Stochastic Computer Approach Applied in the Reliability Assessment of Engineering Structures

K. Frydrýšek and L. Václavek

Abstract The development of computer technology and reliability theory allows for a qualitative improvement of the safety and serviceability assessment of structural components and systems especially in engineering. The random characters of loads, materials, geometries etc. can be considered and evaluated. Using the Simulation-Based Reliability Assessment (SBRA) Method (i.e. Monte Carlo approach based on $>10^6$ of random simulations performed by computers) the application of a fully probabilistic approach to reliability assessment of selected structural components are indicated. Hence, two practical examples are solved (i.e. buckling of a complicated statically indeterminate frame structure and bending of screw implants in bones). Other applications are mentioned. Stochastic transformation models are used in order to express the response of the structural components including the 2nd order theory effects.

Keywords Probability · Reliability assessment · 2nd order theory · Buckling · Frame structures · Beam · Elastic foundation · Medical screws · Engineering · Mechanics · Biomechanics · Simulation-Based Reliability Assessment Method

1 Introduction

Current methods for risk and reliability assessment are mostly deterministic methods (i.e. based on constant inputs and outputs). But in the real world, the typical engineering values as loads, dimensions, material properties etc. are not constant but variable. The developments of computers and reliability theories allow to applied

K. Frydrýšek (✉) · L. Václavek
Faculty of Mechanical Engineering, Department of Applied Mechanics,
VŠB–Technical University of Ostrava, 17. listopadu 15/2172,
708 33 Ostrava, Czech Republic
e-mail: karel.frydrysek@vsb.cz

L. Václavek
e-mail: leo.vaclavek@vsb.cz

A. Abraham et al. (eds.), *Proceedings of the First International Scientific Conference "Intelligent Information Technologies for Industry" (IITI'16)*, Advances in Intelligent Systems and Computing 451, DOI 10.1007/978-3-319-33816-3_13

stochastic/probabilistic approaches for a qualitative improvements of the safety and serviceability assessment of structural components and systems especially in the engineering. In structural reliability assessment, the concept of a limit state separating a multidimensional domain of random (stochastic, probabilistic) variables into "safe" and "unsafe" domains has been generally accepted. For more general information about the history and development of stochastic methods in the branch of engineering, see Refs. [1–5]. There are solved structures on elastic foundations, medical problems, movement of Earth plates, design of machines and its parts, experiments etc.

Hence, the SBRA Method is connected with new innovative engineering. Therefore, the probabilistic applications in the engineering are new scientific trends.

This paper presents some computational stochastic and probabilistic solutions (i.e. SBRA—Simulation-Based Reliability Assessment Method) applied in the branch of engineering. The origins and developments of SBRA Method are described in [2]. The main attention is focused on the "fully" probabilistic approach to reliability assessment using SBRA Method as a suitable tool. Let the resistance of the structure be expressed by random variable R and load effect by random variable S. The failure is defined by $RF < 0$ that is

$$RF = R - S < 0. \quad (1)$$

In SBRA Method, the reliability function (1) is analysed using direct Monte Carlo simulation and Anthill computer programme, see [2]. Let N_f be the number of simulation steps when $RF < 0$ and let N be the total number of simulation steps. Then, the probability of failure can be expressed as $P_f = P_{RF<0} = N_f/N$.

At first, the 2D unbraced steel frames are solved (i.e. buckling; mechanics; elastic transformation models are used in order to express the response of the structural components including the 2nd order theory effects; probabilistic reliability assessment). At second, the femoral screw (made of Ti6Al4V and stainless steel) for treatment of proximal femoral neck fractures of humans are solved (i.e. bending; biomechanics, implant rested in bones is solved as a planar beam on elastic foundation including the 2nd order theory effects; probabilistic reliability assessment). At third, other applications are mentioned.

