Chapter 16
Forecasting Implied Volatilities for Options on Index Futures: Time-Series and Cross-Sectional Analysis versus Constant Elasticity of Variance (CEV) Model

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16.1 Introduction

Forecasting volatility is crucial to risk management and financial decision for future uncertainty. Previous studies have found that the volatility changes are predictable (Engle, 1982; Pagan & Schwert, 1990; Harvey & Whaley, 1991, 1992a, 1992b; Day & Lewis, 1992; Fleming, 1998). In perfectly frictionless and rational markets, options and their underlying assets should simultaneously and properly change prices to reflect new information. Otherwise, costless arbitrage profits would happen in portfolios combined by options and their underlying assets. However, prices in security and option markets may differently and inconsistently change to respond to news because transaction costs vary cross financial markets (Phillips & Smith, 1980). Based on trading cost hypothesis, the market with the lowest trading costs would quickly respond to new information. The price changes of options on index and options on index futures lead price changes in the index stocks because trading costs of index option markets are lower than the cost of trading an equivalent stock portfolio (Fleming, Ostdiek, & Whaley, 1996). Therefore, the dynamic behavior of market volatility can be captured by forecasting implied volatilities in index option markets (Dumas, Fleming, & Whaley, 1998; Harvey & Whaley, 1992a).

In this chapter, we use option prices instead of relying on the past behavior of asset prices to infer volatility expectations of underlying assets. The derivation and use of the implied volatility (called IV hereafter) for an option as originated by Latane and Rendleman (1976) has become a widely used methodology for variance

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J.B. Guerard, Jr. (ed.), *Portfolio Construction, Measurement, and Efficiency*,
DOI 10.1007/978-3-319-33976-4_16

estimation. The IV derived from option prices depends on the assumptions of option valuation formula. For example, IV in Black-Scholes-Merton option pricing model (called BSM hereafter) tends to differ across exercise price and times to maturity, which violates the assumption of the constant volatility of underlying asset in model. The fact that there are as many BSM IV estimates for an underlying asset as there are options on it, as well as the observable nonconstant nature, has attracted considerable attention from practitioner and theoretician alike.

For the academician, previous studies have been proposed to capture the characteristics of implied volatility by either using statistical models or stochastic diffusion process approaches. Statistical models such as autoregressive conditional heteroskedasticity (ARCH) models (Engle, 1982) and GARCH model (Day & Lewis, 1992) have been used to capture time-series nature of IV dynamic behavior. On the other hand, stochastic process models such as constant-elasticity-of-variance (CEV) model (Cox, 1975; Cox & Ross, 1976; Beckers, 1980; Chen & Lee, 1993; DelBaen & Sirakawa, 2002; Emanuel & MacBeth, 1982; MacBeth & Merville, 1980; Hsu et al., 2008; Schroder, 1989; Singh & Ahmad, 2011; Pun & Wong, 2013; Larguinho et al., 2013) and stochastic volatility models (Hull & White, 1987; Heston, 1993; Scott, 1997; Lewis, 2000; Lee, 2001; Jones, 2003; Medvedev & Scaillet, 2007) incorporate the interactive behaviors of an asset and its volatilities in option pricing model. From the practitioner's point of view, the implementation and computational costs are the principal criteria of selecting option pricing models to estimate IV. Therefore, we use cross-sectional time-series regression and CEV model to forecast IV with less computational costs.

The two alternative approaches used in this chapter give different perspective of estimating IV. The cross-sectional time-series analysis focuses on the dynamic behavior of volatility in each option contracts. The predicted IV obtained from the time-series model is the estimated conditional volatility based on the information of IV extracted from BSM. Although the estimated IVs in a time-series model vary across option contracts, this kind of model can seize the specification of time-vary characteristic that links ex post volatility to ex ante volatility for each option contract. In addition, cross-sectional analysis can capture other trading behaviors such as week effect and in/out of the money effect. On the other hand, CEV model generalizes implied volatility surface as a function of asset price. It can reduce more computational and implementation costs rather than the complex models such as jump-diffusion stochastic volatility models because there is only one more variable compared with BSM. Although the constant estimated IV for each trading day may cause low forecast power of whole option contacts, it is more reasonable that the IVs of underlying assets are independent of different strike prices and times to expiration.

The focuses of this chapter are (1) to improve the ability to forecast the IV by cross-sectional time-series analysis and CEV model, (2) to explain the significance of variables in each approaches, (3) compare prediction power of these two alternative methods, and (4) test market efficiency by building an arbitrage trading strategy. If volatility changes are predictable by using cross-sectional time-series analysis and CEV model, the prediction power of these two methods can draw specific implications as to how BSM might be misspecified. If the abnormal returns are impossible in a trading strategy which takes transaction costs into account, we would claim that option markets are efficient.

The structure of this chapter is as follows. Section 16.2 reviews previous option pricing models and related empirical works concerning the viability and use of these models. The data and methodology are described in Sect. 16.3. Section 16.4 shows the empirical analysis and devise the trading and hedging strategies to determine if arbitrage profit can be obtained. Finally, in Sect. 16.5, the implications of the results are summarized from both an academic and practitioner view.

16.2 Literature Review

The amount of option pricing research is substantial. This section briefly surveys the major studies which form the impetus for this research effect. Then we introduce previous literature using time-series analysis as an alternative approach to forecast implied volatilities.

16.2.1 Black-Scholes-Merton Option Pricing Model (BSM) and CEV Model

Option pricing is a central issue in the derivatives literature. After the seminal papers by Black and Scholes (1973) and Merton (1973), there has been an explosion in option pricing models developed over the last few decades (Black, 1975; Brenner et al., 1985; Chance, 1986; Ramaswamy & Sundaresan, 1985; Wolf, 1982; Hull, 2011). BSM formula for a European call option on a stock with dividend yield rate, q, is:

$$C_t = S_t e^{-q\tau} N(d_1) - \mathrm{K}e^{-r\tau} N(d_2) \tag{16.1}$$

where $d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$, $d_2 = d_1 - \sigma\sqrt{\tau}$, $N(\cdot)$ is the cumulative probability distribution function for a standardized normal distribution, τ is time to maturity, r is risk free rate, S_0 is current underlying stock price, K is exercise price, σ^2 is the variance of stock returns, C_t is the theoretical BSM option price at time t.

Black's (1976) model for pricing futures call options is used in this study. His model is:

$$\begin{aligned} C_t^F &= e^{-r\tau}\left[F_t N(d_1) - \mathrm{KN}(d_2)\right] \\ d_1 &= \left[\ln(F_t/K) + \left(\sigma_f^2/2\right)\tau\right]/\sigma_f\sqrt{\tau} \\ d_2 &= d_1 - \sigma_f\sqrt{\tau} \end{aligned} \tag{16.2}$$

where C_t^F is the model price for a call option on future at time t, F_t is the underlying futures price at time t, K is the exercise price of the call option, τ is the option's remaining time to maturity in terms of a year, r is the continuous annualized risk-free rate, ${\sigma_f}^2$ is the instantaneous variance of returns of the underlying futures contract over the remaining life of the option.

Although it is well known that the BSM model exhibits biases in its pricing of deep-in and out-of-the-money options and those with a very short or very long term to maturity, the direction of the bias has not been consistent across studies. Black (1975) found that the BSM model systematically over-priced options which were deep-in-the-money and underpriced those being deep-out-of-the-money. However, MacBeth and Merville (1979) reported an exactly opposite type of systematic bias. To make matters even more imprecise, Merton (1976) notes that practitioners often claim that the BSM underprices both deep-in and out-of-the-money options. In regards to time to maturity, it is generally maintained that the BSM underprices short-maturity and overprices long-maturity options. But again, the evidence contains discrepancies, particularly when the bias relative to both exercise price and maturity are considered. All these authors conclude that, to some degree, the pricing bias is related to the volatility parameter which is typically observed not to be proportionally constant over time. Jarrow and Rudd (1982) focus on the potential effects from distributional misspecification of the underlying return-generating process. Thus, their model takes into account pricing biases which might arise due to differences between the second, third and fourth moments of the assumed and "true" distributions.

Previous studies have shown that the constant volatility assumption is inappropriate, and the evidence of our empirical results presents as well. Several more generalized models have been proposed to overcome the BSM restriction on the volatility parameter. Cox (1975) and Cox and Ross (1976) developed the "constant elasticity of variance (CEV) model" which incorporates an observed market phenomenon that the underlying asset variance tends to fall as the asset price increases (and vice versa). The advantage of CEV model is that it can describe the interrelationship between stock prices and its volatility. The constant elasticity of variance (CEV) model for a stock price, S, can be represented as follows:

$$dS = (r - q)\, Sdt + \delta S^{\alpha} dZ \tag{16.3}$$

where r is the risk-free rate, q is the dividend yield, dZ is a Wiener process, δ is a volatility parameter, and α is a positive constant. The relationship between the instantaneous volatility of the asset return, $\sigma(S,t)$, and parameters in CEV model can be represented as:

$$\sigma\,(S,t) = \delta S^{\alpha - 1} \tag{16.4}$$

When $\alpha = 1$, the CEV model is the geometric Brownian motion model we have been using up to now. When $\alpha < 1$, the volatility increases as the stock price decreases. This creates a probability distribution similar to that observed for equities with a heavy left tail and a less heavy right tail. When $\alpha > 1$, the volatility increases as the stock price increases, giving a probability distribution with a heavy right tail and a less left tail. This corresponds to a volatility smile where the implied volatility is an increasing function of the strike price. This type of volatility smile is sometimes observed for options on futures.

