Intelligent Methods for State Estimation and Parameter Identification in Fuzzy Dynamical Systems

Sergey Kovalev, Sergey Sokolov and Alexander Shabelnikov

Abstract This paper presents a new approach for state estimation and parameter identification in fuzzy dynamical systems. The basis of the proposed approach is adaptive network calculation model of the fuzzy prior and posterior estimates of system state variables taking place in consecutive time steps. The optimization of model parameters based on modified simplex algorithm is also proposed. Presented method for parameter identification has also a set of new properties, such as ability of integration in the expert systems, higher level of potential accuracy and possibility of real-time identification. Example of optimal parameter estimation for fuzzy dynamical system is considered and results of the experiments are provided. Experiments show that estimations of identified parameters obtained on the basis of adaptive network applied in dynamical systems of Sugeno type does not deviate from real values by more than in 10 %.

Keywords Fuzzy dynamical system · Conditional membership function · Prior fuzzy distribution · Posterior fuzzy distribution · Adaptive network model · Parametric identification

1 Introduction

Recent methods for control of complex dynamical objects, which work is connected with uncertainty, are based on the analytical models, which are represented in form of differential and recurrence equations. Analysis of related publications [1–4] shows that the major part of researches uses traditional methods, which has a set of limitations, such as requirement of subordination to normal distribution law, usage

S. Kovalev (✉) · S. Sokolov
Rostov State Transport University, Rostov-on-Don, Russia
e-mail: ksm@rfniias.ru

S. Kovalev · A. Shabelnikov
JSC "NIIAS", Rostov branch, Rostov-on-Don, Russia
e-mail: info@rfniias.ru

A. Abraham et al. (eds.), *Proceedings of the First International Scientific Conference "Intelligent Information Technologies for Industry" (IITI'16)*, Advances in Intelligent Systems and Computing 451, DOI 10.1007/978-3-319-33816-3_29

of traditional mean-square criteria for estimation of the optimality of model parameters, usage of the simplest linear models for measurer systems, etc. Here, the questions, which deal with integration of the empirical knowledges of human experts into poorly formalized process models characterized by incompleteness, imprecision and contradictory [5] as well as presence of fuzzy and subjective factors affected on parameter estimation, are still practically unexplored [6–8].

Nowadays, intelligent models are more preferable for modeling of poorly formalized objects. These models are based on knowledges, the main class of which is fuzzy dynamical systems (FDS) [2–4]. The basis of FDS is formalization of the empirical experience and knowledges of human experts, which is represented in linguistic form via fuzzy logic tools. To make FDS be practically usable in controlling systems, the development of effective construction methods as well as estimation and state correction algorithms is required. It is also important to adapt FDS for the real conditions.

This paper considers the decision of general problems, which are referred to identification, prediction and estimation of FDS states, which describe behavior of poorly formalized dynamical objects.

2 Model of Representation and State Estimation for FDS

Among of many well-known ways for discrete-time FDS representation [9–11], the most simple one is FDS construction in form of recurrence equation [7]:

$$x_{k+1} = F(x_k) \quad (k = 0, 1, \ldots, N), \tag{1}$$

where x is the state from the state space X of FDS, F is the fuzzy mapping $x_k \to x_{k+1}$, which is given by membership function (MF) $\mu_F(x_k, x_{k+1})$. This function is also convenient to be represented in form of conditional MF $\mu_F(x_{k+1}|x_k)$.

Real applications consider both FDS and the measuring system, which realize the transformation of states from X into the external observations from Z taking into account affecting noises. Thus, practically useful model of FDS may be represented considering measurer errors and fuzzy noises in form of the following system:

$$\begin{cases} x_{k+1} = F_k(x_k, \varepsilon_k) \\ z_k = S_k(x_k, \delta_k) \end{cases} \quad k = 0, 1, \ldots, N, \tag{2}$$

where F_k is the state equation for FDS; S_k is the nonlinear function of measurer work; x_k is the internal state of a system; ε_k is the fuzzy system noise described by a defined MF μ_{ε_k}; δ_k is the fuzzy measurer error described by a defined MF μ_{δ_k}; k is the discrete time index.

State space X for (2) is the set of values characterizing the position of a FDS in an observed time step and playing the role of initial conditions for the future system behavior. However, real system has uncertainties and, thus, dependency between current and future values is represented by a fuzzy variable. Estimation and correction of this dependency are very important for dynamical objects control in the area of fuzzy modeling. The task of estimation and correction is defined as follows.

