.

Finally, then, Bakhtin argues that centripetal and centrifugal forces are in tension and shape every utterance. That is, every time we speak, what we say is shaped by centripetal forces (we conform to a great extent to the norms of language, genre, register, etc.) *and* by centrifugal forces (we say things in our own singular way, specific to the moment, to our interlocutor, to the situation; we innovate, invent new ways to say things, etc.). These forces are not entirely neutral, however. In particular, it matters who gets to define the norms and standard forms of language, whether it be the preferred language in which to study a subject like mathematics or the correct way to begin a letter or to write up a proof. (Recall that in the nineteenth century, part of the correctness of replicating a Euclidean proof in a school setting included using the precise same lettering as Euclid—though Netz’s (1998) fascinating study of lettering conventions in ancient Greek mathematical proofs could be called upon for a partial, if retrospective, justification.)

These ideas offer a way to account for the source of the dilemmas observed by Adler in her study. To begin, we noticed an intriguing (though suspiciously neat) connection between Busch’s tripartite characterisation of heteroglossia as concerning the diversity of languages, voices and discourses, and Adler’s three dilemmas of code-switching, mediation and transparency, respectively.

The dilemma of code-switching appears to arise from the tension between centripetal and centrifugal forces relating to the diversity of languages and the need for a shared way of communicating. One of the teachers quoted by Adler, for example, says:

Thandi: In Tswana it becomes a problem because um like if he explains in Tswana, then when it comes to the [terms? unclear] our language is unique; and when you come to 'at most' and 'at least' then what are you going to say? For in our language, 'greater and equals to' and 'greater' there is a little difference. I have to use a long sentence for greater than and equals to.

JA: And for 'at most'? and 'at least'?

Thandi: That is going to be problem to say it in Tswana, 'at most' and 'at least'. That is why I talked of 'not more than' and 'not less than'. I feel if they resort to Tswana, then, when they come to those terms what are they going to do?

JA: Even in English speakers battle with those terms. Would it help to explain the idea in Tswana and then shift to English?

Thandi: I think if I was to explain in Tswana I would run out of words And for my mixed class it would also be a problem because not everyone speaks Tswana. So must I do it again in Xhosa and then Zulu?

(p. 82; transcription slightly simplified)

This example illustrates clearly the dilemma of code-switching: is it better to insist on English but risk the subtleties of a mathematical idea not being understood; or is it better to switch to another language (and if so, which one?) and risk 'running out of words'? The issue of code-switching is clearly related to one of access (Adler, 2001, p. 5; see also Setati, 2008). The teacher wants students to understand the mathematical ideas, but these ideas are not divorced from language. Talking about them in English and talking about them in Setswana is not the same thing at all. Versions of this dilemma can be seen in research in several parts of the world (e.g. Clarkson, 2007; Farrugia, 2009; Halai, 2009). From a Bakhtinian perspective, a tension between language diversity and language uniformity is inevitable whenever multiple languages are present (which, increasingly, is anywhere). The dilemma that arises for the teacher is *pedagogic* in nature, but it arises in response to a tension that is inherent in the linguistic context. In South Africa, a core centripetal force is apparent in the singular national preference for English as the language of learning and teaching (and of much else). Of course, this preference advantages some students and disadvantages others.

The dilemma of mediation can be related to the centripetal–centrifugal tension arising from an inevitable multivoicedness. Teachers in Adler's study wanted to draw on students' ideas expressed in their own words; but they also wanted to guide their students towards particular mathematical understandings. One participating teacher, for example, said:

Sue: the thing that worries me the most is that I am not sure whether, I am not sure to what extent it helps them learn. I think that talking to each other is not unproblematic. I think a lot of the kids don't listen. Maybe they are too young. I think. You can see it with the questions they'll ask a question and say 'I don't understand' and then the one who is up will try to explain and it doesn't really help but they are being polite and they are not quite sure and they say 'OK fine'. I am not sure they understand.

(p. 105; transcription slightly simplified)

This quotation captures the dilemma of mediation: on the one hand, a desire to accommodate students' voices, such as arises through group work; on the other

hand, a fear that these voices do not always lead in a desired direction. Again, this dilemma has been observed in various contexts in research on language diversity in mathematics classrooms (e.g. Khisty, 1995; Moschkovich, 2009). The dilemma for the teacher is about how to mediate these different voices and guide students towards particular mathematical ideas. Again, Bakhtin's ideas allow us to see how a *pedagogic* dilemma has as its source a linguistic tension. In this case, the omnipresent centripetal–centrifugal tension arises in relation to multiple voices.

In a classroom, as in any social situation, there are always multiple voices. In a classroom, however, there is also a curriculum of some form; a sense that students need to learn or understand particular mathematical ideas and not others. Bakhtin's theory, however, is not simply about multiple *speakers*: individual utterances often reflect multiple voices, the words being 'half someone else's'. Thus, when Sue revoices her students' ideas using more mathematical terms, her utterances combine at least two voices, the student's and the teacher's more mathematics-register-informed one. Similarly, when her students reformulate their explanations, they draw increasingly on forms of expression offered by Sue or by other students in the class. Again, their utterances combine plural voices. There is, therefore, a tension even within individual utterances, in which multiple voices are in tension with each speaker's intentions as they struggle to make the words reflect their own intentions.

The dilemma of transparency arises in the context of a particular sense of what counts as mathematical discourse. Teachers in Adler's study wanted their students to be able to express their ideas using standard mathematical discourse. The dilemma they faced was about the extent to which they should shift students' attention from the mathematics to the language used to express that mathematics itself, since extensive attention to language could well disrupt students' mathematical thinking and their very attending to the mathematics. For example, one teacher, Helen, discussed a situation where students who apparently understood some ideas about triangles were then not able to see a flaw in a student-generated sentence on the same topic:

> I think that that sentence came out of something that the group was working with . . . if you actually take a sentence like that which is supposed to be concise, and it carries a whole lot of meaning there is difficulty . . . They can talk to you about it and they can give you a long explanation of what to do . . . so its seems to me to be also a problem of expressing a lot of maths in one clear sentence. For me that is also linked to the issue of how we transmit maths to each other. If you make a mathematical statement you are involved in getting it down to a simple, short-hand language that we can all share. (p. 130)

This quotation again captures the teacher's pedagogical dilemma, which has subsequently been documented in several other studies (e.g. Chval & Khisty, 2009; Monaghan, 2009; Setati, 2005). The quotation is also illustrative of the underlying linguistic tension highlighted by Bakhtin's theory. In this case, it reflects a centripetal–centrifugal tension in a context of multidiscursivity. The students express their mathematical ideas drawing on various discourses available to them. In mathematics, however, there is, at least as an abstract idea, a preferred form of mathematical discourse (codified in Halliday's notion of the mathematics register). For the teacher, it is an important value that students learn to use this form of

discourse. Within language, then, there is a tension between the heteroglossia of the multiple discourses circulating in the classroom and the unitary or monoglossic ideal of a single form of mathematical discourse. As with the other dilemmas, the tension is inherent and is therefore insoluble. It also involves issues of authority, however, since in the classroom, at least, it is generally the teacher who determines what is acceptable as mathematical discourse. Such determinations, however, are linked to broader issues of conformity and marginalisation, of success and failure.

On the Dilemma of 'Resources'

Our examination of Adler's three dilemmas in the light of Bakhtin's metaphor of centripetal and centrifugal forces, and of Busch's distinctions between three aspects of heteroglossia, leads us to a number of observations. The dilemmas that Adler reports—as dilemmas for teachers—can be related to tensions inherent in language. There are always multiple languages, multiple voices and multiple discourses, which must be corralled by the centripetal forces that make communication possible. Schools (and nations) need language policies of some sort—but these policies are always a simplification. Similarly, mathematics needs some kind of 'language', so that it can be communicated and, indeed, thought about. Again, though, this language is necessarily a simplification. New forms of language, new voices, new discourses all make it possible to say and think new thoughts. This tension between centripetal and centrifugal forces is present in every mathematics classroom. It is not surprising, therefore, that teachers, such as the participants in Adler's study, should sometimes feel dilemmaed.

What, though, does this perspective mean, if anything, for the metaphor of resources. One collocation used by Adler has learners' languages as resources and another has code-switching as a resource. Both of these are related to the multiple languages aspect of heteroglossia. Adler, as well as subsequent work by others, seems to give less attention to Busch's other two aspects. If learners' languages or their practice of switching between or among these languages can be considered resources, then presumably so can the discourses learners are familiar with, and the voices available to them, as well as the practices of 'discourse-switching' and 'voice-switching'. This kind of approach usefully diversifies and deepens the notion of resources *tout court*.

There can be a sense that all this multiplicity is automatically a good thing. There are, however, clear relationships among particular sets of languages, discourses and voices, and social (and economic, and political) differences. Language, as Bakhtin says, is stratified (and stratifying). Paradoxically, then, being able to speak multiple languages means that learners have more resources for making mathematical meaning, but some of these languages are valued more than others (usually one, in particular), as Setati (2008) has pointed out. The same point applies to discourses and to voices. Moreover, these differences do not arise from the material nature of language as a resource—it is not simply that Setswana is less valuable in the market

place than English. Rather, Setswana indexes certain kinds of people, activities, ideas, etc., and English indexes others (see Barwell, 2015b). From this perspective, languages, discourses and voices are not resources to be distributed, so much as practices to be acquired. Languages, discourses and voices are infinite both in scope and in flexibility. The possibilities are endless.

As we quoted at the outset, at the end of her book, Adler asks if 'meaningful mathematics conversation as a route to mathematical learning become[s], however unintentionally, the preserve of the privileged few? Expressed in more political terms: in whose interests is the dominant construction of *mathematically rich and meaningful communication*?' (p. 142). Such a situation certainly reflects much of what Adler reports. Why, however unintentionally, might this be so? The tension between centripetal and centrifugal forces is once more relevant. Centripetal force drives language towards uniformity, only held in check by centrifugal counter-forces. Uniformity is reflected in what Bakhtin called 'unitary language'. For us, unitary language refers to ideal forms of language, never actually instantiated, but reflected in common assumptions about language—ideologies of purity, nation-hood, and so on. But which forms of language are taken up as 'unitary' is a political (small p) matter. 'Rich and meaningful communication' in mathematics is, perhaps, a form of unitary language in school mathematics. There is a 'right' way to do it and a 'right' way to say it. Of course, interaction in mathematics classrooms is always much more diverse—heteroglossic—than the unitary ideal. Subtle differences between the languages, discourses and voices used by learners will mean that some approach the unitary ideal more than others—and as such, the speech/writing of some learners will index the mathematically knowledgeable learner more than others.

If tensions are inherent, then there will always be dilemmas. This observation has the potential to be perceived as reactionary, a bit like saying that the poor are always with us, and shrugging as though this state of affairs is inevitable; but actually, it does not mean nothing can be done. Drawing attention to these tensions makes it possible to find new ways to respond. As Adler has shown throughout her career, dilemmas can be productive, both for teachers and for researchers.

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Chapter 3
Dilemmas and the Teaching of Mathematics: A Conversation of Commitments, Obligations, and Ambivalence

Daniel Chazan and David Pimm

When invited to contribute a chapter to this volume, we mulled over various ways of engaging Jill Adler in indirect conversation through her writing (more specifically, her work on multilingual mathematics classroom dilemmas as presented in her book). We envisaged this conversation taking place as we two conversed more directly (ah, Skype!) about what we might say, both to each other and also to such readers as may come across this piece (including, we hope, Jill herself). In attempting to produce various bits of shared or common prose, we fell into the not-uncommon problem of an over-blended voice in places, one that was actually neither of ours (nor anyone else's), while other parts refused to be parted from their original speaker.

We decided that we could have the writing mirror our spoken conversations more directly by presenting it as a transcript (with suitable editing, naturally, to bring out our mutual hyper-articulateness), even if such a presentation sometimes makes for a choppier read. We have eschewed the customary academic apparatus of footnotes and in-text citations, though there is a list of references at the end. Our starting point was with Dan's first-hand engagement with Magdalene Lampert's writing about teaching dilemmas, upon which Jill based some of her work.

DC: I first met the notion of dilemmas as a way to understand teaching in Magdalene Lampert's work, specifically her 1985 piece *How do teachers manage to teach*? I was a practicing teacher just heading to graduate school. I had visited Magdalene's 6th grade mathematics class the year before and had watched her teach. For me, the piece was powerful. It suggested that teaching could be a

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focus of study, something that was not so clear at the Harvard Graduate School of Education at the time. Specifically, the notion that the sorts of dilemmas I had confronted in teaching were something that could be managed, not solved, was a powerful provocation. It suggested to me that one would always be ambivalent about actions taken to manage a dilemma, and that this ambivalence could not be removed. If that is the case, I thought, what is the role of technique and what can mathematics educators provide teachers with to serve as a resource for their work?

DP: I too like the notion of dilemmas needing to be managed rather than solved or even resolved—dilemmas, I feel, are endemic to the enterprise of teaching mathematics (ah, those hardy perennials). David Wheeler has a very nice short piece on "managing" oneself during investigative problem solving (e.g., managing anxiety). In it, he writes: "'I managed to' tells us just enough; I might have tackled whatever I was doing clumsily, inefficiently, long-windedly, unimaginatively, etc., but at least I 'managed to' so I must have 'managed it'." This sense of "managing to" seems apt with respect to teaching.

My touchstone reference for dilemmas and their management is Homer's story of Scylla and Charybdis: in particular, these two mythic sea monsters are quite distinct (as are the perils they personify). Crucially, there is no alternative in two distinct senses if one is to reach one's destination (goal): there is no other way round and there is no ideal path between them that somehow keeps them both at bay. You have to engage to some degree with one or the other (as Odysseus manages to in Homer), though this engagement has costs (for Odysseus, the loss of six of his crew).

DC: It seems to me challenging to represent that sense of an endemic dilemma of practice in writing about teaching. For example, the specific presentation of practice that opens Lampert's article often was not as powerful for readers as the argument about dilemmas in principle. As I read this piece with practicing teachers and others, people (not only teachers!) always wanted to *solve* the dilemma and thought it could be resolved. They thought that there were actions to take about which one would not feel ambivalent. They saw Lampert's dilemma as a problem to be resolved, not a dilemma to be, at best, managed. Interestingly, many of them had techniques that seemed like they might be useful in managing the dilemma.

Some teachers had a different reaction. They were simply not captured by the story; the dilemma did not have quite the same force for them that it did for the author. They saw nothing to be all that ambivalent about; for them, Scylla and Charybdis did not seem so close together; the passage through did not seem fraught or perilous.

DP: According to the OED, the word "dilemma," although it sounds Greek in origin, only dates from about 1520 in English rhetoric and refers to a "double proposition or premise," either one of which (known as a "horn") is unfavorable by itself. In that sense, the connection of dilemma to Scylla and Charybdis seems quite apt, as certainly neither of them is to be embraced!

I have just finished work on a discussion piece for a ZDM special issue on language and communication. There was one article in it, by Richard Barwell, that brought up for me this question of the endemic quality of dilemmas, in this instance in relation to that of "tensions," another word that is in common use about teaching. I think it important to keep these two ideas distinct: perhaps tensions refers to individuals whereas dilemmas are somehow more general or widespread?

DC: Dilemmas as more general than tensions is not my interpretation of Lampert's use of the term "dilemma," but I agree that it seems important to distinguish between something that is more endemic and unavoidable and something that perhaps feels unavoidable only to some.

To continue the story, in 1990, I was hired at Michigan State University into a position Lampert had created with support from a local foundation. The notion was to explore new possibilities for enacting teaching that were designed to be responsive to new theories of learning. The position was structured around the possibility of teaching a grade 9 Algebra I class every morning in a local school as a part of my university teaching load. The position allowed me to talk with Magdalene, with Deborah Ball, with David Wong, and later with Ruth Heaton, Sarah Theule-Lubienski, Tim Lensmire, Suzanne Wilson, and others about doing research on teaching from inside the role of teacher, particularly to understand demands of the new kinds of teaching being envisioned by reformers. Our conversations sent me back to the 1985 piece to try to understand how Magdalene conceptualized dilemmas.

These conversations focused on dilemmas as resulting from "competing commitments," or conflicting aims' of teachers. Though this is in tension with your proposal for tensions and dilemmas, the 1985 piece seems to suggest, though perhaps it is my mis-reading, that the commitments that compete to lead to a dilemma are individually held; that it is as if each teacher has a hierarchy of commitments that can come into conflict. A dilemma is strongly felt if two commitments that are both high in the hierarchy come into conflict around some particular moment of practice. Any action will engage each of the commitments in some way that causes the two to come into conflict. It feels that there are no good choices that can respect both commitments at once, that one has to choose to honor one commitment at the expense of another and will necessarily be ambivalent about the choice made.

DP: I'm interested in the inherent individuality that such a framing suggests: commitments are somehow related to individuals. One of the things that I appreciated about Jill's instances of dilemmas that she identified in her work was that although some teachers may have been more attuned to or aware of them than others, the dilemmas somehow arose from the nature of the language-in-mathematics work that was necessarily taking place in the multilingual mathematics classrooms she explored. Her dilemmas were necessary in the Scylla/Charibdis sense of being inescapable and needed surviving (through technique and awareness, no doubt) if the goal of the successful

teaching of mathematics were to be achieved. But it might be interesting to try to identify the nature of the "competing commitments" in each case.

However, this task of identifying commitments makes it hard for me to connect to Scylla and Charybdis. It is difficult to see how either Scylla of Charybdis in themselves could be seen as a "commitment"! Precisely the opposite, I would have thought, in that they are things to avoid rather than things to cleave to. They lie decidedly outside the individual ship captain (and the myth does not encourage them to be seen as projections). It seems, from what you have said, that teaching dilemmas somehow arise from incompatible good things (presuming commitments to be such). It is almost as if Scylla and Charybdis are *anti*-goal states in Richard Skemp's terminology from 1979, where emotions such as ambivalence signal a lack of progress toward a goal. And it may be that we are realizing teaching produces *anti*-dilemmas!

DC: That certainly flips the Scylla and Charybdis imagery on its head; each of these monsters perhaps represents *not honoring* one of one's competing commitments.

I'll return, though, to the endemic versus personal dimension of dilemmas that we were exploring. As I came to write about the teaching of Algebra 1 the notion of some personal hierarchy of commitments also felt too individualistic for me. It felt connected to the difference between working to improve one's own practice and the goal I had to do research on teaching by teaching. Searching for a way to capture this, I wrote about dynamics of the classroom that would lead to dilemmas regardless of one's particular commitments as an individual or of one's individual level of skill or technique.

Focusing now on work that I'm doing with Pat Herbst on understanding what it is that makes both "reform-minded" teaching and "traditional" teaching kinds of teaching, I believe there are dilemmas between commitments that are inherent in assuming the position of teacher. As Pat has articulated it, there are *obligations* (DP: a new but important extra word, here) that are incumbent on the person who takes on the position of teacher in institutionalized schooling. These obligations are the result of different goals for state-provided, compulsory education held by different stakeholders in a democratic society. Thus, some of the commitments one has as a teacher are not really of one's own choosing, but come about as a function of taking on the position of teacher. They might be thought of as corollaries arising from the didactical contract. Within that framework, there are still options for other commitments that then distinguish teaching of one kind from another.

DP: The work that you and Pat Herbst have done around the practical rationality of teaching mathematics is intriguing and, in its desire to look at obligations and instructional norms (rather than individual teacher beliefs), provides an example of an attempt to "raise" the theoretical discourse about teaching above the level of the individual. And I am interested in the recurring theme of ambivalence in relation to dilemmas. I wonder whether it is possible to conceive of ambivalent *obligations*, irrespective of whether they are viewed as unavoidable bad things or incompatible good things—dilemmas that are somehow at

the heart of the teaching enterprise. But perhaps ambivalence is an emotion concerning the uncertain actions you are going to take, rather than pairs of commitments themselves to which you are trying simultaneously to cleave. So the relation between teacher commitments and teaching actions is also significant—and notoriously hard to examine. And that might be true *vis-à-vis* obligations as well. But the idea of institutional ambivalence is interesting, as is that of professional ambivalence, i.e., the ambivalence of *the* teacher rather than of *a* teacher.

For me, the power of Jill's dilemmas and the endemic quality of them is tied up in the fact of language's central—and, for me, inescapable—implicatedness in the act of teaching mathematics. It also resonates with Bakhtin's thoughts about the centripetal/centrifugal forces of language always both being present (and in tension) in *every* utterance (even if I am a little unhappy about that metaphoric naming): the drive for standardization and the inherent diversity of language practices, respectively. In the Barwell article I mentioned before, he engages in detail with these forces in the context of a second language mathematics classroom in Canada.

Of Jill's three dilemmas, the language transparency one is the strongest of them all in this respect, for it highlights a dual fact about language itself—that it can be both visible and invisible. In passing it has always bothered me a bit that there is no more neutral term than "transparency" to refer to the degree of visibility/invisibility that Lave and Wenger write about, its non-neutrality being evident when the equally appropriate term (that is to say, not very) "opacity" is substituted. Jill nicely teases out how one or other pole can be more evident at a particular time, but neither in themselves is uniformly beneficial. There is the Scylla of always talking "invisibly," which means that the teacher can never meta-comment, a move for me that is one of the core actions of teaching. Then there is the Charybdis of "going on too long" (which Adler's teacher informant worries about), of coming out of the language to talk about some aspect of that language (I vaguely recall Wittgenstein observing somewhere that *English* is the meta-language of English)—a fundamentally teacherly move. These have nothing to do with individual commitments competing—this is far more the clash of Titans that, like Bakhtin's forces, are always with us.

I see a comparable clash with us in other arenas of mathematical activity. For example, in his work on structuring mathematical proofs, Uri Leron made a not-dissimilar point to Jill's about the nature of language in mathematical activity, but in the context of proof-by-contradiction. He articulated a criterion of minimizing the amount of "time" spent in the contrafactual realm by, in the standard proof of the infinity of primes, for example, setting up the construction "multiply together all the primes you have and add one" and then establishing some results about that new number non-contrafactually, before going into the meta-realm in search of a contradiction. Interestingly, this could mean initially withholding from the reader the fact that the proof-form of contradiction is going to deployed, at least until the last possible moment.

DC: Your comment stimulates a large number of responses all at once. First, it is interesting to me that you find the language transparency dilemma the most powerful one that Jill articulates, as I find the dilemma of mediation the strongest. For me, it has the endemic quality we've been discussing. It feels like it can be captured as a clash between validating diverse learner meanings and at the same time intervening so as to work with learners to develop their mathematical communicative competence. This is a very important dynamic that captures tensions between our current theories of learning and the goals of instruction.

DP: Interestingly, I realize that the dilemma of mediation has strong echoes of Bakhtin's language forces, in its acknowledging the different pulls (towards, away?) "official" and "individual" meaning.

DC: I also find the dilemma of "code-switching" a powerful and endemic concern. For me, the commitments that create the dilemma are embedded in conflicting obligations held by the teacher: an obligation to develop student understanding versus an obligation to support student access to English and, through it, to a culture of power. It reminds me strongly of dilemmas experienced by Lisa Delpit as a Black teacher of English in the USA, and techniques proposed by Bob Moses' Algebra Project (DP: or more directly in relation to mathematics in schools in Mamokgheti Setati-Phakeng's work in South Africa).

By contrast, your description of the dilemma of language transparency seems like a clear description of some facts about language, but I don't yet have that sense of inescapable conflict that you referred to earlier. For me, with the dilemma of transparency, of the three dilemmas Jill documents it is *least* apparent just what the competing commitments are. In the way the dilemma is constructed in Jill's text, it is between implicit and explicit language practices. "There is always the problem in explicit language teaching of going on too long" and yet explicit language practice "seems to be a primary condition for access to mathematics." I'm not clear why having explicit language practice go on too long is a problem. What is the commitment that is being violated? Does this reduce down to the dilemma of mediation or is there something else at play? Is there something here that is particular to multilingual classrooms and that has not been articulated yet?

DP: This phenomenon-in-tension about language, I feel, is particularly problematic for teachers, and is present in monolingual classrooms also, but perhaps is heightened in multilingual ones (in that the language of instruction for many, perhaps even all present, is inherently more "opaque"). Explicit language practice, for me, *is* meta-commenting, namely a situation where some previous utterance itself (in some aspect) becomes the object of attention and conversation rather than the meaning it is intended to convey. And, echoing Jill's observation about it being "a primary condition for access to mathematics," for me meta-commenting is a core element of teaching.

The teacher trick of it, if I may use that phrase, is to meta-comment in such a way that the student barely notices it happening, if at all. The time concern, captured so well in the teacher worry about "going on too long," is that the

longer the conversation spends in this meta-sphere (just as with Leron and the contrafactual zone), the more likely the student will notice and feel interrupted and not listened to (and wonder and get confused and ...): "Why are you worrying about *how* I am saying what I am saying and not engaging with *what* I am saying?" And the second worry is that being in the meta-sphere again occludes the mathematical content that both teacher and student (on a good day, at least) want to be the focus of the language.

DC: But, for me, if this really is to be a dilemma, these concerns of the teacher have to be more than worries; these worries feel more like ambivalence about the action to be taken. They don't seem sufficiently Scylla- and Charybdis-like. I'm looking for ways in which these worries are related to central aspects of what it means to be a teacher. So, I would be happier if I could understand how these worries are related to the didactical contract or obligations.

DP: I think there are, in fact, broad obligations in play for this dilemma too. One is conversational on the teacher's behalf to meaningful discussion of mathematics, in other words talking directly and explicitly *about* mathematics. The other has to do with the nature of teaching and the fact that that I see meta-commenting as a core resource for effective and "economic" teaching (if I can use that word—economic of both the teacher's energy and the students' attention). So a commitment to this tool of teaching means running risks, in that it messes fundamentally with the previous commitment, by means of substituting the *form* of the utterances being used to discuss the mathematics for the previous content conversational focus.

DC: I'm still not following because meta-commenting feels more like an action than a commitment, a means to an end. I'm looking for the commitments that meta-commenting as an action is meant to manage. But, first, I feel like I need to know more about what you mean by meta-commenting. It seems to me that you intend something that is more than making a comment that steps back from the content of a conversation. I wonder whether you would think that the "E" in the I–R–E sequence signals meta-commenting?

DP: My immediate reaction in relation to I–R–E is "Generally, no," for although the earlier response is being evaluated (or presumed to be being evaluated, even if it, in fact, is not), it is usually not being commented upon in terms of the language used, which for me is the hallmark of a pedagogic meta-comment.

I think that meta-commenting comes in various forms, depending on what aspect of the language of a previous turn(s) in the conversation is being focused on. For example, "Can you hear me at the back?" is a very familiar and teacherly comment *about* the conversation that reflects a commitment to having much of the classroom conversation public and audible to everyone (a teacher obviously will not make such a comment when attempting an individual interaction with a student). But this is not specifically a mathematical meta-comment. I first came across the idea of meta-commenting as a *teacher* discourse practice in a paper by Michael Stubbs in 1975, and his context was an "English for foreigners" class, a setting where aspects of the

English language was already a central part of the focus and content under consideration.

Another example (this time written) arises from a teacher's interaction with student written work where the teacher's response involves explicit correction and marking. In Bel Kaufman's novel *Up the Down Staircase*, a high-school English teacher is sent a love letter by one of his students. Unsure how to respond to the letter's content, he meta-comments in writing, by correcting the spelling and grammar, and then returns it to her with a grade. This is not a response that someone would be expected to do outside a school context (and, nowadays, perhaps not inside one either).

In a couple of chapters I wrote in 1994, I tried to document some oral classroom examples of meta-commenting (and offer imputed purposes) that seemed to me more about mathematics being taught and involved comments that would not necessarily be carried out by a teacher of any subject. They exemplified for me acute instances of mathematics teaching, of a mathematics teacher taking advantage of an expectation that it is accepted for them to meta-comment in the service of teaching. So meta-commenting of this sort (and order) may not occur very frequently in mathematics classrooms, but when I notice one it really stands out for me. To give a shorthand instance here, Dave Hewitt in a grade eight class was turned to directly by a student working publicly at the board and asked "Is this right?" He responds by turning to the class and meta-commenting, by saying "Zena's asked a question." No one even blinks; the class responds to Zena and the conversation continues.

DC: That's a lovely example of a teacher acting in ways that would seem peculiar outside of the classroom, of how what teachers do in the classroom violates expectations of conversation present in other contexts. Thus, the notion of meta-commenting as an important kind of talk that teachers undertake is appealing to me as an element of a broader description of the work of teaching.

It seems that so much teaching consists of directing, and mis-directing, students' attention. For that reason, meta-commenting, and doing so efficiently, reminds me of Lampert's description of classrooms in which the "problem is not the question" and "the solution is not the answer." And it also reminds me of the research that I've been doing to understand the instructional situation of solving equations. In a situation like that, when students are doing an exercise, we act as if the solution to the problem matters, but we really know that it does not. The knowledge at stake is really a method for solving a class of problems. Students' "work" that they show the teacher is more about showing that they know how to solve an equation than it is about solving that particular equation. In the grand scheme, we don't really care about the problem and its answer. Thus, in many ways, there is important teaching to be done by appropriately shifting students' attention toward aspects of their activity in the context of a particular exercise. In that sense, emphasizing the role of the teacher in directing students' attention serves as an antidote to teaching as telling or conveying information. It makes room for an understanding of the importance of having students be active so that the

teacher can comment on their activity, including on the language in which the activity is couched.

DP: I think the person who writes best about attention-direction in the service of teaching mathematics is Dave Hewitt. In particular, in 1995 while trying to write about what I saw him doing in a sequence of algebra lessons, I talked about his offering "cover stories" (in one instance, this involved the patter of "I'm thinking of a number"). These are plausible narratives-in-the-moment for students as to what is going on, but that purposely (mis-)direct their attention, as a magician might, *away* from what is actually important in a task or situation. I think there is some dangerous nonsense being promulgated about the teacher always having to let the student in on what is being done and why. You likely know what I mean: one instance would be having lesson objectives written on the board from the outset of the lesson. I'm going to be provocative here. It is in a mathematics teacher's interest at times to support a "false consciousness" in the students in terms of directing their attention. I am pushing against the presumption that "lowering the ladder," in Wittgenstein's phrase is always a good thing. We need good cover stories. A teacher helps to establish meaning by controlling and distributing student attention.

DC: But, we have strayed from paying particular attention to comments whose focus is on the nature of the language being used in conversation. So, to return to meta-commenting, I wonder about how you think about students. Can students meta-comment? For example, is Zena meta-commenting when asking "Is this right?" assuming that the "this" is a reference to an earlier utterance?

DP: One of the things I like about meta-commenting is the sense of asymmetry between teacher and learner: who has the right to meta-comment? To what extent is meta-commenting in classrooms seen by students as the primary prerogative of teacher? If so, for example, are there instances of students meta-commenting in small-group work, where at times some student is seen as "taking on the role of the teacher." If there are, this might both ground this observation in the specifics of the talk, as well as provide evidence that students are at some level aware of this as a teacherly action, even if when directly asked what it means to be a teacher they would not list it among the characteristics. So, in terms of Zena, I do not think she is meta-commenting, in that although some idea/discussion is being pointed at with the "this," it is not a specific aspect *of* the language used in that discussion that is being requested be remarked upon. Perhaps if Zena had asked "Did I say that right?" then I might feel differently, as there the language of the utterance is the focus of the request. So, perhaps, with a comment of that sort, a student might meta-question, if not meta-comment.

In terms of talk in mathematics classrooms, one of the best instances of teacher meta-commenting I know comes from Jill's chapter on the dilemma of transparency. The teacher notices an apparent confusion between the words "size" and "side" in relation to angles in a trigonometric discussion. (And in passing, I note that "sine" is also in the mix.) All three of these words are

phonologically very close in English. They are also both pertinent and interact in the discussion in similar ways (e.g., in the expressions "the size of the angle" and "the side of the angle"—and, indeed, "the sine of the angle") and things that may be true mathematically about one of them is not necessarily true about another.

Like the Strait of Messina for Odysseus, the dilemma hoves into view: what is the teacher to do with her noticing? Keeping the discussion at the level of "content" she worries could well disappear into a whirlpool of confusion, while drawing attention to this fact about the words' phonological proximity in English lexis (by meta-commenting) disengages contact with the world of trigonometry and engages instead with the what-you-just-said and aspects of English pronunciation and spelling, and so may be taken as an abrupt shift of topic rather than an attempt at invisible repair.

DC: I can see how meta-commenting as a move teachers make has pluses and minuses in particular contexts, but it is unclear to me when consideration of moves like that are related to a dilemma. So, I'm not sure that it a dilemma that has come into view or simply a decision to make about how to act as a teacher.

DP: One of the things that lies behind any notion of teaching is a sense of what the teacher thinks it means to help someone (which can include, at times, doing nothing). And I think meta-commenting can be seen as an instance of that, namely where one notices something going awry with the language itself that is being used to engage with the mathematics (often because the teacher has a better grasp of the content and so has more attention free to monitor *the way in which* it is being talked about) and has to decide about whether to act on that or not. Meta-commenting is a decision that a particular kind of help would be useful here.

Meta-commenting is like attempting to repair a ship at sea (nautical metaphor alert once more)—ideally, while the repair is being carried out, the ship keeps going even if only with its own momentum. If the ship of conversation stops, then there is a risk of it foundering, being unable to start up again or drifting off in a less helpful direction.

DC: That makes sense to me, even though I am still have trouble articulating the commitments or obligations that, in your description, are competing around the decision of whether or not to meta-comment and, if so, how to comment. I'm starting to realize that there are ways that the images conjured up by Scylla and Charybdis do not link up with dilemmas in quite the way I would like. Maybe it is that the images produced for me by Scylla and Charybdis are more like navigating a path through rapids. What is missing for me in these images is the cost that one pays when managing a dilemma (Odysseus' crew loss). What was powerful for me in Lampert's description of teaching was the sense that there would always be ambivalence; there would always be a cost. I feel that sense of ambivalence and cost in two of the dilemmas that Jill articulates. To try to connect to the nautical image, one might have found a way through, but by definition one would have lost something, a paddle, a crew-mate, ... But, in addition, with teaching, it always seemed to me that while some of the costs might be evident—one might feel ambivalence

because one felt that one had compromised on a cherished ideal—other aspects of the cost, for example, its impact on future interactions, might be impossible to assess. In that sense, the question in Lampert's 1985 title: "How do teachers manage to teach?" echoes nicely the phrase you quoted from David Wheeler at the outset.

DP: One of a number of things our conversation has made clearer to me is the distinction between dilemmas and what I might call "decisions of action," or better "indecisions of action," namely the moment of realization that action of some sort is required. I accept that I have not yet managed to articulate fully the commitments/obligations that sit behind meta-commenting as a teacher action. More thought required (as usual). And with dilemmas, they are not singular or specific. As a teacher, one comes back the next day and they are still there, but, taking up your metaphor of classroom rapids (and bearing Heraclitus in mind), the monsters have moved. Nothing stays still. The dilemma does not go away, simply because I have decided to act in a certain way, *this time*. Nevertheless, spending time on the water can contribute to developing skill.