The formula for pricing a European call option in CEV model is:

$$C_t = \begin{cases} S_t e^{-q\tau}\left[1-\chi^2(a,\ b+2,\ c)\right] - \mathrm{K}e^{-r\tau}\chi^2(c,\ b,\ a) & \text{when } \alpha < 1 \\ S_t e^{-q\tau}\left[1-\chi^2(c,-b,\ a)\right] - \mathrm{K}e^{-r\tau}\chi^2(a,\ 2-b,\ c) & \text{when } \alpha > 1 \end{cases} \tag{16.5}$$

where $a = \frac{\left[\mathrm{K}e^{-(r-q)\tau}\right]^{2(1-\alpha)}}{(1-\alpha)^2\upsilon}$, $b = \frac{1}{1-\alpha}$, $c = \frac{{S_t}^{2(1-\alpha)}}{(1-\alpha)^2\upsilon}$, $\upsilon = \frac{\delta^2}{2(r-q)(\alpha-1)}\left[e^{2(r-q)(\alpha-1)\tau}-1\right]$, and $\chi^2(z,\ k,\ v)$ is the cumulative probability that a variable with a noncentral χ^2 distribution[1] with noncentrality parameter v and k degrees of freedom is less than z. Hsu, Lin and Lee (2008) provided the detailed derivation of approximative formula for CEV model. Based on the approximated formula, CEV model can reduce computational and implementation costs rather than the complex models such as jump-diffusion stochastic volatility model. Therefore, CVE model with one more parameter than BSM can be a better choice to improve the performance of predicting implied volatilities of index options (Singh & Ahmad, 2011).

Beckers (1980) investigate the relationship between the stock price and its variance of returns by using an approximative closed-form formulas for CEV model based on two special cases of the constant elasticity class ($\alpha = 1$ or 0). Based on the significant relationship between the stock price and its volatility in the empirical results, Beckers (1980) claimed that CEV model in terms of noncentral Chi-square distribution performs better than BC model in terms of log-normal distribution in description of stock price behavior. MacBeth and Merville (1980) is the first paper to empirically test the performance of CEV model. Their empirical results show the negative relationship between stock prices and its volatility of returns; that is, the elasticity class is less than 2 (i.e., $\alpha < 2$). Jackwerth and Rubinstein (2001) and Lee, Wu, and Chen (2004) used S&P 500 index options to do empirical work and found that CEV model performed well because it took account the negative correlation between the index level and volatility into model assumption. Pun and Wong (2013) combine asymptotics approach with CEV model to price American options. Larguinho et al. (2013) compute Greek letters under CEV model to measure different dimension to the risk in option positions and investigate leverage effects in option markets. Tsai (2014) applied CEV model to portfolio hedge strategy and found CEV model can reduce replication error of barrier call options.

[1]The calculation process of $\chi^2(z,\ k,\ v)$ value can be referred to Ding (1992). The complementary noncentral chi-square distribution function can be expressed as an infinite double sum of gamma function, which can be referred to Benton and Krishnamoorthy (2003).

Merton (1976) derived a model based on a jump-diffusion process for the underlying security that allows for discontinuous jumps in price due to unexpected information flows. Geske (1979) derived a compound-option formula which considers the firm's equity to be an option underlying the exchange traded option. An interesting feature of Geske's model is that by incorporating the effects of a firm's leverage on its option the model allows for a nonconstant variance. Alternative option pricing models to describe nonconstant volatility is stochastic volatility models which consider the volatility of the stock as a separate stochastic factor (Scott, 1987; Wiggins, 1987; Stein & Stein, 1991; Heston, 1993; Lewis, 2000; Lee, 2001; Jones, 2003; Medvedev & Scaillet, 2007). Heston (1993) assumes the dynamics of instantaneous variance, V, as a stochastic process:

$$dS = \mu S dt + \sqrt{V} S dZ_1 \tag{16.6}$$

$$dV = (\alpha + \beta V)\, dt + \sigma \sqrt{V} dZ_2 \tag{16.7}$$

where dZ_1 and dZ_2 are Wiener processes with correlation ρ. For the complex implied volatility model without closed-form solutions, advanced techniques such as partial differential equations (PDEs) or Monte Carlo simulation are used to estimate the approximation of implied volatility under non-tractable models. Lewis (2000) and Lee (2001) estimate implied volatility under stochastic volatility model without jumps. Jones (2003) extends the Heston model and proposes a more general stochastic volatility models in the CEV class as follows:

$$dS = \mu S dt + \sqrt{V} S dZ_1 \tag{16.8}$$

$$dV = (\alpha + \beta V)\, dt + \sigma_1 V^{\gamma_1} dZ_1 + \sigma_2 V^{\gamma_2} dZ_2 \tag{16.9}$$

where dZ_1 and dZ_2 are independent Wiener processes under the risk-neutral probability measure. The model setting in Jones (2003) allows the correlation of the price and variance processes to depend on the level of instantaneous variance. Recently, Medvedev and Scaillet (2007) deal with a two-factor jump-diffusion stochastic volatility model where there is a jump term in stock price and volatility follows another stochastic process related to stock price's Brownian motion term with constant correlation ρ. Medvedev and Scaillet (2007) empirical results advocate the necessary of introducing jumps in stock price process. They found that jumps are significant in returns. The evidence also supports the specification of the stochastic volatility in CEV model (Jones, 2003; Heston, 1993).

The optimal selection of an option pricing model should be based on a trade-off between its flexibility and its analytical tractability. The more complicated model it is, the less applicable implementation the model has. Although jump-diffusion stochastic volatility models can general volatility surface as a deterministic function of exercise price and time, the computational costs such as parameter calibration or model implementation are high. Chen, Lee and Lee (2009) indicated that CEV model should be better candidate rather than other complex jump-diffusion stochastic volatility models because of fast computational speed and less implementation costs. Therefore, we decide to use CEV model for forecasting implied volatilities in our empirical study.

16.2.2 Time-Varying Volatility and Time-Series Analysis

Several studies have attempted to improve the estimation of the volatility term required by the BSM and Black models. Harvey and Whaley (1992a, 1992b) stated that market volatility changes are predictable by forecasting the volatility implied in index options. Their findings are consistent with the trading cost hypothesis that the index futures and option price changes lead price changes in the stock market (Stephan & Whaley, 1990; Fleming, Ostdiek, & Whaley, 1996). Therefore, we can employ the predicted IV to do hedge strategy and risk management.

All the studies involving IV estimation point out to one degree or another that for any day, the individual IV's for all the options on a particular asset (stock or futures contract) will all be different, and will change over time. Yet as MacBeth and Merville (1979) aptly note, different exercise prices should not imply differing IV's since the IV pertains to the underlying asset itself and not the exercise price. In what might be considered a preliminary basis for this study, MacBeth and Merville (1979) relate systematic pricing differences between market and BSM option prices to the systematic differences that occur among individual IV's relative to exercise price and time to maturity.

Since Latana and Rendleman's (1976) development of the IV concept, numerous researchers have studied different weighting schemes in calculating the IV. The majority of studies, including Schmalensee and Trippi (1978) and Chiras and Manaster (1978), devise weighting schemes which aim at deriving a single weighted IV from among all individual IV's for input into the BSM model. Whaley (1981; 1982) and Park and Sears (1985) utilized an OLS regression procedure to weight and segregate IV's by maturity date. The major finding of the Park and Sears (1985) study, which used option on stock index futures data, was a "time-to-maturity" effect in the pattern of the weighted IV's over time. The authors interpreted their findings as being consistent with Merton's (1973) option pricing model with stochastic interest rate. This is a portion of the IV's instability is due to the diminishing instantaneous variance of the riskless security.

Another rather foreshadowing study conducted by Brenner and Galai (1981) not only found significant divergence between the daily individual IV's and some time-series average IV, but that the distributions of the average IV's were not invariant over time. Finally, Rubenstein (1985) used individual IV's to test five alternative option pricing models versus the BSM formulation, and attempted to explain observed pricing biases. Rubenstein (1985) reported that the direction of pricing bias changed over time. This instability could be a function not only of a time-varying volatility term, but also stochastic interest rates and a changing stock market climate. Harvey and Whaley (1992a, 1992b) utilized OLS regression of the change in IV on S&P 100 index option on lagged IV, week effect dummy variables, and interest rate measures to test if IV is predictable. The significant abnormal returns obtained in Harvey and Whaley (1992a, 1992b) indicated that the market volatility is predictable time-varying variable and can be estimated by time-series analysis.

16.3 Data and Methodology

16.3.1 Data

The data for this study of individual option IV's included the use of call options on the S&P 500 index futures which are traded at the Chicago Mercantile Exchange (CME).[2] The Data is the options on S&P 500 index futures expired within January 1, 2010 to December 31, 2013. The reason for using options on S&P 500 index futures instead of S&P 500 index is to eliminate from nonsimultaneous price effects between options and its underlying assets (Harvey & Whaley, 1991). The option and future markets are closed at 3:15pm Central Time (CT), while stock market is closed at 3pm CT. Therefore, using closing option prices to estimate the volatility of underlying stock return is problematic even though the correct option pricing model is used. In addition to no nonsynchronous price issue, the underlying assets, S&P 500 index futures, do not need to be adjusted for discrete dividends. Therefore, we can reduce the pricing error in accordance with the needless dividend adjustment. According to the suggestions in Harvey and Whaley (1991, 1992a, 1992b), we select simultaneous index option prices and index future prices to do empirical analysis.

The risk free rate used in Black model and CEV model is based on 1-year Treasury Bill from Federal Reserve Bank of ST. LOUIS.[3] Daily closing price and trading volumes of options on S&P 500 index futures and its underlying asset can be obtained from Datastream.

There are two ways to select data in respect to two alternative methodologies used in this chapter. For time-series and cross-sectional analysis, we ignore transaction information and choose the futures options according to the length of trading period. The futures options expired on March, June and September in both 2010 and 2011 are selected because they have over 1 year trading date (above 252 observations) while other options only have more or less 100 observations. Studying futures option contracts with same expired months in 2010 and 2011 will allow the examination of

[2]Nowadays Chicago Mercantile Exchange (CME), Chicago Board of Trade (CBOT), New York Mercantile Exchange (NYMEX), and Commodity Exchange (COMEX) are merged and operate as designated contract markets (DCM) of the CME Group which is the world's leading and most diverse derivatives marketplace. Website of CME group: http://www.cmegroup.com/

[3]Website of Federal Reserve Bank of ST. LOUIS: http://research.stlouisfed.org/

IV characteristics and movements over time as well as the effects of different market climates.