Let the initial information about FDS be presented in form of MF $\mu(x_0)$ and the observed states be presented by vector of observations $Z_k = (z_0, z_1, \ldots, z_k)$ from time interval $[t_0, t_k]$. It is required to predict fuzzy state x_{k+1}, which is defined by MF $\mu(x_{k+1})$. FDS correction is performed by specifying the MF for the determined fuzzy value of x_{k+1}, when z_{k+1} is observed at time step t_{k+1}.

3 Recurrent Algorithm for FDS State Estimation

Estimation and correction for the FDS states are performed based on determination and matching prior information and posterior one characterized by conditional MFs $\mu(x_{k+1}|Z_k)$ и $\mu(x_{k+1}|Z_{k+1})$. The conditional MFs are determined by the following recurrence procedure.

Let the FDS be nonstationary system with discrete time presented by (2). Let initial state MF $\mu(x_0)$ be a priory determined. Measurement errors, noises and states are independent fuzzy variables [12].

Based on assumption that $Z_{k+1} = (Z_k, z_{k+1})$, MF $\mu(x_{k+1}|Z_{k+1})$ can be presented as

$$\mu(x_{k+1}|Z_{k+1}) = \mu(x_{k+1}|Z_k, z_{k+1}). \quad (3)$$

Conditional fuzzy variables $\mu(x_{k+1}|Z_k$ and $(x_{k+1}|Z_{k+1})$, which belong to (3), are independent because fuzzy noises ε_k and δ_k affecting on FDS (determining $(x_{k+1}|Z_{k+1})$) and measurer (determining $(x_{k+1}|Z_{k+1})$), respectively, are also independent. Thus, Eq. (3) can be defined as follows:

$$\mu(x_{k+1}|Z_k, z_{k+1}) = \mu(x_{k+1}|Z_k) \,\&\, \mu(x_{k+1}|z_{k+1}). \quad (4)$$

Conditional MF of fuzzy variable $(x_{k+1}|z_{k+1})$ may be expressed by using measurer function from (2) via the MF of fuzzy noise:

$$\mu(x_{k+1}|z_{k+1}) = \mu(\delta_{k+1}) \quad \left(\delta_{k+1} = S^{-1}_{k+1}(x_{k+1}, z_{k+1})\right). \quad (5)$$

Since nonlinear mapping S^{-1}_{k+1} from (5) is commonly multivalued, the fuzzy estimation for $\mu_{\delta_{k+1}}(S^{-1}_{k+1}(x_{k+1}, z_{k+1}))$ takes maximum possible value via Zadeh extension principle [13]:

$$\mu_{\delta_{k+1}}(S^{-1}_{k+1}(x_{k+1}, z_{k+1})) = \sup_{\Delta = S^{-1}_{k+1}(x_{k+1}, z_{k+1})} \mu_{\delta_{k+1}}(\Delta). \quad (6)$$

Merging (6) and (5), the following equation can be got:

$$\mu(x_{k+1}|z_{k+1}) = \sup_{\Delta=S_{k+1}^{-1}(x_{k+1},z_{k+1})} \mu_{\delta_{k+1}}(\Delta). \tag{7}$$

Conditional MF $\mu(x_{k+1}|Z_k)$ from (4) describes fuzzy mapping $\Phi : Z_k \rightarrow x_{k+1}$, which can be represented in form of the composition of fuzzy mappings:

$$\Phi = (Z_k \rightarrow x_k) \circ (x_k \rightarrow x_{k+1}). \tag{8}$$

Conditional MF $\mu(x_k|Z_k)$ characterizes fuzzy mapping $Z_k \rightarrow x_k$ as well as fuzzy conditional MF $\mu(x_{k+1}|x_k)$ characterizes fuzzy mapping $x_k \rightarrow x_{k+1}$. As a result of composition, we get MF $\mu(x_{k+1}|Z_k)$ for fuzzy mapping $\Phi : Z_k \rightarrow x_{k+1}$:

$$\mu(x_{k+1}|Z_k) = \sup_{x_k}[\mu(x_k|Z_k)\ \&\ \mu(x_{k+1}|x_k)], \tag{9}$$

where $\mu(x_{k+1}|x_k)$ is expressed via fuzzy noise based on the state equation (2):

$$\mu(x_{k+1}|x_k) = \sup_{\Delta=F_k^{-1}(x_{k+1},x_k)} \mu_{\varepsilon_k}(\Delta). \tag{10}$$