Our conversation has been a tale of half a dozen words and their mutual interaction: dilemmas, tensions, commitments, obligations, ambivalence, and, indirectly, technique. And, if I may, water. Magdalene Lampert's focus was on dilemmas and commitments; Jill Adler's identification of dilemmas was rooted in language and multilingual classrooms (but not necessarily only dwelling there). Your and Pat Herbst's focus is on obligations, mine on meta-commenting. One thing I became aware of in the course of our talking is that Jill's study of multilingual classrooms was as an outsider, and that required her to be able to provide evidence, hence a focus on teacher action (and teacher linguistic action in particular), whereas you studied teaching by doing it. The language of dilemmas may actually be polarizing; those two horns. And with commitments more than two might interact and conflict. Yet we don't need a third sea monster—two are quite enough.

The other word we have perhaps not looked at enough is technique (and associated with it, that of repertoire). But this is not to start yet another hare, but simply not to forget it. David Wheeler started teaching in a challenging secondary school in NW London in 1947 and 6 years later I was born less than a mile from where he was still working. Our paths didn't cross until 1980 at ICME, introduced by Bill Higginson across a table in Berkeley with lobsters on it, but for the next 20 years until his death we spent a fair amount of time together. In his final talk, to the CMESG conference in 1998, an organization he had been central in establishing, he spoke on "The commonsense of teaching" and, in passing, about technique. For all the work he had done over his career, he never lost contact with the specifics and the detail of face-to-face classroom teaching. I intend to close here with the words with which he opened. "When I first thought about giving this talk, I intended to give most of the time to technical aspects of teaching, especially the teaching of mathematics, a topic which doesn't seem to me to get the detailed attention and study it deserves."

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Chapter 4
Coming of Age: A Time to Ponder—A Tribute to Jill Adler

Abraham Arcavi and Ronnie Karsenty

Introduction

This chapter is dedicated to our dear colleague Jill Adler. Jill deeply inspires peers, students, teachers, and fellow researchers through her writings, projects and above all through her magnetic and passionate personality, her lively person-to-person interactions and her uncompromising commitment to pristine educational values without eschewing the serious dilemmas involved.

Jill's career and achievements epitomize, in many ways, not only the coming of age of a generation of prolific and creative professionals in mathematics education, but also the boisterous development of our fascinating field. In a sense, mathematics education is almost as old as education itself; as soon as there was mathematics, there was something to teach and to educate about. However, mathematics education is concerned with much more than with the mere act of teaching, and as such it is a young and vibrant discipline (or inter-discipline) of design and inquiry—in fact, younger than the cadre of people who have so much contributed to it. The first and second generation of colleagues who, like Jill, nurtured and shaped the multifaceted profile of our profession are now on the verge of retirement; hence a moment of celebrating accomplishments is in place, as a well as a moment of reflecting back towards reflecting forward..

Borrowing from Dewey (1910, p. 6), reflecting implies to engage in an "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends." Thus we would like to take this opportunity to embark on a wide (albeit brief) look at our discipline, drawing on some heuristics for reflection so often enacted by Jill: observe and then observe again; recognize that everything can be

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questioned; expose your sources of perplexity and articulate your doubts; be open to listening to alternative voices regardless of how far away from mainstream they may be; search until sensing that you are satisfied with what you found but always keep the option to re-question; and most importantly, do not fear to abandon safe grounds in order to critically evaluate yourself and your beliefs. In Jill's words, as conveyed during a lecture she gave in Israel in 2013, "sometimes there is a need to ask difficult questions that move against dominant discourses, particularly when they have the appearance of progressiveness." Accordingly, we aim at abandoning, even if just for a while, our mental comfort zone. The purpose is to engage in the effortful and non-trivial enterprise of overcoming the intellectual inertia so well captured by Dashiell Hammett in *The Dain Curse*: "...people hang on so tight to their beliefs and opinions; because, compared to the haphazard way in which they're arrived at, even the goofiest opinion seems wonderfully clear, sane, and self-evident. And if you let it get away from you, then you've got to dive back into that foggy muddle to wangle yourself out another to take its place."

Critical Questions at a Time of Change

We propose to open (or perhaps re-open) certain issues which merit a serious reflection à la Dewey. By no means do we aim at a systematic and exhaustive review of philosophical issues, thoroughly discussed in many distinguished sources. Rather, we wish to span the scope of worries and questions that our community might find worthy of consideration at a time of generational change. In this section we pose three core questions about our field. In the section that follows we share some of our thoughts regarding these questions, pointing to several heuristics inspired by Jill's work.

Mathematics Education, Who Are You?

"Over the last quarter of a century or so, and for better or for worse, this simple notion of where the core of mathematics education lies has been offset by goals and interests allying it, as an academic field, more closely with psychology of learning, cultural differences, and social justice, among others, than with mathematics itself" (Fried, 2014, p. 3). This sentence echoes voices in our community, as well as some mathematicians' voices, who decry the vanishing of mathematics from mathematics education. Some even claim that due to this drift away from mathematics, "we seemingly lost sight of what our profession should be all about" (Eisenberg, 2014, p. 35). Perhaps this is one reason for the growing divide between many mathematicians and mathematics educators. A possible sign of this divide is that notable figures such as Felix Klein, Jacques Hadamard, George Pólya, Hans Freduenthal, and others are not only rare in mathematics education nowadays, they may even

feel alienated by its present directions (Fried, 2014). This is amply discussed by Fried and Dreyfus (2014), and in our view is one of the challenges that our community faces in this juncture of generational change.

Alongside the discomfort that mathematics-oriented researchers may feel with the directions taken by the field of mathematics education, it is also argued that not only the mathematics is vanishing from this field. Some colleagues claim that it is the education which may no longer be a central concern. What can be the possible sources of such a claim? Dwelling on the possible divide between mathematics education and education may require a delineation of what "education" may be and what are its goals.

The word *education* is derived from Latin and its meaning is to bring up, to rear, to lead forth. It is defined, for example, as: "the act or process of imparting or acquiring general knowledge, developing the powers of reasoning and judgment, and generally of preparing oneself or others intellectually for mature life."[1] Education can be also considered in its widest meaning as the shaping of a person's culture, beliefs, worldviews, values, biases, inclinations, manners, and even the person's moods and affective predispositions. In light of these characterizations, one may wonder whether mathematics education as a discipline is departing from its educational missions. We are concerned about our community becoming more involved with the production and advancement of theories, research designs, and methodologies than with the fundamental objectives of education.

Mathematics Education, for Whom?

It sometimes seems that the final addressee of mathematics education is not yet agreed upon. For example, should the goals of mathematics education be attuned to the individual, to the society, or to both? Are these targets exclusive or compatible? A discussion about this recurring issue is likely to have profound philosophical, political, and practical implications. Some of the concerns involved may not have definite resolutions, but at least they should be explicitly raised time and again (especially at times of change), and so should the stance taken by mathematics educators towards them. These issues are not always explicitly addressed when it comes to designing a syllabus, selecting an approach to a piece of mathematics, establishing a research program, adopting a theoretical perspective, taking a policy decision, or "just" teaching.

Many influential organizations worldwide (e.g., NCTM, 2000, 2014) link mathematics to the practical values of "good citizenship" and to career readiness, emphasizing that mathematics literacy and proficiency are key to employment opportunities as well as to the acquisition of critical thinking. Thus, sound mathematics education is presented as essential to the well-being of both the society at

[1] A definition taken from dictionary.com.

large and the individuals comprising it. In specific politically charged situations, such as social disadvantages associated with race, the access to a viable and sustainable mathematics education is considered to be a civil rights matter (Moses & Cobb, 2001). This is definitely the case with Jill's work, which emphasizes the role that mathematics education needs to assume in promoting democratic values and equal distribution of resources in post-apartheid South Africa (e.g., Adler, 2001; Adler & Davis, 2006). Jill's consistent devotion to this target is one of the many reasons that she is so highly appreciated by the community of mathematics education and that she serves as a role model for her students (see Graven, Phakeng, & Nyabanyaba, this volume). We believe that there is a potential gain in continuing to ask ourselves at whom do we aim as mathematics educators, and why do we choose to do so (in this context the "for whom" and the "why" are intertwined). Moreover, these questions are closely related to bringing back mathematics to the core of the discussion, by considering which mathematical contents and competences are appropriate for which goal. Delineating the positions towards this issue, refining the subtleties and contrasting opposing views should be included in the agenda of our community.

Mathematics Education as an Academic Endeavor: What Are the Potential Risks?

Mathematics education provides a fascinating example of how a nascent and developing cross-disciplinary academic field emerges and flourishes—all the more so when one takes into account how much was accomplished in a relatively short period of time. Many enlightening insights accumulated in just a few decades, and many promising avenues are now being pursued. Exciting discipline, exciting times! However, as an inherent part of conducting any academic endeavor, one of the essential practices is to knowingly question and challenge assumptions and directions. Here are some of our concerns about the further development of mathematics education as an academic domain.

1. **Scientism**. Any academic work aims at systematicity and bases itself on theoretical compasses to set directions, sound methodological tools, and evidence-based findings. Whereas the above may be a truism when it comes to the exact and natural sciences, it is worth revisiting when it comes to education. Over-applying the scientific methods and the scientific standards of the hard sciences to education may preclude the advancement of the field rather than enabling it. Adopting the "scientifically rigorous" approach includes certain elements that may not always be considered relevant to our field, such as:

   - *Pursuit of knowledge for its own sake*. In our view, mathematics education is a highly applied discipline, its raison d'être is to provide guidance, inspiration, insights, experiences, models, original ideas, and tools for reflection

aimed at an ongoing improvement of education. Improvement, in our view, should include opening wide access to mathematics for all, providing affective and cognitive tools for growth, establishing optimal teaching trajectories, instilling motivation and good predispositions towards mathematics, and much more. The pursuit of knowledge per se may not only be sterile (educationally); it runs the risk of overlooking that there are individuals at the other end. One example of some of the possible tensions between the advancement of research and the well-being of students can be found in Tabach (2011). Many of us have experienced these tensions even without making them public.

- *The assumption that everything can be determined by research*. In education, research does not always provide answers or directions for action and thus its role as a basis for decision-making may have serious limitations. Research may turn to be of little help when it comes to value-laden choices or ad hoc contextual circumstances. The illusion that all educational questions can be resolved by a well-designed study based on a sound theory and a tidy methodology can lead us astray. The risks we see in resorting to "exact science" methods are, on the one hand, avoiding the use of values and common sense that are necessary when attempting to reach educated decisions based on data, and, on the other hand, focusing on details in a way that obscures relevance. When the unit of analysis is too small, or when coding schemes are overemphasized, we are in danger of losing the holistic perspective that is essential for shaping an educational worldview, in favor of less-relevant-yet-defendable scientific procedures.
- *The straightforwardness of implications*. Many research studies may have profound and interesting educational implications. It is likely that these implications are acknowledged in the study, however in most occasions they do not receive nearly as much academic effort as invested in the research itself. This is manifested in three types of problematic relationships between the study and its implications. Firstly, there are those studies that state possible implications merely as a lip-service. Secondly, in some studies we find implications that do not follow logically, immediately, and unequivocally from the research results. Sometimes they are mediated by the interpretation of the researcher, which is not the only possible way to read the results. Thirdly, as seen in many studies, even when the results are insightful and convincing, there may be a non-trivial bridge between these results and the suggested ways to apply them, or between these results and the ways they may shape instructional and/or policy decision-making. We claim that the root for most of these problematic situations is the prevalent perspective that translating research into practical implications is less "scientific" and thus perhaps less important than the research itself, or is even beyond the scope of the researcher's work. We argue that such a perspective risks the relevance of mathematics education as an academic domain. A serious consideration of implications requires going far beyond the science; it may imply recruiting

creativity, ingenuity, peer discussions and contextual considerations, and, needless to say, it calls for deep reflection on the study and its results.

2. **Aridness**. When our graduate students embrace the "set of codes" associated with academic work as the central guide for their initial attempts to conduct investigations, we face a possible risk of droughty outcomes; some students approach their research as a sort of a to-do list, lacking the sense of direction about why in mathematics education we do what we do. Intellectual fire that drives genuine academic curiosity may not in itself guarantee good academic work in our field, but it is necessary for an influential study.
3. **Religiosity**. In a lecture given at ICME-9 in 2000,[2] Shlomo Vinner talked about what he called "ritual schemas" that may be activated when students solve mathematical problems. He argued that the need for rituals and the need for meaning are, in a way, in conflict with each other. Borrowing from Vinner, we suggest that similarly to students, we, as members of an academic community have a need for rituals. Sometimes we are unaware that a "religious" spirit may take over our academic endeavors and our professional worldview. We may accept theories on the basis of faith; we may abide by rites (even if we are able to explain and justify them); we may design studies, adopt methodologies, and pursue questions just because of the zeitgeist. A quick look at conference themes, dominant discourses, and research agendas reveals aspects of religiosity which may hinder progress.

To sum up this section, we propose to consider the following question: *How can the risks that we face as a special scientific community be minimized?*

Several Thoughts and Heuristics, Inspired by Jill Adler's Work

One of the most important roles assumed by the experienced members of the mathematics education community is to educate future academics, so that they become those who study mathematics education and educate the following generation of professionals. There are substantive papers (e.g., Boaler, Ball, & Even, 2003) devoted to unpacking the practice of research in our field, its essence, and its skills (e.g., reading, writing, communicating, and formulating questions). These are laudable contributions to understanding our profession. In this section, we would like to share our perspective and discuss some heuristics which are less related to skills and more focused on the spirit of the enterprise. We draw on previous ideas (Arcavi, 2000), and also on ideas inspired by Jill's research. The thoughts and heuristics presented below are offered to the young generation in our community as a way to beware of the possible risks described above.

[2] Later published in the *Journal of Mathematical Behavior* (Vinner, 2007).

1. **Remember the ultimate goal**. If we take a humanistic stance of mathematics education, we should bear in mind that an overarching goal is to contribute to the shaping of better human beings. If we focus on "subjects" rather than on "people," we deviate from our main educational mission. Jill's research always involved working with students and teachers, but most importantly *for* them. In her work one can find enlightening examples of using research as means to improve children's lives. To give just one example, several of Jill's projects in South Africa are concerned with schools that are short of basic conditions to support schooling. In one of these projects, university students were assigned to support learners in such schools. Acting as mentors, they tried to supply disadvantaged learners with the resources that they lack. In that regard, Jill's project aimed at influencing children's lives, providing them with something that middle class students already have. Jill's agenda is clearly about changing practices, changing children's identities and their predispositions towards learning mathematics. Thus, her research contributes to learners and not the other way round.
2. **Consider complementary targets for mathematics education**. We would like to bring forward an alternative view to that of educating towards good citizenship through mathematics: Educating towards an intellectually contented adult, who enjoys knowing and learning; who appreciates mathematics in itself and for its applications; who has developed an aesthetic feeling for mathematical issues; who went through a fulfilling experience of learning mathematics, in which opportunities were given to build self-confidence in a field that is perceived as intellectually prestigious and difficult; who feels respected as a cognizing person; who had access to teachers capable of enacting mathematics as a personality building tool, rather than as a domination tool.
3. **Examine yourself**. Make explicit your beliefs and theoretical predilections about learning and teaching mathematics (including your beliefs about mathematics itself), and then follow questions that puzzle you, that engage your intellectual curiosity, and that you suspect may have an insightful answer; pursue work whose results you believe will have an impact (either by challenging a perspective which was taken for granted, or by confirming, in sound ways, what was assumed to be common knowledge). Consider the importance of possible implications to be derived from your study, and attempt to bear them in mind from the very beginning. To learn another lesson from Jill's work, the objectives of her research often combine the theoretical aims with the potential ways to apply them in the field. For instance, referring to the construct of Mathematics for Teaching (MfT), Jill poses her goals as follows: "What are these emergent forms of MfT? How do they relate to pedagogic practice inside teacher education? How do we explain these and what do they tell us about possibilities for developing and improving mathematics teacher education?" (Adler, 2011, p. 2).
4. **Examine what is available**. Theories may help to make sense of what you see, find, and experiment with. Pursue them not for their own sake, or because they are fashionable; look for them only after you have a burning question or a fascinating experience. Do not become wed too quickly to a single perspective.

Eclecticism and syncretism are not necessarily negative—quite the contrary, they may enrich your work, and possibly prevent you from disregarding what otherwise might be considered as noise. An enlightening example that comes to mind in that sense is how Jill melts together the theoretical concepts of "hybridisation" and "transparency" (borrowed from Lave's and Wenger's work), linguistic insights and data from teacher education research, to create analytical tools with which she offers a conceptualization of resources, shedding new light on seemingly trivial school objects such as the chalkboard (Adler, 2000, 2001).

5. **Question what a "good" question is**. Like in mathematics, there may be no consensus about what a "good" problem or a "good" question is. There may be different aesthetics that are worth knowing and challenging. Don't engage in a question or a problem just because others claim it is good or promising. Probably the strategy advocated for argumentation and proof in mathematics—*convince yourself, convince a friend, convince an enemy* (Mason, Burton, & Stacey, 1985)—is applicable for justifying what is good in the problem posed or chosen. In general, if a research question does not enlighten you in very deep ways, there is a good chance that it will not enlighten the community either.
6. **Broaden the context**. Search for contexts which are broad enough to allow for the emergence of significant research problems (e.g., curriculum design, teacher development, classroom studies). An examination that is too particular and narrow may yield results that are not potentially significant. In "Mathematics for teaching matters" (Adler, 2009), Jill provides a wonderful example on how research can relate to several contexts simultaneously (task design, teacher–learner interaction, teacher education programs, and issues of equity), offering us a thought-provoking contribution.
7. **Be critical with your own expectations**. Imagine the outcomes of an academic study that is meant to address the problems or questions you chose. Are prescriptive answers what you are after? Are theoretical statements what you would like to derive? Would you like to shed light on phenomena previously undescribed? Consider also the possibility that the potential results would be of a lesser importance, yet the process of the work itself is what might add to your understanding about certain aspects of education, thus contributing to its improvement.
8. **Dare to ask**, always to ask: Why? What if? What if not? Can it be otherwise? Is it really relevant? What can be learned from it? Nothing should be immune to questioning. If something is disturbing or puzzling, work hard to make explicit the sources of such discomfort, and try not to allow theory (or peers) to gloss over them. Challenge and question whatever does not make sense, and in contrast, attempt to verbalize why something does make sense to you.

Concluding Words

Reflections on the raison d'être of mathematics education and the roads we take as a community are always timely, certainly in this interesting time of generational shift, and should be put at the forefront more often than not. We should bear in mind that the search for a sense of purpose within a humanistic view of education can be a powerful steering force to be spiced with curiosity, excitement, creativity, and fruitful collaborations.

To conclude, we want to thank Jill for the values and inspiration that she has brought into the field of mathematics education over the years, and wish her a very happy and satisfying future.

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Chapter 5
Establishing a Community of Practice of Leading African Scholars in Mathematics Education: The Significant Contribution of Prof. Jill Adler

Mellony Graven, Mamokgethi Phakeng, and Thabiso Nyabanyaba

Introduction

Jill Adler began supervision of her first cohort of doctoral and postdoctoral students in 1998 at Wits University. In this endeavour she did not proceed with the supervision of six individual students but rather with the establishment of a supportive community of practice in which students actively engaged and participated. Most importantly she navigated ways for the activities within the community of practice to support the development of research trajectories that imagined themselves as African leaders making their mark on the local, African and international landscape and contributing to the many challenges faced by mathematics educators and learners. She inducted us into becoming researchers with a focus on making a contribution to the landscape we researched in.

While many African mathematics education researchers have completed their Ph.D.s and postdoctoral studies at institutions outside the country Prof. Adler's leadership allowed for African mathematics education research to be grounded in an African community of scholars and this grounding provided our research with a powerful edge that our stories, included in this chapter, will point towards. She however made sure that this local grounding was supplemented by international opportunities to work with leading academics in institutions around the world for periods of time and drew on Prof. Stephen Lerman as a key advisor and several other international visiting professors to our research community of practice.

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Prior to 1998 the number of mathematics education doctoral graduates from South African universities was extremely low. This extreme shortage in doctoral level academics in the field not only affected the quality of research and research publications in the field but also inevitably impacted on the mathematics teacher education community in Higher Education institutions in the country. Growing a strong community of mathematics teacher educators, academics and researchers was of national and regional importance.

Supervision styles differ. We have come to realize over time that there are supervisors who expect competence from their students (i.e. meeting the basic requirements) and those who expect and support students in attaining excellence, going beyond the basic requirements, to contributing substantially to the field and the community of educators and researchers. Jill's supervision style clearly falls into the latter and external examiner reports of her students' Ph.D. theses bear testimony to this. For example, examiners consistently pointed to excellence and innovation. For example, comments pointed to theses being among 'the best doctoral dissertations' read and those that extended under researched areas, such as multilingualism in South African classroom practice, 'in such a brilliant way' that enabled reconstruction of interpretations in the field in 'substantial ways'.

Having read through the 'notes of thanks' written to Jill for her supervision in several of Jill's Ph.D. students' theses over the past 20 years we note the following recurring themes:

Theme
Unfailing patience
Strong encouragement and faith in students ability to work independently and to choose their own path
Creating opportunities for working with a wide range of experts in the field
Humility and warmth
Critical insights and especially in relation to the complexities of the South African/African context coupled with pragmatic advice
Genuine interest and development of broader professional being
Excellent listener and detailed reading of student's work

Some examples of the above themes in the words of the students include:
Thank you for your:

'unfailing patience';
'enthusiasm and confidence in the value of this research';
'critical and insightful comments';
'expertise humility and warmth';
'insights into classroom practice in South Africa proved invaluable to my understanding of the complexity of conducting curriculum research in rural schools';
patience to allow the foci to emerge, the independence to choose my own theoretical and analytic tools. . .and your carefully considered insights and pragmatic advice';

'[enabling the] privilege of meeting wonderfully interesting lecturers and researchers ...'

To illuminate and provide a thicker richer description of these themes we reflect on our own stories as students in Jill's first cohort of doctoral students (beginning our studies in 1998 and graduating in 2002). We use Wenger's (1998) framework of communities of practice to analyse these stories and the critical contribution of Jill's 'way of being' and her style of induction into broader overlapping communities of practice, which enabled strong active and centrally participating career trajectories to develop for each of us. We know that many other students can tell stories which will differ as to the specifics of the studies, and the individuality of the student. However, our experience of our continuing engagement with Jill's many other Ph.D. graduates, as colleagues in the field of Mathematics Education, leads us to argue that the themes emerging from those stories would be similar to those we discuss in this paper.

Reflecting on Communities of Practice as a Means of Reflecting on Our Stories

Jill's own Ph.D. (Adler, 1996) and subsequent academic work (e.g. Adler, 1998) drew strongly on Lave and Wenger's (1991) theory of learning and later on Wenger (1998). She embraced this theory in her own teaching/supervision thus exemplifying key aspects of it in her lecturing and supervision practice.

According to Lave and Wenger (1991), learning is located in the process of co-participation and not in the heads of individuals; not located in the acquisition of structure but in the increased access of learners to participation, and it is an interactive process in which learners perform various roles. They prioritize the importance of participation in the practices of a community and identity as primary features of learning:

> As an aspect of social practice, learning involves the whole person; it implies not only a relation to specific activities, but a relation to social communities—it implies becoming a full participant, a member, a kind of person... Learning thus implies becoming a different person with respect to the possibilities enabled by these systems of relations.... learning is not merely a condition for membership, but is itself an evolving form of membership. (p. 53)
>
> In fact, we have argued that, from the perspective we have developed here, learning and a sense of identity are inseparable: They are aspects of the same phenomenon. (p. 115)

Since participation in the practices of a community is essential for the development of identity (and therefore of learning) they refine the notion of community for the purposes of learning and define a 'community of practice' as follows:

> A community of practice is a set of relations among persons, activity, and world, over time and in relation with other tangential and overlapping communities of practice. A community of practice is an intrinsic condition for the existence of knowledge, not least because it

> provides the interpretive support necessary for making sense of its heritage. Thus, participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. (p. 98)

Following from this definition it is clear for us that Jill's cohort of Ph.D. students was a community of practice. We met regularly as a research group, with the common purpose of furthering and strengthening our research, and our engagement overlapped with various other national and international education, mathematics education and research focused communities of practice.

For Lave and Wenger (1991) and Wenger (1998) learning is not located in the acquisition of structure but in increased access of learners to participating roles in expert performances. Thus the notion of access to engaging with members of the community is central in relation to a community of practice. Indeed, as we will see in the stories that follow access to both high quality resources (including leading members of research communities) and opportunities for us to participate in roles of expert performance (such as conference presentations and publications) was a priority for Jill in the supervision of her students.

Wenger (1998) relates communities of practice to the learning components of meaning, practice, community and identity as follows:

> On the one hand, a community of practice is a living context that can give newcomers access to competence and also invite a personal experience of engagement by which to incorporate that competence into an identity of participation. On the other hand, a well functioning community of practice is a good context to explore radically new insights without becoming fools or stuck in some dead end. A history of mutual engagement around a joint enterprise is an ideal context for this kind of leading-edge learning, which requires a strong bond of communal competence along with a deep respect for the particularity of experience. When these conditions are in place, communities of practice are a privileged locus for the creation of knowledge. (p. 214)

The range of many local and international peer reviewed journal publications authored with Jill (e.g. Adler, Pournara, & Graven, 2000; Setati & Adler, 2000) and alone (e.g. Graven, 2002; Nyabanyaba, 1999; Reed, Davis, & Nyabanyaba, 2002; Graven, 2005a; Setati, 2005a; 2005b), both during and following our doctoral studies, is a clear indication that indeed this community of practice that Jill created was a 'privileged locus for the *creation* of knowledge'. Jill encouraged us to share our research in local conference communities and in locally respected journals so as to positively influence and move the field of research forward in our own context. However she also encouraged us to share our work with international communities through conference presentations and publishing in leading journals such as *Educational Studies in Mathematics* and the *Journal for Research in Mathematics Education* (e.g. Graven, 2004; Setati, 2005a) where she supported us to make our African research both visible and influential. The influence of this research is visible in the over 700 citations that the publications in this paragraph have attracted to date.

Wenger (1998, p.5) identifies four components of learning namely: meaning, practice, community and identity. These components of learning are defined as follows:

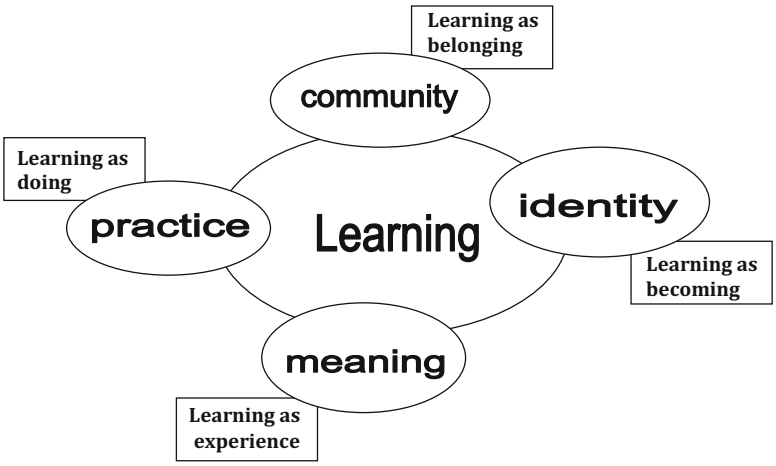

Fig. 5.1 Components of a social theory of learning: an initial inventory

1. Meaning is a way of talking about our ability to experience the world as meaningful
2. Practice is a way of talking about shared historical and social resources, frameworks and perspectives that sustain mutual engagement in action
3. Community is a way of talking about the social configurations in which our enterprise is defined and our participation is recognizable as competence
4. Identity is a way of talking about how learning changes who we are

These four components together provide a structuring framework for a social theory of learning. Wenger (1998, p. 5) summarizes this framework in Fig. 5.1.

Jill's supervision style enabled each of these components to develop strongly and we will refer back to these as we reflect on our stories.

Narrative Vignettes

In the words of Sfard and Prusak (2005, p. 20) what follows is our 'story about stories'.

Kgethi's reflections and story:

> I have had a relationship with Jill since 1989 when I first came to Wits to study towards an Honours degree. That was during apartheid and black students were very few at Wits. In fact I was the only black African woman in our class of about ten students, two of whom were black men. Then I admired her as a teacher, how she interacted with her students and how she cared without the usual patronization that we used to experience from many

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white South Africans. So I was drawn to her and so when she approached me in 1992 to collect data for her Ph.D. research in my Grade 11 mathematics class in Mohlakeng township in Randfontein, I agreed without hesitation despite the political challenges of the time. During the year 1992—2 years after Nelson Mandela was released from prison—black townships were not very friendly to white people. There was still anger and several of my students were members of the Azanian People's Liberation Army (APLA) and so the notice board at the back of my mathematics class had APLA written all over. However given who Jill is, her history of political activism and the manner in which she interacted with people, I had no doubt that my students would accept having her in our class. I did not even have to discuss all these complexities with Jill and she managed well and my students followed her everyday after her recording and helped her carry her video-recording equipment to the car. Jill's way of working inspired me so much that I used to travel about 140 km from Mohlakeng to Johannesburg to meet with other teachers in whose classrooms Jill had also collected data to discuss our lessons. This is how my love for research into mathematics teaching started—with Jill's research into my mathematics teaching.

Jill never pursued me to do my masters degree under her supervision, others did, but I chose her. I had developed a keen interest in the relationship between language and mathematics and what it means to teach mathematics in multilingual classrooms. So my Masters research focused on the use of code-switching in a multilingual mathematics class of Grade 4 learners. Jill's expertise as a supervisor came through for me during the Masters research, which introduced me to the international community. My first publication in an international journal as well as my first invitation to give a plenary lecture outside South Africa came out of this work.

When it comes to mentoring Jill is a master, she knows how to be a strong support without being visible. This is what made her ideal for me as a Ph.D. supervisor. My Ph.D. research was a follow-up from my masters, exploring language practices of intermediate phase mathematics teachers in multilingual classrooms. It started with Jill encouraging me to apply for the Presidential Educational Initiative grant, which I won and it powered the start of my Ph.D. in 1998. While there was an intersection between her research and mine she made sure to keep the two separate. With Jill supervision was not just about getting a degree but about developing a career. She made opportunities available and supported us to pursue them. Of course she never forced anyone to participate but she prepared the ground very well for those who wanted to participate and benefit from the opportunities. She introduced us to the international community in a very special way. Not only did she invite top scholars to come and work with us at Wits, she also had briefing sessions with us when we went to conferences. We looked at the programme and identified

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the people we wanted to meet with and when we got to the conference she would introduce you and move away. She allowed you the student to take the relationship forward without her interference. It is Jill who introduced me to David Pimm, Robyn Zevenbergen, Candia Morgan and Ole Skovesmose among many. When she received invitations from others, which she could not take because she was too busy she passed them onto her students. This is how I got to meet with Richard Barwell who had written to Jill to start an international group on multilingual mathematics education. At the time Richard was like me completing his Ph.D. and so when Jill referred him to me it was just what I needed—a collaborator who was at the same stage of career as I was. While it may be argued that Richard and I would have met anyway given the focus of our research, the truth is that the timing of our meeting was crucial and it has served both our careers very well. Thanks to Jill!

One of the most profound things that she said to me during my Ph.D. studies, which has stuck with me till today, is that when I finish the Ph.D. she will have done a good job if I know more than she does. So when I decided that I would like to go and work with Jim Gee at the University of Wisconsin-Madison for a while during my Ph.D. because I wanted to use Discourse analysis to analyse my data, she encouraged and supported me. When I came back from Wisconsin-Madison she treated me like I was the discourse analysis expert in the group. Of course she did that with each of the students—each one of us had an area, which we were experts in and could introduce the rest of the group to.

Ph.D.s are spoken about as a very lonely period where one works alone and occasionally gets to meet with one's supervisor. This was not the case with us because Jill set up a community of practice that met regularly. During the meetings we got to engage closely with one another's work. Jill set up a programme/time table, which indicated when workshops would happen and then 'in camera' supervision meetings in which she and Stephen Lerman would engage with the student's work. Students went in twos and that is how I got to learn what questions to ask during supervision. It was not surprising that when I visited the University of Wisconsin as a Ph.D. scholar some of the doctoral students thought I was a professor—this was all as a result of the training I had received here at home with Jill.

Co-authoring academic papers with Jill was another instructive experience. She never positioned herself as the knowledgeable other whose voice is the only one that should be heard in the paper. She backed off and oftentimes allowed the student to lead the writing and when that happened she allowed the student to be the first author. It is due to this way of working that it was not difficult to work with her as a colleague after completion of our Ph.D.s. Of course it is important to note that before 2002 Jill was one of only two

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mathematics education researchers at Wits with a Ph.D. and the only one with an NRF rating. After 2002 the community grew and Wits became known as the leader in mathematics education in South Africa.
I got more than a Ph.D. from Jill, I got a career in academia. Truth is that before working with Jill I had no idea what academia was about and I had never considered it as a possibility. Academia was an unthinkable for me. However, on that cold night in June 2002 when I ascended the stage to graduate with a Ph.D. I was not only getting a degree, it was the beginning of a career—I already had publications in international journals and had co-authored two chapters in an edited volume. Jill modelled what it means to be an academic—her knowledge of the field was always current, she worked long hours, won research grants and used them to support her students, led large successful research teams, collaborated with peers, presented papers at conferences and published her work. That I obtained a C1 NRF rating[1] three years after obtaining my Ph.D. is largely due to the quality of the mentoring I got, the research I did during my Ph.D., where I published it and the collaborative relationships I had developed internationally. I now have a B2 NRF rating and I know for sure it is all because of the excellent foundation I got through Jill's mentoring.
While I was a Ph.D. student I also watched how she navigated office politics. I can still hear her voice when she called me into her office every time I got worked up and angry. She would say, 'Is this how you want to be known? You have so much to offer, you are so good and this is not how you should want people here to know you for'. The words still ring in my head whenever I get worked up at work in my current position.