In order to ensure reliable estimation of IV, we estimate market volatility by using multiple option transactions instead of a single contract. For comparing prediction power of Black model and CEV model, we use all futures options expired in 2010 and 2013 to generate implied volatility surface. Here we exclude the data based on the following criteria: (1) BS IV cannot be computed, (2) trading volume is lower than 10 for excluding minuscule transactions, (3) time-to-maturity is less than 10 days for avoiding liquidity-related biases, (4) quotes not satisfying the arbitrage restriction: excluding option contact if its price larger than the difference between S&P500 index future and exercise price, and (5) deep-in/out-of-money contacts where the ratio of S&P500 index future price to exercise price is either above 1.2 or below 0.8.

After arranging data based on these criteria, we still have 30,364 observations of future options which are expired within the period of 2010 to 2013. The period of option prices is from March 19, 2009 to November 5, 2013.

16.3.2 Methodology

In this section, two alternative approaches to estimate IVs are introduced. We first illustrate how to obtain BSM IV for each option contract in MATLAB. Then, based on BSM IVs, we forecast future BSM IVs for each option contract by time-series analysis and cross-sectional regression. Finally, the second method to estimate future IV is based on CEV model. To deal with moneyness- and maturity-related biases, we use the "implied-volatility matrix" to find proper parameters in CEV model. Then, the IV surface can be represented for predicting future IV in different moneyness and time-to-maturity categories.

16.3.2.1 Estimating BSM IV

This chapter can utilize financial toolbox in MATLAB to calculate the implied volatility for futures option that the code of function is as follows:

$$\text{Volatility} = \text{blsimpv}\,(\text{Price},\ \text{Strike},\ \text{Rate},\ \text{Time},\ \text{Value},\ \text{Limit},\ \text{Tolerance},\ \text{Class})$$

where the blsimpv is the function name; Price, Strike, Rate, Time, Value, Limit, Tolerance, and Class are input variables; Volatility is the annualized IV.[4] The

[4]Detailed information of the function and example of calculating the implied volatility for futures option can be found on MathWorks website: http://www.mathworks.com/help/toolbox/finance/blkimpv.html

advantages of this function are the allowance of the upper bound of implied volatility (Limit variable) and the adjustment of the implied volatility termination tolerance (Tolerance variable), in general, equal to 0.000001. The algorithm used in blsimpv function is Newton's method.

When we do the comparison of performance between CEV model and Black model, the implied volatility of Black model for each group at time t can be obtained by following steps:

1. Let $C^F_{i,n,t}$ is market price of the nth option contract in category i, $\widehat{C^F_{i,n,t}}(\sigma)$ is the model option price determined by Black model in Eq. (16.2) with the volatility parameters, σ. For nth option contract in category i at date t, the difference between market price and model option price can be described as:

$$\varepsilon^F_{i,n,t} = C^F_{i,n,t} - \widehat{C^F_{i,n,t}}(\sigma) \tag{16.10}$$

2. For each date t, we can obtain the optimal parameters in each group by solving the minimum value of absolute pricing errors (minAPE) as:

$$\text{minAPE}_{i,t} = \min_{\sigma} \sum_{n=1}^{N} \left|\varepsilon^F_{i,n,t}\right| \tag{16.11}$$

   Where N is total number of option contracts in group i at time t.

3. Using MTALAB optimization function to find optimal σ_0 in a fixed interval. The function code is as follows:

$$[\sigma_0,\ \text{fvalBls}] = \text{fminbnd}\left(\text{fun},\ x_1, x_2\right), \tag{16.12}$$

Where σ_0 is an optimal implied volatility in Black model that locally minimize function of minAPE, fvalBls is the minimum value of minAPE, fun is MATLAB function describing Eq. (16.11). The implied volatility, σ_0, is constrained in the interval between x_1 and x_2, that is, $x_1 \leq \sigma_0 \leq x_2$. The algorithm of fminbnd function is based on golden section search and parabolic interpolation.

16.3.2.2 Forecasting IV by Cross-Sectional and Time-Series Analysis

Time-Series Analysis

Box and Jenkins (1970) time-series model building techniques are used to identify, estimate, and check models describing particular generating processes. These models are of the form

$$x_t - \Phi_1 x_{t-1} - \cdots - \Phi_p x_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} \tag{16.13}$$

where x_t is an observation from a covariance stationary series meaning that

$$\lambda_\tau = \text{cov}(x_t, x_{t-\tau}) \tag{16.14}$$

is independent of t for all τ. The Φ and θ terms represent the autoregressive (AR) and moving average (MA) coefficients and ε_t is white noise.

A developed technique motivated by Hannan and Rissanen (1982) seems to provide a good practical basis for model selection. The process involves two stages of computation. The purpose of the first stage is to obtain estimates of the innovation errors of model. This is accomplished by running successively higher order autoregressive models and using the AIC of Akaike (1969) to determine the optimal order from among them. The innovation errors are estimated by

$$\widehat{\varepsilon}_t = x_t - \widehat{\Phi}_1 x_{t-1} - \cdots - \widehat{\Phi}_k x_{t-k} \tag{16.15}$$

where k is the optimal autoregressive order suggested by the AIC. The second stage involves fitting all different combinations of ARMA (p, q) models where, instead of using full maximum likelihood estimation, the innovation errors estimated in stage one are used as the regressors upon which the moving average parameter estimates are based. This allows use of least squares. The different ARMA (p, q) models are then compared using the AIC of Akaike (1977) and SBC of Schwarz (1978) and the appropriate model is chosen on that basis. A simulation study conducted by Ansley and Newbold (1980) has found that exact maximum likelihood estimation outperforms least squares when the series are of moderate size and moving average terms are involved. An approximation to the full maximum likelihood function has been derived by Hillmer and Tiao (1979).

In addition, alternative simple time-series methods are taken into account to compare with the forecastability indicators from optimal ARMA models. There are five alternative models to generate IV indicators which are used in cross-sectional regression model in next section. These time-series models are as follows:

1. ARMA model (ARMA): $\text{IV}_t = a_0 + \sum_{i=1}^{p} a_i \text{IV}_{t-i} + \varepsilon_t + \sum_{i=1}^{q} b_i \varepsilon_{t-i}$
2. Lag IV method (LIV): $\text{IV}_t = \text{IV}_{t-1}$
3. 5-day moving average method (MAV5): $\text{IV}_t = \frac{\sum_{i=1}^{5} \text{IV}_{t-i}}{5}$
4. 5-day exponential moving average method (EMA5): $\text{IV}_t = \frac{\sum_{i=1}^{5} 2^{i-1} \text{IV}_{t-i}}{\sum_{i=1}^{5} 2^{i-1}}$
5. Regression on lag IV (RGN): $\text{IV}_t = a_0 + a_1 \text{IV}_{t-1} + \varepsilon_t$

The optimal ARMA model is autoregressive-moving-average model with order of the autoregressive part, p, and the order of the moving average part, q where the suitable p and q are based on the goodness-of-fit indicators, AIC and SBC, and the forecastability indicators, RMSE, MAE, and MAPE. The 5-day moving average and the 5-day exponential moving average methods can be expressed as the special cases of the general AR(5) model. Lag IV and the regression on lag IV methods belong to AR(1) process.

Cross-Sectional Predictive Regression Model

A significant amount of information has been shown to exist in a time series of IV. The five alternative time-series models used to describe the generating processes of the IV series examined are all clearly preferred to random walk or "white noise" alternatives. These models do not give the final word on the subject of IV forecasting, however. There are several cross-contract effects that may exist which, if isolated properly, will provide further predictive power. To learn more about these different influences, a large cross-sectional time-series predictive regression model was formulated. The cross-sectional time-series predictive regression model is

$$y_{it} = \beta_0 + \beta_1 x_{1it-1} + \beta_2 x_{2it-1} + \cdots + \beta_{14} x_{14it-1} + \varepsilon_{it} \qquad (16.16)$$

where y_{it} is IV of the i^{th} option contract at time t; x_{1it-1} is the time-series predictor of i^{th} contract for time t based on information known at time $t-1$ and one of forecasting time-series methods; x_{2it-1} is time to maturity of the i^{th} option contract at time $t-1$ which is the unit of year; x_{3it-1} is proportional in-the-money that is equal to the value of (future price at time $t-1$—strike price)/(strike price) if the value is positive, otherwise is zero; x_{4it-1} is proportion out-of-the-money that is equal to the value of (strike price—futures price at time $t-1$)/(strike price) if the value is positive, otherwise is zero; x_{5it-1} and x_{6it-1} are standard deviation of the IV based on previous 5 and 20 observations, respectively; x_{7it-1} and x_{7it-1} are skewness and kurtosis of IV distribution over the previous 20 observations, respectively; x_{9it-1} and x_{10it-1} are the standard deviations of the rate of returns of the underlying future price on previous 5 and 20 observations, respectively; x_{11it-1}, x_{12it-1}, x_{13it-1}, and x_{14it-1} are dummy variables that equals 1 if the trading date at time $t-1$ is Tuesday, Wednesday, Thursday, and Friday, respectively.

The time-to-maturity variable was included because, as was indicated by Park and Sears (1985), there tends to be a certain point close to maturity where the IV's begin to decrease. The third and fourth independent variables have been included to see if deep-in-the-money options and far-out-of-the-money options tend toward higher or lower than expected IV's. Previous studies have had conflicting answers to this important question (see Jarrow & Rudd, 1983). The next two independent variables are included to determine whether or not the standard deviations of the IVs have any positive or negative effect on the IVs themselves. The third and fourth moments of the distribution of 20 previous IV observations were also included in the regression equation to see what, if any, influence they have in determining current IV.

The two measures of the standard deviations of the rate of returns of the underlying future price are of great interest as regressors since these have traditionally been approximations of the variable used in the BSM model to determine the theoretical option price. The final four explanatory variables are weekday effect dummies which are intended to see if certain days give rise to higher IV than others. For example, certain economic announcements are regularly made on particular days of the week and this may have a weekday effect on IV. Note that only four dummy variables are needed to describe the 5 days of the week in order to avoid perfect multi-collinearity with the constant term.