If expression (10) is merged with Eq. (9), MF $\mu(x_{k+1}|Z_k)$ can be calculated as

$$\mu(x_{k+1}|Z_k) = \sup_{x_k}[\mu(x_k, Z_k)\& \sup_{\Delta=F_k^{-1}(x_{k+1},x_k)} \mu_{\varepsilon_k}(\Delta)]. \tag{11}$$

Considering (9) and (11), the final recurrence relations for finding the posterior MF of the fuzzy state of FDS at certain step (k + 1) can be expressed in following form:

$$\begin{cases} \mu(x_{k+1}|Z_{k+1}) = \mu(x_{k+1}, Z_k)\& \sup_{\Delta=S_k^{-1}(x_{k+1},z_{k+1})} \mu_{\delta_{k+1}}(\Delta) \\ \mu(x_{k+1}|Z_k) = \sup_{x_k}[\mu(x_k|Z_k)\& \sup_{\Delta=F_k^{-1}(x_{k+1},x_k)} \mu_{\varepsilon_k}(\Delta)] \end{cases}, \tag{12}$$

Initial information for implementation of (12) is presented by $\mu(x_0|z_0)$, which is taken in the form of initial fuzzy state $\mu(x_0)$ determined by the problem statement.

4 Optimal FDS Parameters Estimation

The task of parameter estimation refers to the problem of parameter identification, when the structure of FDS is a prior determined.

Let the structure of FDS is given in form of system (2), where nonlinear state function F_k and measurer one S_k depend on the set of uncertain parameters at each time step. The parameters are presented by vector A_k and B_k, respectively. In this case, model of non-stationary FDS with uncertain parameters is expressed by the following system:

$$\begin{cases} x_{k+1} = F_k(x_k, A_k, \varepsilon_k) \\ z_k = S_k(x_k, B_k, \delta_k) \end{cases} \quad k = 0, 1\ldots, N. \tag{13}$$

Let the observation set be presented in form of vector $Z = [z_0, z_1, \ldots, z_k]$, which is determined on time interval $[t_n, t_m]$. It is required to calculate parameters of FDS A_k and parameters of measurer B_k, which make the system behavior be similar to experimental observations as more as possible.

To formalize the term "as more as possible", the criterion of identification quality is introduced. This criterion characterizes the rate of correspondence between FDS and experiments. It is calculated based on the matching of prior MF $\mu(x_{k+1}|Z_k)$, which reflects the fuzzy estimation of state from X at the time step t_{k+1} considering the observation of z_k at time step t_k, and posterior MF $\mu(x_{k+1}|Z_{k+1})$, which reflects the fuzzy estimation of FDS state at time step t_{k+1} considering the observation of z_{k+1}. Difference between prior and posterior MF is expressed by fuzzy error e_k of current estimation of FDS. MF of the error has the following form:

$$\mu(e_k, A_k, B_k, B_{k+1}) = \sup_{x_{k+1}} \mu(x_{k+1}|z_k, A_k, B_k)\;\&\;\mu(x_{k+1} + e_k|z_{k+1}, B_{k+1}). \tag{14}$$

Optimality criterion J can be presented by any fuzzy criterion in form of non-linear dependency on both conditional and posterior MFs. Particularly, it is convenient to use minimum of deviation for MF of fuzzy estimation error e_k from its model function defined on interval $[e_{min}, e_{max}]$, i.e.:

$$J_k(A_k, B_k, B_{k+1}) = \int_{e_{\min}}^{e_{\max}} (r(e_k) - \mu(e_k|, A_k, B_k, B_{k+1}))^2\, de_k, \tag{15}$$

where e_k is the current error of the estimation, $\mu(e_k|A_k, B_k, B_{k+1})$ is the MF of fuzzy estimation error (14), $r(e_k)$ is the model function, which is chosen according to the specifications of an identification task.

Estimation problem is concluded in calculation of such vectors A_k and B_k, which minimize criterion J, i.e.

$$\min_{A_k, B_k, B_{k+1}} J = \min_{A_k, B_k, B_{k+1}} \int_{e_{\min}}^{e_{\max}} (r(e_k) - \mu(e_k|A_k, B_k, B_{k+1}))^2 de_k. \tag{16}$$

5 Adaptive Network Model of FDS

Determination of the conditional MF for fuzzy state and optimization of the FDS parameters are made by using both adaptive network model (ANM) and iterative algorithm described below. The basis of ANM is the process of a prior and a posterior MF calculation for each time step from the interval [t_n, t_m] and also their matching based on chosen criterion *J*.