Mellony's reflections and story:

My supervisory/mentee relationship with Jill has been enormously influential across my career and studies. I began my doctoral studies under her supervision in 1998 with little experience of empirical research and only superficial analysis of simplistic 'impact type' data on my development work. This pointed largely to the failure of in-service workshops enabling change. Jill pushed me to delve much deeper into the complex issues at play both in my research and in the development work that I was conducting.
Jill's supervision style involved regular supportive meetings where she would engage with my writing and ideas and ask probing questions. She brought

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[1] A C1 NRF rating indicates that one is an established researcher with some international recognition.

with her a depth and breadth of experience of working with leading international educators and kept us up to date with all the latest work and trends in our field. She also brought leading members of the international community to engage with our cohort of Ph.D. students. So, for example, during our doctoral studies Stephen Lerman, Christine Keitel, John Mason, Candia Morgan, among others, all provided seminar sessions focused on aspects of our research and provided us each with individual time to engage with them on our particular research grapplings. We also were encouraged to socialize with them in evening functions that Jill would arrange. She further arranged, through spearheading funded research collaborations (enabled by joint NSTF-NRF and British Council-NRF partnerships), for each of us to visit with relevant leading academics in their institutions. So, for example, I had the privilege of working with Stephen Lerman at South Bank University and Ken Zeichner of University of Wisconsin Madison at their institutions for a month at a time both during and after my Ph.D. This enabled a powerful supportive space for publishing articles both during and post the Ph.D.

Jill also supported and encouraged us to attend key national and international conferences and always provided input to our papers and presentations prior to conferences. She was always encouraging and her ability to articulate clearly what one was contributing helped to build my confidence. She also drew on our feedback for her own writing which gave us the opportunity to learn from her process of writing and also to reciprocate the relationship of providing critical constructive feedback. This too had an enormous confidence building effect on our work but also enabled a sense of a two-way relationship where we both were learning from our relationship. While she led me into the journey—she did so in a way that developed my confidence, constantly reminding me that I was the expert on my doctoral topic and that I must remain mindful on this while I draw on her insights and experience.

At conferences Jill would make a special effort to introduce me to key mathematics educators in the field especially those related to my work. She would explicitly ask me whom I wanted to meet so that she could introduce me to them. In this way I became inducted to the broader national and international community and Jill's warmth, relaxed and fun-loving way of interacting with scholars in the community enabled me to participate 'centrally' rather than peripherally. A strong sense of my belonging in national and international mathematics education communities developed. The significance of this was critically important—particularly because I had always felt that I did not fit in well with academic research communities. I often felt the competitiveness and academic snobbery that I encountered at conferences and in my university pushed me away from wanting to pursue an academic research career. Jill changed all of this for me. Through her supervision style, that in a relaxed way inducted me into groups of scholars who engaged in

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affirming, supportive and meaningful ways, I made the decision to move from my position as a development officer to an academic position within the university. I have no doubt that I would not have taken this step without Jill's demonstration that within the many traditional challenges of academia—the competitiveness and often verbose rhetoric—there was an alternative 'way of being'. She inducted me into pockets of communities who defied and contrasted such practices with warm supportive collaborative ways of engaging powerfully, meaningfully and deeply with critical issues that demanded our ethical involvement to 'make a difference' in our education contexts and 'make our mark' as researchers both nationally and internationally.

Thabiso's reflections and story

My time under Jill's mentorship was the most fulfilling period of professional growth and certainly very satisfying at a personal level. I joined Jill's Ph.D. group in 1998 following a fairly easy passage through Honours and Masters programmes. Although I had extensive experience as a mathematics teacher and a curriculum developer while in Lesotho, I had very little exposure to the rigours and intensity of Ph.D. studies. Having rapidly gone through the Honours and Masters programmes and enrolling in the Ph.D. programme without much time for considering my career trajectory, I think I was more confident about my ability than I was clear about my direction in the Ph.D. programme. As a result, it took me four proposals before I finally found a line of research that really ticked all the boxes for a credible and sustainable Ph.D. study. All this time, Jill continued to provide very deep and helpful feedback, without ever imposing her preference.

I wonder how conscious Jill was of the things she did that assisted us so well with the Ph.D. journeys she guided us through. First with little research experience, Jill engaged me closely with some of her projects and I gained deep insights into the field of mathematics teacher education and research through my participation in projects involving a number of very experienced researchers.

The relationship with Jill grew in a very balanced and extremely fruitful manner with the highlight being the publication of my very first, and perhaps my best article to date in 1999. The manner of this publication highlights the very unique personality and professional qualities that Jill has. First she would encourage us to attend conferences and provide valuable support including funding. During conferences she would let us develop our own interests, only checking from time to time that we were making the best use of the time. One of these times she then connected me with a journal editor who guided me through the torrid times of publishing for the first time. One of the

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most lasting comments she made during the difficult times when I was trying to deal with the feedback on my draft article was 'a good research report is not necessarily a good journal article'. Both the research assistance in her projects and the support towards publication provided me with some preparation for the Ph.D. studies. However, I was still relatively raw and directionless when I started the journey.

I settled into my Ph.D. research direction rather late and only with the help of one of the expert researchers that I had worked with in a previous research project led by Jill. Jill's manner of supervision provided a wide range of supportive opportunities. She engaged with submissions in a very deep and thoughtful manner before each meeting. During meetings she would listen very attentively, probing and providing useful leads. The most memorable picture of Jill is when she would lean back in her seat and go quiet, followed by a remark: 'Yah, but Thabs, this is very interesting'. This could mean that I had gone completely off or it could be followed by those fulfilling moments when Jill would say 'I had never thought about it that way'. A better listener and a more accommodating professional I have not met. To this day, her manner of conducting meetings with mentees remains the example I try to follow. This extended to seminars where she listened to every input and enriched the engagements with her wide experiences. It took me time to settle into the sometimes robust discussions in these seminars, but Jill had a way of noting when even I had an important point to make. The Ph.D. group of 1998 engaged deeply with a range of issues often with help from seminar presentations by very able researchers raising current and pertinent issues.

Under the NRF funding she had organized for us, I had the privilege of working with Stephen Lerman at South Bank University and Mairead Dunne at Sussex University and both provided phenomenal growth to my Ph.D. studies. However, the studies were not without serious challenges. As a full-time international student, I often had to deal with the responsibilities long distance family life added onto my studies. And when a major research company offered me a senior research position on the third year of studies, it was a choice between finishing in time and the family responsibilities. The intensity of the demands of the new job would not have allowed me to complete the studies otherwise. However, Jill continued to encourage and support me to completion, albeit rather later and with much less opportunity for establishing better research impact of my thesis than I probably would have done under her close supervision. Still I owe my insights to research and teaching mathematics education to Jill's great balance between deep research knowledge and a warm personal touch to her supervision.

Reflecting on our stories several themes emerge across them. These themes resonate strongly with the recurrent comments of other students supervised by Jill, which we shared earlier in the paper. We discuss each of these themes in turn.

Access to High Quality Resources

Across these stories we note Jill's foregrounding of access to high quality resources, including grappling with a broad range of current issues within our Ph.D. cohort community, and opportunities for participating with leaders and members of key overlapping communities (at conferences, in invited seminars, in funded research collaborations in which we could work with these 'experts' in their institutions and so forth). In these ways we were provided access and opportunities for learning as experience; learning as doing; learning as becoming and learning as belonging. Thus in our stories we noted that while Jill 'sets things up for you' she left us 'to do the work'. She acknowledged that it was our journey, and that given access to key resources in our field, we had the knowledge and agency to make our own research decisions. This coheres with Lave and Wenger's (1991) critical emphasis on access to high quality resources. Thus they argue that 'in order to become a full member of a community of practice requires access to a wide range of ongoing activity, old-timers, and other members of the community; and to information, resources, and opportunities for participation. The issue is so central to membership in communities of practice that, in a sense, all that we have said so far is about access' (p. 101).

Career Trajectories: Supporting Learning Trajectories and Respecting Our Agency for Deciding Our Own Research Path

Across our stories we talk of how we got more than a Ph.D. from Jill's supervision and that our careers were launched through our working with her as we became centrally, rather than peripherally, involved in a whole range of academic practices. As noted in our stories, before this we were unsure of what 'being' an academic and a scholar meant. Jill passed on invitations to participate in leading academic research committees and forums to us. She introduced us to leading scholars; supported us to publish in leading journals; supported us in our career decisions and academic work more generally, and in so doing we formed powerful academic career trajectories that sustained and guided us to this day. Furthermore Jill continues to support us in our career trajectories and key decisions we need to make more than a decade after the completion of our doctoral studies.

Respect for Our Expertise and Emphasis on Two-Way Learning: Developing Confidence in Our Potential to Contribute to and Influence the Field

Jill's way of working with us emphasized that we must be the experts on our topic and she supported us in this becoming. She built our confidence by requesting our input into her writing, co-authoring articles with us during the Ph.D. process, and encouraging us (and supporting us) to publish in leading international journals. The positioning of herself as a learner, in relation to our increasing knowledge and expertise of our research, enabled us to develop a confidence in the way we engaged with our work and prepared us for sharing our findings in key spaces.

A Strong Ethical Approach That Foregrounded Making a Difference

Jill demanded high ethical standards of all her students. She enabled this in the way she pushed us to engage and grapple with the many ethical challenges confronted in researching within our post-apartheid and post-colonial contexts. She furthermore encouraged us to share these tensions, and how we managed them, so that others facing similar challenges could benefit from our ethical grapplings (see, for example, Graven, 2005b; Setati, 2005b). She pushed us to our absolute best and held a high standard for us to aspire to. She reminded us that our research was critically important both for moving knowledge forward in addressing the many challenges of mathematics education in Southern Africa and for influencing the international field because our contexts enabled insights that were often absent (or backgrounded) in international research. In this way were encouraged to 'make our mark'. Jill's support enabled our doctoral work to have influence (as evidenced by the extensive national and international citations of her students' work) and this intention to both influence the research field and impact positively on mathematics education in our countries continues to guide our scholarly endeavours.

The Relationship Between Affect and Mastery

While Lave and Wenger (1991) talk of mastery, and indeed as noted in Kgethi's story above 'when it comes to mentoring Jill is the Master', what is not mentioned in their work is the importance of affective qualities of a master that enables and nurtures full participation and access to increasingly central roles within the community and overlapping communities of practice. Across our stories, and the comments of Jill's other students noted earlier, Jill's qualities of *warmth, good*

humour, care and *friendship* stand out as enablers within her mastery at induction into being and becoming influential scholars. As Mellony's story notes it is precisely these qualities that enabled a different, and far more appealing visioning (or imagination as Wenger (1998) would note), and way, of being a scholar. Wenger's framework does however point to the critically important aspect of mutual respect among community members and our stories also foreground the extent to which Jill respected our work, we respected hers and the way in which she encouraged us to share our work among each other in ways that involved critical input and mutual respect.

Concluding Remarks

Our stories above lead us to conclude that we are who we are and where we are today thanks to the relationships we have had with Jill as supervisor, ongoing mentor, colleague and friend. Nobody succeeds on his/her own. There is no such thing as self-made success. We are where we are today in our careers because of the nature of Jill's support for us. She secured funding to support our research and challenged us to give our best. She advocated for us in a range of forums and fought on our behalf when institutional functioning impeded our work and progress. She inspired us by making high level academic participation and scholarly endeavours not only seem doable but worthy of the struggle towards achieving excellence. We are eternally grateful that she is who she is—simply exceptional all round!

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Chapter 6
Successful Postgraduate Mathematics Education Supervision

Renuka Vithal and Busisiwe Goba

Introduction

When the history of Mathematics Education in South Africa comes to be written Jill Adler will have a large and significant place in it. From humble beginnings as a secondary school mathematics teacher, her work spans several decades, from her anti-Apartheid activist mathematics education contribution to her entry into academia and current standing as an "A-rated" education researcher as designated by the South African National Research Foundation[1]—a recognition of "Researchers who are unequivocally recognised by their peers as leading international scholars in their field for the high quality and impact of their recent research outputs". She has been successful in the broad range of research outputs, advancing scholarship, providing leadership and contributing to developments in theory, policy and practice in mathematics education through the chair she presently occupies and acknowledged in the ultimate achievement of the Hans Freudenthal Medal for 2015.

In this chapter we focus on one element of such a successful researcher's work—Jill Adler's postgraduate supervision contribution and reflections—and ask the question what can we learn about supervision from her experiences and practices, which have taken place in a post-Apartheid South Africa? For this chapter we draw on a larger national Project on Postgraduate Education Research (PPER). This research project focussed on masters and doctoral research in the first post-Apartheid decade (1995–2004) across South African Universities that sought to

[1] www. nrf.ac.za/rating

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understand the "education research and the educational turn in South Africa's new century" (Balfour, Moletsane, & Karlson, 2011, p. 195) coinciding with Jill Adler's journey in higher education. In particular, we draw on a doctoral sub-study in the PPER by Busisiwe Goba (Phd in progress),[2] which critically analyses knowledge generation through postgraduate theses in mathematics education. Specifically, the data related to Jill Adler, as a research supervisor, provides the basis for this chapter.

Postgraduate Supervision Literature

Postgraduate education in general, and postgraduate supervision in particular, has come under considerable scrutiny in the recent past generating a wide scholarship in all aspects of policy, programmes, practice and theories. Dedicated conferences and journals have emerged on postgraduate education and supervision as the pressure to increase the numbers of students being recruited and admitted to postgraduate studies, and concerns about their education, experiences, attrition, throughput and completion success rates have come under the spotlight in much the same way as schooling and undergraduate education have been interrogated.

This international phenomenon and global focus on postgraduate education including supervision has been analysed at different system levels. Geographical regional or national studies have been undertaken and comparisons made (e.g. Kehm, 2006). South Africa is no exception. The South African Council for Higher Education (CHE), for example, produced a report on Postgraduate Studies in Higher Education (2009, p. xix) in which they refer to the notion of "the burden of supervision" because

> the number of postgraduate students has more than doubled over the past fifteen years, whilst the number of permanent academics has only increased by 40 % over the same period. In addition the pile-up effect of postgraduate students places more demands on the supervisory capacity of the system.

Doctoral education, specifically, has been investigated in several recent studies given its key role in knowledge generation and the rise of the knowledge economy (e.g. Nerad & Heggelund, 2008). In 2010 the Academy of Science of South Africa released a "consensus study" on doctoral education in South Africa using both quantitative and qualitative data

> to generate evidence-based advice on the status on the South Africa PhD and what concretely needs to be done to escalate the numbers and quality of doctoral graduates to meet the high-level skills demands of this emerging economy. (p. 15)

[2] Goba, B. (PhD in progress). *A critical analysis of knowledge produced through postgraduate mathematics education research in South Africa (1995–2004)*. University of KwaZulu-Natal, South Africa.

Some of the findings relevant to this chapter were that "the traditional apprenticeship model may not be an efficient approach for the purpose of rapidly increasing the production of doctoral graduates in South Africa"; poor student–supervisor relations is one of the risk factors for non-completion of doctoral degrees; "there seems to be a balance between those students who portray their experience of doctoral education as positive and those who portray it as negative"; and limited supervisory capacity is one of the primary barriers to increasing productivity in PhD programmes in the South Africa Higher Education system (pp. 16–17). It is against this background that Jill Adler's views, approach and efforts in postgraduate supervision in the South African context needs to be viewed.

Research on postgraduate supervision engages a broad range of topics and questions. At least one aspect that is widely reported on are different supervision models, approaches, frameworks and pedagogies proposed and tried (Adkins, 2009; Burnett, 1999; Fataar, 2012; Johnson, Lee, & Green, 2000; Lee, 2010; McCallin & Nayar, 2012; Nulty, Kiley, & Meyers, 2009; Olson & Clark, 2009; Parker, 2009; Price & Money, 2002; Samuel & Vithal, 2011). Another area that has received much attention may be broadly described as research supervision experiences (of both students and supervisors) and relationships, especially issues of power (Hemer, 2012; Krauss & Ismail, 2010; Lee, 2010; Madikizela-Madiya, Goba, & Nkambule, 2013; Manathunga, 2007; Zeegers & Barron, 2012).

Many of the studies cited refer to postgraduate research supervision in different disciplines or fields of which mathematics education is one and has generated relatively recent publications. One of the earlier volumes focussing on mathematics (and science) education research supervision is by Mallone, Atweh, and Northfield (1998) exploring many of the themes and challenges which have been engaged elsewhere, for example, a chapter on "critical reflections on a problematic student–supervisor relationship" (Taylor & Dawson, 1998) and descriptions of why particular supervision "worked" (Leder, Forgasz, & Landvogt, 1998). Similar studies have continued reporting on mathematics and mathematics education students' and supervisors' conceptions and experiences of supervision and have identified obstacles such as programme structure, student participation in and outside class, and faculty beliefs about teaching and learning (Herzig, 2002); and the need for measures like providing comprehensive information, monitoring funding and fostering networks for students that can improve research supervision (Morton & Thornley, 2001). Models for doctoral supervision in mathematics education using group supervision approaches have been presented in the literature particularly from the Scandanavian context. Grevholm, Persson, and Wall (2005) analysed a "dynamic model" developed and implemented in doctoral education and supervision in a Swedish context; and Vithal (2010) described a group supervision approach that sought to "democratise mathematics educational doctoral research teaching and learning" in a Danish-South African doctoral project. Both these resulted in all doctoral students in these graduating.

The brief literature review of this topic of research supervision in postgraduate programmes suggests that this is an emerging area of inquiry and exploration in general but also within mathematics education. It is against this background that

this chapter, which provides an analysis from one exceptional mathematics education supervisor's perspective and experiences, adds to the conversation and understanding of research supervision.

Context and Methodology

For this chapter we analyse data that has been produced by the Project on Postgraduate Education Research (PPER) hosted by a research team at the School of Education at the University of KwaZulu-Natal and in which Busisiwe Goba was a researcher and doctoral student focussing on mathematics education postgraduate studies. PPER surveyed and developed a data base of 3774 masters and doctoral theses across South Africa's 23 Universities from 1995 to 2004 and included interviews with a selection of supervisors (Balfour et al., 2011). From these data Goba (PhD in progress) extracted the mathematics education thesis and interviews with mathematics education supervisors. Following further verification and validation conducted across universities to ensure as complete a data base as possible for the mathematics education theses, the study yielded a total of 190 masters (150) and doctoral (40) theses (see Table 6.1). Four mathematics education supervisors were interviewed. It is from this data that we analyse one exemplary case of postgraduate supervision. Jill Adler was one of the supervisors that was interviewed by Busisiwe Goba and another researcher. Although the interview was conducted with assurances of confidentiality and anonymity and this has been preserved in all publications generated from the PPER and in Goba's doctoral study, special permission was sought and granted by Jill Adler to refer to her by name in this analysis. This chapter offers an analysis based primarily on this interview to generate themes to explain Jill Adler's approach and practices related to research supervision.

Jill Adler began her academic life at the University of Witwatersrand (Wits) and has continued there to her current position as professor and occupying a chair in mathematics education. This is relevant since Wits generated by far the highest number (54 or 28%) of education masters (43) and doctoral (11) theses from among all 15 (of 23) South African universities that produced mathematics education masters and doctoral graduates during the first decade of democratic South Africa (1995–2004) as Table 6.1 shows. In fact Wits produced more than double the second placed institution with 23 mathematics education graduates and theses.

Jill Adler accounted for 16 (29.6 %) of the 54 mathematics education masters and doctoral theses from Wits in the period from 1995 to 2004, this while having graduated with her own doctorate only in 1996. The 16 includes 4 doctoral studies she supervised and an additional 3 in which she assisted in the South African-Danish doctoral project (Vithal, 2010). This figure of 16 masters and doctoral graduates was more than what was produced from each of nine other universities in the same period (see Table 6.1). It should be mentioned that mathematics education theses accounted for only 5 % of all postgraduate education studies in the PPER study. Given the difficulties of recruiting and scarcity of mathematics

Table 6.1 Mathematics education postgraduate studies in South African universities (1995–2004)

South African Universities offering Mathematics Education M&D 1995–2004	Masters	Doctoral	Total
1. Witwatersrand	43	11	**54**
2. Pretoria	17	6	**23**
3. Johannesburg	17	4	**21**
4. Cape Town	20	0	**20**
5. South Africa	10	9	**19**
6. KwaZulu-Natal	14	1	**15**
7. Stellenbosch	8	2	**10**
8. Nelson Mandela	6	2	**8**
9. Northwest	5	3	**8**
10. Rhodes	6	1	**7**
11. Western Cape	1	1	**2**
12. Tshwane Technology	1	0	**1**
13. Free State	1	0	**1**
14. Zululand	1	0	**1**
Total	**150**	**40**	**190**

Adapted from Goba (PhD in progress)

education postgraduate students, it is remarkable that Jill Adler's career total (from 1993 to mid-2014) of supervised masters and doctoral students is 40 of which 26 are masters (M.Ed. and M.Sc.) and 14 are doctoral graduates (excluding the 3 she assisted with). This high level of research supervision offers an opportunity to learn about successful supervision by analysing her philosophy, approach and practices related to supervision, especially in the South African context characterised as a developmental and transforming society, and which can provide insights for others to benefit from and emulate.

Discussion of Themes

The analysis of the interview with Jill Adler yielded three main paired themes: creating community and being in community; becoming expert and connected; and projects and problems of practice. Each of these is discussed below with excerpts from the data and with reference to the literature.

Creating Community and Being in Community

Traditional models of research supervision involving a student and a supervisor characterised as one-on-one or master–apprenticeship relationships have come

under much criticism and been replaced or supplemented in recent years by other models of supervision, most notably forms of group supervision. Various combinations of groups of students supervised by groups of supervisors are often described and theorised in terms of notions of "community". A strong first theme evident in the interview with Jill Adler was the importance of "community" in how she approached her research supervision:

> *No, I didn't work on a one-to-one basis. Let me just say there were a couple of reasons why I didn't work on a one-to-one basis. One was, when I set it out, it was practical. If I'm going to take all of you on, you have to agree to work together, to help yourselves as well as . . . there was no such thing as 'community of practice' in those days. People hadn't worked using those notions, certainly not for graduate study. It was more that it both resonated with our work and it was also this is the way we'll manage. It just felt like the right thing to do.*

In their review of current practices in postgraduate research supervision McCallin and Nayar (2012) point to the rise of "group supervision" or "the community model" of supervision as an alternative model and "some evidence suggesting that group supervision improves the supervisory process" (p. 68). In outlining new approaches to doctoral supervision, Lee (2010) concurs in identifying this as one of the emerging models of supervision and alludes to the underpinning theorising also as mentioned by Jill Adler:

> A . . . conceptual approach to teaching and supervising at this level is to look at practices implied by the model of 'community of practices' (Lave & Wenger, 1991), which is, in effect a decentralized version of the master/apprenticeship role. Lave and Wenger's work has had great impact in highlighting sociological issues . . . and they explore the way in which the student is helped (or not) to move through legitimate peripheral participation to an understanding and mastery of the tacit knowledge required to participate fully in an academic community. (p. 18)

Community models have taken many different forms in postgraduate supervision as the literature attests such as "a learning community approach" to support scholarly writing (Parker, 2009) or to promote peer learning (Stracke, 2010); or for instance, as "leader-scholar communities" in professional doctorates to support students to undertake research in local educational contexts (Olson & Clark, 2009). The vast majority of studies that theorise postgraduate supervision in terms of notions of community do so with reference to Lave and Wenger's (1991) "community of practice" and "legitimate peripheral participation". Jill Adler also referred to "community of practice" in the interview. For her creating and being in community was a "practical" way in which she could "manage" the large numbers of students who were keen to be supervised by her, but also because it "resonated" with her work and "felt right" since it also echoed her own theoretical stance and engagements in her mathematics education research (e.g. see Adler, 2001).

Group supervision can take many different forms. However, one type of supervision that has emerged is as cohort models, which are variously described, for example, as "collaborative cohort model" (Burnett, 1999) or "seminar-based cohort model" (Samuel & Vithal, 2011). When probed on how she develops and maintains collaborative working relations with her research students, Jill Adler also described a cohort model approach that she used in her supervision:

For the first cohort it was structured and it was . . . if you going to do this you have to agree this is how it's going work. You come six times a year, and we worked intensively for a week together. One (student) was full time the others were all part time so it was structured. The first cohort was very nice because it was structured so we looked at setting the questions . . . But after they were on their way, new students then started to come so I took the next group in by 2001. I took in three new students. Now I had students who were nearly finished and students who were starting. Then I had to learn how do I set this community up where there were new comers and then people that are at different stages and the community is bigger. But it was still structured.

They had to come in and now the way we run the programme in the centre is that there are four PhD weekends a year. All PhD students have to come and the PhD weekends are on a Friday afternoon and Saturday all day . . . those that are not in Johannesburg, they come in from the Wednesday afternoon, we work all of Thursday, Friday. I've started the same sort of thing that I was doing before. We work together all day, they have to present work to each other and to me. They each have to take responsibility for thinking about each other's work. So there is a structure and now the next one is coming in May and I have to go to the US so they going to run (it) by themselves.

The need to provide "structure" featured strongly in the interview in explaining how supervision is organised and carried out. In her models of supervision, Lee (2010) too identifies in one of the alternate models of research supervision as constituting two axes of "support" and "structure". Samuel and Vithal (2011, p. 83) similarly argue that the cohort model offers a "strong structured support" that serves to scaffold students' research learning as they progress through the doctorate. Another observation from Jill Adler's account is how the cohort model itself evolved as different challenges needed to be addressed, which is also evident in other cohorts (Samuel & Vithal, 2011). Although group or cohort models have emerged in response to the increasing demands of having to manage larger numbers of research students, they have changed the nature of the student–supervisor relationship and interactions (Govender & Dhunpath, 2011; McCallin & Nayar, 2012; Samuel & Vithal, 2011).

Group or cohort supervision lends itself to the formation of communities. A strong assertion evident in the data is that the notion of being part of a community as a researcher is very important to Jill Adler. Her drive to create communities for her students was strongly articulated and is reflected not only in her supervision approach in how she constructs supervision experiences for her students but was a significant aspect of how she chose to become part of a community to enable her own doctoral study in how she constructed her supervision experiences as a research student.

I was on my own, there was no community, I had to set up my own community and PME became my community. It was an established conference where there were people interested in socio-cultural theory. I went every year to PME when I was doing my PhD so that I had a community to interact with in my work. I didn't have it with colleagues who were studying and I didn't have it in my supervisors that were close at hand. So I had to construct it. I suppose I knew how important that was. It wasn't how I was supervised, it (was) what happened while I was doing my PhD. I made sure that was part . . . that the students got involved, that my students were part of . . .set up a community themselves and became part of the community so they had to go to PME, go to SAARMSTE, they had to engage and interact.

Being part of a community, actively seeking out communities to advance one's research and thinking and creating communities for others is the message that stands out in sharpest relief in the interview with Jill Adler. Although cohort or group supervision is a recent phenomenon and still under research, it is being adopted in several disciplines including mathematics education research supervision. For example, Grevholm et al. (2005) describe a "dynamic model" for doctoral supervision in research groups in mathematics and mathematics education. They found high levels of student satisfaction, no attrition, completion in the regulation and successful transition into academic careers in a programme in the Swedish context. Similar success is also reported in a mathematics education doctoral project that used a group supervision approach that speaks to working across North-South contexts of Denmark and South Africa. Vithal (2010) describes how mathematics education doctoral research teaching, learning and supervision in groups can be set up to generate a more democratised supervision environment that led to all student completing their studies.

Becoming Expert and Being Connected

For Jill Adler this group or cohort approach to research supervision was a means for inducting research students into becoming supervisors themselves and supporting them in their journeys into academic life.

> *I was very conscious that I wasn't only working with them to get their PhDs; they were also all going to be academics. And so they were learning not only to become a PhD, but to become supervisors themselves. So the whole model that was set up was that they had to interact with each other and with me and that we were learning about supervision as they were learning about their PhDs.*

There was an even bigger agenda and vision that Jill Adler deliberately pursued than simply supervising research students to completing their studies or to become supervisors or academic, it was to also contribute to mathematics education itself:

> *I was trying to do two things when I was supervising . . . my concern was not just to be a supervisor . . . we needed to create the next generation. That was absolutely clear to me and that if we wanted to develop the field at all in Maths Education there had to be two ways—one with the PhDs, one with the Honours to help produce the next generation.*

However, developing supervisors and the next generation of academics to grow Mathematics Education itself was difficult because of having students

> *who were each doing very, very different studies—Language, Realistic Maths, Tertiary . . . that made my job very hard because I wasn't necessarily the expert in all of them. I certainly was knowledgeable. I made sure they all went to see and to be with an expert. Each one went somewhere else for three months to be with somebody. . . so that they become the expert in their field. Even now when I work with my students, I don't think they have done a full PhD if they haven't moved beyond me in their study.*
>
> *They must have become the expert. And that's a very hard corner to turn, but it's a very important one. So you have to encourage them to become themselves. I've not had a problem with that.*

A second dominant theme that is discernible in the interview with Jill Adler was her concern not only that her research students become academics or supervisors themselves but that they be put on the path to becoming "experts" through the supervision process. This was explicitly expressed as part of her philosophy of research supervision:

> *You have to provide the space for others to become experts. Create the space and the environment for others to take on the serious problem they are interested in and to become the experts*

This awareness and consciousness to meet the double challenge of not only supervising students to achieve their degrees but enabling them to become independent and expert can be shown empirically in the number of Jill Adler's students now themselves occupying research chairs and in leadership positions such as Heads of Departments or Schools, Deans and Deputy Vice-Chancellors.

This very aspect was also identified as being one of the most challenging in research supervision which varied from student to student:

> *What I found challenging and difficult is the intellectual direction. Being able to think about . . . being able to know when you know enough, and to know and feel confident that you don't know, that they must go and find it out somewhere else, and to feel confident enough to do that. That's a challenge. . . the challenge for people, who when they get into a difficult patch—and most do—and lose their confidence, is to help them not to rely on you. So it's to be a support and a serious support, always be a support, but do not take the project over. In fact refuse to take the project over.*

Supervisors play many different roles in the course of supervising students as mentor, advisor, friend, colleague, counsellor and so on. A growing literature on the different facets of the relationship between students and supervisor has emerged as attested to in the earlier section on the literature in this area. Johnson et al. (2000, p. 135), for instance, point to the "paradoxical nature of the processes of the production of the autonomous scholar self" that underlie practices of different models of supervisory relationships. Jill Adler's emphasis on providing "support" points to how seriously she takes this aspect in the supervisor–students relationship, but at the same the challenge and difficulty to also let go and to allow each student to journey through the struggle to find their own path.

Jill Adler articulates the caring and the challenges in getting to know her postgraduate students, and ways that it carried its own cost for students to become academics in their own right:

> *It's different for Masters and PhD. PhD it's a long journey. You get to know your students quite intimately. You live with them in some ways, it's hard, PhD is hard. I don't know any of my students who haven't gone through personal trauma of some kind while I was supervising. Some it might be illness themselves or some loss, many of them experienced a death in their family in some way. You have to write those stories and you can't sort of say leave those personal stories behind. So how do you not let the relation get so intimate that you stop doing your job. So that's hard, you get to know people deeply. I think there are instances, where people have got so dependent in a psychological way on the supervisor emotionally that they can't stand on their own two feet. You can see instances of that, people who write and criticize the supervisor's work. That's the way they manage to break free.*

Each supervisor and student navigates their unique research relationship which may endure over several years in the case of doctoral degrees, and as noted earlier, this relationship and experience has become the subject of research itself. As Jill Adler alludes, it can be a difficult and fraught relationship. How supervisors conduct themselves in the student–supervisor relationships and the practices they engage is shaped, in part, by their own histories as research students, which can be positive or negative. Certainly the ways in which Jill Adler conceptualised and went about supporting her own doctoral study was reflected in the approach she deployed within her supervision practices. That she put so much emphasis on developing her students into experts, she drew on and learnt in her own postgraduate studies, the need for and value of seeking out relevant experts:

> *I was already an established professional by the time I was doing my PhD. I needed someone to give me the confidence to do the work and who would help me conceptually, so I had a philosopher. She didn't know the field (mathematics education) at all so I had somebody who was quite far away, who was a Vygotskyian. There were very few maths educators who were working in a Vygotskyian framework and she was. So she had the expertise. Then I had to pull the two together. I had to go outside the country to find somebody to supervise me so she was more at a distance. I spent a little bit of time in Australia.*

A strong feature of Jill Adler's research supervision practice is the ways in which her commitment to seeking out experts led to her building and working within networks. These links that she establishes in turn enable her students to also becoming networked. When she set up the PhD cohort, she referred to drawing in a colleague from the UK. This valuing and connecting with scholars from different places also serves her well in her research projects in which there was a need to cross disciplinary boundaries. The importance of networks in research supervision is documented in the literature (Lieberman & Dorsch, 2005; McCallin & Nayar, 2012), and has also been identified as one of the measures that can improve research supervision in mathematics education (Morton & Thornley, 2001). Becoming expert is facilitated by becoming connected; and being connected contributes to being expert and recognised in the discipline or area of study. Jill Adler consciously attends to this in her supervision practice and approach as part of her supervision philosophy.

Building Projects and Problems of Practice

A third theme that emerged from the interview and was found to inform her supervision of research students is the way in which Jill Adler retained a sharp focus on problems of practice, and built research projects around these, which in turn attracted students into these projects and created a particular supervision environment and imperatives.

This focus on problems of practice is deeply rooted and was identified by her as something that stood out for her from her Masters and Doctoral studies:

I think my experience is different. All the time I was studying a problem of practice.

Her supervision approach and practices need to be seen in the context of the problems of practice that drove her selection of focus within mathematics education—these being teacher professional development as a practitioner, on the one hand, and on language as a researcher, on the other hand:

So the focus has got two strands to it that has been in the background and in the foreground at different times. Coming into the university, coming into the college, I was doing Teacher Education. One of the things that started to be absolutely clear to me in the first three years and which is why I say all my research has been informed by practice and a problem of practice was specifically related to my own practice as teacher-educator. What was clear to me is that the reform curriculum, if you think back to 1989, 1991, in the US at that time the standards were being developed, and constructivism was everybody's word. A lot of the research that was starting to impact here was from the UK. They were influential people and people came here, were running workshops for teachers. It was very clear that this new methodology meant a communicative classroom and they were ignoring the fact that many of the kids couldn't speak the language they were meant to be talking in.

So for me it was a real problem that all this research took absolutely no cognisance of the fact that for many kids they were learning maths in a language that they were not fluent in. And so what does this mean in a communicative classroom? What language do they talk in? How do they get access to English? And, so I started courses like that for my students. I had a whole range of students and everybody would say, "Of course it's a problem," "of course it's harder if you're learning a language while you're learning mathematics." Then I'd say what's hard about it? I thought I need to find out more. And that's how I set up my PhD. So it was secondary teachers' knowledge of their practices of teaching in multilingual classrooms. I started with the assumption that there are hundreds of teachers who have been teaching in multilingual classrooms all their lives. They are very experienced. What do they do? What do they think the problem is? How do they understand the problem? What can we learn from their experience? So it was language, and the book that came out was 'teaching mathematics in multilingual classrooms'.