16.3.2.3 Forecasting IV by CEV Model

To deal with moneyness- and expiration- related biases in estimating BSM IV, we use the "implied-volatility matrix" to separate option contracts and estimate parameters of CEV model in each category. The option contracts are divided into nine categories by moneyness and time-to-maturity. Option contracts are classified by moneyness level as at-the-money (ATM), out-of-the-money (OTM), or in-the-money (ITM) based on the ratio of underlying asset price, S, to exercise price, K. If an option contract with S/K ratio is between 0.95 and 1.01, it belongs to ATM category. If its S/K ratio is higher (lower) than 1.01 (0.95), the option contract belongs to ITM (OTM) category. According to the large observations in ATM and OTM, we divide moneyness-level group into five levels: ratio above 1.01, ratio between 0.98 and 1.01, ratio between 0.95 and 0.98, ratio between 0.90 and 0.95, and ratio below 0.90. By expiration day, we classified option contracts into short-term (less than 30 trading days), medium-term (between 30 and 60 trading days), and long-term (more than 60 trading days).

Since for all assets the future price equals the expected future spot price in a risk-neutral measurement, the S&P 500 index futures prices have same distribution property of S&P 500 index prices. Therefore, for a call option on index futures can be given by Eq. (16.5) with S_t replaced by F_t and $q = r$ as Eq. (16.17)[5]:

$$C_t^F = \begin{cases} e^{-r\tau}\left(F_t\left[1-\chi^2(a,\ b+2,\ c)\right] - K\chi^2(c,\ b,\ a)\right) & \text{when } \alpha < 1 \\ e^{-r\tau}\left(F_t\left[1-\chi^2(c,-b,\ a)\right] - K\chi^2(a,\ 2-b,\ c)\right) & \text{when } \alpha > 1 \end{cases} \tag{16.17}$$

where

$$a = \frac{K^{2(1-\alpha)}}{(1-\alpha)^2\upsilon},\ b = \frac{1}{1-\alpha},\ c = \frac{{F_t}^{2(1-\alpha)}}{(1-\alpha)^2\upsilon},\ \ \upsilon = \delta^2\tau$$

[5]When substituting $q = r$ into $\upsilon = \frac{\delta^2}{2(r-q)(\alpha-1)}\left[e^{2(r-q)(\alpha-1)\tau} - 1\right]$, we can use L'Hospital's Rule to obtain υ. Let $x = r - q$, then

$$\lim_{x\to 0}\frac{\delta^2\left[e^{2x(\alpha-1)\tau}-1\right]}{2x(\alpha-1)} = \lim_{x\to 0}\frac{\frac{\partial\delta^2\left[e^{2x(\alpha-1)\tau}-1\right]}{\partial x}}{\frac{\partial 2x(\alpha-1)}{\partial x}} = \lim_{x\to 0}\frac{(2(\alpha-1)\tau)\delta^2\left[e^{2x(\alpha-1)\tau}\right]}{2(\alpha-1)} = \lim_{x\to 0}\frac{\tau\delta^2\left[e^{2x(\alpha-1)\tau}\right]}{1} = \tau\delta^2.$$

The procedures to obtain estimated parameters of CEV model in each category of implied-volatility matrix are as follows:

1. Let $C_{i,n,t}^{F}$ is market price of the nth option contract in category i, $\widehat{C_{i,n,t}^{F}}(\delta_0, \alpha_0)$ is the model option price determined by CEV model in Eq. (16.17) with the initial value of parameters, $\delta = \delta_0$ and $\alpha = \alpha_0$. For nth option contract in category i at date t, the difference between market price and model option price can be described as:

$$\varepsilon_{i,n,t}^{F} = C_{i,n,t}^{F} - \widehat{C_{i,n,t}^{F}}(\delta_0, \alpha_0) \tag{16.18}$$

2. For each date t, we can obtain the optimal parameters in each group by solving the minimum value of absolute pricing errors (minAPE) as:

$$\text{minAPE}_{i,t} = \min_{\delta_0, \alpha_0} \sum_{n=1}^{N} \left| \varepsilon_{i,n,t}^{F} \right| \tag{16.19}$$

   Where N is total number of option contracts in group i at time t.

3. Using optimization function in MATLAB to find a minimum value of the unconstrained multivariable function. The function code is as follows:

$$[x,\ \text{fval}] = \text{fminunc}\,(\text{fun},\ x_0) \tag{16.20}$$

where x is the optimal parameters of CEV model, fval is the local minimum value of minAPE, fun is the specified MATLAB function of Eq. (16.19), and x_0 is the initial points of parameters obtained in step (1). The algorithm of fminunc function is based on quasi-Newton method.

16.4 Empirical Analysis

In the empirical study section, we present the forecastability of S&P 500 index option price for two alternative models: time-series and cross-sectional analysis and CEV model. First, the statistical analysis for time-series futures option prices of the contracts expired on March, June and September in both 2010 and 2011 is summarized. Then we use time-series and cross-sectional models to analyze each individual contract and compare their forecastability of IV. Finally, we estimated IV by using CEV model and compare its pricing accuracy with Black model.

Table 16.1 Distributional Statistics for Individual IV's

Option Series[a]	Mean	Std. Dev.	CV[b]	Skewness	Kurtosis	Studentized Range[c]	Observation
Call Futures Options in 2010							
MAR10 1075	0.230	0.032	0.141	2.908	14.898	10.336	251
JUN10 1050	0.263	0.050	0.191	0.987	0.943	6.729	434
JUN10 1100	0.247	0.047	0.189	0.718	−0.569	4.299	434
SEP10 1100	0.216	0.024	0.111	0.928	1.539	6.092	259
SEP10 1200	0.191	0.022	0.117	0.982	2.194	6.178	257
Call Futures Options in 2011							
MAR11 1200	0.206	0.040	0.195	5.108	36.483	10.190	384
MAR11 1250	0.188	0.027	0.145	3.739	25.527	10.636	324
MAR11 1300	0.176	0.021	0.118	1.104	4.787	8.588	384
JUN11 1325	0.165	0.016	0.095	−1.831	12.656	10.103	200
JUN11 1350	0.161	0.018	0.113	−0.228	1.856	8.653	234
SEP11 1250	0.200	0.031	0.152	2.274	6.875	7.562	248
SEP11 1300	0.185	0.024	0.131	2.279	6.861	7.399	253
SEP11 1350	0.170	0.025	0.147	2.212	5.848	6.040	470

[a]Option series contain the name and code of futures options with information of the strike price and the expired month, for example, SEP11 1350 represents that the futures call option is expired on September, 2011 with the strike price $1350 and the parentheses is the code of this futures option in Datastream

[b]CV represents the coefficient of variation that is standard deviation of option series divided by their mean value

[c]Studentized range is the difference of the maximum and minimum of the observations divided by the standard deviation of the sample

16.4.1 Distributional Qualities of IV time series

A summary of individual IV distributional statistics for S&P 500 index futures call options in 2010 and 2011 appears in Table 16.1. Comparing the mean IV's across time periods, it is quite evident that the 2011 IV's are significantly smaller. Also, the time-to-maturity effect observed by Park and Sears (1985) can be identified. The September options in 2011 possess higher mean IV's than those maturing in June and March with the same strike price.

The other statistical measures listed in Table 16.1 are the relative skewness and relative kurtosis of the IV series, along with the studentized range. Skewness measures lopsidedness in the distribution and might be considered indicative of a series of large outliers at some point in the time series of the IV's. Kurtosis measures the peakedness of the distribution relative to the normal and has been found to affect the stability of variance (Lee & Wu, 1985). The studentized range gives an overall indication as to whether the measured degrees of skewness and kurtosis have significantly deviated from the levels implied by a normality assumption for the IV series.

Using significance tests on the results of Table 16.1 in accordance with Jarque–Bera test, the 2010 and 2011 skewness and kurtosis measures indicate a higher proportion of statistical significance. We also utilize simple back-of-the-envelope test based on the studentized range to identify whether the individual IV series approximate a normal distribution. The studentized range larger than 4 in both 2010 and 2011 indicates that a normal distribution significantly understates the maximum magnitude of deviation in individual IV series.

As a final point to this brief examination of the IV skewness and kurtosis, note the statistics for MAR10 1075, MAR11 1200, and MAR11 1250 contracts. The relative size of this contract's skewness and kurtosis measures reflect the high degree of instability that its IV exhibited during the last 10 days of the contract's life. Such instability is consistent across contracts.

However, these distortions remain in the computed skewness and kurtosis measures only for these particular contracts to emphasize how a few large outliers can magnify the size of these statistics. For example, the evidence that S&P 500 future price jumped on January 18, 2010 and plunged on February 2, 2011 cause the IV of these particular contracts sharply increasing on that dates. Thus, while still of interest, any skewness and kurtosis measures must be calculated and interpreted with caution.

16.4.2 Time-Series and Cross-Sectional Analysis for IV Series

The optimal ARMA models for the IV series are based on the goodness-of-fit indicators, AIC and SBC, and the forecastability indicators, root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

Table 16.2 shows the results of the regression model in Eq. (16.16). The time-series predictor variables calculated by different forecasting models are all significant, which should come as no surprise—IV depends on past IV value. However, the fact that other regressors were found to be significant indicates that not all of the variation in IV series is explained by the past. Time-to-maturity has the predicted positive effect. The closer an option is to expiration, the lower the IV. The in-the-money effect is significantly positive; however, the out-of-the-money give mixed insignificant influence on IV series. Merton (1976) shows that large deviations from the strike price tend to bias the BSM theoretical price downward. Therefore it is logical to expect the IV of the deep-in-the-money and far-out-of-the-money contracts to be higher because the writer of these calls runs a greater risk of being stuck in his position. However, in this study, the selected IV time series calculated by BSM model cannot show the downward characteristic obviously because the longest trading data is the option contract with the strike price close to the underlying asset.