Let output signal (or observed state of a system) z_k is observed at certain step $t_k \in$ [t_n, t_m]. Then, the fuzzy state of FDS at time step t_{k+1} can be calculated based on composition of fuzzy relations $S_k^{-1} : z_k \to x_k$ and $F_k : X_k \to X_{k+1}$ determined by measurer equation and state one (13), respectively. Conditional MF $\mu(x_k|x_k)$ is determined for fuzzy relation S_k^{-1} based on measurer equation taking into account fuzzy noise δ_k:

$$\mu(x_k|z_k, \mathrm{B}_k) = \mu_{\delta_k}(S_k(x_k, \mathrm{B}_k) - z_k). \tag{17}$$

Conditional MF $\mu(x_{k+1}|x_k)$ is defined for fuzzy mapping F_k based on state equation taking into account fuzzy noise ε_k:

$$\mu(x_{k+1}|x_k, \mathrm{A}_k) = \mu_{\varepsilon_k}(F_k(x_k, \mathrm{A}_k) - x_{k+1}). \tag{18}$$

According to (17) and (18), MF $\mu(x_{k+1}|z_k)$ has the following form for composition $S_k^{-1} \circ F_k$:

$$\mu(x_{k+1}|z_k, \mathrm{A}_k, \mathrm{B}_k) = \sup_{x_k} \left[(\mu(x_k|z_k, \mathrm{B}_k) \,\&\, \mu_{\varepsilon_k}(F_k(x_k, \mathrm{A}_k) - x_{k+1}) \right] \tag{19}$$

Expression (19) is a prior MF characterizing the fuzzy value of state from *X* considering observed state z_k at step t_k.

To calculate posterior MF $\mu(x_{k+1}|z_k)$, output signal z_{k+1} should be assumed. The posterior MF for fuzzy state x_{k+1} is calculated based on measurer equation (13) considering fuzzy noise δ_{k+1}:

$$\mu(x_{k+1}|z_{k+1}, \mathrm{B}_{k+1}) = \mu_{\delta_{k+1}}(S_{k+1}(x_{k+1}, \mathrm{B}_{k+1}) - z_{k+1}) \tag{20}$$

Calculations for the MFs and criterion *J* can be shown in form of network structure representing feedforward calculation illustrated by Fig. 1.

Figure 1 shows that each element implements separated stage of transformations for input signals z_k and z_{k+1} into conditional MF $\mu(x_{k+1}|z_k, \mathrm{B}_k, \mathrm{A}_k)$ and $\mu(x_{k+1}|z_{k+1}, \mathrm{B}_{k+1})$.

Output block calculates *J* according to the given input signals. Identification of the FDS parameters is performed by handling parameters of A_k of FDS and both B_k and B_{k+1} of measurer using backpropagation method [14] and modified Nelder-Mead simplex method [15].

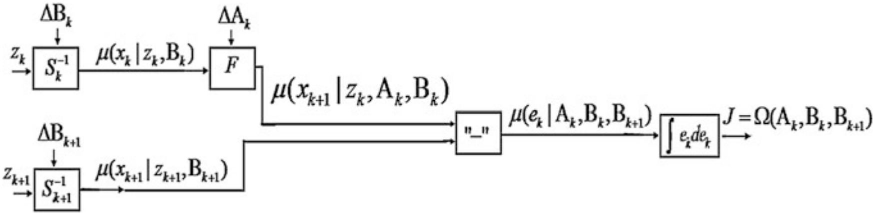

Fig. 1 General structure of ANM

6 Example

Implementation of above described method can be illustratively shown on the example of identification of the following FDS:

$$\begin{cases} x_k = f(x_{k-1}, a_{k-1}) + \varepsilon_k = a_{k-1} \cdot x_{k-1}^{1.7} + \varepsilon_k \\ z_k = s(x_k) + \delta_k = b_k \cdot x_k^{1.2} + \delta_k \end{cases}$$

where a_{k-1} is the identified parameter of the system; b_k is the parameter of an observer; ε_k is the fuzzy noise presented in the system, which is represented by Gauss MF with zero mean and variance $\sigma_\varepsilon = 0.05$; δ_k is the fuzzy noise presented in the measurer, which is represented by Gauss MF with zero mean and variance $\sigma_\delta = 0.22$.