2 Unbraced Planar Frame (Buckling)

The common approach to the stability (buckling) assessment of structural steel components and systems is based so far mainly on the conventional models using buckling length, buckling coefficients, compressive strength and other characteristics, see [6]. However, the computer technology allows for qualitative improvement using probabilistic approach and advanced transformation models based on 2nd order theory.

The unbraced planar steel frame shown in Fig. 1a and Table 1 consists of two cantilevered columns (*1*, *2*), two leaning columns (*3*, *4*) and three crossbars. The supports of cantilevered columns are not rigid, but elastic with stiffness k_j.

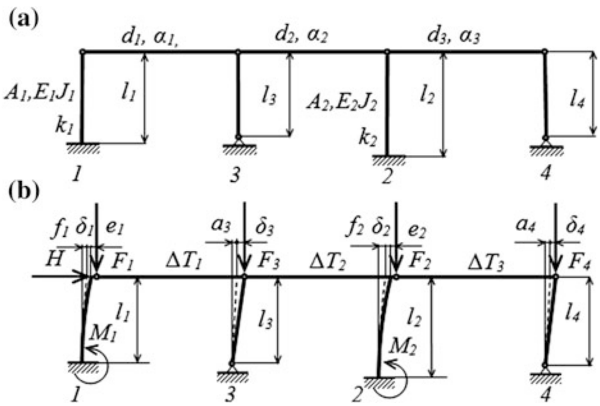


Fig. 1 Planar frame **a** undeformed situation, **b** deformed situation

Table 1 Nomenclature in Sect. 2 (Planar frame)

A_j	Area of cross-section	J_j	Quadratic moment of area
a_i	Initial imperfection	j	Index = 1 and 2
d_i	Length of cross-bar	k_j	Elastic stiffness
E_j	Modulus of elasticity	l_i, l_j	Length of column
e_j	Eccentricity of vertical force	M_j	Reaction bending moment
EQ	Earthquake load	α_i	Thermal coefficient of expansion
F_i	Vertical force	δ_j	Horizontal displacement
f_j	Amplitude of initial crookedness	θ_j	Angle of relative rotation
H	Horizontal force	ΔT_i	Temperature difference
i	Index = 1, 2, 3 and 4	W	Wind load

The frame is loaded by forces F_i, H, and by forced deformation caused by temperature differences ΔT_i (with reference to the temperature during erection), see Fig. 1b. Initial curvatures with amplitudes f_j, were considered. Unavoidable eccentricities of forces F_i are e_i. Imperfections a_i represent the initial deviations.

Analytical transformation model of the frame is expressed by following equations

$$\delta_1 = \frac{H + \sum_{i=3}^{4} \frac{F_i a_i}{l_i} + \sum_{i=1}^{2} \frac{F_i V_i}{G_i l_i} + \frac{F_3}{l_3} d_1 \alpha_1 \Delta T_1 - \frac{F_2}{G_2 l_2} \sum_{i=1}^{2} d_i \alpha_i \Delta T_i + \frac{F_4}{l_4} \sum_{i=1}^{3} d_i \alpha_i \Delta T_i}{\frac{F_1}{G_1 l_1} + \frac{F_2}{G_2 l_2} - \sum_{i=3}^{4} \frac{F_i}{l_i}}; \omega_j = \sqrt{\frac{F_j}{E_j J_j}};$$

$$\delta_2 = \delta_1 + \sum_{i=1}^{2} d_i \alpha_i \Delta T_i; M_j = F_j[(1 + \frac{1}{G_j})\delta_j - \frac{V_j}{G_j} + e_j + f_j]; P_j = 1 - \frac{F_j l_j}{k_j} \frac{\tan(\omega_j l_j)}{\omega_j l_j};$$

$$G_j = \frac{\tan(\omega_j l_j)}{P_j \omega_j l_j} - 1; V_j = e_j[\frac{1}{P_j \cos(\omega_j l_j)} - 1] + f_j\left\{ \left(P_j[1 - (2\omega_j l_j/\pi)^2]\right)^{-1} - 1 \right\}. \quad (2)$$