The coefficients on the standard deviation of the IV variables give the significantly positive signals based on previous 20 observations, but show the negative effect based on previous 5 observations when the short term effect is of significance.

Table 16.2 The results of the cross-sectional time-series predictive regression model

Year	2010	2011	2010	2011	2010	2011	2010	2011	2010	2011
Variable	ARMA		Previous IV		5-day moving averages		5-day exponential moving averages		Regression	
Intercept	0.0068 (3.13)***	0.0037 (1.61)	0.0146 (6.34)***	0.0260 (11.08)***	0.0049 (1.62)	−0.0067 (−2.31)**	0.0024 (0.95)	0.0016 (0.65)	0.0088 (3.70)***	0.0137 (5.51)***
Time-series predictor	0.9362 (68.26)***	0.9634 (62.36)***	0.8798 (60.80)***	0.7827 (52.38)***	0.9576 (48.08)***	1.0546 (52.08)***	0.9650 (58.85)***	0.9582 (58.33)***	0.9064 (61.01)***	0.8574 (54.03)***
Time to Maturity	0.0038 (2.43)**	−0.0013 (−1.12)	0.0068 (4.03)***	0.0048 (3.67)***	0.0010 (0.48)	−0.0054 (−3.93)***	0.0007 (0.41)	−0.0013 (−1.00)	0.0057 (3.35)***	0.0023 (1.78)*
Proportion in-the-money	0.0718 (5.81)***	0.0914 (4.83)***	0.0880 (6.56)***	0.1536 (7.38)***	0.0904 (5.72)***	0.0507 (2.34)**	0.0694 (5.01)***	0.0732 (3.67)***	0.0757 (5.63)***	0.1383 (6.74)***
Proportion out-of-the-money	−0.0088 (−1.41)	0.0143 (1.84)*	−0.0160 (−2.36)**	−0.0056 (−0.65)	−0.0004 (−0.05)	0.0204 (2.34)**	−0.0029 (−0.42)	0.0048 (0.59)	−0.0141 (−2.08)**	0.0173 (2.04)**
σ_{IV} (5 obs.)	−0.0054 (−0.10)	−0.2240 (−5.61)***	0.0366 (0.61)	−0.1059 (−2.38)**	0.0273 (0.39)	−0.4047 (−9.02)***	−0.0242 (−0.40)	−0.3028 (−7.25)***	0.0308 (0.52)	−0.1324 (−3.03)***
σ_{IV} (20 obs.)	0.2355 (3.59)***	0.1244 (2.01)**	0.3268 (4.59)***	0.3887 (5.74)***	0.2889 (3.42)***	0.1995 (2.89)***	0.2123 (2.88)***	0.2498 (3.90)***	0.3385 (4.78)***	0.4061 (6.12)***
Skewness	0.0004 (1.60)	0.0005 (1.44)	0.0005 (1.79)*	0.0006 (1.80)*	0.0001 (0.24)	0.0002 (0.68)	−0.0001 (−0.19)	0.0000 (0.10)	0.0007 (2.27)**	0.0011 (3.12)***
Kurtosis	−0.0002 (−1.78)*	0.0003 (2.15)**	−0.0001 (−0.55)	0.0001 (0.63)	−0.0001 (−0.71)	0.0006 (4.65)***	−0.0001 (−0.67)	0.0003 (2.49)**	−0.0001 (−0.93)	0.0002 (1.40)
σ_{R_future} (5 obs.)	0.0450 (0.73)	−0.0885 (−1.24)	0.0212 (0.32)	−0.0959 (−1.21)	−0.0407 (−0.52)	−0.2633 (−3.31)***	−0.0146 (−0.21)	−0.1846 (−2.48)**	0.0266 (0.40)	−0.1116 (−1.43)
σ_{R_future} (20 obs.)	0.1285 (1.46)	0.2018 (1.85)*	0.3202 (3.37)***	0.7288 (6.11)***	0.0536 (0.46)	0.1396 (1.13)	0.0433 (0.43)	0.2973 (2.61)***	0.3259 (3.44)***	0.5721 (4.85)***

(continued)

Table 16.2 (continued)

Year	2010	2011	2010	2011	2010	2011	2010	2011	2010	2011
Variable	ARMA		Previous IV		5-day moving averages		5-day exponential moving averages		Regression	
Tuesday	−0.0001 (−0.15)	0.0002 (0.23)	−0.0003 (−0.29)	0.0010 (0.99)	−0.0009 (−0.76)	−0.0004 (−0.42)	−0.0007 (−0.70)	0.0001 (0.12)	−0.0003 (−0.30)	0.0009 (0.91)
Wednesday	0.0014 (1.48)	0.0010 (1.12)	0.0015 (1.47)	0.0015 (1.53)	0.0001 (0.10)	0.0000 (−0.01)	0.0009 (0.82)	0.0007 (0.77)	0.0015 (1.47)	0.0015 (1.52)
Thursday	0.0009 (0.96)	0.0005 (0.55)	0.0010 (1.02)	0.0008 (0.82)	0.0000 (−0.02)	−0.0005 (−0.50)	0.0008 (0.77)	0.0004 (0.38)	0.0010 (1.02)	0.0008 (0.78)
Friday	0.0009 (0.99)	0.0025 (2.70) ***	0.0010 (1.01)	0.0031 (3.08) ***	0.0006 (0.49)	0.0016 (1.58)	0.0010 (0.92)	0.0026 (2.68) ***	0.0010 (0.99)	0.0030 (3.05) ***
R^2	0.9341	0.8313	0.9219	0.7913	0.8936	0.7899	0.9182	0.8163	0.9223	0.7986
Adjusted R^2	0.9334	0.8302	0.9211	0.7899	0.8926	0.7885	0.9175	0.8151	0.9215	0.7972
S_e	0.0116	0.0132	0.0127	0.0147	0.0148	0.0147	0.0130	0.0138	0.0126	0.0144
F-value	1532.75***	733.25***	1276.67***	564.08***	908.61***	559.51***	1214.92***	661.33***	1283.62***	589.98***
Chow test	2.125***		6.108***		3.763***		2.725***		2.985***	

For the forecasting methods used in Table 16.2 are as follows: ARMA is autoregressive-moving-average model with order of the autoregressive part, $\mathbf{p}$, and the order of the moving average part, $\mathbf{q}$ as the form $\mathbf{IV_t} = \mathbf{a}_0 + \sum_{\mathbf{i=1}}^{\mathbf{p}} \mathbf{a_i IV_{t-i}} + \boldsymbol{\varepsilon}_\mathbf{t} + \sum_{\mathbf{i=1}}^{\mathbf{q}} \mathbf{b_i} \boldsymbol{\varepsilon}_{\mathbf{t-i}}$. The suitable $\mathbf{p}$ and $\mathbf{q}$ are based on the goodness-of-fit indicators, AIC and SBC, and the forecastability indicators, RMSE, MAE, and MAPE. Previous IV is simply an AR(1) process of the form $\mathbf{IV_t} = \mathbf{IV_{t-1}}$. The 5-day moving averages can be expressed as an AR(5) model of the form $\mathbf{IV_t} = 0.2\ \mathbf{IV_{t-1}} + 0.2\ \mathbf{IV_{t-2}} + 0.2\ \mathbf{IV_{t-3}} + 0.2\ \mathbf{IV_{t-4}} + 0.2\ \mathbf{IV_{t-5}}$. The 5-day exponential moving averages is another special case of the general AR(5) model where the parameters are restricted as follows: $\mathbf{IV_t} = \frac{16}{31}\mathbf{IV_{t-1}} + \frac{8}{31}\mathbf{IV_{t-2}} + \frac{4}{31}\mathbf{IV_{t-3}} + \frac{2}{31}\mathbf{IV_{t-4}} + \frac{1}{31}\mathbf{IV_{t-5}}$. The Regression is the general case of AR(1) as $\mathbf{IV_t} = \mathbf{a}_0 + \mathbf{a}_1\mathbf{IV_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t}$. Proportion in-the-money is equal to the value of (future price at time $\mathbf{t}-1$—strike price)/strike price if the value is positive, otherwise is zero. Proportion out-of-the-money is equal to the value of (strike price—future price at time $\mathbf{t}-1$)/strike price if the value is positive, otherwise is zero. σ_{IV} (5 obs.) and σ_{IV} (20 obs.) are the standard deviations of the IV based on previous 5 and 20 observations, respectively. Skewness and Kurtosis are the skewness and kurtosis of the IV distribution over the previous 20 observations, respectively. σ_{R_future} (5 obs.) and σ_{R_future} (20 obs.) are the standard deviations of the rate of returns of the underlying future price on previous 5 and 20 observations. Tuesday, Wednesday, Thursday, and Friday are the dummy variables that equal to one if the trading date of IV observation at time $\mathbf{t}-1$ is equal to that weekday and otherwise equal to zero. The value in table is represented the estimation coefficient with the t-value in parentheses

*, **, and *** in parenthesis indicate the significance at 90, 95, and 99 % level. Total observations for 2010 and 2011 are 1530 and 2098, respectively. The chow test is therefore follow $F(15, 3598)$ distribution with critical value 2.04 at 99 % level

The skewness and kurtosis terms have consistently slight effects over two sample periods even though sometimes the effects have statistical significance. Perhaps what can be said about the lower relationship between these two statistic measures and predicted IV is that the influence of the outliers bringing about the skewness and kurtosis is already captured by other independent variables such as the time-series predictor estimated by forecasting model or the standard deviation of IV series. The coefficient on the standard deviation of the rate of returns of the underlying future price only has significantly large positive effect on IV for the 20-day measure. The strong relationship to historical standard deviations of underlying assets seems that the IV series not only response to market deviation from the functional specification of the BSM model but also reflect the market assessment of the standard deviation of underlying assets.