The problems of practice both contextualised her research and defined its content. The context and content embedded in the problems of practice shaped the research projects she succeeded in having funded and shaped the ways in which she conceptualised and organised her research supervision. Her supervision approach developed from the way problems of practice were cast in the form of projects and could be tackled within the projects that were set up. As observed before, this too was evident from early on, soon after completing her Ph.D.

I'm fresh out of my PhD and I thought ok, I need, we need to do some research, so I set up a project. I got money from what was then the FRD. We had a group of ten people working on the programme, some in maths, some in Science, some in English Language. I was the only PhD; I was the most experienced researcher of everybody; I had just finished my PhD; and we undertook this project. I learnt a huge amount, but I took ten people with me. Some people came in as researchers; . . . research assistants. They helped on that project. They finished their Masters. So people started to come in to work with me and they come in on that project. And as soon as they finished and my PhD was finished they came in to do their PhDs. So, what happened in that project was that it had a language focus. And . . . that's what I was interested in.

The projects created a particular supervision ethos and environment and Jill Adler's supervision discourse is one in which people worked together as she

endeavoured to take her students along with her. Not only does this setting allow for a wide engagement in areas of mathematics education but also multi- and interdisciplinary research was enabled by the group or cohort supervision model she adopted. Strongly featuring supervision as co-learning in joint team work with groups of students and groups of supervisors facilitated making the problem of practice in the project the central feature around which students could be supervised in ways that connects them to her research interests and connects each project member.

Constant reflections on what the research in each project reveals about the problem under investigation lead Jill Adler to identifying new projects, new teams and new students:

> *. . .the problems started to come, the problem here is with subject knowledge. We hadn't got the design well enough. There's no question about it, they learned a lot of maths, because there were maths courses, but they didn't talk about that when they talked. When they talked about their teaching, they talked about group work, they talked about language, they talked about the resources they used, they talked about the activities that they did. When we pushed them in the interviews, about what did you want them to learn, mathematically, of course they had those goals, but they couldn't articulate them. We didn't know whether it was the design or whether in fact this is much harder to see, to know and to understand and yet it was our goal in the programme; that were helping them deepen their knowledge. Some of the primary teachers whose knowledge was poor. . . they didn't make progress and in some instances we thought they went backwards. Because they took on reform practices on a very weak knowledge base, they didn't strengthen it well enough. . . So they were trying new activities and they didn't have the grasp of mathematics to deal with what the kids were doing, or to push the activities. So it became a real problem both empirically, ethically that we hadn't got to grips with what it means to actually deepen their knowledge.*
>
> *The secondary teachers were a little different. They took up more of the maths . . . what I leant from that is that just like children in class, teachers are different. They learn different things. They come to the programme with different backgrounds and foregrounds. We don't build that and tap into it. We don't understand what it means to actually deal with teachers who are diverse. So it's interesting working in diverse contexts with diverse histories. And so we set up the next project, that's how QUANTUM got going. I raised money for that to develop special maths courses for teachers. That's how our Honours programme got going and that started the next round of research which then opened up the question: what do . . . we think we are doing maths and teaching, we're doing them together. Sometimes they're parallel; sometimes they are integrated, what's happening? That's the question and we've got the beginning of some insight into what's happening; that's why it's become more about professional knowledge and it's become more focused on the content.*

Through her research journey what is evident is that Jill Adler moved from one project to the next as she identified a new emergent problem of practice that needed investigation. In setting up each project, a community is needed and was created. But for communities to flourish, expertise is sought from both within and outside mathematics education, and from both within and outside South Africa. In this way the three themes identified in the analysis of the interview data are linked and together offers an explanation for Jill Adler's success in mathematics education research supervision.

Concluding Remarks

The question of what is successful postgraduate research supervision or that which could be characterised as "supervisory excellence" (Nulty et al., 2009) cannot only be the number of students who graduate or the time they take to complete, although these are important and critical measures. In this respect Jill Adler is without doubt successful in terms of the students she has graduated. Successful supervision is much more than quantitative measures, spanning a range of aspects including the quality of the supervision and the knowledge generated.

The importance of the notion of community for herself and for her students, so centrality embedded in her philosophy and approach makes her supervision successful through the enduring nature and quality of the student–supervisor relationships that invariably come to be constructed and the kinds of learning experiences and outputs arising from community settings. Yet another measure of successful supervision would be what each student and the supervisor takes away and manifests in what she and her students achieve and to where each progresses in their intellectual and career journeys from the encounter.

Her strong awareness and focus on the need to create experts and independent scholarship in her students; and the way these efforts took form in projects in which problems of practice were investigated; and in turn spawned wide networks that enabled students and supervisors as groups to leverage each other in generating different research outputs and outcomes that benefit all, are yet a further indicator and testimony of successful supervision. This is not to say there were not those who might have struggled in their relationship with her as their supervisor. However, in this too, her deep reflection and understanding of how supervisors and students might struggle through the supervision and to let go is acknowledged and therefore a strength in supervision awareness and understanding.

This analysis opens the question about the need for more research not only about successful supervision but equally also about successful supervisors. At least one lesson gleaned from the themes explored is the strong influence of supervisors' own experiences as research students in shaping their supervision practices and approaches as supervisors. Moreover, they can be positive or negative:—positive in that they may reproduce particular supervision practices that worked for them, or negative in the sense that they would refrain from certain practices or deliberately engage these because they were absent in their own experiences as research students.

Jill Adler has created a significant and formidable legacy in many areas: from her early contribution as a mathematics teacher in providing access to mathematics to adult learners through newspapers during Apartheid to her post-Apartheid seminal research in multilingualism and teacher education and the exceptionally large number of mathematics education leaders and researchers who have come through postgraduate studies with her and been supervised by her. Her success in supervision has also significantly benefited mathematics education in her university as the data shows, contributing to making the University of Witwatersrand a leading institution in Mathematics Education.

This chapter is both an analysis of her perspectives about mathematics education postgraduate supervision and a tribute to her. It provides insights into how she has arrived at her current standing as a leading international mathematics education researcher and practitioner in South Africa. Perhaps most importantly, the outcomes of her successful supervision collectively captures and reflects the contribution Jill Adler and all the students whose lives she touched through her supervision, have made to mathematics education knowledge, practice, policy and theory in South Africa and internationally.

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Chapter 7
Researcher as Activist: A Conversation

Anna Sfard and Hamsa Venkat

South Africa is not just one among many comparable settings for mathematics education research and Jill Adler is not an ordinary mathematics education researcher who, her special prominence in the field notwithstanding, does basically the same type of work as all other members of this worldwide community. For the last two decades, South Africa has been trying to shed its old identity and to design a new one, and Jill is an activist deeply involved in this process. She was an activist even before being a researcher and this former activity colors whatever she has been doing since then. In fact, Jill turns everything into activism, research included.

"Researcher as activist?" one may wonder. According to Wikipedia, "[a]ctivism consists of efforts to promote, impede, or direct social, political, economic, or environmental change, or stasis."[1] This certainly fits: Jill has always been driven by a burning wish for radical social change in the country whose history is replete with wrongs and injustice. The Wikipedia authors also say that the term activism "connotes a peaceful form of conflict" (ibid). Both words, *conflict* and *peaceful*, seem apt. Jill is always in conflict with the reality around her, as she sees it, and with those whom she holds responsible for whatever abuses she spots. Her work has been concerned with issues of access and equity across language, race, and class, with specific attention to mathematics education. But as emotional as she is about those aspects of reality she is trying to mend, there is no violence in her methods: her strong emotions are channeled into research, where they serve as an inexhaustible source of creative energy.

[1] http://en.wikipedia.org/wiki/Activism, retrieved on 26 May 2014.

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All this said, the question may still arise whether research can ever be seen as activism. It does not appear among the different incarnations of activism listed in Wikipedia: "Various forms of activism range from writing letters to newspapers or politicians, political campaigning, economic activism such as boycotts or preferentially patronizing businesses, rallies, street marches, strikes, sit-ins, and hunger strikes" (ibid). Research, on the other hand, as defined by the same source, comprises "creative work undertaken on a systematic basis in order to increase the stock of knowledge, including knowledge of man, culture and society, and the use of this stock of knowledge to devise new applications."[2] As an activity geared toward "increasing the stock of knowledge" research may appear far removed from *direct* efforts to shape the political and social reality. Well, it does not seem so to Jill. For her, research is a perfect weapon to use in the face of conflict, especially if you wish your struggle to be peaceful.

Jill's activism, her constant wariness of academic detachment and her ongoing effort to stay close to the reality-in-need have been attracting both of us to her and her work for several years now. In South Africa, both of us are outsiders, albeit to different degree and in slightly different ways.[3] Jill's body of work was crucial to our initial understanding of the South African scene. Our intersections with her professional oeuvre became personal interactions, with issues of language, race, and class infusing conversations across our personal and professional lives. These interactions have been enriched by engagements that have worked holistically across the personal, professional, and the mathematical. Now, after a few years of ongoing contact, we are in a position to reflect on both the special needs of South Africa in the domain of mathematics education and on Jill's research and activism. As foreigners, we may be able to see things that remain unnoticeable to insiders. As famously observed by Mikhail Bakhtin, "[i]n the realm of culture, outsideness is a most powerful factor in understanding" (Bakhtin, 1986, p. 7). In the conversation that follows we tell the story of our encounters with Jill, while also reflecting on the uniqueness of both the South African scene and Jill's work. The main focus will be on the special challenges with which mathematics education researchers in South Africa are faced. We will pay particular attention to the professional identity that Jill has chosen for herself: that of being the researcher who refuses to play the role of the "neutral observer" and insists on changing the reality under study as she goes.

[2] http://en.wikipedia.org/wiki/Research, retrieved on 26 May 2014. This definition of research is quoted from OECD (2015) Frascati Manual: proposed standard practice for surveys on research and experimental development, 6th edition (retrieved 27 May 2012 from www.oecd.org/sti/frascatimanual).

[3] Hamsa comes from an Indian family that moved to England when she was 5 years old, before moving to South Africa in 2005, and Anna who grew up in a Jewish family in Poland, has lived in Israel since she was eighteen.

Meeting Jill, Meeting South Africa

Anna: I remember my first meeting with Jill, back there in 1997 at PME in Lahti, Finland. I was surprised by the closeness and familiarity which I sensed from the very first moment. Jill was coming from a place that, for all I knew, was different in every possible respect—politically, socially, geographically, and historically—from those that shaped me all along the way. So where was this familiarity coming from? This experience repeated itself with an even greater force when we met again in 2007 in London. I then realized that I should probably not be surprised, after all. The immediacy with which we found common language, the similarity of our tastes and ways of thinking, all this could be accounted for with our common Jewish East European roots. You and your family, Hamsa, have a different history. What about your first encounter with Jill?

Hamsa: Jill was the first academic that I spoke to at the University of the Witwatersrand. She was to me, as she has been to so many others, welcoming, supportive, and fun. I recall my early conversation with Jill, related to finding my way to her house, in a Johannesburg suburb called Melville, for an evening with other students and staff in the Marang Centre for Mathematics and Science Education at the University of the Witwatersrand soon after moving to South Africa. "Do you know Melville at all?" asked Jill. "The only place I know in Melville is the Country Club," I said. And then Jill, screaming with laughter, exclaimed, "Oh my god, you've moved from England and you think you're the Queen, don't you!"

We talk more. She tells me that I am much posher than she is. I protest, on the basis that she is the one who listens to classical music (a proxy of some certainty in relation to class in my opinion), and tell her that I went to a "sink" comprehensive in London. "And anyway" I tell her, "Melville is all liberal shabby chic anyway."

Anna: But perhaps our sense of familiarity, our tendency to use old lenses while looking at the new and foreign blinds us to what is really important. I recall Jill telling me one day about how disastrous the situation in South Africa was with regard to mathematics education. To this, I replied that it was so also in Israel. She then said that I probably didn't know what I was talking about. And indeed, my visits in SA schools convinced me that as bad as the situation might be in other countries, South Africa had its own kind of "bad." Once I started visiting schools in Johannesburg, and especially those in townships, I had to admit that Jill was right. It was now clear to me that what Tolstoy said about unhappy families can also be said about failing education systems: each case is unique, each of them expresses itself in its own way. Ever since then I have been collaborating with Jill in trying to pin down the particulars of the South African problem. Based on our conversations, I believe that this has always been the first aim of her work. I recall Jill scanning examinations scores of students from schools she had been working with and then exclaiming in utmost exasperation "Why, on earth are these kids failing so badly in spite of all these efforts ?"

Hamsa: Yes, Jill's work over three decades has been concerned with delineating the South African landscape of mathematics teaching and learning—a landscape

that was, and continues to be, riven with the brute inequities of apartheid. And, of course, this is not just "delineating." Whatever she does is only as good as the tangible difference it will be able to make.

Look at Jill's current work as one of the South African Mathematics Education Chairs. This project involves leading a longitudinal research and development project in ten government secondary schools. The remit is to research the terrain and design and implement interventions that have the potential to work at scale. As such, it explicitly requires the combination of activist and academic orientations. I lead a similar project working with ten government primary schools. Both Jill and I have chosen to work with teachers as the key levers of change and we are particularly interested in teachers' mathematical discourse in the context of instruction. We work in adjacent offices. We talk frequently—often excitedly, sometimes despondently—about what we have seen in classrooms where we are working alongside teachers. We deliberate on what we have seen and about what else we can do to change things for the better.

Anna: And this is the project that has brought the three of us together. It was the questions of possible ways to improve mathematics education in South Africa that dominated our conversations during my 5 week long stay in Johannesburg in 2012. Those encounters provided me with a great opportunity to observe Jill's "peaceful struggles."

Struggling for a Language in Which to Talk as Researcher

Hamsa: Because our lenses as Chairs are on the teachers, the mathematical discourses of interest in our work have been at the level of instruction rather than at the level of learning. A key aspect of our collective conversations around mathematics education has been with describing what Jill and I have called "mathematical discourse in instruction" (MDI) (Adler & Venkat, 2014; Venkat & Adler, 2012). This notion has been developed to talk about some of the particularities of the South African mathematics education context. One salient aspect of this particularity is that the incomplete, fragmented mathematical discourses known to be relatively common at the level of learning in the developed world seem to prevail at the level of teaching in South Africa.

In developing the construct of MDI, Jill and I drew on your language, Anna, for recognizing and describing mathematical discourse—an archeology that involves mathematical objects, visual mediators, narratives, and routines (Sfard, 2008). Your studies provide examples of mathematical discourse that "stray" in different ways from a discourse that the community of mathematicians might describe as mathematical. But in your work, these "nearer" and "further" discourses are generally exemplified with excerpts of *learners*' conversations and interactions around mathematics. What we need are tools for understanding the work of *teachers*. The notion of MDI that Jill and I are developing uses your terminology in the context of teachers' mathematical work, and thus extends the sphere of consideration to the

tasks of teaching—incorporating examples, representations, their transformations, and explanations.

Anna: I certainly have a lot of sympathy for this project. After all, I came to Johannesburg with exactly this aim in mind: to think with you together about the language in which to speak about the relevant phenomena. I also share your belief that the discursive conceptualizations and methods used so far mainly in investigating students' learning embrace the promise of being helpful also in studying the work of teachers. Call this belief my personal bias, but now, after we've been joining forces in thinking about Jill's data, it has earned some tangible support. While looking at what was going on in the classrooms through a discursive lens, and thus in minute detail, we got a better sense of some South African versions of mathematical "unhappiness" and of their historical sources.

Struggling to Capture the Gist of the Unique South African Problem

Hamsa: Our focus on the teacher's discourse and its specific features soon began paying off. We started noticing phenomena that had gone unnoticed until then. In our school visits and in our postgraduate students' early data from primary and secondary classrooms, we saw frequent examples of what we marked with the label "disconnections in teaching." We had the name, but we still struggled to find a language to describe these disconnections accurately and in detail. Part of the problem was that disconnections were seen at different levels in different portions of data. In a few instances, there were mathematical errors within teachers' handling of specific examples, but more often, disconnections were seen in teachers' explanations of the mathematics involved. These disconnections could not be straightforwardly ascribed to the low level of teachers' mathematical content knowledge. Although this explanation can often be heard in the public discourse on education in South Africa, Jill and I view it as overly simplistic.

In the search for a deeper insight, we worked on MDI in the context of specific examples. We focused on teachers' "accompanying explanations" and the sequences of representations and transformations that were presented. Two vignettes of disconnections that we have discussed and written about are described below.

Vignette A. Venkat and Mhlolo (2011) analyze an episode in Grade 11 where a teacher introduces and discusses a table depicting the relative frequencies of families with various numbers of children. Following up on a learner's earlier mention of "tally tables" as a way of presenting data, the teacher subsequently proceeds to add a third column to her frequency table, titles it "tallies" and then produces the tally from the frequency value. She emphasizes the diagonal line convention for the fifth stroke, and reminds learners: "Your tally and your frequencies must be of the same number."

Vignette B. "Halving" is the topic being dealt with in a Grade 2 class. The task in focus is working out "Half of 26." Each pair in the class is asked to make 26 balls from clay—which they do. The teacher's explanation proceeds as follows: "I want us to count to 13, and move those balls aside. How many balls are on the other side? 13 as well. So 13 is half of 26" (Venkat, 2013, with the vignette based on fieldnotes from classroom observation).

In these vignettes, Jill and I focused on the explanations and the sequences of representations and transformations "accompanying" the examples. In vignette A, we noted the lack of connection between the stated problem, which relates to teaching tallying, and the signifier (the frequency table) that was presented as a basis for the tallying process. The teacher made no reference to tallying as a way of organizing and collating raw data, which could then be summarized further into frequency values. Instead, frequency values were endorsed as appropriate signifiers for tallying processes to operate on. We concluded with a comment on the extent of the problem in this episode:

> "whilst equivalence between the tally graphic produced for each frequency is maintained, connections that could serve to establish the purpose of tallying processes in mathematics are not simply made invisible, but actively disrupted." (Venkat & Adler, 2012, p. 31)

In vignette B, the endorsing narrative for the value 13 that arrives mid-stream in the teacher's explanation consists of a verification involving a check for "13 on the other side as well." The only routine made available to a child who does not know the answer at the outset is thus to "guess and check" a range of options until the verification step produces alignment. There is an absence of a derivational narrative that connects "halving 26" with "13." And yet, although the ordering of the table in the first episode with frequencies preceding tallies is a little odd, in neither of the two examples can the final representations on the board be said to be mathematically wrong. Rather, they are pedagogically problematic, and it is teaching discourses that do most of the disrupting work.

Jill tended to approach the episodes with an explicit theoretical frame. I tended to have a more inductive take on the data. We noted areas where the well-known distinction between procedural and conceptual understanding (e.g., Hiebert & Lefevre, 1986; Star, 2005) seemed to be helpful. But we also saw that much of our data, while certainly falling outside the remit of work that could be called "conceptual," seemed also to fall outside of some of the descriptors of "procedural" working.

Underlying the descriptors of "conceptual" working in the literature, there is recurring reference to the notion of "connection." We delved into this literature in more detail, but found, once again, that the "grain size" or "zoom lens" of connection was frequently longer or broader than the grain size one had to consider to fathom the problems we were seeing. This literature discussed connection as being at the heart of "coherence" within mathematics teaching—with comparative studies indicating the usefulness of backward reference across lessons (Sekiguchi, 2006). Former research also points to the need for purposively designed "example sequences," with teacher discourse drawing attention to invariances across example

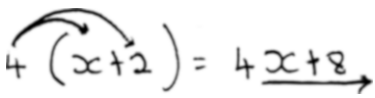
spaces. This kind of focus is necessary to help the learners towards abstractions and generalizations (Watson & Mason, 2006). In the examples above, connection breaks down *within* examples, resulting in our need to look at teachers' mathematical narratives in shorter timeframes than the multiple examples within lessons/multiple lessons that are in focus in the international literature.

In deliberating about this and similar problems, Jill and I spent much of our time interrupting each other. Each of us finished the other's sentences in ways that the instigator did not agree with it. And we laughed at ourselves in the process. But while doing so, we also sharpened the tentative common language and started to notice more clearly the differences at the level of our empirical studies that necessitated moves and adaptations from the international literature at the theoretical level.

Another empirical vignette drawn from the follow-up work that Jill and I did on MDI (Adler & Venkat, 2014) is useful at this point. In one of the excerpts from Jill's project we saw a ninth grade class doing some work on rules for multiplying expressions with exponents, and then the teacher's setting of the following two examples for whole class work:

$$5.\ 4(x+2) =$$
$$6.\ 4x(x+2) =$$

In contrast to the earlier two vignettes, the teacher's narrative within each of these examples was not overtly disruptive. In fact, in many ways, it was comfortingly familiar. This was true also for the symbolic presentation of instructions, which in the case of $4(x+2)$ looked like this:



The broader example sequence used a structured pattern of variation with attention across examples to gradually expanding the example space of products of expressions. But we remained dissatisfied. Our reasons had to do with subtle aspects of the teacher's discourse, visible particularly in those cases when a learner offered an incorrect answer. For instance, when $4(x+2)$, after being presented as $4x+8$, was said to be equal $12x$, the teacher offered this narrative:

Vignette C: "We can't add $4x$ and 8 because 8 does not have the variable of x." She stated that because of this, $4x$ and 8 were not like terms.

In our analysis, we interpreted this explanation in terms of "'actions on symbols' based on perceptually visible features (how these 'look')," and noted that in your terms, Anna, this was "a marker of ritual rather than exploration" (Adler & Venkat, p. 140). Similar recurring narratives involved phrases such as "we multiply each and every term inside the bracket by …" and reference to "numbers" that "have or carry variables" (Adler & Venkat, 2014, p. 143).

Using the language characteristic of a discursive approach (drawing from the one you use, Anna) Jill turned our attention to the teacher's word use and substantiations across this example sequence. With this apparatus, we focused on the nature of the teacher's narratives and noted the following:

> "while mathematical words like *variable*, *power* and *exponent* are used [. . .] they are in phrases that refer to actions on disconnected symbols. Exponents are disconnected from their bases [..]; and variables are disconnected from coefficients [..]. Symbols are acted on (things are 'done' to them—'we did the exponents'; [..] as parts, and not as holistic algebraic objects" (p. 143)

Anna: I think I would go even further and say that the teacher was manipulating the signifiers (signs on paper), not the mathematical objects that were signified (quadratic functions signified by the two expressions).

Hamsa: Right. In the recurring phrases that we pointed to, there was evidence of the actions on disconnected symbols, dependent on perceptual cues. In this lesson we also saw that the "continuity" of the focus on products of expressions across the example space was disrupted by a MDI which described later examples as sites for something "new" rather than a mere expansion of the current procedure of "opening the brackets" (applying the Distributive Law, that was already in use in Questions 5 and 6). A case in point was Question 9, $(x + 2)(x + 3)$, which was preceded by the teachers remark that the class was now going to do something "completely different." This talk was accompanied by the writing up of a new heading: "Distributive Law."

Anna: So a monomial expression multiplying a bracketed binomial expression is not part of the "Distributive Law" for this teacher?

Hamsa: Yes, that is what the MDI suggests. The Distributive Law is described as applying only to a binomial expression multiplying another binomial expression, and this in spite of the fact that the $(x+2)$ expression recurs in Question 9 following its presence across Questions 5, 6, and 7.

Looking across the vignettes allowed us to draw some interesting contrasts. In this last excerpt, we noticed that although MDI that presented all the tasks as symbolic manipulations had its *limitations*, it was not *disruptive* in the same way that the MDI in the first two vignettes were. Students in this class, imitating the teacher's narrative, could produce the correct answer to similar problems. This is not possible in vignette B, where the disruption allows no progress at all on similar problems (beyond guessing and checking), unless the child (like the teacher) knows the answer at the outset. Vignette A, perhaps even more seriously, presents tallying as a process that acts on frequency values, and thus presents tallies and frequencies simply as disconnected elements to be covered within data handling work rather than as useful compression processes. As Jill had pointed out in her analysis of student responses to testing done early in her project, guessing seemed to be a highly prevalent activity in many of the classrooms she and her team were working in Adler (2012). So instead of the "disruptions within examples" seen in the first two excerpts, here we saw "limitations within examples" and "disruptions between

examples." The teacher was breaking or fragmenting potential continuities between examples.

The three vignettes are all problematic, and yet, they are each problematic in different ways. In vignette A, the process enacted is mathematically inappropriate in relation to frequency values. In vignette B, the correct answer appears in the absence of a deductive process. In vignette C, the processes are correct in spite of being ritualized, allowing example sequences, rather than single examples to be brought within the frame of consideration of MDI. In this sense, there is greater global coherence in the teaching sequence in vignette C as MDI has greater temporal and thematic scopes.

Some of the themes in the international mathematics education literature have been useful to us—like your notion of ritualized discourse Anna. So were the numerous publications that have criticized mathematics teaching for "fragmenting" mathematics—e.g., Haberman's (2010) description of a "pedagogy of poverty" in which students come to be "merely engaged in the pursuit of isolated facts" (p. 85). But as your paraphrase of Tolstoy's famous saying points out, Jill's search has been into the particulars of these occurrences in the South African terrain. And this has required both invention and adaptation of a theoretical language—for good research related to the phenomena and good activism that works from the "ground up."

Anna: Yes, I agree with both parts of your story. In particular, I second your stress on the importance of the language you speak in as researcher. In my professional life, I am guided by the simple maxim: 'If in spite of all your efforts you don't mange to change the reality, try to change the language in which you think about it.' This is exactly what Jill and you have been doing: you have been working on the new lens through which the mathematics classroom would present itself differently, hopefully revealing the roots of the problems with which, so far, you have not been able to cope to your satisfaction. The multi-level disconnections in mathematics teachers' discourse, which became so clear to you thanks to your new lens, is, no doubt, a symptom to which one should pay closest attention while planning for a better future.

It seems that in the course of our "triangular" collaboration, your MDI was subject to yet another extension, one that generalizes your observations beyond those regarding example spaces. You have briefly mentioned the phenomenon of *ritualization* that can often be observed in classrooms. For me, ritualization is the gist of the matter. It is a kind of feature that, in spite of everybody's best intentions, is being transmitted from one generation of teachers to another, preventing the students from being exposed to the type of mathematics they are supposed to learn. This phenomenon is crucial to my understanding of the South African problem with school mathematics.

But let me begin from where all this started for me, that is, from my first visit to a school in one township. Prior to this visit, I expected a huge difference between what I was going to see there and what I had known from classrooms in other parts of the world. I was thus surprised by the sense of familiarity that overwhelmed me after a few minutes. True, the 11th grade class I was visiting was bigger than those I had been used to, and the old caravan in which the lesson was taking place was

clearly too small for this large group of young people. It is also true that these learners, unlike their peers in many other places, had only a small number of textbooks and worksheets to share between them. Other than that, everything looked and sounded like mathematics lessons with which I was so well acquainted from other parts of the world: the teacher was explaining some rules,[4] giving examples and, from time to time, probing learners' ability to follow, recapitulate, and reapply the procedures in new examples. Some of her questions were answered as expected, some of them required the teacher's intervention before the "correct" answer could be formulated. I left this classroom with a sense of *dejá vu*. This story repeated itself during my visits to other schools and other classrooms. It was not until the fourth or fifth such encounter that I started having doubts about the mathematics discourse of even the highest achieving learners. And then, when I began to study video recordings of the lessons, my doubts extended to the mathematical discourse of the teachers.

The turnaround happened when I started paying attention to the unwritten rules the teacher and the students were following in their mathematical storytelling. *Ritual* was the first word to come to my mind when I attempted to describe—to oneself and then to others—what it was that I was seeing. In my work, I speak about ritualized discursive activity when I have evidence that the learner is just retelling stories heard from others, narratives she has stored in her memory as separate items, without connecting them one to another. More often than not, these stories are summoned from memory by superficial linguistic clues that the person has associated with them for some reason. Only too often, they are retold in imprecise or outright incorrect forms. The other end of the spectrum of possibilities constitutes those discursive activities that can be described as *explorative*. One can use this latter descriptor whenever there are signs showing that a person makes a sustained effort to *derive* new narratives from stories already told. The relation between the idea of ritualized discourse and your notion of disconnection is thus clear: ritualized discourse is one in which every narrative is a stand-alone element, unrelated to any other; this, as opposed to its being a part of a system of narratives, deductively interconnected with the others.

Of course, one's mathematical activity is rarely just ritualized or just explorative. For instance, one can be an eager explorer of numbers and numerical operations but act in the ritualized manner while dealing with functions. Moreover, nobody can escape temporal ritualization. Indeed, I believe that ritualization is the necessary first step toward one's participation in a new mathematical discourse. It is from here that the explorative discourse will eventually develop, if at all. The problem begins when the move from ritual to exploration is blocked, for one reason or another. This seemed to be the case in the majority of Johannesburg classrooms that I had an opportunity to observe. Once I realized this, it was not too difficult to concur with you and Jill on the source of the problem: The blockage of ritualization had its roots in the teacher's mathematical discourse, which constituted the model

[4] The topic of the lesson was translating numbers from decimal to scientific notation.

for the learners to emulate and which, in itself, was uncompromisingly ritualized. Indeed, the classroom I was observing seemed free from any signs of explorative activity, and not only on the part of the students, but also on that of the teacher. The learners were merely reproducing the memorize-and-retrieve routines which the teachers were explicitly encouraging them to master.

But let me forestall possible misinterpretation: The story just told is not to be read as yet another narrative on teachers' deficit and guilt. To those who haste to dismiss the above observations as one of those patronizing accounts on teaching and teachers with which we are only too familiar these days, let me say this: While I can agree with the assertion about the "lack," I cannot accept the claim about the "guilt." Yes, the insufficiency of teachers' mathematical knowledge may well be one of the central causes for South African learners' insufficient achievements. But these teachers are not to blame. They are merely victims of the centuries long educational depravation. The results of such deprivation would not go away in just a decade or two, however determined one is to make the change happen. The teachers whom I met in Johannesburg did strike me as truly and deeply—in fact exceptionally—committed to their educational mission. If in spite of this their students were learning ritualized mathematics, the evidence suggests that it was because this was the mathematics of the teachers themselves. It would appear that ritualized mathematics was the only one to which the teachers were given an access within the deeply inequitable apartheid system. I concluded that it was this self-perpetuating nature of educational deprivation that constituted the primary source of the South African problem.

Struggling to Make a Difference

Hamsa: So we agree on the conclusion, but what are its practical implications? Does it even have any? Because if not, what is the worth of all this? Research and its outputs are insufficient from the perspective of the activist.

Jill has taken her findings forward into interventions that seek to provide openings for teachers to develop their MDI. She has focused on functions, a topic where school exit performance in South Africa shows considerable problems. These analyses have led Jill to instigate interventions that seek to support the development of teachers' MDI. She has used the "learning studies" model (Lo & Pong, 2005) to set up openings for groups of teachers in the Chair project high schools to work with an experienced mentor to collectively discuss and analyze the kinds of errors learners make on items dealing with functions, and use discussions about the "object of learning" to iteratively plan and adapt a lesson, drawing explicit attention to example sequences and explanations. Helping teachers to develop their word use in ways that align with mathematical, rather than colloquial discourses, through seeing the limitations of the more colloquial discourses across example spaces has been a key part of this work.

Anna: Yes, as researcher, Jill is trying to change the reality as she goes. In the face of centuries-long history of social injustice and wrongdoing, she does not have patience to wait for "the final" results of her studies, nor has she time for mere storytelling. She feels the urge to act here and now. Fortunately, Jill's research, although ongoing and still far from completed, leads to immediate operational conclusions. The fact that in some schools, not only the learners of mathematics, but also the teachers, may be unaware of the possibility of explorative mathematical discourse, makes it clear that probably the best thing an activist can do is to provide these teachers with formative explorative experience. Not surprisingly, this is what Jill seems to be aiming at while inviting her participants to mathematical conversations with herself and her Ph.D. students.

Activism in Academia: A Concluding Remark

It is not by accident that the word *struggle* features prominently all along this chapter and constitutes the leading motif in all the subtitles of this text. Struggling seems to be the story of Jill's life. She and her husband struggled against apartheid before and after they met, and ever since 1994[5] they have been struggling to create a new South Africa, free from the deeply rooted prejudice and social injustice of the past. In parallel, Jill has also been struggling to reconcile her conflicting identities of the impartial researcher on the one hand and of deeply engaged activist on the other. Another type of reconciliation she has been struggling for was between her academic commitments and her utmost dedication to the betterment of mathematics education practice.

Even if she has not yet emerged fully victorious from any of these struggles—and the truth is, there may even be no such thing as unequivocal victory in any of them—she has certainly made an enormous contribution to the future of her country by the very activity of deepening our understanding of South African problems and by promoting, in parallel, the vision of a new, better South Africa. In this context, one cannot but recall the words of another South African activist, the great writer Nadine Gordimer[6]: "The gap between the committed and the indifferent is a Sahara whose faint trails, followed by mind's eye only, fade out in sand." We can say with full certainty that Jill has been spending her life on just one side of Sahara—the one that belongs to the committed.

[5] This is when, under the presidency of Frederik de Klerk, apartheid was officially abolished.

[6] As it happens, Nadine Gordimer passed away just as we were finishing writing this text, on 13 July 2014.

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Chapter 8
Defining the Problems of Mathematics Education, Getting the Description Right and Making It Count: Jill Adler's Contribution and Beyond

Kate le Roux, Willy Mwakapenda, Craig Pournara, Nancy Chitera, Vasen Pillay, and Bruce Tobias

Introduction

Jill Adler has been active in mathematics education during a time of radical change in the sociopolitical context of South Africa; from the apartheid years and the struggle for democracy, to the establishment of a democratic government in 1994, to structural, policy and curriculum changes in schooling and mathematics education, and through the ongoing struggle for equitable access to quality mathematics learning in the first 20 years of democracy. This change has opened up spaces for collaboration with mathematics educators and researchers in other southern African countries such as Botswana, Lesotho, Malawi, Mozambique, Namibia, Swaziland and Zambia. These countries share with South Africa many of the challenges of providing quality mathematics education to all in contexts of poverty. Mathematics education reforms in this region have taken place in a broader context of

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international contestation over mathematics curriculum. In the process, the reforms of some in the political North,[1] such as a move towards more relevant and participatory mathematics, have gone "global" and been adopted as "local" in the political South (Adler, Ball, Krainer, Lin, & Novotna, 2005, p. 373). Describing her empirical interests, Jill explains the particular meanings that democracy and development take on in the latter context:

> In more macro terms, I am concerned with both the growth and development of mathematical knowledge and of democracy [. . .] Given this macro-framework, teaching mathematics successfully to all in school is a complex task. It includes: enabling *epistemic access* for all to *appropriate mathematical knowledge* in school, and enabling the *participation and inclusion* of diverse voices in the mathematics curriculum. (Adler, 1998a, p. 24, emphasis in the original)

In 2003, Jill and her colleague Stephen Lerman published a chapter entitled "Getting the Description Right and Making It Count: Ethical Practice in Mathematics Education Research" in the *Second International Handbook of Mathematics Education*. Their perspective on ethics in this chapter built on traditional views of "rightness" as systematic, rigorous and valid research and of ethics as informed consent, confidentiality and anonymity. It also built on more recent work (such as that of Howe and Moses, and Sowder) that recognises research as ideological and the need for researchers to have respect for truth, for research participants, and for democracy. The aim of the 2003 chapter was to generate a conversation within the mathematics education research community about the ethical (and simultaneously methodological, theoretical and political) challenges in mathematics education research. These challenges relate to defining what counts as a problem in mathematics education, making the research count both locally for the participants and more broadly for the mathematics education community, and getting the description right. "Right" here means conducting rigorous and valid research using theories and methods of the community in ways that produce a comprehensive and respectful description of the empirical space. The researcher's ethical duty extends to dissemination and consideration of how one's research may be taken up in the academic and public domains. Adler and Lerman emphasise that their focus is on the ethics of *mathematics* education research; their use of italics signals a key concern that being honest to the context may mean foregrounding the social and political and backgrounding the mathematics, and thus taking the focus beyond what some in the community might consider the scope of mathematics education research.