Table 2 Stochastic input values in Sect. 2 Planar frame

Input random variables		Extreme value or range	Histograms
F_1	Dead load	200 kN	Dead1
	Long lasting load	50 kN	Long2
	Short lasting load	50 kN	Short2
F_2	Dead load	400 kN	Dead1
	Long lasting load	150 kN	Long2
	Short lasting load	150 kN	Short2
F_3	Dead load	200 kN	Dead1
	Long lasting load	100 kN	Long2
	Short lasting load	200 kN	Short2
F_4	Dead load	100 kN	Dead1
	Long lasting load	100 kN	Long2
	Short lasting load	100 kN	Short2
W	Wind	±40 kN	Wind1
EQ	Earthquake	$0.02\Sigma F_i$, or $0.03\Sigma F_i$	Gumbel02
ΔT	Temperature difference	From −21 to +40 °C	Gamma08
Input random variables		Mean value or range	Histograms
f_1	Amplitude of initial curvature, columns *1*, *2*	±20 mm	Normal2
f_2		±25 mm	Normal2
e_1	Eccentricities of forces F_1, F_2	±30 mm	Normal2
e_2		±38 mm	Normal2
a_3	Equivalent geometrical imperfections, columns *3*, *4*	±27 mm	Normal2
a_4		±30 mm	Normal2
A_1	Area of cross section, columns *1*, *2*	13.1×10^2 mm^2	N1-04
A_2		17.1×10^2 mm^2	N1-04
S_1	Section modulus, columns *1*, *2*	138×10^4 mm^3	N1-08
S_2		216×10^4 mm^3	N1-08
I_1	Moment of inertia, columns *1*, *2*	193×10^6 mm^4	N1-08
I_2		367×10^6 mm^4	N1-08
E_1, E_2	Modulus of elasticity, columns *1*, *2*	210 GPa	N1-15
F_{y1}, F_{y2}	Yield stress, columns *1*, *2*	248–500 MPa	A36-m
k_1	Elastic stiffness, flexible support, columns *1*, *2*	8×10^7 Nm/rad	N1-30
k_2		1.5×10^8 Nm/rad	N1-30

Input random variables are summarized in Table 2. Names of histograms are the same as in [2] (i.e. Anthill software). Except EQ, all input random variables are mutually uncorrelated. Force EQ is correlated with forces F_i. Force H is the sum W and EQ, both of them may act to the left or to the right. The coefficient of temperature expansion is $\alpha_i = 12 \times 10^{-6}$ K^{-1}. Dimensions $l_1 = 6$ m, $l_2 = 7.6$ m, $l_3 = 5.4$ m, $l_4 = 6$ m and $d_1 = d_2 = d_3 = 10$ m. Numerical results are summarized

Table 3 Calculated probabilities of failure P_f—safety (carrying capacity, Planar frame)

Failure probability P_f—refered to the onset of yielding			
$EQ = 0.02\Sigma F_i$*Gumbel02		$EQ = 0.03\Sigma F_i$*Gumbel02	
Column *1*	Column *2*	Column *1*	Column *2*
$<10^{-6}$	$<10^{-6}$	23.5×10^{-6}	1×10^{-6}

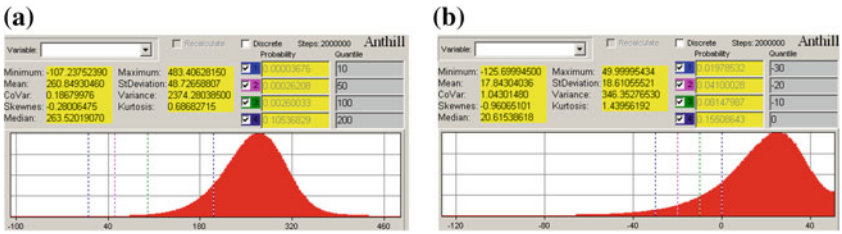


Fig. 2 Histograms of the safety function **a** Carrying capacity, **b** serviceability—Planar frame, Anthill software

in Table 3 and [6]. Two million simulation steps were used for both of the safety and serviceability assessment.