The weekday effect dummies indicate a significantly small Friday effect where the IV are slightly higher. This may be related to the fact that certain economic announcements are made on Friday such as employment situation or lag response to the announcements made on Thursday such as money supply and jobless claims. These economic announcements will alter the market perception of asset price volatility, especially currently the situation of economics that just came through the financial crisis and is suffering from European sovereign-debt crisis. The Friday effect might also be related to option market inactivity the day before the weekend. Further study may investigate this apparent weekday effect to explain why Friday's market may be out of line with that of other days.

Whether the estimated models change significantly over time is an important question. The parameter estimates obtained for this cross-sectional time-series model seems not consistent in 2010 and 2011 sample periods. A Chow test[6] statistic indicating structural change based on five forecasting methods are obtained in Table 16.2 for the 2010 and 2011 regressions. These values exceed the table value of 2.04 for an F random variable at the 99 % level. The chow test indicated the significant change of structure in the cross-sectional time-series predictive regression model on 2010 and 2011. It would therefore be wise for the practitioner to update parameter estimates periodically even though both 2010 and 2011 sample periods are suffering from global financial crisis.

16.4.3 Ex-Post Test for Forecastability of Time-Series and Cross-Sectional Regression Models

In this section, the practical monetary value of the IV estimates versus more naive methods is tested, to determine which might be superior from a trader's point of

[6]Chow test $F_{q,n-k} = \frac{e'_* e_* / \alpha}{e'e/(n-k)}$ where $e'_* e_*$ is restricted SSE, $e'e$ is unrestricted SSE, α represents the number of restrictions, and k is number of regression coefficients estimated in unrestricted regression.

view. In addition, we hope that these results will further support the theoretical and practical superiority of using individual IV estimates versus some weighted-IV measure.

Trading rule tests in this chapter utilize seven different estimates for IV as follows: (1) a 5-day equally weighted moving average of the IV (MAV5); (2) a 5-day exponentially weighted average of the IV (EMA5); (3) a 1-day lag of the actual IV for the option (LIV); (4) 1-day ahead simple regression forecasts of the IV (RGN); (5) 1-day ahead ARMA forecasts of the IV (ARMA); (6) 1-day ahead cross-sectional time-series predictive regression forecasts of the IV based on Eq. (16.16) (CSTS); (7) a simple-constant mean of an individual IV time series for the estimated IV of that option (MEAN).

The trading rule used is simply to buy underpriced and sell overpriced options, while taking an opposite position in the underlying futures contract according to the hedge ratio computed by the estimated IV. The holdout periods for each option are 20 trading days. Here the day count convention in Black option pricing model is used actual/actual basis. Mispricing will be identified by comparing the market price for an option with the price calculated by Black option pricing model using one of the seven IV estimates. The overpriced (underpriced) options are defined as the situation that the theoretical price calculated by Black option pricing model is smaller (larger) than the market price. The trading behavior is buying (selling) the underpriced (overpriced) future option and selling (buying) S&P 500 index future for hedge. In order to magnify the mispricing as might be seen from the eyes of a trader, ten options and ten times the hedge ratio of futures are sold or bought in opposite position respectively in each transaction. Positions are closed out once the absolute value of mispricing diminishes to a predetermined minimum level equal to 0.1. If the mispricing has reversed and is of a great enough significance larger than 0.1, the trading rule is utilized again.

In order to ascribe as much realism as possible to these tests, the following market trading costs are considered. Transaction fee per transaction of \$2.3 is determined by CME group which provides CME Globex trading platform for 24-h global access to electronic markets. Total transaction fees is transaction costs of option position + transaction costs of future position:

$$\sum_{i=1}^{n} (\$2.3 \times 10) + \sum_{i=1}^{n} (\$2.3 \times 10 \times \text{hedge ratio}_{T_i})$$

where n is the total number of times a position is opened at time T_i.

Although a portion of the margin required of a trader enter into a futures position can be put up in the form of interest earning T-bills, a substantial portion required for maintaining the margin account by the clearinghouse must be strictly in cash even for a hedge or spread position. Consequently, there is a real interest cost involved, for which we will further reduce gross trading income:

$$\text{Margin Interest Costs} = \sum_{i=1}^{n} \left(\text{RMM} \times \text{NF}_{T_i} \times R_{T_i} \times \tau_i\right)$$

where RMM is required maintenance margin from CME group,[7] NF_{Ti} indicate the number of futures contracts entered into trading which is equal to ten times hedge ratio at time $T_{i,}$ R_{Ti} is the risk free rate defined as the 3-month T-bill rate used in Black option pricing model, τ_{i} is the length of futures position holding until maturity in annual terms, and n is the total number of times a futures position is entered.

Furthermore, there is little assurance that one could buy or sell these contracts and expect to receive the closing prices reported in the paper when the market reopens the next morning. To approximate such market costs the position is penalized each time a futures position is entered and existed by "one tick" equal to 0.1 index points = $25 per contract[8]:

$$\text{Futures Liquidity Costs} = \sum_{i=1}^{n} (\$50 \times \mathrm{NF}_{T_i})$$

where $50 = 2 × $25 represented the entered and existed cost by one tick, the market value of two price ticks; NF_{Ti} is defined as the number of futures contracts entered into trading which is equal to ten times hedge ratio at time $T_{i,}$, and n is the number of times a futures position is entered. More severe liquidity and timing costs are calculated and deducted for each option transaction:

$$\text{Option Liquidity costs} = \sum_{i=1}^{n} [\$250 \times (\mathrm{NEPA}_{T_i} + \mathrm{NMMO}_{Ti})]$$

where $250 = 10, (number of options bought or sold) × $250 (the market value multiplier for the option premium) × 0.1 (one tick price as the correspondingly liquidity), NEPA represented the number of exercise prices in out-of-the-money options are $5 away from underlying future prices at time T_i, and NMMO represented the percentage of maturity months out. For example, a option assumed to be expired on September 2010 and this option start to be traded on February 2010, then the NMMO on June 2010 is equal to the number of month of the period between February and June divided by the number of month of the period between February and September, that is, (6–2)/(9–2) = 4/7.

The test results are summarized in Tables 16.5. We use seven alternative methods, a cross-sectional time-series regression and six time-series models, to compute tomorrow's IV for each contract. The cross-sectional time-series (CSTS) model

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[7]The minimum required maintenance for S&P 500 index futures is various in different period. For example, from Jan 28, 2008 to Oct 1st, 2008, the maintenance cost is $18,000 per future contract. However, the period during Oct 1st, 2008 to Oct 17, 2008, the required maintenance is changed to $20,250. The maintenance costs are $22,500, $24,750, $22,500, and $20,000 for other periods Oct 17, 2008–Oct 30, 2008; Oct 30, 2008–Mar 20, 2009; Mar 20, 2009–Jun 2nd, 2011; and Jun 2nd, 2011 until now.

[8]The detailed contract specifications for S&P 500 futures and options on futures can be found in CME group website: http://www.cmegroup.com/trading/equity-index/files/SxP500_FC.pdf

Table 16.3 Cumulative survey of trading results for samples in holdout period

IV estimate	Gross value of all trades	Total trading costs	Net value of all trades	Number of trade made	Net profit or loss per trade
(a) 2010					
MAV5	**1,673,339**	785,469.1	887,869.8	95	9,345.997
EMA5	1,185,108	735,197.1	449,910.6	95	4,735.901
LIV	1,325,712	**405,671**	**920,041.3**	95	**9,684.645**
RGN	1,077,990	432,747.3	645,243.1	95	6,792.033
ARMA	535,833.8	462,131	73,702.82	95	7,75.8192
CSTS	413,830.8	618,588.4	−204,758	95	−2,155.34
MEAN	454,006.3	1,714,191	−1,260,185	95	−13,265.1
(b) 2011					
MAV5	840,926.2	706,118.6	134,807.6	152	886.8921
EMA5	−794,276	784,070.1	−1,578,346	152	−10,383.9
LIV	2,433,862	**500,012.3**	1,933,850	152	12,722.7
RGN	**3,170,605**	1,090,987	**2,079,618**	152	**13,681.7**
ARMA	679,499.3	786,602.5	−107,103	152	−704.63
CSTS	−4,119,967	600,665.3	−4,720,633	152	−31,056.8
MEAN	4,168,410	2,752,602	1,415,807	152	9,314.522

The holdout period is the last 20 days of each S&P 500 index futures option contracts. There are seven IV estimates for the trading rule test: MAV5 is the 5-day moving averages method, EMA5 is the 5-day exponential moving averages method, LIV is Previous IV method, RGN is the Regression method, ARMA is autoregressive-moving-average model, CSTS is the cross-sectional time-series predictive regression model represented in Eq. (16.16) where using ARMA as predictor method, and MEAN is the constant value over the entire period equal to the mean of individual IV series. The definitions of first five IV estimates are indicated in Tables 16.2 and 16.3. The gross value of all trades are included the bought and sold price of options plus the value in the end of maturity if the trades are not closed out before maturity. Total trading costs are included the total transaction fees, margin interest costs, future liquidity costs, and option liquidity costs. The net value of all equals to gross value of all trades minus total trading costs. The net profit or loss per trade is the value of net value of all trades divided by number of trade

utilized some of the insights of time-series analysis as would be impounded in the optimal time-series predictors, ARMA model. Also, it takes into account the historical 5-days and 20-days standard deviation of the continuous return for the underlying futures contract, the short-term variability and skewness and kurtosis of the IV, the time-to-maturity, and weekday effects. Table 16.3a, b summarize the cumulative trading results for the selected options contract in Table 16.1. For both years, EMA5, LIV, and RGN perform better than the sophisticated model such as cross-sectional time-series predictive regression. The results implied that the ARMA model may have over-fitting problem and thus make CSTS model perform worse. The worse prediction is using MEAN model to estimate IV. MEAN model's IV is constant for entire period of contract; thus, MEAN model neither deal with the fluctuation of option market nor response to everyday's new important information. It also implied that the constant volatility setting in BSM model may be misspecified.