Adler and Lerman use the southern African context to illuminate these ethical challenges. They draw on their experience as postgraduate supervisors and on three research areas in the field (in-service professional development for teachers,

[1] Adler and Lerman (2003) express unease with their use, in the absence of better alternative, of the terms *developed* and *developing* to describe differences across countries. In this chapter we choose to use the terms *political North* and *political South* proposed by Janks (2010). These terms keep in view the histories that have produced the differences.

international comparative assessments, and research on technology in mathematics education) to ask, How does respect for practice, democracy, truth and persons play out as descriptions are produced? Are they right? For whom and how do they come to count? While they argue that the inequalities in this region illuminate ethical issues, they insist that these issues should not be marginalised to this context and challenge all mathematics education researchers to join their conversation:

> We insist, though, that whilst inequalities might be more stark in Southern Africa than in many other places, inequalities and injustices are just as pervasive and ubiquitous in every part of the world and within every society, if sometimes less obvious. (2003, p. 465)

More than a decade later in South Africa itself, inequities in access to quality mathematics and its associated symbolic and material resources persist, as does the dominant discourse of failure at the level of teachers and learners. We argue, therefore, that the conversation generated by Adler and Lerman needs to be carried forward. The six authors of this chapter are teachers, teacher educators and researchers who practise in southern Africa. Our own masters, doctoral and/or postdoctoral work was supervised by Jill and we have worked with her in other spaces she has opened up for us in the community. In this chapter we take the conversation forward in two ways. First, we look back at Jill's published research from the past 20 years, focusing on her books, book chapters and journal articles (published individually or with collaborators) that have come to count in the community. We trace what she has constituted as mathematics education problems in the southern African context and the field more generally, identify the ethical and related theoretical, methodological and political choices she has made in order to get the description right, and consider how and for whom her work has come to count. In the second part of this chapter we use our own research histories to respond to the challenge posed by Adler and Lerman to make explicit our decision-making with regard to getting descriptions right and making our research count.

Part 1: Getting the Description Right and Making It Count—A Review of Jill's Research

The Contextual World of Jill's Research

Adler and Lerman argue that researchers in the political South act in a world in which what counts as a research problem is defined by the political North; both empirically through a focus on "good practice" such as relevant mathematics and learner-centred pedagogy, and theoretically through a focus on the mathematics of these practices. The adoption of these problems in the political South, they argue, may produce descriptions of failure and deficit in this context. For example, the revoicing of mathematical English in contexts of limited English-language infrastructure may be interpreted negatively as teacher-centred. In addition, the choice

of empirical and theoretical problems has implications for the researcher's positioning relative to the researched and to the mathematics education community.

Jill's empirical and theoretical choices emerge from her understanding of the context of her work in mathematics education in South Africa. These choices are contextual and personal, like the work of teachers in her research (Adler, 2001a; Adler & Reed, 2002). It is also the answers to empirical questions about this context that have driven the development of her work over time. Thus we begin by sketching how Jill represents this context.

Jill consistently uses words such as "complex", "contradictory", "diverse" and "challenging" to describe the sociopolitical and economic context of post-apartheid South Africa in which the body of her research is located:

> The extent of diversity, inequality and poverty in the country presents South Africa with enormous challenges—challenges that can easily overwhelm. At the same time, each day, the many steps forward reinforce realistic possibilities of a more humane and democratic future, and within this, a qualitatively better education for all South Africa's children. The South African landscape that I paint, therefore, is complex, difficult and contradictory. However, I paint it with a brush tinged with optimism... (Adler, 2001a, pp. 17–18)

In this context, change means simultaneously working to repair the damage done by apartheid and to redress inequities of the past and looking forward to work with tensions between democracy and development in a globalised and technological future (Adler, 2002, 2005).

When describing micro-level mathematics classrooms in post-apartheid South Africa, Jill chooses similar words to those she uses for the wider context. These classrooms are complex spaces; the majority are multilingual, many are resource-poor (with resources being material, sociocultural and mathematical), and all are challenged to recontextualise a mathematics curriculum that takes up reform from the very different context of the political North. Mathematics classrooms in South Africa are also diverse in terms of resources, learner population and English language infrastructures. Development and democracy are in tension in these classrooms, the former focused on moving ahead with new mathematical ideas and the latter concerned about access and participation for all (Adler, 2001b).

The dialectic relationship between the wider ideological, socio-economic and political context and what comes to be in mathematics education is always in view in Jill's research. On the one hand she recognises the context as constraining, that is, context matters in teacher education and school classrooms (Setati & Adler, 2001). Yet the wider context is also enabling, offering possibilities, new identities and spaces in which to work (Adler, 2001a; Parker & Adler, 2005). Jill represents teacher educators and teachers as having agency and acting intentionally to manage the challenges in these spaces (Adler, 2001a; Adler & Reed, 2002; Hossain, Mendick, & Adler, 2013).

As we move beyond the advent of democracy, Jill recognises that we continue to hold the past (Adler, 2005). Yet, as the context changes, she foregrounds particular aspects of the context that require attention. For example, she stresses the importance of understanding relations between the macro- and micro-level contexts (Adler, 2000a) and in particular the impact of material poverty on the latter

(Adler & Venkat, 2014). These shifts over time point to Jill's growing concern that well-intentioned reforms that are driven by a social justice agenda and aimed at challenging the historical elitism of mathematics may, in fact, exacerbate the inequities of that past:

> Precisely because socio-economic inequality persists and is pervasive in South Africa, vigilance is required with respect to who has opportunity to learn what in the context of teacher education as much as in school itself. (Adler & Davis, 2011, p. 141)

Adler and Lerman note that the context of research itself, for example, multilingual classrooms, is "productive of particular accounts" (2003, p. 455) and are concerned that such accounts may portray the participants as being in deficit. Thus we end this section by commenting on how Jill chooses to present the challenging context of her research to her local and international audiences.

For Jill, getting the description "right" begins with a comprehensive description of the complexity of the context in which she works. Rather than choosing, for example, between development and democracy, or between explicit mathematical language teaching and giving learners opportunities to make mathematical meaning, she is interested in how to manage the inherent tensions and dilemmas (Adler, 1995, 1997, 1998a).

However, while Jill represents the South African context as unique in its complexity, she simultaneously represents this context as offering insights for mathematics education elsewhere:

> In the first instance, the South African context itself gives rise to questions and insights specific to prevailing local conditions. A consideration of the context throws a spotlight on the particular challenges in teacher education, which are nevertheless not unique to South Africa. (Adler & Davis, 2011, p. 141)

Indeed, we will argue in this chapter that asking questions in a range of complex settings in South Africa enables Jill, in turn, to ask unsettling questions to the political North about reform practices that have become commonsense in the latter context, for example, mathematical communication (Adler, 1997, 1999), resource-use (Adler, 2001b, 2012), and descriptions of procedural teaching (Venkat & Adler, 2012). Thus, while Jill's research has fed back into her localised context (Venkat, Adler, Rollnick, Setati, & Vhurumuku, 2009), she has also made the empirical sites of her research in South Africa matter (Adler, 2006) in such a way that her local research has come to count in the broader community.

The Constitution of Empirical Research Problems

Jill describes the problems she works on as empirical, or "problem[s] in the world" (Adler, 2000a, p. 31). Her broad concern is ongoing inequities in access to quality mathematics learning—or who has access to what mathematics—in the light of the wider sociopolitical and economic context. Jill views the teacher as both the object and the agent of change in this context (Adler, 1998a), and thus the teacher and

teaching are the focus of the five research projects we discuss in this section. However, this interest requires a dual orientation, for understanding how to work with teachers requires investigating what emerges in teacher education *and* in classrooms, across contexts (Adler & Pillay, 2007).

Jill's 1996 doctoral research focused on how teachers in a range of multilingual classrooms in South Africa give meaning to the concept of mathematical communication. This work is specific in that it has multilingual classrooms in view, as well as the mathematical nature of the discourse mediated by the teacher. How do teachers promote access to the language of learning, access to mathematics, and access to cultural practices of classrooms where talk is privileged? Jill was explicit about her intention to problematise taken-for-granted practices like code-switching (Adler, 2001a). Having answered her empirical questions about teachers' work, Jill in turn asked a question about teacher education; How can teacher educators empower themselves and their students to manage the tensions in teaching mathematics in multilingual classrooms (Adler, 1998a)?

Jill's collaborative project with mathematics, science and English-language education colleagues at the University of the Witwatersrand was located in an in-service teacher professional development programme (a Further Diploma in Education, FDE) aimed at both redress and development. The empirical research focus was on understanding how teacher education comes to count in context, by characterising subject, pedagogic and contextual knowledge across levels (primary and secondary), and contexts (rural and urban schools with different English-language infrastructures). For Jill, it was important to keep the specificity of mathematics teaching in view in a project that worked across school subjects. While this project was located in international discourses about curriculum reform and teacher education, it asked questions about take-up in the South African context of multilingual classrooms, severe lack of and inequalities in material, sociocultural and human resources, and rapid change to a reform curriculum (Adler, 2002). For Jill and her colleagues, answers to empirical questions about what teachers need to know to enable participation in mathematics inevitably lead to questions about what in-service teacher education should look like.

The FDE research focused on resources like the chalkboard, textbooks, time and language. These were the only resources available to some teachers in the programme, and are available to all teachers across contexts. The researchers' interest was not in labelling these resources as good or bad, for example, linking the chalkboard to transmission teaching. Rather, they asked questions about how resource-use across contexts opens or closes opportunities for participation. This empirical focus was an ethical choice, and can be contrasted with research focusing on technology as a resource, research problematised by Adler and Lerman as aimed "at the few, across the world as well as across schools, within technology-rich countries" (2003, p. 464).

The empirical focus of the Qualifications on the NQF for Teachers Underqualified in Mathematics (QUANTUM) project in South Africa emerged from answers to the questions posed in Jill's doctoral work and the collaborative FDE research. That research describes all teaching as dilemma-filled, with teachers acting with the material and sociocultural resources available in their contexts.

However, the choice of research questions in QUANTUM was also contextualised in a concern about who was learning what in teacher education in post-apartheid South Africa. What mathematical work do teachers do to manage the language related dilemmas? How and when do teachers and teacher educators use the resource of mathematical content knowledge, and what is constituted as a result? How do teachers and teacher educators manage the tensions when mathematics and teaching are the dual objects of learning in teacher education?

Thus in QUANTUM the empirical focus shifted to how the two objects of mathematics teacher education—mathematics and teaching—are constituted. Again the zoom was cast across diverse contexts and with an interest in what is made available to learn in both classrooms (e.g. Adler & Pillay, 2007) and in teacher education (Adler & Davis, 2006) and how these may be related. While this QUANTUM project suggests a shift in Jill's empirical focus to the *mathematical* content knowledge of teachers and her engagement with the broader body of research on mathematical knowledge for teaching, this does not mean that the social and political are out of focus. Rather, the mathematical work of teachers is contextualised; it is specifically mathematics *for teaching*, and is both personal and contextual work related to who the teachers are, what they are teaching, and where they are teaching (Adler, 2006; Adler & Huillet, 2008). Again, understanding mathematics for teaching was underpinned by questions about how teacher education should respond:

> If we know more about 'what' and 'how' mathematics is used in and for teaching across contexts, we will then be able to grapple with whether, how and where these practices are teachable, and then too who (what expertise) is required for this teaching. (Adler, 2006, p. 10)

In her position as Chair of Mathematics Education at Kings College London from 2007 to 2014, Jill turned the questions from QUANTUM South Africa to the context of teacher education in the United Kingdom in QUANTUM-UK. Like her research in South Africa, her empirical focus was on mathematically underqualified teachers and on reforms that emphasise deep understanding of mathematical concepts as against surface procedural knowledge. However, working now in a context in which research on mathematical knowledge for teaching has "mushroomed" (Adler & Patahuddin, 2012, p. 19), Jill broadened the scope of her empirical questions. Rather than just asking what it means to understand mathematics in depth or how such understanding can be developed in teacher education, the questions also focused on what the discourse of understanding mathematics in depth does in the context of the multicultural United Kingdom (Hossain et al., 2013). How does this discourse work to include/exclude participation in teacher professional development? What is possible for different participants?

To end this section we focus on the ongoingWits Maths Connect Secondary (WMCS) project headed by Jill in her position as a FirstRand Foundation Chair in Mathematics Education.[2] The empirical focus of this project on teachers' work in

[2] The Chairs in Mathematics Education are a private-public partnership aimed at improving teacher education and learner performance in South Africa.

school and on what forms of professional development can deepen teachers' knowledge of mathematics for teaching builds on Jill's earlier research, but is located in shifts in the sociopolitical and educational context of South Africa. Acknowledging again the substantial challenges that enduring poverty and inequality pose to democracy and development, Jill recognises the school as the only site of learning for the majority of learners in South Africa. Thus, in a context of increasing curriculum prescription, the teachers' mathematical discourse in instruction (MDI) and what mathematics is made available to learn during instruction matters deeply (Adler & Ronda, 2015; Adler & Venkat, 2014; Venkat & Adler, 2012). Jill and her co-researchers have thus developed analytic tools for describing mathematics teaching in detail *and* for interpreting differences in this teaching. In her most recent work, Jill continues to ask questions about practices that are taken-for-granted across contexts, in this case focusing on exemplification, explanatory talk, and learner participation. As in other projects, Jill has practice in view, for it is these practices, she argues, that are a productive starting point for work with teachers (Adler & Ronda, 2015). Indeed, under her leadership the WMCS project is engaging directly with the impact of the project's professional development programmes on learning gains in local classrooms, and initial evidence indicates some promising results (Pournara, Hodgen, Adler, & Pillay, 2015).

This discussion of Jill's empirical focus points to her ongoing advocacy both for equal access for all learners to opportunities for learning quality school mathematics and for equal access for all teachers to quality mathematics teacher education. Yet, as she notes in her discussion of the FDE research in Adler and Lerman, this advocacy cannot be separated from ideological conceptions about what counts as quality learning, mathematics and good teaching. Jill does not, however, accept these ideological conceptions as inevitable. Rather, as noted, she argues that it is precisely when practices appear as commonsense that they should be critiqued (Hossain et al., 2013). Hence her ongoing attention to making strange familiar practices such as code-switching, using a chalkboard and choosing examples and providing explanations.

The Constitution of Theoretical Research Problems

For Adler and Lerman the choice of theory and associated methodology is an ethical one related to both getting the description right (describing the empirical problem in a respectful and comprehensive way) and making one's research count (allowing one to talk in the language of the wider community). They argue that having the zoom of the lens tightly on the mathematical activity has come to count in the *mathematics* education research community. However, a tight focus on the mathematics in an investigation of reform practice in certain contexts may yield

only absences and descriptions of failure, and hence do damage to that context. While Adler and Lerman challenge the community to be more open to the wider social and political issues, they note that such research does not easily count in the field.

Jill regards theories as the tools for understanding her empirical problems in mathematics education (Adler, 2001a). Her theoretical and methodological questions discussed in this section are posed both to herself as a researcher and to the wider mathematics education research community to which she belongs as, Do we as a community know how to look at these empirical problems? (Adler, 1998a, 2000a, 2005; Adler & Huillet, 2008). Jill's earlier writing points to an additional use of her theoretical language; she proposes that teachers themselves use theoretical concepts like dilemmas, hybridity and transparency to talk and think about their own practice (Adler, 1998a, 2000b). This proposal is consistent with her representation of teachers as professional, intentional actors in their contexts of work.

The discussion of Jill's empirical focus in this chapter points to a shift over time to foregrounding the mathematical work of teachers at the micro-level of the classroom. However, we argue in this section that her choice of tools has allowed her to keep in view the social and political context of this mathematical work. We note that Jill recruited social and discursive perspectives from outside of the community before such shifts were named as "turns" in the mathematics education literature (e.g. Lerman, 2000; Morgan, 2006), and that she has continued to select and adapt perspectives from these turns as appropriate for her empirical focus. These theoretical choices are necessitated by the changing context of the political South; not only do her perspectives allow a comprehensive view of this context, but they also permit a respectful, non-deficit account thereof (Adler, 1998b, 2000a, 2002; Setati & Adler, 2001).

In her early work in the review period, Jill drew on social theories of mind, for example, Vygotsky's sociocultural theory, Mercer's theory of practice and Lave and Wenger's social practice theory, and adapted these for her mathematics education research. These choices allowed her to view school mathematics and teacher education as social practices, learning as social, and teachers and learners as constitutive of and constituted by their social contexts (Adler, 2000a). The interest in what is constituted as mathematics in and for teaching and teacher education in QUANTUM and QUANTUM-UK saw a shift to using Bernstein's sociology of pedagogy and to Davis' Hegelian interpretation of Bernstein's notion of evaluation. This perspective views mathematical knowledge as shaped by the institutions of schooling and curriculum and by the activity of teaching within these, but also has in view how the object of learning comes to be at the micro-level (Adler, 2012; Adler & Davis, 2011). In the QUANTUM-UK project, Jill also broadened the zoom of the research lens through her use of poststructuralist ideas on structure and agency (Hossain et al., 2013). On the other hand, her research in the WMCS project zooms in on the micro-level production of mathematics in the classroom with, for example, Sfard's concept of mathematical discourse and Marton's variation theory as tools (Adler & Venkat, 2014). Although zooming in, Jill acknowledges what is in the background:

> Of course, teachers' MDI is only part of a set of practices and conditions through which performance is produced, not least of which is social class and related material and symbolic resources in the school. (Adler & Venkat, 2014, p. 133)

Jill is thus explicit about what aspects of the empirical are foregrounded and backgrounded in her accounts, and acknowledges that these are a function of the tools that she and her co-researchers choose (Adler & Davis, 2011). She also challenges herself and the readers of her international publications to meta-reflection on the constraints of theoretical choices and whether these are indeed productive of comprehensive and respectful descriptions in their contexts of use. For example, her early turn to the social was accompanied by a concern that the community may draw unproblematically on theories from outside of the field, with the use of Lave and Wenger's community of practice identified as problematic for viewing the learning of school mathematics in classrooms (Adler, 1998b). Jill's view of mathematics teacher education as a social practice and of mathematics for teaching as living in institutions, dynamic and context-bound has allowed her to challenge more static perspectives on pedagogic content knowledge (PCK), subject matter knowledge (SMK) and mathematical knowledge for teaching (MKT) that have gained ground internationally (Adler & Huillet, 2008; Venkat & Adler, 2014). We return to these challenges to the community in the discussion of Jill's political work.

Adler and Lerman argue that the choice of empirical and theoretical research problems has implications for the researcher's positioning relative to the researched and to the academy. We now turn to how Jill has worked with the implications of her particular choices.

Getting the Description of the Teacher Right

Jill's descriptions of school mathematics in South Africa are a reminder of the substantial and ongoing challenges of improving access to quality mathematics for all. Twenty years after the first democratic elections, this context is saturated with deficit views of teachers and learners. Adler and Lerman highlight the tension between "tell[ing] it like it is" (2003, p. 454) in such a context and taking care of those researched. Working with this tension requires attention to how one describes the research and also how the research might be used, for example, to feed a political agenda of blaming the teacher. The FDE project used as an example in the 2003 paper produced accounts of how "more resources" does not always translate into better practice, accounts that could be used in defence of poor policy delivery and thus, unintentionally, result in the oppression of teachers.

With the teacher as the focus of her research in this context, how does Jill communicate her sometimes unsettling findings (Adler, 2001b) but at the same time take steps to avoid "hold[ing] them [teachers] up for criticism" (Adler & Lerman, 2003, p. 465)? We ask not only how respect for research practice, democracy, truth

and persons figure in her descriptions of the objects of her research, but also how she comes to advocate for what teachers have to offer the mathematics education community.

Jill's first resource in this respect is her theoretical work, since her choice of sociocultural theories enables descriptions of teacher' work as both contextual and personal. While teachers' action is recognised as shaped, for example, by their deep rural location or the local language-in-education policy, they are depicted as acting intentionally and as agentic in this context, being reflective and knowledgeable about their practice and as having something to offer the broader community. Using verbs rather than nouns, Jill represents teachers as "people in action" rather than as objectified people with fixed attributes (Sfard, 2008, p. 43). For example, we read about a teacher's re-sourcing in a particular context, that is, how the teacher recontextualises and appropriates resources (including his/her own knowledge) in a way that is both personal and contextual (Adler, 2001b). Jill's choice of the word "knowledgeability" is deliberate:

> Knowledgeability, contains within it, a sense of being knowledge-able. As a positive statement, I believe, it provides a conceptual tool that could assist us to shift away from a deficit discourse in teacher professional development. (Adler, 2000a, p. 3)

Jill draws on a number of methodological resources for her description of teachers' work. We note, first, her attention to both operationalising theoretical concepts used by the mathematics education research community in the particular empirical context of South African classrooms and to communicating this detailed analytic work in her publications (Adler & Ronda, 2015; Pillay & Adler, 2015).

In addition, Jill's choice of the "teacher-in-context" as the unit of analysis is given further meaning by her selection of teachers located in a range of schools. In South Africa she has worked in severely under-resourced schools in rural areas and in townships, but also in well-resourced private schools. In her writing with colleagues in the UK (Adler et al., 2014; Hossain et al., 2013), Jill chooses to focus on prospective teachers with a range of mathematical, cultural and educational backgrounds and different orientations to the dominant discourses. With these choices Jill brings into view what teachers in a range of contexts have to offer, while also challenging the mathematics education community to interrogate features of mathematics education that may have become commonsense.

Jill draws on multiple data sources such as interviews and observations to provide detailed case studies of teachers' work in these different contexts. Although she emphasises that her research products are written in her research voice (Adler, 1995, 1998a, 2001a), Jill provides rich descriptions of the complexity of the contexts in which teachers act and uses the teachers' voices to surface their knowledgeability in these contexts. Thus we read how individual teachers manage the tensions of using language-based practices in their multilingual classrooms, use the available resources, and recontextualise their learning from in-service development. We also have a view of how teachers talk and think about this work, for example, the teacher Helen in Jill's doctoral study:

> In retrospect, when I look at that lesson, I went on *but much too long* (laughter), on and on and on, and I keep saying the same thing and I repeat myself, on and on. . .. But the thing is then if you have a sense that there is shared meaning amongst the group, can you go with it? Um . . . when the sentence is completely wrong? . . . Can you let it go? Can a teacher use a sense of shared meaning to move on? (Adler, 1999, p. 61, emphasis in the original)

In summary, Jill's theoretical and methodological choices allow her to hold teachers up for understanding of their personal work in a complex context, rather than for criticism. She also holds up her ethical choices for scrutiny by the research community. Like the work of Cooper and Dunne discussed by Adler and Lerman, Jill's response to an ethical challenge is to make her choices and position explicit in her writing.

The Political Work of Making theResearch Count and Positioning Oneself in the Community

Adler and Lerman argue that getting the description right and "making a new perspective count in an unequal world" (2003, p. 456) are political acts. We end the discussion of Jill's research by considering how she has acted politically to make her empirical and theoretical work, emerging from the empirical problems of classrooms in the political South, count both locally and internationally.

We focus, first, on Jill's empirical interest in reform practices. These practices are promoted as good practice internationally and in her socially and politically charged local context have been taken up as offering solutions to the challenges of democracy and development. Yet, as noted, Jill uses her research on these reforms in her complex local context to pose unsettling questions about their use in the political North. She also casts teacher educators and researchers in South Africa in a political role of challenging official messages such as those on group work in the school curriculum, language-in-education policy, resourcing policies and teacher professional development (Adler, 2001b, 2006; Adler & Reed, 2002).

We believe that Jill is able to challenge her local and international colleagues in this way precisely by positioning herself as having the same goals as these constituencies, that is, to move beyond practices in schools and in teacher education that exacerbate inequalities. In addition, Jill's conversations with these constituencies are enabled by the theoretical language she provides for teachers, policy makers and researchers to talk about the practices in context, for example, the language of dilemmas, resources and teacher knowledgeability. Finally, our review gives a sense of the range of Jill's contributions (original research, reviews of the field, theoretical arguments) and the range of spaces in which she converses (national and international publications in the fields of general education, mathematics education, teacher education and language and education).

Next, we consider Jill's theoretical and methodological work in the international community. We have noted that Jill questions this community about whether the theoretical and methodological tools used by this community do, in fact, lead to

descriptions that are right in particular contexts. She opens up the space for these challenges by recognising the contribution to the community made by research that has preceded her and on which she builds, for example, research using cognitive theories of language, communities of practice, and PCK, SMK and mathematical knowledge for teaching. She challenges this work by highlighting limitations of these theories for viewing the specifics of her empirical context (Adler, 1998b, 2006; Adler & Davis, 2006; Setati & Adler, 2001) and by explicitly offering her adaptation of these tools for this context for use by the wider community (Adler, 1998a; Adler & Ronda, 2015; Pillay & Adler, 2015).

In addition, Jill engages this community in critical conversations about the development of the field of mathematics education research itself, for example, what constitutes *mathematics* education research (Adler & Lerman, 2003), and the nature of the subfield of research on mathematical knowledge for teaching and the extent to which this subfield has come to count in education research more generally (Adler, 2015).

Thus, in her empirical, theoretical and methodological work Jill positions herself within the mathematics education community by "engagement with others and with history" (Adler & Lerman, 2003, p. 465). She not only "meets" (p. 465) what is valued by this community, but also offers constructive critique of mathematics education reform and research from within.

Part 2: Getting the Description Right and Making It Count—Jill's Students Reflect

Most of the research projects discussed in this review were conducted in teams under Jill's leadership. While these projects sought to answer empirical and theoretical problems, they had the additional goal of building research capacity in the global South. We—the authors of this chapter—have all benefited from Jill's research leadership, and thus in the second part of this chapter, we write about the ethical aspects of our research stories. How did we, under Jill's supervision and postdoctoral support, strive to get our research descriptions right and to make them count? Our empirical sites in the political South vary, as do the specific ethical challenges that we choose to illustrate. Each story is personal and contextual, and thus written in the first person of the researcher. However, taken together, they illustrate our thinking at "fork-in-the-road" moments in our research and thus how Adler and Lerman's perspective on ethics in mathematics education research "lives" in our research processes and products.

Craig: Making My Doctoral Study Count in Two Communities

I began my doctoral study with clear intentions that the outputs of my research should contribute to the mathematics education research community and count for teachers – typically two very distinct communities. The empirical site of my doctoral study was a financial mathematics course, which I designed and taught, for pre-service secondary mathematics teachers at a South African university. The study focused on teacher knowledge and learning which involved revisiting school mathematics and learning new mathematics, both in relation to financial mathematics. Deep into the study and overwhelmed by the volume of data and by difficulties in refining my focus, I was tempted to narrow the focus of the study to students' learning of annuities because the data on students' learning of annuities was rich and varied, and sufficient for a doctoral study. I could therefore abandon the focus on revisiting school mathematics with little consequence for meeting the criteria of the thesis. However, I was never comfortable with that decision because it would reduce my work to a study of students' learning of annuities in which the students happened to be pre-service teachers, and this would likely eliminate the focus on pre-service teacher education. By contrast, retaining the focus on revisiting school mathematics raised the study beyond financial mathematics to explore teacher knowledge and learning in pre-service mathematics teacher education more broadly. While this decision made the analysis and write-up more difficult, the outputs have the potential to count in the community in relation to both financial mathematics and mathematics teacher education.

The challenge to make the research count for teachers is captured in a question posed to me by Palesa, one of the student teachers participating in my study: "The research, right . . . how is it helping? It is just a research being done and submitted or . . . is it going to help in schools?" Palesa's question, posed in the midst of my teaching of the course and data collection, was a sobering reminder of my initial deliberate intention to make my *research* count for *teachers*. This meant putting the research out in places where teachers are, where they read, and in ways that are accessible and meaningful for their practice. So publications in research journals, while the expectation of the academy, would not count in this regard. Whilst, I could merely upload a collection of resources on a website, this would not communicate the learnings from the *research*.

Given the empirical focus on teachers' knowledge for teaching financial mathematics, I wanted to share what my research offered about knowledge of mathematics, finance and teaching. I did not want to offer "teaching tips" or "exciting lesson ideas". While workshops provide opportunity to share my insights with teachers, I wanted something more permanent and with the potential to count more widely. So I chose to write four articles for the local South African mathematics teachers' journal, *Learning and Teaching Mathematics*. These articles focus on the use of spreadsheets for unpacking annuities, the compound interest formula as a model of the growth of money in banks, conventions in the use of timelines, and effective interest rates and their application to adverts of banking products

(Pournara, 2011, 2012, 2013, 2015). In these articles I explain the connections between mathematics and the financial context in ways that are linked to the curriculum. I do not expect that teachers will necessarily teach the background knowledge about these connections to learners, but rather that this knowledge will count for teachers by providing them with deeper insight, leading to greater confidence and more meaningful teaching of financial mathematics.

Willy: The Space to Develop My Research Interests in the Context of the Political South

My interaction with Jill and her work has helped me to identify and clarify my empirical and theoretical interest in a research space concerned with a conversation between and within contexts. My interest in doing mathematics across and within contexts emerged in a learning theories course presented by Jill in my masters studies. This was my first introduction to the notion of situated cognition, which is linked to the claim that the nature of a person's knowledge is inextricably tied to the contexts in which that knowledge was acquired. This exposure was the motivation for my empirical interest in how street vendors in the deep urban context of Hillbrow in Johannesburg use mathematics in their street-selling practices. My research brought into view how mathematics served as an organisational tool for the vendors and that they demonstrated great success in the use of mathematics in their selling practices, while they may not have demonstrated similar success in mathematics in school. These findings opened up spaces to publish, for example, Mwakapenda (1995), thus making my research count in the community, and also flamed my interest in further research on contexts in mathematics education.

The empirical focus of my doctoral research was the use of everyday experiences in teaching secondary mathematics in Malawi, a country that like South Africa is located in the political South. The key question was: To what extent can everyday experiences be used as a vehicle for changing the learning and teaching of secondary mathematics in Malawi? The focus emerged from a perspective that using everyday experiences in the classroom has the potential to change aspects of student learning such as interest and confidence in mathematics, participation in classroom discussions, engagement in critical thinking and reflection on the use of mathematics in everyday life. Like the scenario painted by Adler and Lerman in the introduction to their chapter, my doctoral research highlighted that trying to implement "real-life" approaches to teaching and learning, as promoted in international reforms, may be inconsistent with the contexts and conditions of schooling in a particular country. In-school and out-of-school (everyday-life) practices are inherently different in terms of goals and intentions of their participants, and the identities that are being promoted. In particular, Jill has elaborated that in school "students remain students until they leave", that only a few students may become mathematics teachers or mathematicians, and that teachers in the context of schooling are not practising mathematicians (Adler, 1998b, p. 169).

In my postdoctoral research supervised by Jill, I explored the use of concept mapping in researching student understanding of mathematics, this time in the South African context (Mwakapenda & Adler, 2003). The study explored first-year university students' understanding of mathematics, focusing on the extent to which students were able to make connections among key mathematical concepts in the South African school curriculum, a curriculum that has as a critical outcome that the student "demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation" (Department of Education, 2006, p.18). Adler, Pournara, and Graven (2000) identified in this curriculum various levels of integration, namely, integration across components of mathematics, between mathematics and everyday knowledge, and across school subjects.

Researching connections between mathematics and contexts has opened up a deeper awareness of the multidisciplinary nature of mathematics (e.g. Mwakapenda, 2010), and particularly to connections between mathematics and language. The latter more recent development in my research draws on Jill's grounding work on talk in the mathematics classroom as both visible and invisible (Adler, 1999). The fact that the language dimension is increasingly coming to the fore in my research is a response to the context of my current work as a mathematics teacher educator and researcher in South Africa, a context in which explicit emphasis on the role and complex diversities of language are recognised.

Bruce: Getting the Description Right Through On-Going Theoretical and Methodological Choices

Establishing neutrality and objectivity is intrinsically problematic to research, particularly when an individual researcher undertakes small-scale case studies. One response, proposed by Adler et al. (2005) is to:

> . . .develop strong and effective theoretical languages that enable us to create a distance between us and what we are looking at. (p. 372)

Seeking answers to my empirical interest in why secondary students generally struggle with word problems, I needed to access student mathematical experience. I turned to James Gee's discursive perspective, one that is largely oriented outside of the mathematics education research community. This theoretical perspective and associated tools offered the possibility of looking at an old problem in mathematics education through a fresh lens. That is, it allowed me to approach the problem from the students' point of view through an analysis of the text that they put forward about the mathematics.

I was soon to discover that I was dealing with two difficulties in my search for neutrality and objectivity. Firstly, the qualitative nature of my research meant that I faced the challenges of what Adler and Lerman (2003) identify as traditional views of rightness like validity and rigour. Secondly, getting the description right required

accessing the midlevel situated meanings (Gee, 2005) "residing" in the students' heads. In addition, Gee's linguistics-based framework had the potential for describing the Discourse of school mathematics, but developing an associated analytic framework in the context of *mathematics* education research emerged as problematic. While certain analytic tools arising from the theory neatly described aspects of a linguistic nature, they fell short in enabling commentary on mathematical activity, particularly in the form of students doing mathematical problems, what they wrote down, how they were able to express themselves symbolically, and what they did within the constraints of mathematical discourse. This mathematical activity, of course, was central to understanding my empirical problem, that is, what it is that students do that speaks to us about why they so often get it wrong.