Load carrying capacity is evaluated with reference to the onset of yielding (most exposed fibers of columns *1*, *2* are considered, see Fig. 2). Serviceability assessment of the structure refers to the lateral displacement limit value of the upper end of column *1*. Computed probabilities of failure (probability of exceeding the lateral displacement value, see Fig. 2) are summarized in [6].

To assess the safety and serviceability of this unbraced planar frame using current codes (i.e. different methods LRFD or ASD approach, see [7]) and considering its complexity would not be an easy task, see, e.g. the discussion on the assessment of such system in [8]. However, in this article, the authors offer better stochastic/probabilistic solution which is based on own analytical model applied in Anthill software (SBRA Method).

3 Probabilistic Reliability Assessment of Femoral Screws Intended for Treatment of "Collum Femoris" Fractures (Bending)

Proximal femoral neck fractures, see Fig. 3a, remain a vexing clinical problem in traumatology and orthopaedics (i.e. common type of trauma), see [9–11].

One possible treatment method for femoral neck fractures, is the application of femoral screws made up from stainless steel or Ti6Al4V material, see Fig. 3b, c.

The analytical model for strength analyses of femoral screws is based on the theory of beams on an elastic foundation, where the bone is approximated by the elastic foundation prescribed by stiffness k/Pa/, see [1, 4, 10]. Quite large and

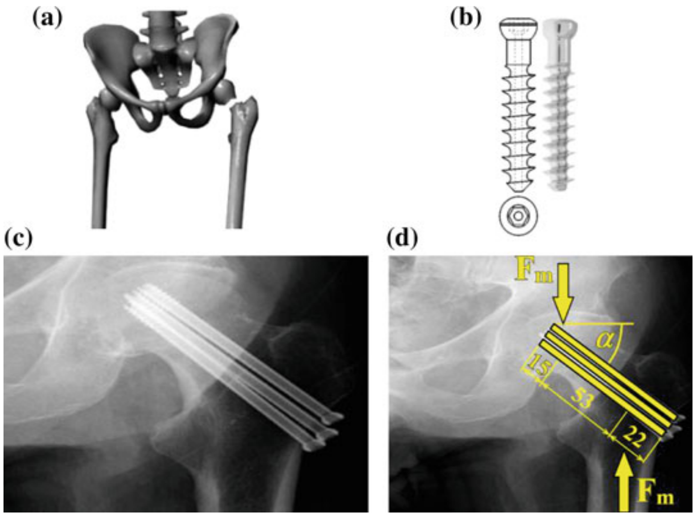


Fig. 3 **a** Femoral neck fracture. **b** Femoral screws. **c** Treatment of fracture—X-ray snapshot. **d** Beams on elastic foundation

complex review about the theory and practice of elastic foundation is performed in author's book [1].

Three screws are applied in parallel positions on the elastic foundation (i.e. in the bone). The force F/N/ acting in one screw can be defined via total loading force F_m/N/, see Fig. 3d, by the equation $F = F_m/n = m\ k_m k_{dyn} g/n$. The variables are as follows: m/kg/ is the entire mass of a patient; k_m/1/ is the coefficient of mass reduction; k_{dyn}/1/ is the dynamic force coefficient; g/ms^{-2}/ is the gravitational acceleration; and n/1/ is the coefficient of inequality in the division of force F_m into three screws. These variables are defined via truncated histograms. The force F can be decomposed into forces $F_1 = F \cos \propto$ and $F_2 = F \sin \propto$, see Fig. 4a. The femoral screw angle α/deg/, which is defined by the limits of adduction and abduction, see Fig. 4b, and the yield limit Re/MPa/ for material, are likewise defined via truncated histograms, see Fig. 5a. According to the 2nd order theory and the theory of beams on an elastic foundation, three linear differential equations for the intervals $x_{1,2,3}$, can be written as $EJ_{ZT} \frac{d^4 v_i}{dx_i^4} + F_2 \frac{d^2 v_i}{dx_i^2} + k v_i = 0$ together with 12 boundary conditions. EJ_{ZT}/Nm^2/ is flexural stiffness, v_i/m/ is displacement and x_i/m/ are coordinates. Hence, bending moments M_{oi}/Nm/, and maximum stresses σ_{MAX}/MPa/, see e.g. Fig. 4a, can be calculated.