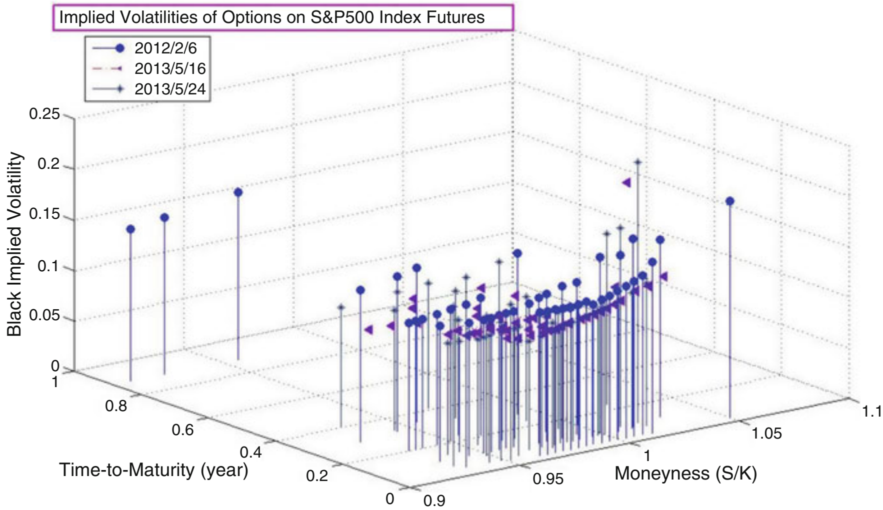

Fig. 16.1 Implied volatilities in black model

16.4.4 *Structural Parameter Estimation and Performance of CEV Model*

In Fig. 16.1, we find that each contract's Black IV varies across moneyness and time-to-maturity. This graph shows volatility skew (or smile) in options on S&P 500 index futures, i.e., the implied volatilities decrease as the strike price increases (the moneyness level decreases).

Even though everyday implied volatility surface changes, this characteristic still exists. Therefore, we divided future option contracts into a six by four matrix based on moneyness and time-to-maturity levels when we estimate implied volatilities of futures options in CEV model framework in accordance with this character. The whole option samples expired within the period of 2010 to 2013 contains 30,364 observations. The whole period of option prices is from March 19, 2009 to November 5, 2013. The observations for each group are presented in Table 16.4.

Since most trades are in the futures options with short time-to-maturity, the estimated implied volatility of the option samples in 2009 may be significantly biased because we did not collect the futures options expired in 2009. Therefore, we only use option prices in the period between January 1, 2010 and November 5, 2013 to estimate parameters of CEV model. In order to find global optimization instead of local minimum of absolute pricing errors, the ranges for searching suitable δ_0 and α_0 are set as $\delta_0 \in [0.01, 0.81]$ with interval 0.05, and $\alpha_0 \in [-0.81,\ 1.39]$ with interval 0.1, respectively. First, we find the value of parameters, $(\widehat{\delta}_0, \widehat{\alpha}_0)$, within the ranges such that minimize value of absolute pricing errors in Eq. (16.19). Then we use this pair of parameters, $(\widehat{\delta}_0, \widehat{\alpha}_0)$, as optimal initial estimates in the procedure of

Table 16.4 Average daily and total number of observations in each group

Time-to-maturity(TM)	TM < 30		30 ≦ TM ≦ 60		TM > 60		All TM	
Moneyness (S/K ratio)	Daily Obs.	Total Obs.	Daily Obs.	Total Obs.	Daily Obs.	Total Obs.	Daily Obs.	Total Obs.
S/K ratio >1.01	1.91	844	1.64	499	1.53	462	2.61	1, 805
0.98 ≦ S/K ratio ≦ 1.01	4.26	3, 217	2.58	1, 963	2.04	1, 282	6.53	6, 462
0.95 ≦ S/K ratio < 0.98	5.37	4, 031	3.97	3, 440	2.58	1, 957	9.32	9, 428
0.9 ≦ S/K ratio < 0.95	4.26	3, 194	4.37	3, 825	3.27	2, 843	9.71	9, 862
S/K ratio <0.9	2.84	764	2.68	798	2.37	1, 244	4.42	2, 806
All Ratio	12.59	12, 050	10.78	10, 526	7.45	7, 788	27.62	30, 364

The whole period of option prices is from March 19, 2009 to November 5, 2013. Total observations is 30, 364. The lengths of period in groups are various. The range of lengths is from 260 (group with ratio below 0.90 and time-to-maturity within 30 days) to 1,100 (whole samples)

Table 16.5 Initial parameters of CEV model for estimation procedure

Time-to-Maturity (TM)	TM < 30		30 ≦ TM ≦ 60		TM > 60		All TM	
Moneyness (S/K ratio)	α_0	δ_0.	α_0	δ_0	α_0	δ_0	α_0	δ_0
S/K ratio >1.01	0.677	0.400	0.690	0.433	0.814	0.448	0.692	0.429
0.98 ≦ S/K ratio ≦ 1.01	0.602	0.333	0.659	0.373	0.567	0.361	0.647	0.345
0.95 ≦ S/K ratio < 0.98	0.513	0.331	0.555	0.321	0.545	0.349	0.586	0.343
0.9 ≦ S/K ratio < 0.95	0.502	0.344	0.538	0.332	0.547	0.318	0.578	0.321
S/K ratio <0.9	0.777	0.457	0.526	0.468	0.726	0.423	0.709	0.423
All Ratio	0.854	0.517	0.846	0.512	0.847	0.534	0.835	0.504

The sample period of option prices is from January 1, 2010 to November 5, 2013. During the estimating procedure for initial parameters of CEV model, the volatility for S&P 500 index futures equals to $\delta_0 S^{\alpha_0 - 1}$

Table 16.6 Total number of observations and trading days in each group

Time-to-Maturity (TM)	TM < 30		30 ≦ TM ≦ 60		TM > 60		All TM	
Moneyness (S/K ratio)	Days	Total Obs.	Days	Total Obs.	Days	Total Obs.	Days	Total Obs.
S/K ratio >1.01	172	272	104	163	81	122	249	557
0.98≦S/K ratio≦1.01	377	1,695	354	984	268	592	448	3,271
0.95≦S/K ratio < 0.98	362	1,958	405	1,828	349	1,074	457	4,860
0.9≦S/K ratio < 0.95	315	919	380	1,399	375	1,318	440	3,636
S/K ratio <0.9	32	35	40	73	105	173	134	281
All Ratio	441	4,879	440	4,447	418	3,279	461	12,605

The subsample period of option prices is from January 1, 2012 to November 5, 2013. Total observations is 13, 434. The lengths of period in groups are various. The range of lengths is from 47 (group with ratio below 0.90 and time-to-maturity within 30 days) to 1,100 (whole samples). The range of daily observations is from 1 to 30

estimating local minimum minAPE based on steps (1)–(3) in Sect. 16.3.2.3. To compare with the option pricing performance of Black model, we set the interval between 0.01 and 0.08 to find optimal implied volatility via estimation procedure in Sect. 16.3.2.1. The initial parameter setting of CEV model is presented in Table 16.5.

In Table 16.5, the average sigma are almost the same while the average alpha value in either each group or whole sample is less than one. This evidence implies that the alpha of CEV model can capture the negative relationship between S&P 500 index future prices and its volatilities shown in Fig. 16.1. The instant volatility of S&P 500 index future prices equals to $\delta_0 S^{\alpha_0 - 1}$ where S is S&P 500 index future prices, δ_0 and α_0 are the parameters in CEV model. The estimated parameters in Table 16.9 are similar across time-to-maturity level but volatile across moneyness.

Because of the implementation and computational costs, we select the sub-period from January 2012 to November 2013 to analyze the performance of CEV model. The total number of observations and the length of trading days in each group are presented in Table 16.6. The estimated parameters in Table 16.7 are similar across time-to-maturity level but volatile across moneyness. Therefore, we investigate the

Table 16.7 Average daily parameters of in-sample

Time-to-Maturity (TM)	TM < 30				30≦TM≦60				TM > 60				All TM			
Moneyness (S/K ratio)	CEV			Black	CEV			Black	CEV			Black	CEV			Black
Parameters	α	δ	IV	IV	α	δ	IV	IV	α	δ	IV	IV	α	δ	IV	IV
S/K ratio >1.01	0.29	0.19	0.188	0.200	0.14	0.18	0.183	0.181	0.29	0.21	0.204	0.196	0.25	0.19	0.1890	0.1882
0.98≦S/K ratio≦1.01	0.34	0.16	0.162	0.1556	0.30	0.16	0.154	0.147	0.14	0.16	0.155	0.155	0.39	0.17	0.151	0.150
0.95≦S/K ratio < 0.98	0.22	0.13	0.137	0.135	0.30	0.13	0.134	0.131	0.24	0.14	0.141	0.139	0.37	0.14	0.136	0.132
0.9≦S/K ratio < 0.95	0.05	0.15	0.159	0.152	0.25	0.13	0.133	0.128	0.26	0.14	0.136	0.131	0.38	0.14	0.135	0.129
S/K ratio <0.9	−0.23	0.22	0.252	0.243	−1.67	0.14	0.193	0.159	0.25	0.15	0.145	0.142	0.23	0.15	0.157	0.152

The in-sample period of option prices is from January 1, 2012 to May 30, 2013. In the in-sample estimating procedure, CEV implied volatility for S&P 500 index futures (CEV IV) equals to $\delta(S/K\ \text{ratio})^{\alpha-1}$ in according to reduce computational costs. The optimization setting of finding CEV IV and Black IV is under the same criteria performance of all groups except the groups on the bottom row of Table 16.8. The performance of models can be measured by either the implied volatility graph or the average absolute pricing errors (AveAPE). The implied volatility graph should be flat across different moneyness level and time-to-maturity. We use subsample like Bakshi et al. (1997) and Chen et al. (2009) did to test implied volatility consistency among moneyness-maturity categories. Using the subsample data from January 2012 to May 2013 to test in-the-sample fitness, the average daily implied volatility of both CEV and Black models, and average alpha of CEV model are computed in Table 16.7. The fitness performance is shown in Table 16.8. The implied volatility graphs for both models are shown in Fig. 16.2. In Table 16.7, we estimate the optimal parameters of CEV model by using a more efficient program. In this efficient program, we scale the strike price and future price to speed up the program where the implied volatility of CEV model equals to $\delta\left(\text{ratio}^{\alpha-1}\right)$, ratio is the moneyness level, δ and α are the optimal parameters of program which are not the parameters of CEV model in Eq. (16.17). In Table 16.8, we found that CEV model perform well at in-the-money group.