One of Gee's primary tools, called "building activity", refers to how students talk about and convey their experience (through texts of varying form, even gesture and emotion) regarding the activity in which they are involved. Applied to the solving of mathematical word problems, this communication was only evident in what the students produced on paper. Thus during my analysis I became aware of the analytic problem of combining texts of how students "spoke" about a problem in retrospect with what they had produced in the written solution. In a pilot for my study an attempt to have students do a problem and simultaneously talk about what they were doing had proved unproductive. Now, having students do a problem and then reflect on it verbally also did not provide any resonance.

Had I persevered with my choice of theoretical framework (knowing or not knowing that it was inadequate), would I have generated a description of the problem that might have been deemed trustworthy by the mathematics education community? In Adler and Lerman's terms, would I have got the description right?

My response was to return to Gee's theory and to refine the analytical tools to more appropriately highlight student mathematical experiences in their doing of mathematics. Because of the rift between linguistic-based data and mathematics-based data this process was lengthy. However, a new analytic model resulted, the strength of which lay in how the data was processed so as to give insight into the mathematical procedures that the students produced. It went further than this. It gave a model that allowed me to process the data in systematic steps, and hence enabled me to place that data in a space that helped to distance me as the researcher from the direct context of that data. This extended analytic work thus contributed to my attempts to get the description right and make my research count in the *mathematics* education community.

Kate: Responding to and in Turn Introducing Ethical Challenges

The empirical context of my doctoral research differs from the varied contexts of Jill's own research, but the different contexts present similar empirical and theoretical problems. Located at an elite South African university, the first-year calculus

course in my study aims to support science students regarded by the institution as, on account of their educational background, needing additional support in the transition from school to advanced mathematics. Historically, such "foundation" courses have been a key response of higher education to the multiple national challenges of redress, access, democracy and development. In its promotion of relevant mathematics and a learner-centred pedagogy, the course in my study draws on international reforms in school and undergraduate mathematics. Theoretically, my study was positioned at the intersection of perspectives; the psychological perspective that has dominated undergraduate mathematics research and in which mathematics is an essential feature, and the turn to the social, discursive and political in school level research.

The empirical problem emerged in my practice as a new lecturer on this relatively established course. While researchers, including Jill and her students, were asking unsettling questions about the relationship between reform practices and access to school mathematics (e.g. Adler, 1997; Dowling, 1998; Nyabanyaba, 2002; Sethole, 2003), the relationship between these reforms and epistemic access to advanced mathematics was not visible either in my practice or in the undergraduate mathematics education research. My decision to ask critical questions in the site of my teaching practice required ongoing attention to ethical issues, with each response on my part seemingly opening up additional ethical challenges. I share my questioning and responses in this section. Since I continue to grapple with these questions as I publish my doctoral and postdoctoral research, I ask the questions in the present tense.

Firstly, what does it mean to adopt a critical research perspective in a space in which academic identities of lecturers (including my own) are underpinned by a strong social justice agenda and in which participating students have already been harmed by the complex history of the country and labelled as "educationally disadvantaged"? Challenged to present a comprehensive and respectful account of this context rather than a deficit view of lecturers and students in my doctoral research, I recruited theoretical and methodological tools from Norman Fairclough's critical linguistics, tools that were not developed specifically for studying mathematical practices. The sociopolitical practice perspective and tools of critical discourse analysis allowed me to describe and, crucially, explain the context as the interplay between micro-level action and the agency of students and lecturers, and macro-level sociopolitical structures.

However, this widening of the research lens raised the additional question, Can my research description be right and count if the *mathematical* action is out of view? Hence, my move to recontextualise constructs from mathematics education (those of Candia Morgan, Judit Moschkovich and Anna Sfard) for use within Fairclough's perspective. This time-consuming move produced a refined description of the micro-level action as a complex interplay of *mathematical*, discursive and sociopolitical action. While this description had the potential to count by using the language of other mathematics education researchers, it placed restrictions on the detail with which I could focus on one form of action. Furthermore, zooming out to describe the macro-level context rendered the course, with a poor record of

supporting students to graduation in the mathematical sciences, and the participants recognisable. Thus, how do I tell it like it is but at the same time take steps to do no harm to the lecturers and students?

I end by noting that, while the theoretical and methodological work of my study allowed me to talk in the language of other mathematics education researchers, Jill also opened up spaces for more direct engagement with these researchers. Not only did she include her doctoral students in visits of international researchers to South Africa, but she also made our research visible to the community through her careful choice of examiners for our dissertations. This support for participation in the wider community continues beyond the dissertation itself, in the form of joint publications and the opening up of conversations at international conferences.

Conclusions

Adler and Lerman position their 2003 paper as the start of a conversation within the mathematics education community about what it means to get the research description right and make it count locally for the participants and globally for the community. In the first part of this chapter we have continued this conversation by reflecting on Jill's research over the past 20 years. Our review of her ethical choices has been enabled by her constant attention to making her decisions explicit. This attention brings into view the interplay between her empirical, theoretical, methodological and political choices in, firstly, producing valid, rigorous, respectful and comprehensive descriptions. These comprehensive descriptions may be unsettling and "can easily overwhelm" (Adler, 2001a, p. 17), particularly with respect to the apparent inertia in addressing inequities in access to quality learning in school mathematics. However, Jill's respect for participants allows her to paint the context "with a brush tinged with optimism" (Adler, 2001a, p. 18). Secondly, she uses the various empirical, theoretical, methodological and political choices available to her to make her research count both for the teachers for whom she advocates and in the knowledge producing community in which she participates.

We have continued this conversation in the second part of this chapter, with Craig, Willy, Bruce and Kate responding to Adler and Lerman's challenge to make our ethical choices explicit. These personal responses illustrate the impact of Jill's respect for practice, democracy, truth and persons on the postgraduate supervision space, and hence her students. Our grappling with theoretical and methodological choices shows that Jill is not satisfied when her students' descriptions do not do justice to the interrelated social, political, discursive and mathematical aspects of the context. Our struggles identifying empirical research problems and places to publish point to how Jill challenges her students to make their research count locally, even if this increases the complexity of the research itself. Yet our review of Jill's work points to how her own research and engagement with the international community, simultaneously, lays the political groundwork for her students'

research in the global South to count beyond this context. Indeed, under Jill's supervision, her students have had the courage to enter this space opened by Jill and others—including by her co-author Stephen Lerman who is a silent partner in this chapter—and to challenge what has come to count globally.

We end by raising two challenges related to how we "engage with others and with history" as we attempt to "meet [and] … challenge, sets of socially constructed standards and values" (Adler & Lerman, 2003, p. 465). Firstly, if we are to make our research count locally, we need to communicate to stakeholders—who may only seek neat "sound-bite" strategies—the complexity of the context as well as doable responses. Secondly, if we are to make our research count globally, we need to challenge commonsense views not from a southern periphery but from the base laid by Jill, and not just with respect to ideology, but with evidence that we have acted on these challenges. We, as members of the mathematics education community, need to follow Jill's example in taking these issues forward in the conversation.

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Chapter 9
Variation and Contingency in Mathematics Teacher Education: Considerations for the Development of a Knowledge Base for Teaching

Paula Ensor

Introduction

This chapter engages with a key issue in mathematics teacher education (and in teacher education more generally): how we develop a knowledge base for teaching, and linked to this, how we advance teacher professionalism and teaching as a profession. Central to the discussion here is the contribution that Jill Adler, Zain Davis and their colleagues have made to the development of Mathematics for Teaching in the context of mathematics teacher education, a contribution which highlights in productive ways the complexity of developing mathematics teacher knowledge and professionalism.

The chapter begins by highlighting features of variation and contingency in teacher education, which I have discussed in previous work (Ensor, 1999, 2000a, 2000b, 2001). It then moves on to describe the research projects of Adler, Davis and others in order to foreground key manifestations of variation and contingency in mathematics teacher education, in relation to knowledge, pedagogic identity, content and recontextualisation. I conclude with the suggestion that the issue of developing a knowledge base for teaching may be much more complex than current commentary suggests.

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Variation and Contingency in Teacher Education

Two recent, interesting interventions in the area of teachers' professional knowledge, by Shalem (2014) and Taylor (2014), usefully foreground the notions of variation and contingency in teacher education. In different and complementary ways they attempt to get at the issue of teaching as a profession, teachers' knowledge and teachers as professionals. Shalem focuses on pedagogic judgement, through which teachers mobilise knowledge and experience in order to address the regularities and idiosyncrasies of teaching in classrooms. She uses the example of teaching English developed by Christie and Macken-Horarik (see Shalem, 2014: 101) who recruit Systemic Functional Linguistics to develop a theoretical framework for explicating the subject English in classrooms. Shalem argues for the need to codify professional knowledge such as the subject English in order to "bind" theoretical knowledge with pedagogic judgement. This codification holds the potential to systematise and specialise teacher education and lift it from its weakly institutionalised status as a craft. Shalem sees this codification as the pathway to developing a knowledge base for teaching.

Taylor (2014) concentrates on mathematics teaching, and looks at the different aspects of knowledge that might constitute a knowledge base for teaching. Both Shalem and Taylor regard the development of such a knowledge base as the prerequisite for the development of teaching as a profession, and echo a thorough and conceptually insightful review on teaching as a profession by Jeanne Gamble (2010). There is much to commend the argument of Shalem and Taylor, but both leave open the question: why is such an approach to codification not already in place, and why has it proved so difficult to achieve?

Taylor makes the point that the knowledge bases of social work, teaching and clinical psychology are incipient and growing, and along this path lies the potential for their professionalisation. But there are crucial differences here. Clinical psychology and social work exhibit national (and perhaps even international) regularities of curriculum and practice, and both are overseen by professional bodies. This is not the case with teacher education and teaching. Variation and contingency permeate teacher education practice, from the design of the teacher education curriculum to the uptake of this, if any, in school classrooms. As empirical studies described later in this chapter show, teacher education programmes manifest considerable variation in the content of both "theory" and "method" courses; in the ways in which students respond to the courses on offer; and in the way in which they recontextualise these practices into schools. This applies to both pre-service and in-service education. This lack of regularity is imbricated in the contingent character of teacher education practice, in that each aspect emerges as the localised intersection of knowledge, identity and context. This variation and contingency forms the focus of the discussion in this chapter.

The Research in Focus

Adler has been particularly prolific in research and development in mathematics education in South Africa, as a leading researcher in her own right, and as the supervisor of many graduate students. Three research projects are of particular interest for the present discussion: two Ph.D.s supervised by Adler, that of Mellony Graven[1] and the publications that emanate from this (Graven (2002, 2004)) and that of Parker (2008)[2] and the publications that ensue from this (Parker (2006) and Parker and Adler (2005, 2012)); and most significantly for my purposes here, the QUANTUM project led by Jill Adler and Zain Davis (Adler, 2008, 2009, 2012; Adler & Davis, 2006a, 2006b, 2011; Adler & Huillet, 2008; Davis, 2005; Davis, Adler, & Parker, 2007). It is not possible to engage with these projects in great detail and I have drawn selectively from them in order to highlight the themes and issues of contingency and variation which are of central interest in this chapter.

QUANTUM was a research and development project initiated in 2003 with a central focus on qualifications for teachers underqualified in mathematics. It had "as its central concern the question of *what* is constituted as *mathematics in and for teaching* in formalised in-service teacher education in South Africa and *how* it is constituted" (Adler & Davis, 2011: 139). Emphasis is placed on what the researchers term a principled description of what is going on in practice in teacher education, commenting that "(i)t is not an idealised or advocated set of contents or practices, but rather a description of what is recognised as content through our gaze" (Adler & Davis, 2011: 158). Putting this differently, Adler, Davis and their colleagues may be interested ultimately in the codification of mathematics for teaching (in much the way in which Shalem advocates) but in the first instance they are concerned to describe extant practices and how teacher educators (and teachers) constitute mathematics for teaching.

The theoretical and methodological framework for the QUANTUM project was developed substantially from work undertaken by Zain Davis in the course of writing his Ph.D. (Davis, 2005). In the first phase of the research, the research team focussed on higher education institutions that offered formal mathematics teacher education programmes. The data consisted of written assignments and tests gathered from 16 in-service offerings across 13 institutions in five of the provinces

[1] Graven (2002) locates her study in an in-service teacher education project intended to develop leader teachers in mathematics with the capacity to interpret, critique, and implement curriculum innovations taking place in mathematics education in South Africa at the time. Her purpose was to investigate mathematics teachers learning through an INSET programme in the context of very rapid curriculum change.

[2] Parker (2008) (see also Parker, 2006; Parker & Adler, 2005, 2012) conducted her research during the same period of rapid change in South Africa. In her case she focussed on pre-service mathematics teacher education and the implications of a changing policy environment for institutional arrangements (the merging of teacher education colleges with universities) and curriculum. Parker's Ph.D. is a wide-ranging study that captures teacher education, locationally, organisationally and educationally.

in South Africa, which had been collected in order to build "a comprehensive and robust description of how and what mathematics was being privileged across contexts of practice" (Davis, 2005: 279).

Four programmes were then selected for detailed analysis. A fine-grained apparatus for analysing data is presented in Adler and Davis (2006b), which enabled the categorisation of all formal assessment tasks across the sampled programmes. The research team was interested in the extent to which either mathematics and/or teaching was privileged in the tasks chosen for analysis, and the forms of reasoning ("unpacking") demanded by it. It was found that the mathematics privileged in the main was "the ability to demonstrate mastery of procedures and underlying concepts", the recall and reproduction of procedures taught on the teacher education programmes and the use of what the researchers describe as "compressed" or "unelaborated" mathematics (Adler & Davis, 2006b: 289). In other words there was little "unpacking" of mathematical ideas.[3]

Images from the QUANTUM Project

Adler and Davis (2011) develop three images or models of mathematics teacher education developed from three different teacher education sites which enrol teachers for in-service professional development programmes. They do this by identifying the legitimating strategies used by teacher educators in making pedagogic judgements. That is, they analyse the appeals made by teacher educators to legitimate their pedagogic arguments: appeals to mathematics, mathematics education, the everyday (also referred to as "metaphor"—the use of everyday metaphorical and visual representations of mathematical objects), experience of teaching, the school curriculum and authority of the adept teacher (see Adler, 2008).

On the basis of the legitimating strategies used by teacher educators, Adler and Davis (2011) construct three images, or identities, for mathematics teacher educators, which they describe in the following way: firstly "look at my practice" (students are encouraged to look at the practice of the teacher educator), secondly "look at your practice" (students are encouraged to reflect on their own practice to provide an image of teaching) and thirdly "look at (mathematics teaching) practice" (students are encouraged to reflect on the teaching practices of other teachers of mathematics, which are variable, to provide an image of teaching) (Adler & Davis, 2011: 140).

In the first case, "look at my practice", the researchers argue that mathematics teaching is presented as a practical accomplishment to be acquired tacitly, with

[3] The second part of the project considered five case studies of teachers in schools and the production of Mathematics for Teaching in those contexts, which is does not form part of the focus of the present discussion.

emphasis placed upon what is described as the "sensible" or "imaginary" (Adler & Davis, 2006a). This identity is achieved through the extended use of metaphor, visualisation and everyday contexts, and the privileging of mathematics as an inductive practice in which heavy reliance is placed on the empirical testing of routines and procedures. The teacher educator offers her own practice as an image of best practice. Adler and Davis argue that the principles that regulate the teacher educator's practice remain implicit and that the framing of criteria with respect to mathematics teaching is weak (Adler & Davis, 2011: 153). Putting this differently, this image of teaching is constructed through the dominant use of localising (or particularising) strategies (see Dowling, 1998).

The second model, "look at your practice", encourages student teachers to reflect on their own practice and expertise. Again the authors argue, the principles that regulate the teacher educators practice are not made explicit, but rely rather on shared experience. Legitimating appeals are made mainly to the experience of the student teachers, or the experience and authority of the teacher educator as an adept. This is similar to the first model in its predominant use of localising strategies.

The third model, described as "look at (mathematics teaching) practice", contrasts strongly with the first two in that in this case the teacher educator attempts to make visible to student teachers the principles that structure her practice. The focus here is upon "intelligibility" (rather than "sensibility") and the "symbolic" (rather than the "imaginary"). Legitimating appeals are largely drawn from what is described as "mathematics education" in order to interrogate practice. Mathematics is presented as a reasoned activity in terms of which "(t)he teachers were required to describe, justify and explain their thinking in relation to both what they brought to the discussion or observed and what they had read" (Adler & Davis, 2011: 150). Adler and Davis conclude that this reflects "strong pedagogic framing (control of the criteria by the lecturer)" (Adler & Davis, 2011: 150). In relation to the case described as "look at mathematics teaching practice" Adler and Davis (2006a) suggest that this model provides for "a specialisation of consciousness that is at once practical *and* theoretical, empirical *and* principled" (Adler & Davis, 2006a: 2 emphasis in the original). In this instance, use is made of what Dowling (1998) would term generalising and specialising strategies. In this third modality the teacher educator makes use of references to mathematics education literature and her own practice to make available to students as explicitly as possible the principles that regulate her own practice.

The deployment of legitimating strategies by Adler, Davis and Parker as a way of "getting at" the nature of teacher education discourse is potentially productive insofar as the appeals, or grounds, can be described further in terms of their abstracting or particularising dimensions. The research reported intimates a range of generalising and localising strategies being deployed; generalising strategies which entail discursively elaborated arguments from mathematics and mathematics education which extend beyond the confines of the teaching site; and localising strategies which focus on the "here and now"; upon the practices and experience of the teacher educators, and the students themselves, and which place emphasis on

the tacit dimension of teacher education. In the following sections of the chapter I use these three images to open up a discussion on teacher education more broadly.

Moments of Variation and Contingency

A number of important features emerge from the QUANTUM research which highlight the variation and contingency I pointed to earlier. I have suggested (Ensor, 1999) that there are three key moments in teacher education: transmission by teacher educators, acquisition by student teachers and transmission by student teachers as teachers in classroom. These three key moments in turn suggest three interrelated issues of theoretical interest through which the notions of contingency and variation might be considered.

- *Knowledge and practices* that are drawn on to constitute the pedagogic discourse of teacher education (either pre-service or in-service), that is, the discursive and non-discursive (tacit) resources that are selected and combined in constructing a teacher education curriculum (or, in Shalem's terms, pedagogic judgement), and the degree of specialisation of the discourse made available to student teachers.
- *Pedagogic identity*, the identities of teacher educators, of student teachers, and of the latter as they move as teachers into schools. Identity holds particular salience in teacher education, shaping and augmenting its contingent and variable nature. The effect of teacher educator professional identity is more far-reaching than in other professions, as it uniquely shapes the nature and form of the teacher education curriculum, in terms of what is selected, and how it is implemented. Student identity frames how student teachers position themselves in relation to what they are taught, and later, as teachers, how they put together their own teaching repertoires. Issues of identity are constitutive of teacher education practices, contributing to and consequential upon its contingent nature.
- *Context and recontextualisation*, in that teacher education characteristically entails two contexts: the teacher education programme context where teaching and learning takes place; and the school site of practice. Recontextualisation inserts itself at two stages in the teacher education process. In the first instance, mathematics teacher educators recontextualise from mathematics and mathematics education in constructing and delivering the discourse which they privilege as best practice (which I refer to as the *privileged repertoire,* Ensor, 1999, 2001). The second stage is the recontextualisation from the teacher education course by teachers in schools.

While I have separated out for analytic purposes the issues of knowledge, identity, context and recontextualisation, within teacher education practice all of these issues are deeply intertwined. Identity and context shape recontextualisation, and in turn the constitution of curricula and pedagogy. In the discussion that follows

each is considered in turn, drawing on the research of Adler and her colleagues to illustrate some of the key issues at stake.

Knowledge

Variation in Content

The purpose of any mathematics teacher education course is to communicate to student teachers (in the case of pre-service teacher education) or practicing teachers (in the case of in-service teacher education) a representation of best practice, or a privileged repertoire. This can refer to a mathematics method course, or a full mathematics teacher education curriculum, comprising mathematics, mathematics education and mathematics for teaching. Typically such a repertoire entails mathematics, approaches to teaching and assessing mathematics, the organisation of classrooms and the use of resources. This repertoire is communicated discursively (by verbal or textual engagement with arguments about the best approach to teaching) and also usually tacitly, via the modelling of best teaching practice. Each of the three images of teacher education generated by the QUANTUM project, discussed above, displays different privileged repertoires and this variation is evident in all of the studies under discussion in this chapter, including the studies of Mellony Graven and Diane Parker. As Parker comments:

> Teacher education is delivered at the institutional level, and the constitution of pedagogic discourses for specializing teachers are localized within the institution, determined by the specificity of the context and the operation of the distributive rule within that context, in relation to both the student teachers and their lecturers. (Parker, 2008: 560)

This variation in provision is strongly tied to the identities of teacher educators and contrasts significantly with professional education in other areas, such as engineering, medicine, and law, where provision is strongly regulated. Variation in teacher education curriculum is manifest in two ways: variation in content, as referred to above; and variation in the degree of specialisation of this content.

Variation in Degree of Specialisation of Knowledge

Mathematics teacher education programmes vary considerably in content, and also in the balance between the discursive and non-discursive (or tacit) aspects and the specialisation of both sets of practices (Ensor, 2000a, 2001, 2000c).

Shalem (2014) and Taylor (2014) prioritise the discursive aspects of teacher education discourse, those aspects that are linguistically retrievable and which hold the potential for codification. Shalem makes reference to the tacit dimension as one to be overcome, rather than as a necessary part of teacher education knowledge.

Both she and Taylor background the charismatic, intuitive, embodied, craft aspects of teacher education, the home to habitus, other than to indicate that these aspects should be restricted if not entirely overcome. A polarity between the discursive and non-discursive emerges which sets them up as competing options, rather than as two necessary aspects of teacher's knowledge.

Jamous and Peloille (1970, see also Delamont & Atkinson, 1995) in an interesting history of medical education in France, suggest that *all* professions entail what they term a technical/indeterminate ratio, a relationship between that which is codified and that which is tacit, or, as they put it, those aspects that are attributed to the "virtualities of the producers" (p.112). They trace the contestation over French medical education from the beginning of the nineteenth century to the late 1960s, showing how both the technical and the indeterminate form a battle ground to establish the boundaries for professional inclusion and exclusion, and how the comportment of a "good" medical practitioner is recognised. As Delamont and Atkinson (1995: 96) put it, "*(i)ndeterminacy* refers to elements of occupational competence and practice that are dependent on tacit knowledge. They are not susceptible to codification and representation as explicit rules or recipes. They are thought of as part of the personal quality of the competent practitioner" (emphasis in the original).

The work by Jamous and Peloille is useful in two respects here: they highlight the dual nature of professional knowledge through their technical/indeterminacy ratio, and through their investigation of medical education in France they contrast, by implication, the contingent, variable, volatile and charismatic nature of teacher education in South Africa, and in many other parts of the world.

In the work of Adler, Davis and their colleagues the issue of variation in the specialisation of teacher education discourse is exemplified in the three models described above, with a relatively higher level of specialisation in the third model than in the other two. Parker (2008) pays considerable attention to the formation of teacher education pedagogic discourse, and uses Bernstein's notions of classification and framing to describe these. Classification and framing are helpful in tracking curriculum and pedagogic variation synchronically and diachronically but are limited in their capacity to capture variation in specialisation of content and the specificity of context independence and abstraction. Discourses may be strongly classified, but this does not in itself imply that they are highly specialised. In other words, classification and framing alone do not enable us to grasp adequately the significance of the explicit and the tacit for teacher education pedagogy. This is an issue to which I will return later in the chapter.

Theorising Teacher Education Knowledge

What predisposes teacher education to the wide variation described above?

This issue can be addressed in a number of ways. Lerman (2001), Parker (2008), Adler (2012) and others use Bernstein (2000) to describe mathematics education as

a horizontal knowledge structure (rather than a hierarchical one) with a weak grammar. As Moore points out:

> Hierarchical knowledge structures are those that have a strong capacity to subsume knowledge at increasingly higher levels of theoretical synthesis and generality and horizontal ones are those that have a weak or limited capacity to do so. Physics is an example of the former and the sociology of education of the latter. This difference between and within knowledge structures is described in terms of strong and weak '*grammars* ' (this can be understood in terms of a capacity for 'meta-dialogue'.) (Moore, 2013: 144)

Horizontal knowledge structures have a tendency to grow through serial expansion. As Bernstein (1996) comments:

> A horizontal knowledge structure is not motivated by an integrated code but more by what we have called a collection code or serial code. The constraints on the production of this knowledge (a crucial feature of this code) create a series of expanding, non-translatable, specialized languages with non-comparable principles of description based on different, often opposed, assumptions. Difference in assumptions is not in *itself* a criterion for distinguishing *within* vertical discourses. Horizontal knowledge structures develop by *addition* of another specialised language. (Bernstein, 1996: 173)

Bernstein describes a strong grammar as "a capacity for 'meta-dialogue'", and those languages which have strong grammars "have an explicit conceptual syntax capable of relatively precise empirical descriptions and/or generating formal modelling of empirical relations" which are differentiated from horizontal knowledge structures with weak grammars "where these powers are weaker" (Bernstein, 2000: 163).

In general, teacher education discourse is not specialised to the same extent as mathematics (or linguistics or economics) and in this sense exhibits a "weak grammar". As Lortie (1975) has commented, teaching, unlike other professions, lacks a "technical vocabulary". Teacher educators tend to make available what I have described as a *professional argot* (Ensor, 1999, 2001), a way of discussing teaching practices that are usually lacking in specialisation and evoke signifiers that are ambiguous with respect to their referents. By professional argot I refer to the circulation of such expressions as "learner-centredness", "constructivist classrooms", "mediation" and so forth. The terminology used ubiquitously within teacher education and education more broadly cannot be unambiguously defined independently of context. Parker (2008: 403) discusses this in her thesis with reference to the "floating signifiers" that characterise teacher education discourse.

Lerman and others suggest that mathematics education (unlike mathematics, or physics) exhibits horizontality in that it lacks strong hierarchical integration and that mathematics education knowledge grows through the proliferation of theories and approaches that are usually incommensurable, rather than through internal accretion and reconfiguration. Much of this variation is tied to the preferences (or identities) of individual teacher educators, which explains, in part at least, the wide variation in the content of mathematics teacher education programmes.

Taylor (2014) takes a somewhat different tack, following Bernstein (2000) in describing teacher education as a region, "where knowledge is drawn from various disciplines and combined to serve a new professional purpose. Regions look

backwards to the field of knowledge production and forward to the field of practice: "Regions are the interface between disciplines (singulars) and the technologies they make possible (Bernstein 1990: 65)" (Taylor, 2014: 174). Teacher education curricula (educational psychology, sociology, philosophy, etc.) are constituted via recontextualisation from their associated disciplines or singulars.

We can draw on the contributions of both Taylor and Lerman to suggest that teacher education could be regarded as a region (in Taylor's terms), which is constituted segmentally by a series of horizontal approaches, which exhibit many of the structural features of horizontal knowledge structures and which, like them, have a tendency to proliferate (following Lerman). In other words, as a region teacher education can be regarded as internally differentiated into a set of distinctive approaches, usually identified with individual authors, which behave in much the way that Bernstein describes in relation to horizontal knowledge structures.

We can illustrate this point with reference to research on Mathematics for Teaching internationally. Adler (2008) and Davis et al. (2007) position their own work on Mathematics for Teaching within an international research domain, associating themselves broadly with other mathematics educators who have drawn on Shulman's work on pedagogic content knowledge. They show that over time a division in research focus has emerged between those researchers working specifically on the mathematics required for teaching (and they cite Ma and Even in this regard) and those that focus on teaching and the appropriate use of mathematics in this context (exemplified in the work of Deborah Ball and her colleagues). So we see here a proliferation of different approaches to a central problem, each approach linked to the name of its sponsor. The question then is how this proliferation is contained in order to stabilise a knowledge base, a point to which I will return later in the chapter.

Variation in teacher education can be described in another way, using the language of Dowling (1998, 2009). Dowling takes issue with the notion of hierarchical and horizontal knowledge structures, and makes two different moves. He firstly introduces the notion of discursive saturation—a claim about the extent to which a discourse (such as mathematics or mathematic education) can be grasped within language. The second move is to consider the degree of specialisation of content and mode of expression. Taking these two dimensions together Dowling argues that in relation to any pedagogic discourse there is a domain of practice which is generative, that encapsulates the principles, specialised comportments and modes of argument, which if made available to students and if successfully acquired, enable a novice to gain access to the means of contributing meaningfully to the activity. So, for example, access to mathematical principles within the context of school mathematics, understanding the links between topics and how legitimate mathematical texts are to be produced, gives students access to the ability to produce mathematical texts themselves. Dowling refers to this generative domain as the "esoteric" domain. I suggest that how we constitute an esoteric domain for any knowledge domain varies according to the form of its knowledge structure, and the strength of its grammar (or in Dowling's terms, the level of discursive saturation). So the issue arises: can we constitute a generalised esoteric

domain for mathematics teacher education, as we can for school mathematics, and if not, why not? This goes to the heart of the project outlined by Shalem and Taylor. If we are to generate a stable knowledge base for teaching, a generalised esoteric domain should be able to stand relatively independently of pedagogic identity or context.

Pedagogic Identities

Lerman (2001) makes the point that the most productive way of approaching research in mathematics teacher education is "to argue for a focus on identities and the settings in which those can change, as a way of conceptualising mathematics teacher development" (p.49). The creation of pedagogic identities is manifest in a number of key ways in teacher education. Teacher educators put together the mathematics teacher education courses they teach, drawing on a range of resources from mathematics, from mathematics teacher education research, from classroom teaching and so forth, and in so doing they display their own identities, and at the same time project identities for mathematics teachers in schools, school learners, and the student teachers on their courses. This knitting together of curriculum, pedagogy, forms of assessment and so forth, constitutes different and distinct views of best practice, projecting simultaneously images of good teachers, good learners and successful student teachers, as well as images of their opposites.

The QUANTUM project, and more especially the research of Diane Parker, illustrates in interesting ways the different identities of teacher educators, the differential positioning of student teachers in relation to teacher educators (whether they be pre-service student teachers, or teachers engaged in in-service teacher education) and the ways in which identities for school mathematics teachers, and learners, are projected.

Parker considers how official mathematics teacher education discourse constructs and projects identities for teacher educators, teachers and learners of mathematics, and in her study of two institutions offering mathematics teacher education, shows how different teacher educators prioritise different aspects of practice. Parker demonstrates the partitioning of practice by student teachers in both of the institutions in which she conducted her study, in terms of what "good" students should aspire to be, and what in turn they are expected to repudiate. How "success" as a student is constructed articulates with different social class and gender positions, and enters into the cycle of social reproduction.

Parker also focuses in detail on how student teachers in the two institutions position themselves in relation to the teacher educators they are engaged with. Parker uses the expressions "positive and negative identification" to refer respectively to those student teachers who identify strongly with the message of the teacher educators, and seek to apprentice themselves to it, and those who negatively identify, and act selectively on what is offered to them.

The work of Adler and Davis, and Parker, brings out strikingly the variable and contingent nature of pedagogic identities in mathematics teacher education, and their effects on curriculum and pedagogy.

Context and Recontextualisation

The discussion to this point has highlighted the ways in which teacher education programmes are differentially constructed in different higher education institutions through processes of recontextualising. Recontextualising is also at work as participants in in-service and pre-service teacher education courses draw from their teacher education courses in constructing their own teaching repertoires in classrooms.

In a study undertaken prior to the QUANTUM investigation, Adler (2002) describes how teachers recontextualise from an in-service programme in different ways. She highlights the partial, selective and uneven manner in which teachers appropriate resources from the course, such as code switching, the use of resources, the organisation of the classroom using group work, subject knowledge, and so forth.

In my own study (Ensor, 1999), I found that recontextualising from the pre-service course I analysed was contingent and emerged as the resultant of a number of intersecting factors: student teacher identity and opportunities to acquire the teacher education course on offer, educational biography, and school setting. Insofar as elements from the programme were recontextualised, beginning teachers drew on a number of discrete tasks and professional argot, which provided a way of describing processes in the mathematics classroom.

In their study of classrooms Adler and Pillay (2007) and Kazima, Pillay, and Adler (2008) again underline the variable and contingent nature of teaching, and the ways in which Mathematics for Teaching is constituted across contexts. As Adler and Pillay (2007: 99) comment: “The point we are making here is that the general elements of MfT as originally described by Shulman, and more specifically in mathematics by Ball et al. can be identified and interpreted in a range of classroom contexts, but these take on a specificity in particular practices”.

Conclusion

Engaging with the work of Adler, Davis and their colleagues has opened up a space for discussion of variation and contingency in mathematics teacher education (and teacher education in general). It has illuminated the vexed issues of teacher professionalism and the development of the knowledge base for teacher education and teaching, both of which are tied in with the development of teaching as a profession. I have suggested that variation and contingency is manifest at a number

of levels: in the constructing of teacher education curricula by teacher educators, in the positioning of student teachers with respect to the curriculum they are offered, and in the recontextualisation of these practices into school mathematics classrooms. These three moments display the highly contingent nature of mathematics teacher education, in contrast to those more strongly institutionalised professions such as medicine, engineering and law. Official regulation (via the state and/or professional bodies with robust accreditation processes) goes some way to constituting an "esoteric domain" in each of these professions. My own research and that of Adler, Davis and their colleagues suggests that the constitution of an esoteric domain for mathematics teacher education is contingent (and often highly idiosyncratic) and varies across institutions, programmes within institutions, and individual teacher educators. It is much more susceptible to the individual charisma, priorities and interests of teacher educators than is the case in other professions. The research of Adler, Davis, Parker and Graven very productively underlines this, highlighting the lack of specialisation of mathematics education as a knowledge area, both in terms of content and mode of expression, and the implications this has for how we develop a knowledge base, how we induct student teachers into the teaching of mathematics, and consequently for how classroom teaching unfolds.

Gamble (2010) argues that a precondition for the professionalisation of teacher education is "working towards the formalisation of an abstract knowledge base" (Gamble, 2010: 26). Yet how do we escape from the seemingly endless proliferation of new approaches (such that characterise horizontal knowledge structures) in order to achieve this? What is it about the knowledge and practices entailed in teacher education that this proliferation continues? The contributions of Adler, Davis and their colleagues have potentially opened the way for codification of Mathematics for Teaching to develop, but we already witness this is one of a number of approaches in the field. It is hard to see a way out of this interminable expansion without at some point the standardisation through professional regulation of all teacher education programmes. Taylor hints at the need for this in relation to the construction of student teacher identities. "... there is a choice to be made regarding the identity to be shaped. An endogenous approach assumes that the answer to the identity question is not arbitrary but that one particular knowledge persona is preferable for the profession, in that it gives adepts a key advantage over rival contenders" (Taylor, 2014: 179).