Probabilistic reliability assessment can be carried out via SBRA Method by means of the reliability function (1), depending on load capacity, compared to the extreme stress values with yield limit. The probability that plastic deformation will occur in the beam is $P_f = 2.51 \times 10^{-5}$ i.e. $P_{f\%} = 0.00251\,\%$ (calculated for 5×10^6 Monte Carlo simulations, see Fig. 5b). Hence, the femoral screws are safe and suitable for patient treatment and the surgeons can use them for treatment.

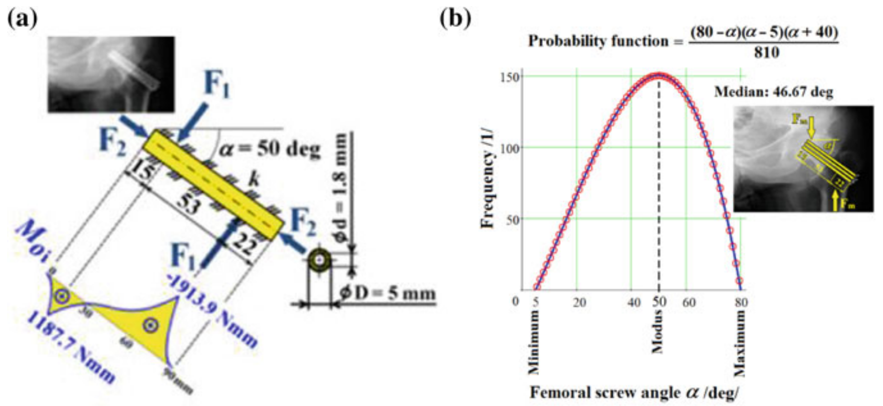
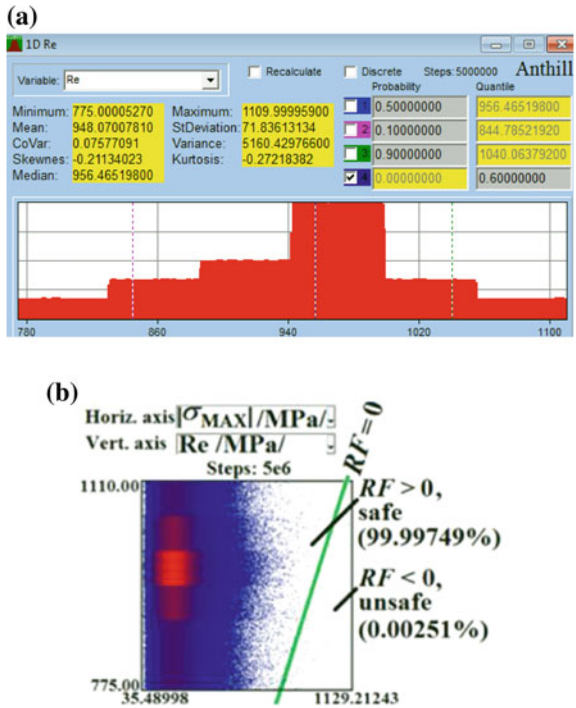


Fig. 4 **a** Beam on elastic foundation. **b** Histogram of femoral angle

Fig. 5 **a** Histogram of yield limit for Ti6Al4V material (Anthill software); **b** Probabilistic assessment of femoral screw (i.e. result of SBRA Method, Anthill software)