Let a_{k-1} equal 2, c_k equal 1.2 for all k. Then, MFs have the following forms:

$$\mu(\varepsilon_k) = \exp\left(-\frac{1}{2 \cdot 0,5} \cdot \varepsilon_k^2\right).$$

$$\mu(\delta_k) = \exp\left(-\frac{1}{2 \cdot 0,22} \delta_k^2\right)$$

To describe the optimality criterion, the minimization of the MF deviation for fuzzy estimation error e_k is used:

$$J = \min_{a_k} \int_{e_{\min}}^{e_{\max}} \left(r\,(e_k) - \mu(e_k, a_k)\right)^2 de_k.$$

Model function of error is determined on interval $[e_{\min}, e_{\max}] = [-1, 1]$ of its changing as follows:

$$\begin{cases} r(e) = e + 1 & \mathit{if} \quad -1 \le e < 0 \\ r(e) = -e + 1 & \mathit{if} \quad 0 \le e \le 1 \end{cases}$$

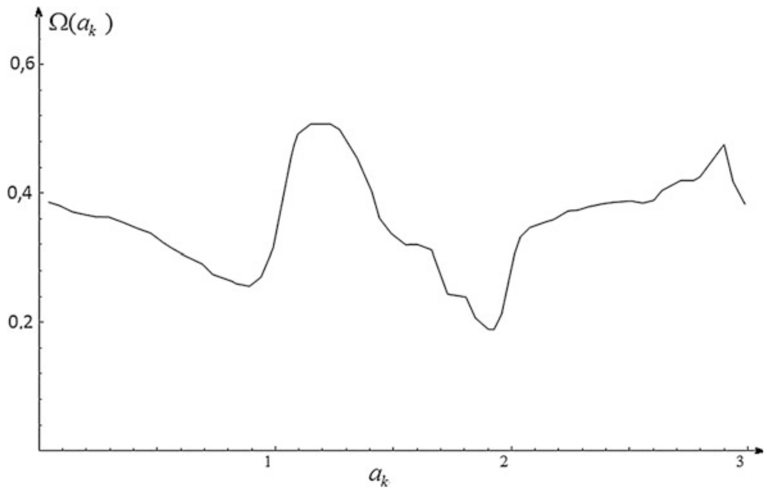

Fig. 2 Dependency between J and a_k for k = 37

Let initial state x_0 equal 0.8 and the calculation of $\sup_{x_k}(*)$ be provided with discretization step Δ = 0.05 for x. Let also the integral from (15) be performed numerically utilizing quadratic formulas with step Δ = 0.05 and infinite limits be changed by finite ones satisfying finite requirements for estimation algorithm (x_{min} = 0, x_{max} = 4). Here, the Nelder-Mead method [9] together with ANM optimizing a_{k-1} plays the main role for providing both the satisfactory computational speed and the required accuracy of results. Presented example consider the imitation of noises be generated programmatically using standard package of *Mathematica* software.

Calculation results are presented on figures below. Figure 2 illustrates curve of dependency between J and identified parameter a_k, when k = 37. The curve shows that the minimum of J is placed near to real value of a_{k-1} = 2.

Figure 3 presents the dependency between parameter a_k, which is required to be determined, and iteration step k of Nelder-Mead algorithm. Curve illustrates that a_k come around its real value and it deviate from this no more than by 10 %, when k increases.

The set of 400 experiments was performed to experimentally test efficacy of proposed approach. In the experiments, fuzzy Sugeno models with various numbers of unknown parameters (from 3 to 9) represent the FDS. Results show that the calculated estimations of a_k differ from their real values no more than by 10 % in the major part of samples (more than 95 %). Moreover, results proved that identified parameters of FDS are approximately converged to their real values after 20 −30 iterations of algorithm in the major part of samples. Small number of required iterations shows the practical possibility of using ANM for FDS identification in real time mode.