Figure 16.2 shows the IV computed by CEV and Black models. Although their implied volatility graphs are similar in each group, the reasons to cause volatility smile are totally different. In Black model, the constant volatility setting is misspecified. The volatility parameter of Black model in Fig. 16.2b varies across moneyless and time-to-maturity levels while the IV in CEV model is a function of the underlying price and the elasticity of variance (alpha parameter). Therefore, we can image that the prediction power of CEV model will be better than Black model because of the explicit function of IV in CEV model. We can use alpha to measure the sensitivity of relationship between option price and its underlying asset. For example, in Fig. 16.2c, the in-the-money future options near expired date have significantly negative relationship between future price and its volatility.

The better performance of CEV model may result from the over-fitting issue that will hurt the forecastability of CEV model. Therefore, we use out-of-sample data from June 2013 to November 2013 to compare the prediction power of Black and CEV models. We use the estimated parameters in previous day as the current day's input variables of model. Then, the theoretical option price computed by either Black or CEV model can calculate bias between theoretical price and market price. Thus, we can calculate the average absolute pricing errors (AveAPE) for both models. The lower value of a model's AveAPE, the higher pricing prediction power of the model. The pricing errors of out-of-sample data are presented in Table 16.9. Here we find that CEV model can predict options on S&P 500 index futures more precisely than Black model. Based on the better performance in both in-sample and out-of-sample, we claim that CEV model can describe the options of S&P 500 index futures more precisely than Black model.

Table 16.8 AveAPE performance for in-sample-fitness

Time-to-Maturity (TM)	TM < 30			30 ≦ TM ≦ 60			TM > 60			All TM		
Moneyness (S/K ratio)	CEV	Black	Obs.	CEV	Black	Obs.	CEV	Black	Obs.	CEV	Black	Obs.
S/K ratio >1.01	**1.65**	1.88	202	1.81	**1.77**	142	5.10	**5.08**	115	5.80	**6.51**	459
0.98 ≦ S/K ratio ≦ 1.01	**6.63**	7.02	1290	**4.00**	4.28	801	4.59	**4.53**	529	**18.54**	18.90	2620
0.95 ≦ S/K ratio < 0.98	2.38	**2.34**	1560	4.25	**4.14**	1,469	3.96	**3.89**	913	14.25	**14.15**	3942
0.9 ≦ S/K ratio < 0.95	0.69	**0.68**	710	1.44	**1.43**	1,094	3.68	**3.62**	1,131	**7.08**	7.10	2935
S/K ratio <0.9	0.01	0.01	33	**0.13**	0.18	72	0.61	**0.60**	171	0.69	**0.68**	276

The in-sample period of option prices is from January 1, 2012 to May 30, 2013

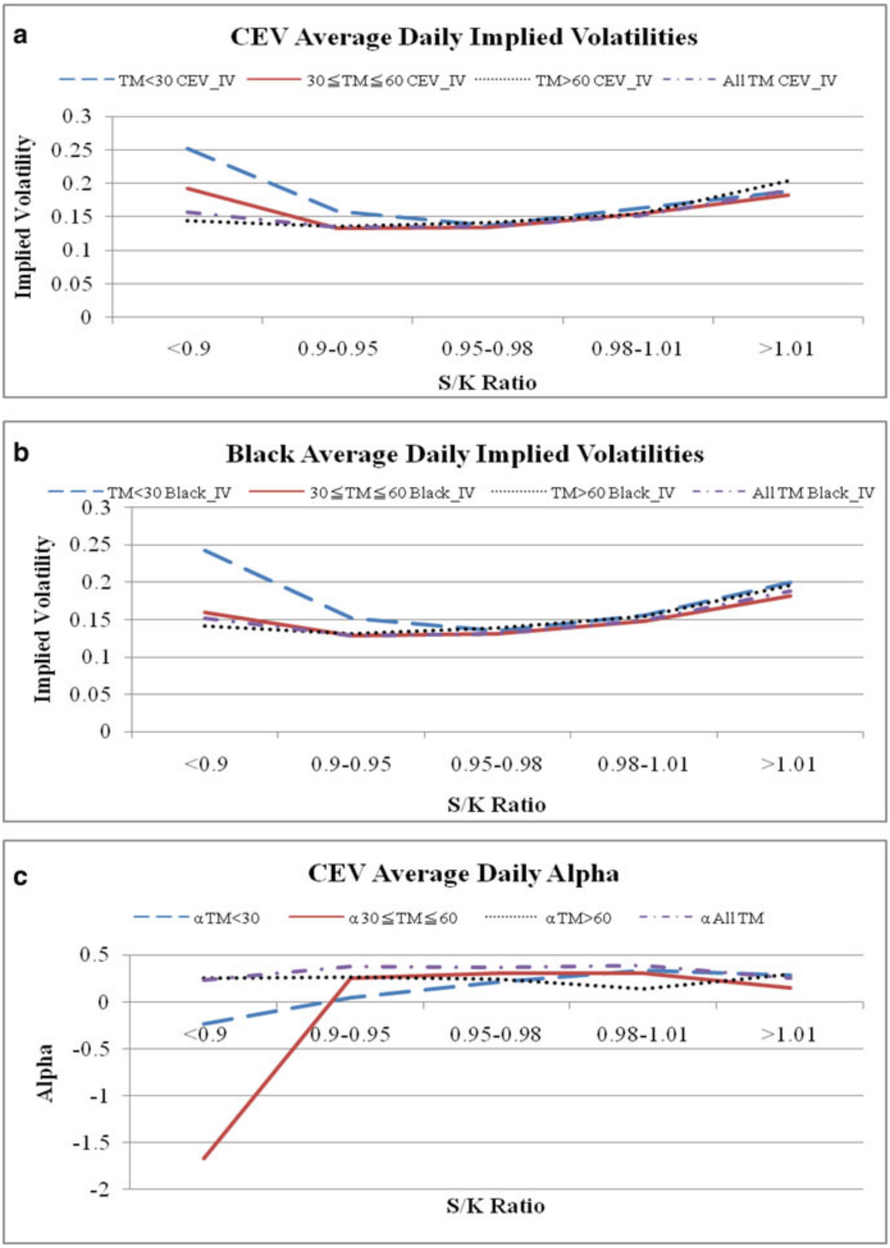

Fig. 16.2 Implied volatilities and CEV Alpha Graph

Table 16.9 AveAPE performance for out-of-sample

Time-to-Maturity (TM)	TM < 30		30 ≦ TM ≦ 60		TM > 60		All TM	
Moneyness (S/K ratio)	CEV	Black	CEV	Black	CEV	Black	CEV	Black
S/K ratio >1.01	**3.22**	3.62	**3.38**	4.94	**8.96**	13.86	**4.25**	5.47
0.98 ≦ S/K ratio ≦ 1.01	**2.21**	2.35	2.63	**2.53**	**3.47**	3.56	**2.72**	2.75
0.95 ≦ S/K ratio < 0.98	**0.88**	1.04	**1.42**	1.46	1.97	**1.95**	**1.44**	1.45
0.9 ≦ S/K ratio < 0.95	**0.34**	0.53	**0.61**	0.62	1.40	1.40	**0.88**	0.90
S/K ratio <0.9	**0.23**	0.79	**0.25**	0.30	1.28	**1.27**	**1.03**	1.66

16.5 Conclusion

The purpose of this essay has been to improve the interpretation and forecasting of individual implied volatility (IV) for call options on S&P500 index futures in 2010 to 2013. The two alternative methods used in this essay are cross-sectional time-series analysis and CEV model. These two alternative approaches give different perspective of estimating IV. The cross-sectional time-series analysis focuses on the dynamic behavior of volatility in each option contracts and captures other trading behaviors such as week effect and in/out of the money effect. On the other hand, CEV model generalizes implied volatility surface as a function of asset price.

By empirically explaining the composition through time-series analysis and cross-sectional time-series regression models, the disadvantages to evaluating an option IV by Black model have been demonstrated. More importantly, the results based on our trading strategy provide some evidence as to how the Black option pricing model might be misspecified, or jointly, how the market might be inefficient. Though the original model implicitly assumes a frictionless market and a constant volatility term, market realities along with past studies would not be able to substantiate these types of assumptions. The forecasting performances of seven time-series regression models based on our trading strategy show that the simple regression models perform better than sophisticated cross-sectional time-series models because of over-fitting problem in the advanced models. In addition, although our trading rules based on the prediction of these models can make profit, the net profit depends on the transaction costs. Therefore, the setting of trading strategy should be necessarily adjusted to the transaction costs.

We also show that CEV model performs better than Black model in aspects of either in-sample fitness or out-of-sample prediction. The setting of CEV model is more reasonable to depict the negative relationship between S&P 500 index future price and its volatilities. The elasticity of variance parameter in CEV model captures the level of this characteristic. The stable volatility parameter in CEV model in our empirical results implies that the instantaneous volatility of index future is mainly determined by current future price and the level of elasticity of variance parameter.

In sum, we suggest predict individual option contract by using simple regression analysis instead of advanced cross-sectional time-series model. Even though the moneyness and week effect have significant influence on index future option prices, the over-fitting problem in an advanced cross-sectional time-series model will decrease its pricing forecastability. With regard to generate implied volatility surface to capture whole prediction of the future option market, the CEV model is the better choice than Black model because it not only captures the skewness and kurtosis effects of options on index futures but also has less computational costs than other jump-diffusion stochastic volatility models. In future research, we can apply CEV model and its Greek measures to other liquid option markets to test market efficiency based on our trading rules.

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