4 Conclusion

The transition from the pre-computer era reliability assessment to modern computers technology leads to re-engineering of the entire assessment procedures. The probabilistic SBRA Method is connected directly with this transition. Methods such as SBRA can be considered and applied in order to evaluate the safety and serviceability of structural components and systems taking into the account variabilities of inputs. One of the main prerequisites, related to the application of qualitatively higher reliability assessment methods using the potential of modern computers, is the transition of thinking of designers from deterministic to probabilistic. In case of the stability (buckling) problems (i.e. unbraced planar frame), the current assessment criteria based on buckling length and compressive strength, buckling coefficients, etc. can be gradually replaced by a fully probabilistic approach, using transformation models and 2nd order theory. In case of the design of femoral screws intended for treatment of “collum femoris” fractures in traumatology/orthopaedics (i.e. bending of planar beam rested in femur), the probabilistic assessment criteria can be based on fully probabilistic approach, using transformation models and 2nd order theory. Other examples are mentioned too.

Acknowledgments This work was supported by the Czech projects TA03010804 and SP2016/145.

References

1. Frydrýšek, K., Tvrdá, K., Jančo, R., et al.: Handbook of Structures on Elastic Foundation, pp. 1–1691. VŠB - Technical University of Ostrava, Ostrava, Czech Republic (2013). ISBN: 978-80-248-3238-8
2. Marek, P., Brozzetti, J., Guštar, M., Tikalsky, P., et al.: Probabilistic Assessment of Structures Using Monte Carlo Simulation Background, Exercises and Software, 2nd extended edn. ITAM CAS, Prague, Czech Republic (2003). ISBN: 80-86246-19-1
3. Kala, Z.: Sensitivity analysis of steel plane frames with initial imperfections. Eng. Struct. **33**(8), 2342–2349 (2011)
4. Frydrýšek, K., Čada, R. Probabilistic reliability assessment of femoral screws intended for treatment of “Collum Femoris” fractures. In: BIOMECHANICS 2014—International Conference of the Polish Society of Biomechanics, pp. 61–62. Łódź (2014). ISBN: 978-83-7283-628-1
5. Lokaj, A., Vavrušová, K., Rykalová, E.: Application of laboratory tests results of dowel joints in cement-splinter boards VELOX into the fully probabilistic methods (SBRA method). Appl. Mech. Mater. **137**, 95–99 (2012). doi:10.4028/www.scientific.net/AMM.137.95. ISSN: 1660-9336
6. Václavek, L., Marek, P., Konečný, P.: SBRA approach to the stability assessment of structural components and systems. In: Proceedings of McMat2005—2005 Joint ASME/ASCE/SES Conference on Mechanics and Materials, Article no. 519, pp. 1–6. Baton Rouge, Louisiana, USA (2005)
7. Specification for Structural Steel Buildings: ANSI/AISC 360-05 Standard. AISC Inc., Chicago (2005)

8. Geschwindner, L.F.: A practical approach to the “Leaning” column. Eng. J. **31**, 141–149 (1994)
9. Frydrýšek, K., Jořenek, J., Učeň, O., Kubín, T., Žilka, L., Pleva, L.: Design of external fixators used in traumatology and orthopaedics—treatment of fractures of pelvis and its acetabulum. Procedia Eng. **48**, 164–173 (2012)
10. Frydrýšek, K.: Strength analyses of full and cannulated femoral screws made up from stainless steel and Ti6Al4V, Calculation report, FME VŠB-Technical University of Ostrava, Ostrava, Czech Republic, pp. 1–43 (2014)
11. Farhad, N., Bradley, E.J., Hodgson, S.: Comparison of two tools for the measurement of interfragmentary movement in femoral neck fractures stabilised by cannulated screws. Robot. Comput.-Integr. Manuf. **26**(6), 610–615 (2010)