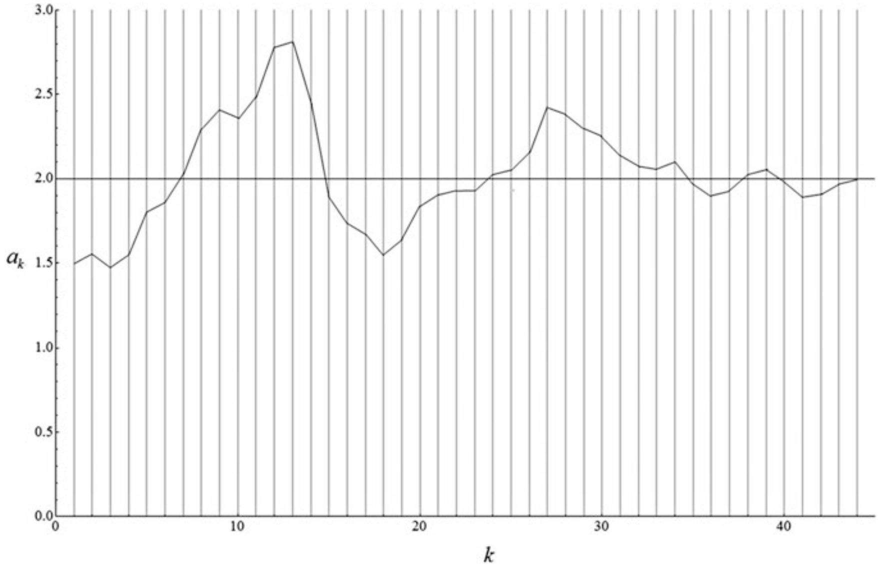

Fig. 3 Dependency between a_k and iteration number

7 Conclusions

This paper presents the new approach for estimation of states and identification of parameters in fuzzy systems describing the dynamics of poorly formalized processes. The proposed method utilizes adaptive network model and modified simplex algorithm. Experimental test of the method shows that determined parameters deviated no more than by 10 % from their real values in the major part of samples. Proposed approach for parameter identification has also the set of new properties, among which possibility for integration in the system of expert knowledges, higher level of accuracy versus traditional techniques and possibility of identification in real time are presented.

Acknowledgments The reported study was funded by RFBR, according to the research project (Grants No. 10-07-00158 and No. 10-01-00058).

References

1. Grop, D.: Metody identifikatsii system (The methods for system identification). Mir, Moscow (1979)
2. Lyung, L.: Identifikatsiya sistem. Teoriya dlya pol'zovatelya (System identification. Theory for user. Nauka, Moscow (1991)

3. Pashchenko, F. F.: Vvedeniye v sostoyatel'nyye metody modelirovaniya sistem. Autentifikatsiya nelineynykh system (Introduction to wealthy methods of system modeling. Autentification of nonlinear systems). Finansy i statistika, Moscow (2007)
4. Guo, R., Qin, Z., Li, X., Chen, J.: Interacting multiple model particle-type filtering approaches to ground target tracking. J. Comput. **3**(7), 23–30 (2008)
5. Pospelov, D.A.: Logiko-lingvisticheskiye modeli v sistemakh upravleniya (Logical-linguistic models in control systems). Energoizdat, Moscow (1981)
6. Gordon, N.J., Salmond, D.J., Smith, A.F.M.: Novel approach to nonlinear/non-Gaussian Bayesian state estimation. In: IEE Proceedings F (Radar and Signal Processing), vol. 140, no. 2, pp. 107–113. IET Digital Library, London (1993)
7. Doucet, A., Godsill, S., Andrieu, C.: On sequential Monte Carlo sampling methods for Bayesian filtering. Stat. Comput. **10**(3), 197–208, Springer, Heidelberg (2000)
8. Shteynberg, S.Y.: Identifikatsiya v sistemakh upravleniya (Identification in control systems). Energoatomizdat, Moscow (1987)
9. Kudinov, Y.I., et al.: Nechetkiye modeli dinamicheskikh protsessov Fuzzy models of dynamical processes). Nauchnaya kniga, Moscow (2007)
10. Terano, T., Asai, K., Sugeno, M.: Applied Fuzzy Systems. Academic Press, New York (2014)
11. Petrov, B.N., Ulanov, G.M., Goldenblat, I.I., Ulyanov, S.V.: Teoriya modeley v protsessakh upravleniya. Nauka, Moscow (1978)
12. Liu, B., Liu, B.: Theory and Practice of Uncertain Programming. Physica-verlag, Heidelberg (2002)
13. Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning. Inf. Sci. **8**(3), 199–249 (1975)
14. Werbos, P.J.: Beyond regression: new tools for prediction and analysis in the behavioral sciences. Ph.D. thesis, Harvard University, Cambridge (1974)
15. Nelder, J.A., Mead, R.: A simplex method for function minimization. Comput. J. **7**(4), 308–313 (1965)