This all begs the question I raised at the beginning of the chapter: what is it about teacher education as a knowledge structure that elides the stability that would enable a coherent knowledge base to coalesce? If the many approaches in mathematics education to Mathematics for Teaching, or some variant of it (Adler, Ball, Even, Ma to name a few) are so prolific, and behave in the ways of horizontal knowledge structures, is professionalising teacher's knowledge possible? The chapter has not attempted to address this question, but rather to open up further the nature of the problem that requires resolution through a productive engagement with the work of Jill Adler and her colleagues.

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Chapter 10
Teachers Editing Textbooks: Transforming Conventional Connections Among Teachers, Textbook Authors, and Mathematicians

Ruhama Even, Michal Ayalon, and Shai Olsher

Introduction

Jill Adler began her academic work at a time when research in mathematics education mainly focused on studying the individual student's cognition and knowledge, and development work was mainly associated with curriculum development. Recognizing the shortcomings of this narrow focus for improving mathematics education, Jill was a leading driving force in advancing the field of mathematics education by expanding inquiry to include aspects related to mathematics teachers and teaching, as well as incorporating a sociocultural approach to research in mathematics education, in order to capture, rather than eliminate, the complexity of classroom teaching and learning, and of the professional development of mathematics teachers.

Shared interest and commitment to research and development work in the area of the professional development of mathematics teachers, and in incorporating a sociocultural perspective to this work, created opportunities for Jill Adler and Ruhama Even (the first author of this chapter) to interact at an international level during the last two decades. Like her writing (e.g., Adler & Lerman, 2003; Adler & Ronda, 2014), these interactions reflected Jill's deep respect and concern for teachers, her strong commitment to making a contribution to the community in which her research is situated, attending to policy and institutional context.

Acknowledging the need for research studies that are concerned with textbook use (e.g., Fan, Zhu, & Miao, 2013), and troubled by the small number of such

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studies in South Africa—a country in which the textbook is highly regarded as a useful and important resource for teaching and learning—a current focus of Jill's work is teachers' use of textbooks. Together with her doctoral student Moneoang Leshota, they investigate the relationship between the affordances of a textbook, and teachers' pedagogical design capacity in the mediation of the object of learning in the classroom. As part of the wider study, Leshota and Adler (2014) investigate teachers' mobilization of the textbook, demonstrating that the insertions and omissions that teachers make in the textbook play an important role in the kind of mediation of the object of learning that takes place in the classroom.

> While *impotent omissions* do not harm the object of learning, *critical omissions* on the other hand detract from the object of learning and therefore affect the end result of mediation. With respect to insertions, *distractive insertions* have been shown to have potential for being harmful to the object of learning, as they may lead to erroneous mediation; thus, if insertions to the content have to be made, they should be *robust insertions* which serve to enhance the object of learning (p. 295).

The *M-TET* (*Mathematics Teachers Edit Textbooks*) project[1] in Israel also acknowledges the importance of attending to teachers' mobilization of the textbook. But in contrast to Leshota and Adler that analyze it, teachers' mobilization of the textbook is used by us as a point of departure for examining how the conventional connections among teachers, curriculum developers, and mathematicians could be transformed into more productive ones, while contributing to teachers' professional development and to building a professional community of teachers. The M-TET project attempts to do so by inviting mathematics teachers to collaborate in editing the textbooks they use in their class in a work environment that is characterized by aspects that are usually not part of teachers' practice. These include designing a textbook for a broad student population instead of focusing on the specific student population taught, producing a textbook by making changes in a textbook designed by expert curriculum developers, and consulting with professionals who are not part of the teachers' usual milieu: mathematicians and curriculum developers.

This chapter presents the rationale underlying the M-TET project; it describes the unusual work environment offered to participants, and explores the nature of the connections of the participating teachers with a textbook author and a mathematician that participation in the project made possible. Finally, it discusses what could be gained by offering such a work environment.

[1] The M-TET project is part of the Rothschild-Weizmann Program for Excellence in Science Teaching, supported in part by the Caesarea Edmond Benjamin de Rothschild Foundation.

Teachers, Curriculum Developers, and Mathematicians

Conventional connections between mathematics teachers and curriculum developers are limited and mainly unidirectional—originating from curriculum developers and proceeding to teachers. The prevalent views and assumptions about the teacher's role commonly regard mathematics teachers as curriculum enactors and users of curriculum materials (e.g., textbooks) furnished by expert curriculum developers. In contrast to their central role in enacting the curriculum and in using textbooks, teachers usually play a rather insignificant role in developing textbooks. Indeed, some textbook authors are also teachers, and as part of the process of curriculum development, selected teachers are often recruited by curriculum developers to teach an experimental version of a new curriculum program in order to gather information about how students deal with the tasks posed, to estimate the time needed to work on tasks in class, and to construct a conjectured learning trajectory (e.g., Clements, 2002; Hershkowitz et al., 2002). Still, obviously, only a small number of selected teachers can actually participate in developing textbooks in the ways described.

This view of the teachers' role as enactors of the curriculum and users of curriculum materials is also reflected in research on the relationships between teachers and textbooks, which typically focuses on how textbooks influence classroom instruction and how teachers use curriculum materials (e.g., Eisenmann & Even, 2011; Haggarty & Pepin, 2002; Remillard, Herbel-Eisenmann, & Lloyd, 2009; Stein, Remillard, & Smith, 2007; Thompson & Senk, 2014). Consequently, the voice of the vast majority of teachers remains unheard and most teachers rarely influence textbook design and development. Teachers' aspirations about desired textbooks and adjustments that they make in textbooks—based on their experiences, knowledge, and beliefs about mathematics and its teaching and learning, as well as their acquaintance with the system in which they teach and with their own students—often remain unknown to curriculum developers and to the community of mathematics educators at large.

Conventional connections between teachers and university mathematicians are also limited. They occur mainly during the teacher preparation stage, when prospective teachers study advanced mathematics in courses taught by university mathematicians. However, teachers rarely have opportunities to consult with mathematicians about the mathematics they teach in class during their teaching career. Professional development courses and workshops for practicing mathematics teachers are usually designed and conducted by mathematics educators, and not by university professors whose main activity is mathematical research (of course there are a few exceptions).

Moreover, school teachers rarely initiate and lead interactions with mathematicians and textbook authors; interactions in which the teachers make decisions about the content, timing, and format of the interactions, as is illustrated in numerous publications dealing with the professional education of mathematics teachers (e.g., Even & Ball, 2009). Usually it is non-teachers (e.g., teacher educators, supervisors,

policy makers) who initiate and lead such activities, and they are the ones who make decisions regarding the content and format of the connections of teachers with mathematicians and with curriculum developers.

The M-TET project examines how the conventional connections among teachers, curriculum developers, and mathematicians might be transformed into connections that are more productive. Below we first present a general description of the project. Then we demonstrate the nature of the connections of the participating teachers with a textbook author and a mathematician that the M-TET work environment made possible.

General Description of the M-TET Project

Background

As a country with a centralized educational system, the Israeli school curriculum is developed and regulated by the Ministry of Education. Like in South Africa, in Israel the textbook is also highly regarded as a useful and even a central resource for teaching and learning. In 2009 the Ministry of Education launched a new national junior-high school mathematics curriculum (Ministry of Education, 2009). The new national curriculum emphasizes problem solving, thinking, and reasoning for all students as well as connections among mathematical concepts, topics, and domains. In response to the introduction of the new curriculum, the mathematics group in the Department of Science Teaching at the Weizmann Institute of Science began developing a new comprehensive junior-high school mathematics curriculum program entitled *Integrated Mathematics* (*Matematica Meshulevet*). The textbooks are developed in regular/extended and limited scope versions. The *Integrated Mathematics* textbooks are used in more than 250 schools throughout Israel.

The M-TET project, now in its sixth year, uses the *Integrated Mathematics* textbooks as a point of departure. During the first 3 years of the project (2010/2011–2012/2013 school years), teachers were invited to collaborate in editing the textbooks they use in their classes and to produce, as a group product, revised versions of those textbooks that would be suitable for a broad student population. A mathematician, the textbook authors, and researchers in mathematics education were made available to the participating teachers for consultation during the editing process. The activity in the last 3 years (2013/2014–2015/2016 school years) shifted to teachers' collaborative editing of the teacher guides for the textbooks used in their classes, maintaining a similar work environment. This chapter focuses on the first 3 years of the M-TET project, when groups of teachers collaborated in editing textbooks. The first author is the head of both the *Integrated Mathematics* project and the M-TET project; the other two authors are leading team members of the M-TET project.

Project Objectives

The main objective of the M-TET project is to examine how the conventional connections among teachers, curriculum developers, and mathematicians could be transformed into ones that are more productive, while contributing to teacher's professional development and to building a professional community of teachers.

Project Operation

Teacher's participation in the M-TET project consists of the following: (1) ongoing distance work and (2) monthly face-to-face whole-group meetings. During the first 3 years of operation, the ongoing distance work included textbook editing of various types (e.g., adding tasks, changing the phrasing or the order of the textbook tasks), reacting to other participants' suggestions (e.g., supporting, opposing, debating, and elaborating), and discussing mathematical and pedagogical issues (e.g., what approach is suitable for students with difficulties). The monthly whole-group face-to-face meetings were built on the preceding teachers' distance work of textbook editing, and these meetings also served as departing points for subsequent distance work. They consisted of collaborative work on textbook editing, instruction on the technological tool used in the project (e.g., editing, reacting, viewing the change history), discussions of mathematical and pedagogical issues (e.g., the role of technological tools in mathematics lessons), and discussions of community working norms (e.g., the issue of amending another teacher's editing suggestion).

To enable collaborative textbook editing and the production of a joint revised textbook, we used, with some modifications, the MediaWiki platform and Wikibook templates for constructing the project's website. This website serves as an online platform for collaborative work on a common database (i.e., a textbook) and for discussions in a forum-like fashion (for more information on the technological platform used in the M-TET project, see Even and Olsher, 2014).

Participating teachers were provided with two kinds of support that accompanied both the distance work and the face-to-face meetings. One was technical support in using the technological platform for textbook editing. The aim of this support was to provide a smooth, efficiently running work environment that enables teachers to perform the desired editing without having to deal with, or be constrained by, technological difficulties. The other kind of support was related to conceptual issues that emerged as part of the editing work. To this end, the participating teachers were offered an opportunity to consult with various professionals throughout their ongoing distance work and during their monthly face-to-face meetings. The professionals made available for consultation included the authors of the *Integrated Mathematics* textbooks, a research mathematician, and researchers in the field of mathematics education.

During the first year, the project team purposely avoided any intervention with, commenting on, or evaluation of the teachers' work, besides instructing the teachers on how to use the project website. The role of the project team during that year was to ensure a smooth running work environment and to moderate, but not direct, the monthly face-to-face meetings. Similarly, during that year, the consultants associated with the project were explicitly instructed not to initiate any intervention with, comment on, or evaluate the teachers' work. Instead, the consultants were instructed to respond only when explicitly approached by the teachers, and to address only queries related to the following areas: the reasons for specific choices made in the textbook by the textbook authors, the mathematics in the curriculum, and research in mathematics education. Findings from a study that examined the changes that the first-year participants in the project suggested to make in the seventh grade textbook they were using were reported in Olsher and Even (2014).

From the second year onwards, the participating teachers continued to receive an autonomous work environment wherein they could freely edit the textbooks as they wished. However, the consultants associated with the project were allowed to freely comment on the teachers' editing suggestions and could freely address any query raised by the teachers. In addition, a sizable part of the monthly face-to-face meetings was devoted to discussions with the textbook authors and with the mathematician on issues chosen by the teachers and by the project team. Next, we will use the editing work on a unit of the Pythagorean Theorem that took place during the second year of the project to exemplify the nature of the teachers' connections with a textbook author and a mathematician that the M-TET work environment made possible.

Editing a Unit on the Pythagorean Theorem

During the second year of the project the 20 participating teachers worked on editing two textbooks from the *Integrated Mathematics* curriculum program; both textbooks were intended for eighth grade: a regular/extended textbook (Buhadanah et al., 2011a) and a limited scope textbook (Buhadanah et al., 2011b). These textbooks, which basically covered the same mathematics topics, were approved by the Ministry of Education for students in the upper two-thirds and lower one-third achievement levels, respectively. The teachers worked in two (occasionally overlapping) small groups: one group focused on editing the regular/extended textbook, and the other on editing the limited scope textbook. During the face-to-face monthly meetings the whole group discussed the editing proposals and dilemmas encountered by each small group.

The teachers devoted a considerable amount of time during that year to editing a unit on the Pythagorean Theorem in the limited scope textbook: about 6 weeks of distance work and two face-to-face whole-group meetings. Next, we will first present the textbook approach, and then the main stages of the editing work,

highlighting the connections among the teachers, the unit author, and the mathematician, which were made available for consultation.

The Textbook Approach

The Pythagorean Theorem comprised seven lessons in the textbook, a total of 46 pages. The first two lessons in the textbook were the focus of most of the teachers' editing work. In the first lesson in the textbook the students were requested to find the lengths of the sides of several right triangles by measuring them, then to organize their results in a table, and finally, to determine whether a hypothetical student's (false) claim about the connections between the lengths of the sides is true (see Fig. 10.1). (The text in this, as well as in all other figures, is a translation to English of the original Hebrew text.)

The textbook then presented a claim of another student—this time it was the Pythagorean Theorem (i.e., in a right triangle the sum of the areas of the squares built on the legs equals the area of the square built on the hypotenuse)—and asked students to determine whether it was true in several cases. The lesson concluded by explicitly stating the Pythagorean Theorem, followed by tasks intended for student work.

The second lesson in the textbook began with a reminder of the connection between the sides of right triangles found in the previous lesson (i.e., the Pythagorean Theorem), stating, "We will check, using additional examples, whether this claim is indeed true" (see Fig. 10.2).

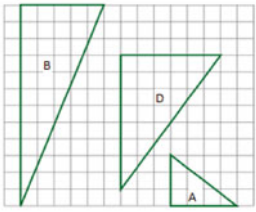
The Teachers' Initial Approach

A lively discussion developed among the teachers concerning two central issues regarding the way the textbook introduced the Pythagorean Theorem. The first issue dealt with the decision to begin the teaching of the Pythagorean Theorem with a false statement (Udi's claim in Fig. 10.1). Two contrasting approaches were raised: Some of the teachers wanted to revise the textbook, driven by the concern that the false statement would be adopted by the students. Instead, they suggested to start by correctly phrasing the Pythagorean Theorem, and only later asking students to examine relationships, like the one Udi suggested in Fig. 10.1, which are true for some, but not for all right triangles. In this way, those teachers argued, students would appreciate the beauty of the Pythagorean Theorem, which is always true. Other teachers supported the textbook's approach, and argued that one way of dealing with students' mistakes is to purposely start by examining a false statement that appears to be true.

The second issue that the teachers discussed was that the textbook first stressed the idea that making a generalization, based only on checking a few cases, might

Connections between the lengths of the legs and the length of the hypotenuse

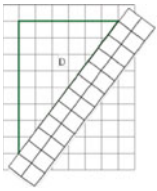
2. a. In the following table the lengths of the sides of one triangle (C) are given and the lengths of the sides of the drawn triangles (A, B, D) are missing.



Use the squares to complete in the table the lengths of the legs of the three triangles.

Triangle	Short leg a	Long leg b	Hypotenuse c
A			
B			
C	7	24	25
D			

b. Use the squares to complete in the table the length of the hypotenuse of each triangle. (The length of the hypotenuse of a triangle can be measured by placing a graph paper alongside the hypotenuse and counting the squares. See the figure.)



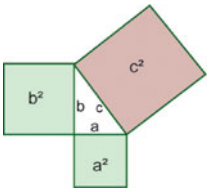
c. Udi observed the table and said: "If we multiply the length of the short leg by itself, we will get the sum of the long leg and the hypotenuse" ($a \cdot a = b + c$). Is Udi right?

Fig. 10.1 Excerpt from the first textbook lesson on the Pythagorean Theorem (Buhadanah et al., 2011b, p. 236)

lead to wrong conclusions (Udi's claim in Fig. 10.1). However, the textbook later justified the Pythagorean Theorem by relying only on a few examples and even explicitly suggested it as a legitimate means of checking whether a claim is true (see Fig. 10.2): "We will check, using additional examples, whether this claim is indeed true" (Buhadanah et al., 2011b, p. 243). The teachers felt that the textbook approach was problematic but were not sure how to go about resolving it.

In the previous lesson we found a connection between the areas of squares built on the sides of a right triangle. The sum of the areas of the squares built on the legs equals the area of the square built on the hypotenuse.

$$a^2 + b^2 = c^2$$



a and b are the lengths of the legs, c is the length of the hypotenuse.

We will check, using additional examples, whether this claim is indeed true.

Fig. 10.2 Excerpt from the second textbook lesson on the Pythagorean Theorem (Buhadanah et al., 2011b, p. 243)

Consulting with the Textbook Author

The teachers decided to consult with the author of the textbook unit on the Pythagorean Theorem—one of the co-authors of the textbook—and she was invited to the following face-to-face monthly meeting. When meeting with her, the teachers first presented their contrasting approaches regarding beginning the lesson on the Pythagorean Theorem with a false statement. The author responded by explaining her view on the potential of such an introduction to create a feeling of surprise that the Pythagorean Theorem is true, and the need to find a way to prove it. Below is an excerpt from the discussion:

Teacher A: I'm afraid that the error [the erroneous formula] will stick with the students.

Teacher B: Why? . . . We need to put the mistakes on the table. This is overwhelming. It creates a conflict. It requires them to use critical thinking.

Teacher C: It's not good to start a new topic with a mistake. I think that we need to change this.

. . .

Teacher A: It is similar to the Pythagorean formula and it's confusing.

Author: The idea is to illustrate that you can't depend on examples in order to generalize and to reach conclusions. The Pythagorean Theorem is a surprising theorem. However, it won't be surprising if we just introduce it in class. Therefore, this is a golden opportunity to encourage students to evaluate another formula that works in some cases and suddenly does not work, and to build up the need for a different sort of justification, not generalization from examples.

Teacher C: I agree. This is a wonderful opportunity.

The conversation then moved to the second issue. The teachers suggested adding a proof to the Pythagorean Theorem. This suggestion gained the author's support:

Teacher D: I still have a problem. The goal is to prevent reaching conclusions from examples. We are showing them first that it's prohibited and then that it's okay. What are we really showing them?
Teacher E: Why, then, isn't a proof added, even a visual one?
Teacher F: Good. I have an actual proof.
. . .
Author: Concerning what you said (turning to Teacher D), you're right. . .. I think we should include a proof. . .. Perhaps if we revise the textbook we will include a proof—maybe the visual side of the proof in order to justify why the theorem is true.

The Teachers' Intermediate Approach

Following this conversation, the teachers decided to leave the introduction to the topic as is, and to add a proof for the Pythagorean Theorem. They decided to adopt the proof that appeared in the regular/extended textbook (Buhadanah et al., 2011a) that is based on a visual demonstration (as in Fig. 10.3) accompanied by a full deductive proof based on triangle congruence. However, the teachers found it difficult to decide whether a full deductive proof would fit the current learning stage of students in the lower one-third achievement level or whether an informal visual demonstration would better suit these students, and whether a visual demonstration could be considered as proof at all.

Consulting with a Mathematician

The teachers decided to consult with the mathematician made available for consultation. They met with him and presented their thoughts about adding a proof to the Pythagorean Theorem. The mathematician supported the teachers' suggestion:

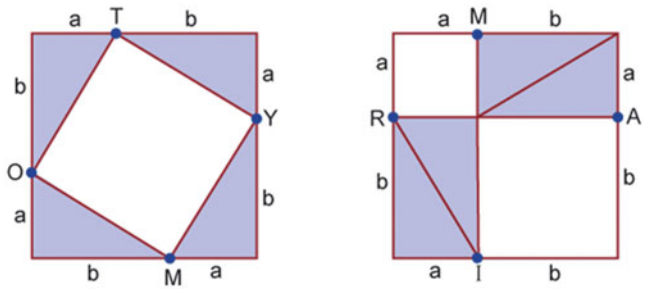


Fig. 10.3 Visual proof of the Pythagorean Theorem suggested by the teachers (adapted from Buhadanah et al., 2011a, p. 241)

"More and more examples do not convince or prove... It is against mathematical thinking."

He then expanded his response and shared with the teachers the role of proof in his own everyday mathematical work:

> How do I work, how do real mathematicians solve problems: we do not think in terms of theorems. As mathematicians, when we develop a mathematical theory, we do not think using axioms. We look at several examples and look for a pattern. Once I see the pattern, then I have a feeling that maybe I can develop a theorem. Then I have to find a proof, a general proof. I can no longer base things on examples. This is against mathematical thinking.

The mathematician emphasized the importance of coherently building the idea of proving in mathematics, beginning already at this stage, also for students with low achievements. Yet, he added that he leaves the decision about whether to add a proof, and which kind, to the teachers, who are "experts on pedagogy." However, if the teachers decide not to add a proof or to add a partial proof, he strongly recommended adding a comment in the textbook indicating that a complete proof will be presented in the future.

The Teachers' Final Decision

Eventually, the teachers added to the textbook that they edited the full deductive proof based on triangle congruence that appeared in the regular/extended textbook, and a link to an applet that makes the Pythagorean Theorem more tangible (by dragging the squares) directed at classes where the presentation of a proof was not appropriate.

The Author's Revision of the Textbook

Later, when preparing a revised version of the textbook, the author of the unit on the Pythagorean Theorem added a proof of the Theorem, as the teachers had suggested. However, after negotiating with the Ministry of Education during the long process of having the textbook approved for use in class—the introductory part of the unit on the Pythagorean Theorem was completely changed. The new version of the textbook includes a more straightforward presentation of the Pythagorean Theorem and its illustrations; in particular, the beginning of the lesson and the proof were omitted in the revised version.

Discussion

Leshota and Adler's (2014) study points to the important role that the insertions and omissions that teachers make in the textbook play in the kind of mediation of the object of learning that takes place in the classroom. The M-TET project enables the insertions, omissions, and other changes that teachers make in the textbook to become the object of learning for teachers in a work environment characterized by aspects that are usually not part of teachers' practice. These include designing a textbook for a broad student population instead of focusing on the specific student population taught, producing a textbook by making changes in a textbook designed by expert curriculum developers, and consulting with professionals who are not part of the teachers' usual milieu: mathematicians and curriculum developers. As was illustrated in the Pythagorean Theorem case, the unique work environment of the M-TET project created an authentic setting for establishing novel connections between teachers and textbook authors, and between teachers and a mathematician.

Working with colleagues in designing a textbook for a broad student population facilitated the development, clarification, and articulation of shared ideas regarding the teaching of mathematics, which could then be confidently presented and discussed with the textbook authors and mathematicians. This includes, for example, whether textbooks should present false claims (e.g., Udi's claim in Fig. 10.1) and whether proofs are needed in textbooks for students with low achievements. Being well thought of by a group of teachers the teachers' questions, ideas, and proposals were worth listening to, thinking about, and responding to, by the textbook author and a mathematician.

Moreover, as demonstrated in the case of the Pythagorean Theorem, in contrast to common practice, in the M-TET work environment, interactions among teachers, textbook authors, and mathematicians were initiated by the teachers themselves, who were also the ones who determined the content, timing, and format of these interactions, based on their needs and goals. For example, the textbook author of the unit on the Pythagorean Theorem was invited by the teachers to meet with them when, as a group, they could not reach an agreement regarding the beginning of the lesson on the Pythagorean Theorem. Similarly, when hesitating which proof would better fit students with low achievements, and whether a visual demonstration is really a mathematical proof, the mathematician was consulted.

Such connections with textbook authors and mathematicians were well appreciated by the teachers. This is illustrated in the following statement made by one of the teachers who also revealed how the interactions—which she termed collaborations—with the textbook authors and the mathematician contributed to her classroom teaching:

> The talks, the collaboration with the authors and the mathematician, there are not such things anywhere. It makes me feel important that they want to listen to me and to work with me. They talk to me eye-to-eye… It changed the way I see myself and the way I use the curriculum in class. I now ask myself: What is the aim of this task? What would the author say about this part of the lesson? Is the mathematical concept in this lesson used correctly?

The shortcomings of conventional connections between teachers and textbook authors, and those between mathematics teachers and mathematicians are of a different nature, as are the strengths of the transformed connections made possible by the M-TET work environment. Conventional connections between mathematics teachers and textbook authors usually occur via teaching materials written by curriculum developers (e.g., textbooks and teacher guides), and when curriculum developers conduct workshops for teachers aimed at improving the implementation of the curriculum program they develop. In such situations, textbook authors become "teachers" and teachers become "students" whose role is to learn from the experts. The M-TET work environment facilitates interactions in which the teachers and textbook authors have more equal positions and authority. As in traditional interactions, in the M-TET work environment the teachers have opportunities to learn from the textbook authors about their intentions and ideas (e.g., the issue of false claims). However, such learning occurs on the teachers' terms. Moreover, teachers have opportunities to deliberate with textbook authors' ideas and principles related to the teaching of mathematics, to be heard by textbook authors, and to influence textbook design (e.g., a missing proof).

The shortcomings and strengths related to the conventional and transformed (respectively) connections between teachers and mathematicians differ from those of the teachers and textbook authors portrayed above. The Conventional connections between mathematics teachers and university mathematicians usually occur only during the teacher preparation stage. Practicing teachers rarely have opportunities to consult with mathematicians about the mathematics they teach in class. The M-TET work environment provides such opportunities for practicing teachers, again, on the teachers' terms. The M-TET work environment facilitates interactions in which teachers and research mathematicians discuss issues that are of interest to the teachers and are authentic to their teaching practice (e.g., aspects of mathematical proofs). Teachers are provided with opportunities to increase their confidence (e.g., the essential role of proofs), and to improve their understanding of what mathematics actually is by hearing first hand from an active research mathematician about the nature of the work he engages in as part of his everyday professional life.

As demonstrated here, the unique characteristics of the M-TET work environment provide opportunity to transform the conventional connections of teachers with textbook authors and mathematicians into more productive ones, while contributing to the professional development and building of a professional community of teachers, as one teacher said:

> I feel that I am in a continuous process of growth. The project empowers me, being part of a group who works together on something important... The ability and the motivation to test my intentions all the time, not to surrender to the routine assignments of teaching, but instead, to stop, to analyze the lesson and the tasks, to reflect on the lesson and to consider a change... The interactions with the other teachers and the project team, listening, talking, and sometimes even arguing with other teachers, learning from different people having different opinions—this is all part of me now. It is difficult for me to think of myself, who I was had I not been here.

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Chapter 11
Jill Adler: Biographical Contributions

Stephen Lerman

This book marks some of the huge contributions Jill has made and continues to make in mathematics education research and development in South Africa and internationally, and her service to the mathematics education research community in South Africa and internationally. The chapters are written by people who have known Jill and her work, and many have collaborated with her on research and development, or have drawn on her work having been inspired by her writing and also by her energy and commitment, and her ideas, including equity, in particular, but also to language, to teacher education, to developing theoretical frameworks.

The purpose of this chapter is different, however; it is to sketch a picture of Jill through a brief biography, one that shows her to have been outstanding in all kinds of ways throughout her life. We could take this into her family relationships and into her friendships, wherein she is greatly loved and appreciated. But we will stay with her history. This chapter contains a number of contributions from people who knew her in her earlier years as well as testimonials from schools and other places where she made a mark. Other information was provided by Jill's husband Taffy and by Jill's colleagues.

Jillian Beryl Smidt was born on 31st January 1951 to Edna and Oscar in Johannesburg, South Africa. Her parents were both born in South Africa to immigrants who escaped from the pogroms in Lithuania at the end of the nineteenth century, as is the case for many of the Jewish community in South Africa. Jill was the second of three children, the first-born being her sister Sharon and the youngest her brother Steven.

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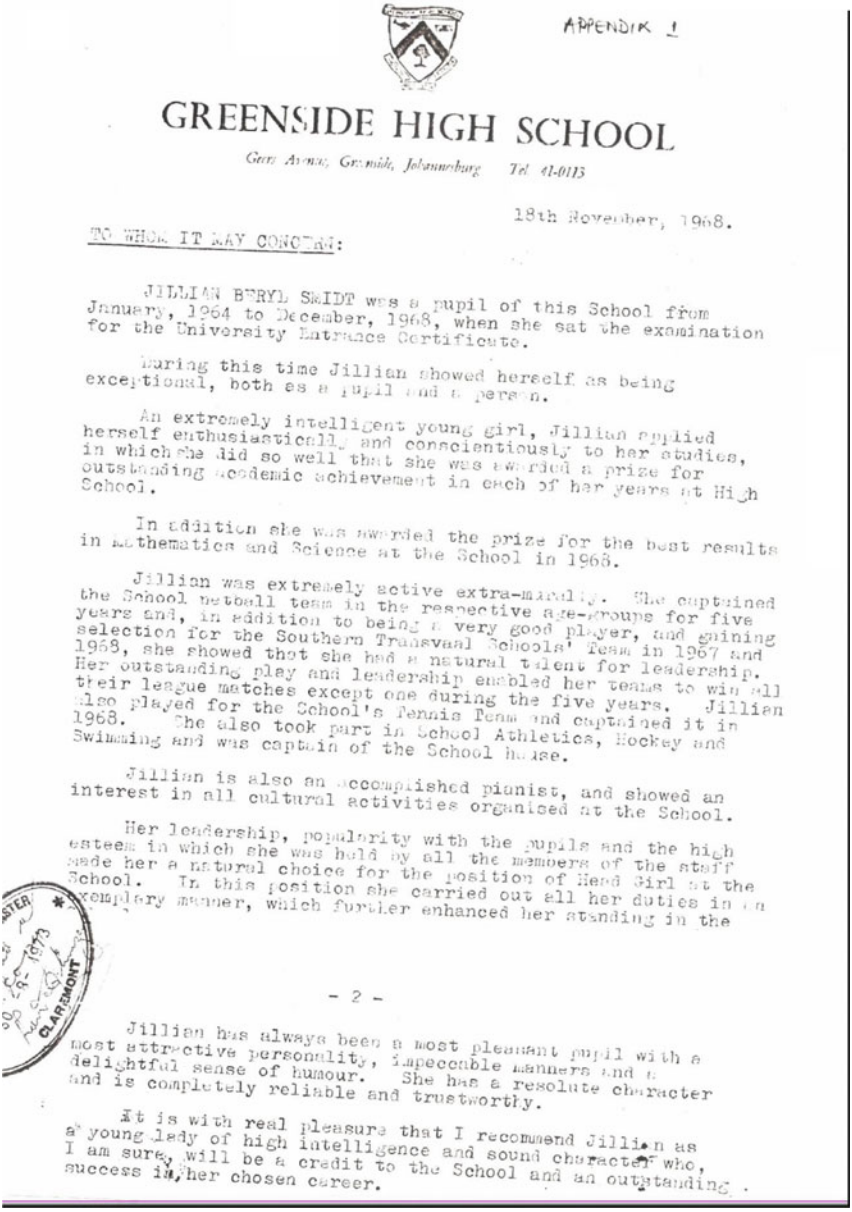
Education and Early Years School Teaching

Jill attended Emmarentia Primary School and then Greenside High School. On leaving the latter, the following reference was written, that describes her academic and other achievements:



APPENDIX 1

GREENSIDE HIGH SCHOOL

Gem Avenue, Greenside, Johannesburg Tel. 41-0113

18th November, 1968.

TO WHOM IT MAY CONCERN:

JILLIAN BERYL SMIDT was a pupil of this School from January, 1964 to December, 1968, when she sat the examination for the University Entrance Certificate.

During this time Jillian showed herself as being exceptional, both as a pupil and a person.

An extremely intelligent young girl, Jillian applied herself enthusiastically and conscientiously to her studies, in which she did so well that she was awarded a prize for outstanding academic achievement in each of her years at High School.

In addition she was awarded the prize for the best results in Mathematics and Science at the School in 1968.

Jillian was extremely active extra-murally. She captained the School netball teams in the respective age-groups for five years and, in addition to being a very good player, and gaining selection for the Southern Transvaal Schools' Team in 1967 and 1968, she showed that she had a natural talent for leadership. Her outstanding play and leadership enabled her teams to win all their league matches except one during the five years. Jillian also played for the School's Tennis Team and captained it in 1968. She also took part in School Athletics, Hockey and Swimming and was captain of the School house.

Jillian is also an accomplished pianist, and showed an interest in all cultural activities organised at the School.

Her leadership, popularity with the pupils and the high esteem in which she was held by all the members of the staff made her a natural choice for the position of Head Girl at the School. In this position she carried out all her duties in an exemplary manner, which further enhanced her standing in the

- 2 -

Jillian has always been a most pleasant pupil with a most attractive personality, impeccable manners and a delightful sense of humour. She has a resolute character and is completely reliable and trustworthy.

It is with real pleasure that I recommend Jillian as a young lady of high intelligence and sound character who, I am sure, will be a credit to the School and an outstanding success in her chosen career.

I have a copy of one of her school reports showing a mark of 100 % in mathematics. The Head Teacher referred to Jill's musical abilities. In fact I understand that her standard of piano playing was such that had she pursued her musical studies she could have been at concert performing level. Other interests grabbed her imagination however, to the benefit of our community!

Readers will recognise so many attributes that have served her well throughout her career: outstanding academic; excellent leader; resolute character; sense of humour; reliability and many others, though I don't think she still plays netball.

Jill began her studies at the University of the Witwatersrand in 1969, majoring in Mathematics and Psychology. Her studies were interrupted half way through the first year when she gained an American Field scholarship that took her to the USA. Two fellow students, Judy Favish and Anton Johnson, wrote the following:

Jill, Learner Teacher (1969–1973)

Jill's all-round outstanding performance at school laid the foundation for her love of education. She excelled academically and served as Head Girl both at Emmarentia Primary and Greenside High Schools. Her matric results were splendid and she was a highly competent sportswoman in netball, captaining the team which won the provincial championships for a number of years. She was awarded an American Field Service Scholarship during her matric year, which was a testament to her special qualities. The scholarship meant that she would have to break her studies at the University of Witwatersrand (Wits) after 6 months, and complete her first year after spending a year in America.

Jill spent the year in Seattle where she thrived in the new environment and built very close relationships with her American family. On her return she threw herself into the subjects which would determine her future career. She obtained her Bachelor of Science in 1972, majoring in Mathematics and Psychology, at which she acquitted herself with above average results. She chose to do subjects in science and humanities because of her interest in people and her love of figures.

At Wits her friends started seeing the signs of someone who was destined to make a big impact in whatever field she would become involved in. Jill was hard working and conscientious; quiet and orderly too. She was also an accomplished guitarist and singer, soulfully rendering the protest songs of her generation: Pete Seeger, Joni Mitchell, Joan Baez etc.

Her future teaching skills were put to the test just before a Statistics test in third year Psychology. Her classmates had not managed to attend many Statistics lectures, as these were held at the grim time of 8:00 a.m. on Wednesdays. Jill tutored her classmates so well that they passed the test, but sadly for her, her deeper knowledge of the subject matter meant that she detected many ambiguities in the questions and did very badly in the test herself. She was, however, not prepared to take this lying down and along

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with other students challenged the lecturer to demand a better test. This time she did very well. Her friends chose not to risk writing again.

Jill was part of a small group of her advantaged peers who chose to get involved in activities designed to improve living and working conditions of black South Africans. She joined the South Africa Volunteer Service (SAVS) and participated in several building camps. One year she spent 3 weeks building a clinic in Liphiring, a tiny rural village in Lesotho. The conditions were not luxurious, but full of the luxury of freedom. Thirteen volunteers slept for a month in a single mixed dormitory, without hot water or flush toilets, and worked each day in full equality mixing cement, carrying bricks and the like. The evenings were companionable and restful, often mingled with the strains of a Joan Baez song rendered by Jill. Several weekends were spent building school toilets in the Bronkhorstspruit area, about 150 km north east of Johannesburg.

Jill bridged the divide between the "apolitical" hard scientists and the left-oriented social scientists, the latter constantly conscious of the injustices of their racist society. She was well read in the praxis of the radical educational thinkers that moved them: Freire, Illich, A S Neill and more. Most of the friends around her were activists in the student movement and in its community work. She lived with care on the line between academic success and political involvement, between being part of the work for a new society and not courting the attention of the security police. She herself was never jailed or banned, but she supported the commitments of her friends, some of whom were incarcerated. She kept her integrity, was prepared to challenge her seniors on issues of principle, but was always reluctant to hurt people, especially her parents.

After completing her degree she decided to enrol for teaching qualifications and headed off to Cape Town to live in a collective with two of her close friends from Wits, who had also been members of SAVS, plus two others. They lived in a modest old house at 12 Queen Victoria Road, Claremont. Jill set her mark on the household from the outset, having decided to be a strict vegetarian, so the other four learned rapidly to turn out vegetarian evening meals. This was surely good for all of them too.

The house was full of activism and activists streaming through all the time. The lounge was mostly occupied both by temporary guests and excited discussion groups. Jill had the front room, very nicely furnished, where (as always before) she would sit and conscientiously do her assignments, or play music. Sometimes, as the flow got too strong, she would shut the door, and then everyone knew, 'Do Not Disturb'! She would sigh deeply over the philosophies practiced by the lecturers with their emphasis on fundamental pedagogics, designed to reproduce the authoritarian structure of the surrounding (white) society; but she held on, enjoying the teaching practice and

(continued)

accepting that to become a teacher required that secondary teachers' diploma. Her own crowd also flowed in and out through the door. Some came from the little half-hidden local gay constellation, who existed precariously on the edge of a society which did not approve of them either. Liberation and rights were a major part of their discourse too.

The house also had its tiffs, its personality clashes, and its share of incompatibilities. Jill lived quietly (if uncomfortably) through all that, the model of a socially conscious idealist with a strong work ethic. Jill, a compassionate and gentle person, was always ready with a sympathetic ear and good counsel. She was very central to calming the house dwellers and keeping them together! By the end of that year, the inmate mix had changed quite a bit, if not the tempo, but she stayed on there and cast herself further along the path she had been following through all the previous years.

Her teaching diploma in hand, licenced to teach mathematics at all levels, she applied—most unusually for her graduate group—and was accepted to become a teacher at the Harold Cressy High School, a school reserved for secondary level pupils defined by the regime as Coloured. This step, way outside of the usual frames of the apartheid system, clearly revealed her commitment to the people, the rights and the needs of her broader society.

Her university friends all remember Jill with great affection. She was always a really good friend and supporter, and helped a lot in keeping her friends and their relationships on a somewhat even keel!

The degree mentioned above was a B.Sc. in Mathematics and Psychology in 1972 followed by her teaching qualification in 1973, the Secondary Teacher's Diploma from the University of Cape Town in mathematics and school guidance.

As Judy and Anton indicate above, the experience of Harold Cressy school, near Cape Town, brought Jill into active participation in work against the apartheid regime. One of Jill's colleagues, Isaac Abrahams, wrote the following:

Jill and I meet 40 years ago in 1974. Jill was fresh from university and I had been teaching for 3 years. At the outset it was just amazing how Jill adapted to the situation at Harold Cressy teaching mathematics to standards 7, 8 and 9. It seemed as if she had been at the school for years. I clearly recall how Jill introduced me to 'group work'. I was excited with this new approach of collaborative learning as a teaching methodology. It required much more careful preparation and planning but I took on the challenge of this new approach in teaching Afrikaans and General Science. It made for much more lively and interactive debate amongst the students in contrast to the 'chalk and talk' approach. Mr. Victor Ritchie (principal) and Mr. Peter Meyer were

(continued)

the two senior maths teachers at the time. They too observed the healthy relationship that was developing between Jill and her charges. I also found that putting students in groups with the teacher in a facilitating role, had huge benefits not only for the group leaders but the group constituents benefitted equally. It really was encouraging to see live debate within the group dynamics.

Harold Cressy High School at the time was one of the more progressive schools, despite being hugely under-resourced. It was blessed with a committed and well-qualified team of teachers many of whom ascribed to the New Unity Movement ideas and ideologies and many were members of Teachers League of South Africa. I think Jill thrived working in an environment where she was not only stimulated intellectually but also politically. I know that Jill was an avid reader of the Educational Bulletin of the Teachers League of S.A. and it provided topics for many in depth political discussions that Jill and I engaged.

But Jill also played a significant role in leadership development at the school. After many discussions Jill and I decided to speak to the principal Mr. Ritchie on the formation of a Student Body at Harold Cressy. We were then tasked to draw up a 'draft constitution' and provide guidelines for the functioning of a Student Representative Council in collaboration with the students. The early years of the 1970s were the turbulent years especially at disadvantaged schools and SRC's at schools were discouraged and frowned upon. Jill and I were elected the teacher representative on the SRC and our roles were to nurture, develop, guide, aid and assist the students during this formative period. It was in this climate that the first SRC was constituted at Harold Cressy. The school will always be indebted for the pioneering work that Jill did in this regard. The long hours she spent in aiding and assisting students, is testimony of her sincerity of purpose and genuine concern for the wellbeing of the students at the school.

On a personal level, Jill and I grappled with and debated many of the pertinent political issues of the time. Through her efforts and subtle motivation, I was inspired to continue the ground-work done at student and SRC level. 1976 is clearly seen as a 'watershed' year in the struggle for freedom and democracy in this country. During this period the students at most of the disadvantaged schools throughout the country came out in their thousands in a show of anger and rejection of the inequalities that permeated our South African society. SRCs were being established at the majority of high schools. I served as an advisor on a student organization that was known as the 'Super SRC' comprising of representatives from 40 high schools. Within 6 months this body grew to an organization of 81 affiliated schools from throughout the Western Cape. Through this body student protest action was facilitated and co-ordinated at a sophisticated level.

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Why I say all of this? Jill fresh from university, filled with her own ideas and ideologies, sees the need for the establishment of SRCs. Also, I don't know if Jill is aware of this: September 16 and 17 1976, declared 2 days of strike and protest. Huge stay-aways from schools, colleges, factories and most work-places. I was arrested and detained for a short spell on Sept. 17 1976. What is interesting is that during my first spell of interrogation, it was made quite clear to me, I quote: "Julle soort wat met die kinders se minds werk ,is meer gevaarlik as die klipgooiers en die wat plekke afbrand." (translated: you guys that work with the children's minds are more dangerous than the stone-throwers and rioters). Another interesting point, Jill and I continued communicating by writing to each other after she left in 1975. In our correspondence we would clearly articulate our anger and condemnation of the Nationalist Party, the justice system and the brutality of the security police. Imagine, at the time of my detention some of these letters amongst other incriminating documents were removed from my work desk by the police, Of course I felt the full wrath of the riot police during interrogation wanting to know "wie is die wit vrou van Johannesburg wat sulke #$#^ skryf". (translated: who is the white woman from Johannesburg who writes such #*$#^).*

But Jill, here we are 2014, 40 years after I first met you when you came to Harold Cressy. You left after 2 years and I stayed at Cressy for a spell of 18 years before moving on to take a post at a training college where I taught Environmental Education and Geographical Science for a period of 8 years. Then early retirement as a result of closure of colleges. I then taught Maths and Science at Eros School, a school for Cerebral Palsy and Physically Challenged learners for about 8 years. How ironical!

Lastly, thanks Jill, for those social evenings we enjoyed at your shared flat in Orangezicht, was it Le Fleur Street? How different to me were those vegetarian dishes you and your flatmate prepared for us considering that came from a very traditional Muslim upbringing stooped in our own customs. And that little white VW beetle of yours! Finally I recall once saying to you :

"You will probably settle overseas with your Taffy and become a professor of Mathematics somewhere in the world."

The following testimonials were written when Jill left Harold Cressy in 1975, taking up a position at the King David High School in Johannesburg for 6 months, to the end of 1976.

Harold Cressy High School.

Principal - Teacher:
MR. V. J. RITCHIE
Phone : 47-2878

ROELAND STREET
CAPE TOWN

5 December 1975

Dear Miss Smidt

On behalf of the S.R.C. and thus on behalf of every student in H arold Cressy, I would like to extend my sincere gratitude for the way in which you aided the S.R.C. during its first year of existence at this school. It is obvious that the society would not have met with its few successes in the way that it did if your advice and help had not been there to guide us.

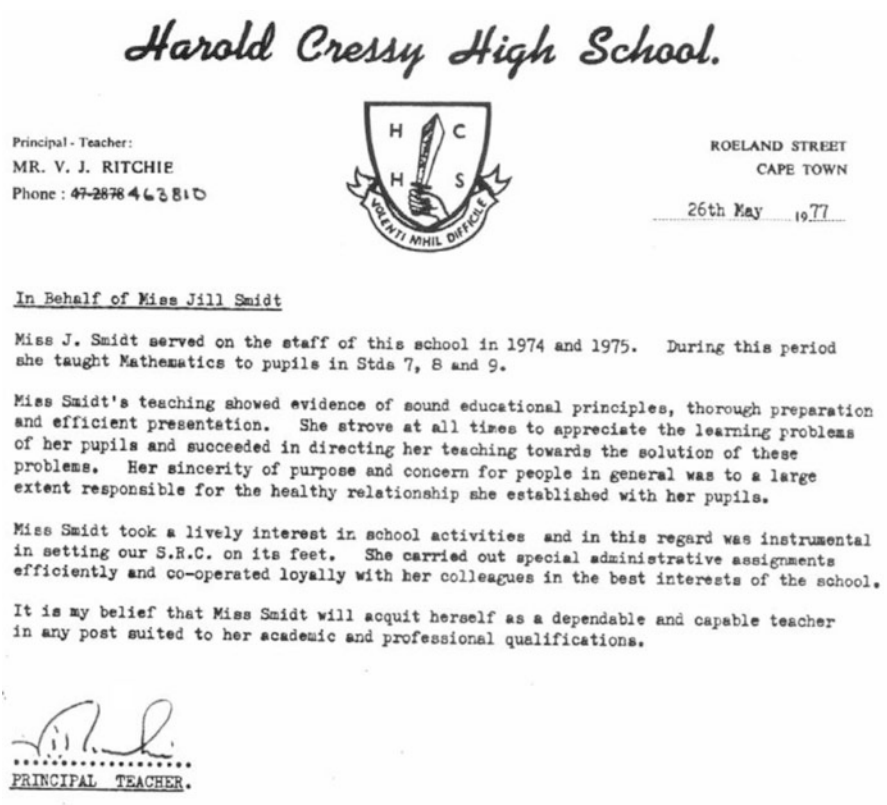
We regret your departure, but I am sure that this will not discourage us in our efforts next year. W e hope that you have personally been satisfied with the way in which the society has functioned this year and we welcome any further criticism from you at any time.

As for your trip, I wish you everything of the best and lots of happiness and luck. On your return to South Africa (if you do), please inform us as we would be glad to send you copies of our newspaper through which you should be able to follow what we are getting up to.

Thank-you once again.

Yours faithfully

M.S. JASSIEM
(CHAIRMAN OF INTERIM COMMITTEE 1975-76)



Harold Cressy High School.

Principal - Teacher:
MR. V. J. RITCHIE
Phone : 47-2878 46 38 10

ROELAND STREET
CAPE TOWN

26th May 1977

In Behalf of Miss Jill Smidt

Miss J. Smidt served on the staff of this school in 1974 and 1975. During this period she taught Mathematics to pupils in Stds 7, 8 and 9.

Miss Smidt's teaching showed evidence of sound educational principles, thorough preparation and efficient presentation. She strove at all times to appreciate the learning problems of her pupils and succeeded in directing her teaching towards the solution of these problems. Her sincerity of purpose and concern for people in general was to a large extent responsible for the healthy relationship she established with her pupils.

Miss Smidt took a lively interest in school activities and in this regard was instrumental in setting our S.R.C. on its feet. She carried out special administrative assignments efficiently and co-operated loyally with her colleagues in the best interests of the school.

It is my belief that Miss Smidt will acquit herself as a dependable and capable teacher in any post suited to her academic and professional qualifications.

.....
PRINCIPAL TEACHER.

Jill met her husband-to-be Taffy Adler in Cape Town in 1975 and they married in 1978. Their first child Joshua was born in October 1979 and their second child, Michelle, November 1982. It is interesting that both her children have followed careers in within the general field of education.

The SACHED Years

In January 1976 Jill began work at SACHED, the South African Committee for Higher Education, which was founded in 1959 as an innovative response to the crisis in university education brought about by the National Party government's move to enforce apartheid at university level and thus complete their apartheid design for education (see link http://www.c3l.uni-oldenburg.de/cde/support/readings/nonyo98.pdf). SACHED was an adult education organisation and Jill worked on a number of areas including Bophuthatswana Teaching Upgrade project (BTUP); curriculum development; and various teaching projects, such as a winter school in 1977. The following reference was given after that school:

EIC

601 DUNWELL HOUSE, 35 JORISSEN STR. BRAAMFONTEIN, 2001
TEL.(011) 39-2476/7/8

TO WHOM IT MAY CONCERN

I have known Jill Adler since 1977 when she was the session leader for Maths at our annual Winter School.

The Winter School, which is attended by over 1000 students, aims to provide black matriculants with help for their final exams. In each subject, difficult areas of the syllabus are used as the foundation for the study of broader skills and approaches to the subject - so that the School has more than 'cram' value.

Ms Adler's handling of the Maths sessions, which involved teaching a group of 500 students of different levels of competence (some of whom had no maths teachers at their schools) proved her to be a teacher of the highest quality and flexibility which the demands made by teaching in such circumstances obviously call for.

In 1978 Ms Adler took charge of the entire Maths course. This involved the writing of study material for the course, recruitment of other teachers and tutors, planning the Maths timetable as well as presenting a number of the sessions herself. In all this, I found her to be extremely efficient - in several years of running the Winter School, Ms Adler was the only teacher I have worked with who was able to take total responsibility for a course.

I therefore have no hesitation in saying that the contribution she has made to black education, through her involvement in the Winter School, has been extremely valuable.

Marion McNair
MANAGER

Jill came into SACHED when their newspaper project was expanding into a 12 page supplement, called Peoples College, to be carried by the World newspaper and was responsible for the mathematics content in the newspaper articles as well as the outreach programme, a position that took her to work with teachers in outlying areas. The World newspaper was banned by the apartheid regime in 1977, primarily because it was seen as supporting students who were boycotting the segregated "Bantu" education system following the June 1976 Soweto student revolt. This is what she wrote about the newspaper project as part of her cv at the end of her time there:

CONCLUSION

The medium of my course development has in the main been the newspaper – a cheap way of reaching many people. The response to the newspaper maths/arithmetic courses has been small relative to the circulation of The World and then Post. (At this stage we are not able to say what response constitutes a good response.) However, the response has increased, in fact more than doubled from 1977 to date – an encouraging sign.

The problems of distance learning programmes in general are commonly known and have been explored elsewhere outside of South Africa. South Africa is different from most other places that run distance learning programmes in that they are often largely state supported.

The work done by SACHED is pioneering work in South Africa. SACHED's newspaper project is certainly pioneering in a wider field than just South Africa. It . ran for practically three years and appears to offer a positive input into distance education. However, there is no academic study to support this claim.

The teaching of maths/arithmetic through the medium of a newspaper in South Africa needs to be intensively and academically studied. In this way only can one establish its real potential and decide on its future direction.

Jill's master's degree studies would produce the systematic study of the newspaper project that she called for here. The following reference was provided for Jill when she left in 1985.

5 / 6th Floors, Colstaven Building
54 Simmonds Street, Johannesburg
2001. P O Box 11350, Johannesburg
2000. Tel (011) 834-1341/836-4638
Telex: 4 — 87413 SA

SACHED Trust
incorporating Turret Correspondence College

1 October 1985

TO WHOM IT MAY CONCERN

Ms Jill Adler has served the SACHED Trust, an independent organisation committed to the development of non-racial education in South Africa, in a number of invaluable ways over the past nine years.

She joined the R&D section of SACHED, of which I was then head, as a Mathematics course writer in 1979. The task was to develop, for adults, distance learning materials which were to appear in a pioneering educational newspaper. She displayed great flexibility and ingenuity in translating her extensive skill as a face-to-face teacher of adolescents to the unexplored and challenging task of teaching adults through the newspaper. Her work was always imaginative, meticulous and delivered according to a tight schedule.

She was quickly promoted to become a trainer of other course writers and a quality controller of their work. She finally replaced me as head of R&D. In all these roles, she worked extremely well with staff members in her team, providing the necessary leadership and treating them with consideration, while demanding high standards.

In recent years, she embarked upon research for the Trust into the mathematics material which had been published in the newspaper. This work was not only of high academic quality (she was awarded her Masters degree cum laude) but proved also to be of great value to SACHED.

In general, Ms Adler will only undertake tasks which she can predict she will be able to perform to her demanding standards. It is therefore a pleasure to work with her as, inter alia, one can always rely on the high quality of what she does.

J A Glennie

J A GLENNIE
Assistant Director: Finance & Administration

Higher Education

Jill obtained her M.Ed. in 1985 from the University of the Witwatersrand, gaining a distinction. As was signalled above, her dissertation was entitled: "Mathematics by Newspaper in South Africa: Junior Secondary Mathematics for Adults Through the Medium of a Newspaper". She left SACHED for a position as Senior Lecturer at Johannesburg College of Education in January 1986. The College was absorbed into the University of the Witwatersrand in 1989, and Jill was Lecturer there until March 1994 when she was promoted to Senior Lecturer. Jill has already embarked on her doctorate degree, being supervised externally by Dr. Kathy Crawford at the University of Sydney. Kathy was the perfect supervisor. Jill was moving towards sociocultural research on teaching and learning, a field in which Kathy was a world leader.

Kathy has written the following:

Memories of Jill as a Ph.D. Student

I had a history of my own as a participant in protests against apartheid education policies in South Africa and a developer of bi-cultural programs for traditionally oriented Aboriginal teachers in central Australia. Indeed, Marike de Klerk was a student of mine while F W de Klerk (a future South African President) was the South African ambassador to Canberra in the 1980s. World opinion was strengthening against the South African regime and this complicated the student/supervisor relationships. Once, I was embarrassed to receive an invitation to attend cocktail party at the South African embassy on the night following my participation in a large protest outside the embassy. Only mutual respect and honesty got us through the awkward phone call to Marike at the embassy to explain that the security guards would recognise me from the night before and enquire if I was still welcome.

I was involved as a co supervisor while Jill conducted her Ph.D. research while I was at the University of Sydney some years later during the latter part of de Klerk's presidency. She arrived to visit us with her usual strength and ethical clarity. We set out to see how we might invent a new way to analyse the dynamics of mathematics teaching in South African classrooms in ways that would be beneficial to both teachers and students. At the time several students in the Novae research laboratory were experimenting with some new video-annotation software, invented by Jeremy Roscelle at SRI. The new techniques allowed us to develop a multilayered and textured analysis of different aspects of an activity.

As a supervisor, I had a personal interest in multicultural teaching from my earlier work in Aboriginal communities. In particular, I had been made very aware of the enormous, and largely ignored, impact of cultural access and experience in learning in mathematics. Students in traditionally oriented cultures in central Australia start school with a language base and a culture to support topology but have difficulty with school based expectations for the development of number concepts and Euclidean geometry. I was also convinced that one weakness of all observational research, and in particular cross cultural research, stemmed from the fact that researcher observations and analysis were

(continued)

not objective. According to Activity Theory consciousness is subjective. The 'object' of any shared activity, what individual people think they are doing, is ideational and stems from cultural historical interpretations of the possibilities of the situation. To understand observable behaviours one needs to understand the interpretations and intent of the participants. Thus, in any situation, observable behaviours mean different things to different participants. In a multicultural situation there are obvious shared differences between the socio-cultural history of members of different ethnic groups. In the case of Jill's research, there was a clear case of different cultural historical experiences between her and the teachers she was investigating.

Jill devised a method of analysing video tapes of class room interaction that respected the fact that she could not know what teachers thought they were doing—their objects. Her method involved an initial annotation of videotapes of classroom interaction as an expert mathematics educator. This was followed by a second session with the teachers where the video tapes were used to stimulate recall of the events by the teachers and as the basis of an inquiry about their intentions and interpretations. Not only did this form of 'stimulus recall' provide information about teacher interpretations of the classroom activity, and related student behaviour, but the ensuing discussions and explanations had the additional developmental effect of helping teachers to become more aware of previously routine aspects of their teaching.

When Jill arrived we at Novae were considering a layered analysis of video data by different expert researchers or from different perspectives, Jill extended this idea to include interpretation by the teachers who were the objects of the research. I am forever indebted to her contribution and have used similar processes ever since to research multicultural situations and in complex projects involving stakeholders from traditionally separated disciplines and groups.

Jill was awarded her Ph.D. in 1996 from the University of the Witwatersrand (Rand). Her thesis was entitled: "Secondary Teachers' Knowledge of the Dynamics of Teaching and Learning Mathematics in Multilingual Classrooms".

Before I move to her most recent history, there is an incident that needs to be reported as it demonstrates Jill's commitment to anti-apartheid activities and beyond into a life-long concern for equity.

The 1986 Charge of Misconduct

On 16th June 1976 a series of protests led by high school students in South Africa began as a response to the introduction of Afrikaans as the medium of instruction in local schools. Briefly, tens of thousands of students took part, to be met with massive police brutality. Police gave the number of dead as 176; others estimated it as possibly as many as 700.

Ten years later, still under the apartheid regime, amongst the many protests to commemorate that event a group of eight lecturers at the Johannesburg College of

Education, including Jill, went on strike that day. They were charged with misconduct by the Transvaal Education Department. Here is the charge document:

Verwysings No. Reference 21414800
Navrae Enquiries .....
Tel. No. .....

T.O.D./T.E.D. 200

Alle mededelings moet geadresseer word aan die Direkteur van Onderwys.
All communications must be addressed to the Director of Education.

TRANSVAALSE ONDERWYSDEPARTEMENT
TRANSVAAL EDUCATION DEPARTMENT

Privaatsak X76
Private Bag X76
PRETORIA
0001

1986-10-20

REGISTERED MAIL
AND ADVICE CARD

Mrs J.B. Adler
Johannesburg College of Education
27 St Andrews Road
PARKTOWN
2193

CHARGE OF MISCONDUCT

You, Jillian Beryl Adler, a teacher in the employ of the Transvaal Education Department in a permanent capacity at the Johannesburg College of Education and therefore a teacher in respect of whom the provisions of Chapter V of the Education Ordinance, 1953, apply, are hereby charged with misconduct in terms of Section 90 of the said Ordinance in that -

CHARGE ONE

You, as contemplated in Section 89(j) of the said Ordinance, absented yourself from your post or duty without leave or valid cause on or about 1986-06-16;

CHARGE TWO

You, as contemplated in Section 89(a) of the said Ordinance, disobeyed or disregarded, or made wilful default in carrying out a lawful order given to you by a person having authority to give the same or by word or conduct displayed insubordination by not reporting for service on or about 1986-06-16 as instructed by the Rector.

You are hereby, in terms of section 90(5) of the said Ordinance, instructed to send or deliver to the Director of Education within fourteen (14) days from the date of this letter, a written admission or denial and if you so wish, a written explanation of the misconduct with which you are charged.

For DIRECTOR OF EDUCATION

Jill wrote a four-page defence, the first page being the following:

STATEMENT RE MISCONDUCT CHARGE OVER 16 JUNE 1986

1. FACTUAL BACKGROUND

The 16th of June 1986 was the tenth anniversary of the Soweto Revolt. It was widely recognised as a symbol of remembrance by representative groupings in industry, trade unions, businesses, educational institutions and their academic staff associations. The call for the expression of that remebrance in the form of a national day of mourning was respected or at least tolerated in many quarters (many companies agreed to either paid or unpaid leave for staff on that day.) Furthermore, it was clearly anticipated that the response to the call would be of national and historical significance.

2. REASONS OF CONSCIENCE PROMPTING STAND TAKEN

I am a senior lecturer in the professional studies department at JCE, responsible for the professional mathematics training of the B.Prim Ed students. As a professional educator and educationist, I consider myself part of and responsible to the broader education community in South Africa. My role embraces inter-alia, awareness of and participation in in-service courses, academic development through attendance at conferences and the presentation of papers, research work and teacher organisations.

My actions and professional conduct are informed and motivated by the underlying assumption that education by definition must be egalitarian and non-authoritarian. Education that is oppressive and confined cannot be good education. My working life has been dedicated to addressing the educational crisis in this country.

As an educationist concerned with educational progress and development in this country, I regard 16 June, and particularly its 10th anniversary, as a day of particular symbolic importance. It represents the disruption and inequality in schooling in South Africa and the need to put an end to such. As a result I felt it important ~~that~~ to respond to the call for a national day of mourning in my professional role as an educator. More simply I identified with the position that the 10th anniversary of the Soweto Revolt be a time of remembrance and also symbolic protest against educational inequality.

On a more personal and individual level I felt it important to remember the young people who have lost their lives in trying to improve both the educational system in South Africa and the country as a whole.

3. WHAT I DID

On Monday, 16 June 1986 I mourned for the children who died and continue to die in South Africa. I spent the day at home with my family. To help my children participate in an active way we

The finding was the following:

DEPARTEMENT VAN ONDERWYS EN KULTUUR
DEPARTMENT OF EDUCATION AND CULTURE

ADMINISTRASIE: VOLKSRAAD
ADMINISTRATION: HOUSE OF ASSEMBLY

TRANSVAALSE ONDERWYSDEPARTEMENT
TRANSVAAL EDUCATION DEPARTMENT

Navrae: Allocated Tasks
Enquiries:
Verw.: TOIn 407-8
Ref.:
Tel.: (012) 201-3114

Provinsiale Gebou
Provincial Building
Privaatsak X76
Private Bag X76
Pretoria 0001

REGISTERED MAIL AND CONFIDENTIAL

Mrs J.B. Adler
Johannesburg College of Education
27 St Andrews Road
PARKTOWN
2193

1988 -09- 29

CHARGE OF MISCONDUCT: FINDING

With reference to this Department's charge of misconduct dated 20 October 1986, I wish to advise you that, after consideration of the report by the Commission of Inquiry, the Director of Education has, in terms of charge one, found you guildy as charged in terms of section 93(2)(b) of the Education Ordinance, 1953. In terms of charge two, you have been found not guilty in terms of section 93(2)(a) of the Ordinance.

The Director has decided, in terms of section 93(2)(b)(i) of the Ordinance, to caution you. The fact that more drastic action has not been taken against you does not mean that the Director has condoned your action. Please note that a recurrence of a similar charge may have a serious effect on your future as a Lecturer.

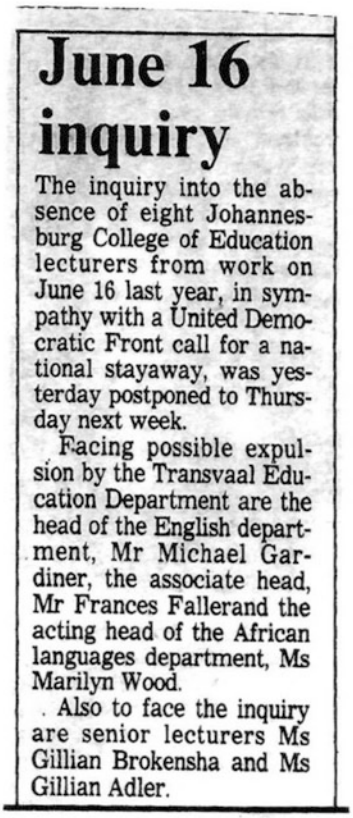
In view of the finding of the Director, leave without pay will be granted to you for 16 June 1986 in terms of Regulation 29(2) of the Regulations prescribing the Conditions of Service of Teachers.

For DIRECTOR OF EDUCATION

Rig korrespondensie aan die Direkteur: Transvaalse Onderwysdepartement en meld verwysingsnommer
Address correspondence to the Director: Transvaal Education Department and quote reference number

TRANSOND TRANSED
FAX: (012) 2014326

We also have some newspaper cuttings that report this event:

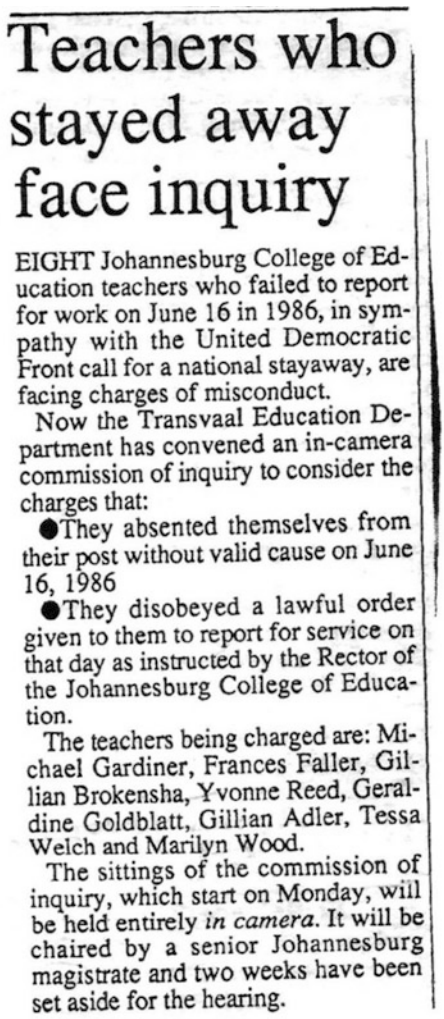


June 16 inquiry

The inquiry into the absence of eight Johannesburg College of Education lecturers from work on June 16 last year, in sympathy with a United Democratic Front call for a national stayaway, was yesterday postponed to Thursday next week.

Facing possible expulsion by the Transvaal Education Department are the head of the English department, Mr Michael Gardiner, the associate head, Mr Frances Fallerand the acting head of the African languages department, Ms Marilyn Wood.

Also to face the inquiry are senior lecturers Ms Gillian Brokensha and Ms Gillian Adler.



Teachers who stayed away face inquiry

EIGHT Johannesburg College of Education teachers who failed to report for work on June 16 in 1986, in sympathy with the United Democratic Front call for a national stayaway, are facing charges of misconduct.

Now the Transvaal Education Department has convened an in-camera commission of inquiry to consider the charges that:

- They absented themselves from their post without valid cause on June 16, 1986
- They disobeyed a lawful order given to them to report for service on that day as instructed by the Rector of the Johannesburg College of Education.

The teachers being charged are: Michael Gardiner, Frances Faller, Gillian Brokensha, Yvonne Reed, Geraldine Goldblatt, Gillian Adler, Tessa Welch and Marilyn Wood.

The sittings of the commission of inquiry, which start on Monday, will be held entirely *in camera*. It will be chaired by a senior Johannesburg magistrate and two weeks have been set aside for the hearing.

This was, of course, potentially very dangerous for Jill and the others. The 16th of June is now a public holiday, Youth Day, in South Africa, in remembrance of the events of 1976.

Coming to the Present Day

In 1997 Jill was appointed to the University of the Witwatersrand/Sentrachem Foundation Chair of Mathematics Education Development in the School of Mathematics, Faculty of Science. She set about building a strong group in mathematics

education, and a large and quite outstanding group of doctoral students. I was privileged to have been invited by Jill to help in their supervision and quite a challenge they were too, in the very best of senses: Kgethi (Setati) Phakeng; Mellony Graven; Thabiso Nyanbanyana; Irene Broekmann (deceased); Philip Dikgomo; and Margot Berger. Readers will know of the careers of some of these people and will appreciate the significance of Jill's work with them. They were followed many other excellent Ph.D. graduates and they have marked their debt to Jill in chapters of this book. Between them they have made and continue to do hugely significant work in South Africa and the impact of their work on the international field is equally significant and impressive.

Jill now holds the First Rand Foundation in Mathematics Education Chair and is doing hugely important work in schools in the Gauteng Province of South Africa. A substantial number of the people who work with her have gained Masters and Doctorates from researching what is leading to disadvantaged students not achieving mathematics qualifications and how to improve performance. The Chair is charged with finding ways to extend the successes beyond those ten schools to all of South Africa, in partnership with the other five such Chairs in the country. This work is marked in chapters of this volume.

Jill and I served together on the International committee of PME, the International Group for the Psychology of Mathematics Education, and she was a tremendous support through my period as President. This period included the PME meeting in Stellenbosch in 1998. Jill has been a Vice President of ICMI, the International Commission on Mathematical Instruction; she still holds that position at the time of writing this chapter, but that involvement is by no means over at the time of publication of this volume.

Furthermore, this volume records also the significance of her work way beyond mathematics education within South Africa. She is, for example, one of the only two A rated education researchers in the country and until recently was the only one. Jill was Member of Board of Directors for the Gauteng Institute of Education Development (GIED) from 1998 to 2003, a curriculum development initiative funded jointly by the Gauteng Department of Education and the British Government aid agency DFID. She has served the National Research Foundation (NRF) in several capacities over the years. And we could say much more.

Jill has received many awards, including the University of the Witwatersrand Research Award in 2003; with her colleagues she also received the Vice-Chancellor's Academic Citizenship Award in the same year; the Academy of Science of South Africa 'Science-for-Society Gold Medal Award' 2012; the Svend Pederson Award from Stockholm University for research in Mathematics Education in 2015. She is a Visiting Professor at King's College London.

Jill is held in the highest esteem around the world. She is also loved and appreciated by so many, many people, not least her family. This book marks Jill's history and achievements up to the date of the completion of this book. *There is more to come, for sure!!*

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M. Phakeng, S. Lerman (eds.), *Mathematics Education in a Context of Inequity, Poverty and Language Diversity*, DOI 10.1007/978-3-319-38824-3