Chapter 4
Transforming Mathematics Teaching with Digital Technologies: A Community of Practice Perspective

Alison Clark-Wilson

Abstract Dynamic mathematical digital resources promise a transformation of the teaching and learning of mathematics by enabling teachers and learners to experience and explore difficult mathematical ideas in more tangible ways. However, reports of classroom practice reveal an underuse of such technologies—particularly by learners—and research findings articulate the complexities of the process of classroom integration by teachers. The work described in this chapter is set in the context of a large-scale multi-year study, *Cornerstone Maths* (CM), which aims to overcome known barriers to technology use in lower secondary mathematics with the professional development of the participating teachers as a central tenet. Here, the design and implementation of the CM professional development as experienced by a group of four teachers from one school's mathematics department is examined from a Wengerian perspective as a means to understand the trajectories of teachers' growth in both their mathematical knowledge for teaching and their associated emerging mathematical pedagogic practices with technology.

Keywords Transformation • Mathematics teaching • Digital technologies • Community of practice • *Mathematics* • Learners • Classroom integration • Teachers • Learning environment • Barriers • Professional development • Wenger • Subject content knowledge • Pedagogical practice

Introduction

The advent of dynamic mathematical digital resources in the early 1990s promised a transformation of the teaching and learning of mathematics as the technology enabled teachers and learners to experience and explore difficult mathematical ideas in more tangible ways. A host of digital environments and resources has resulted, but as research studies and school inspection reports ensued, it was soon evident

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that this process of transformation was far more complex than originally anticipated. The early wave of innovative practices and the enthusiasm of the innovators were not sufficient to bring about long-lasting changes in the prevailing classroom practices of many countries.

The Cornerstone Maths (CM) project (2010–2013) was conceived to respond directly to this situation by adopting a *design-based research* approach (Penuel, Fishman, Cheng, & Sabelli, 2011) to produce a set of curriculum units that exploit the dynamic and multi-representational potential of digital technology to address known "hard to teach" topics in 11–14 mathematics: linear function; geometric similarity; and algebraic patterns and expressions (Hoyles, Noss, Vahey, & Roschelle, 2013). The resulting curriculum units comprise: specially designed web-based software; student workbooks; teacher guides; and a mandatory professional development (PD) programme. This paper describes outcomes from an ongoing Nuffield Foundation-funded CM project that is being co-directed by my colleague Celia Hoyles and I. The study aims to analyse the development of teachers' *mathematical knowledge for teaching* (Hill & Ball, 2004) and associated mathematics pedagogical practice as they engage in professional development and teaching of the CM curriculum unit on algebraic patterns and expressions using an adapted *lesson study* approach.

Transforming Mathematics Teaching with Digital Technologies: Key Ideas from the Literature

It is important to note from the outset that when using the word technology, I am not referring to general technology "hardware" such as interactive whiteboards, mobile 'phones, the internet or iPads, but to device agnostic digital environments that require the learner to engage and interact with mathematical ideas in very particular ways. Such environments may have been created within available mathematical software (i.e. dynamic geometry, dynamic graphing, spreadsheet or statistical software) or they may be embedded within a web-page or application. A general feature is that the environment is designed such that the users (learner and/or teacher) are required to change a mathematical variant and observe the resulting outputs such that they can construct a deeper mathematical understanding of how different mathematical ideas are dynamically related.

The example shown in Fig. 4.1 shows a task where students are required to edit either the graph (by dragging "hotspots") or the function (by varying the values of m or c in the general equation $y=mx+c$) so that the character in the simulation reaches a specified distance in a specified time, which is provided within the task narrative.

These teaching approaches are far from new and the research literature includes multiple findings that conclude positive impact on students' mathematical understandings (Borba & Confrey, 1996; Godwin & Sutherland, 2004; Hoyles, Kent, Noss, & Smart, 2012; Hoyles & Lagrange, 2009; Kaput, 1986; Romberg, Fennema,

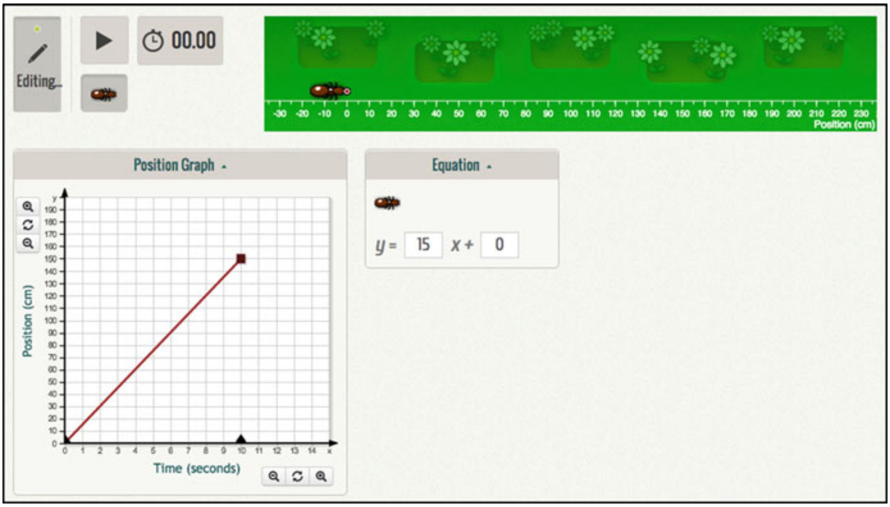

Fig. 4.1 Cornerstone Maths Software: Linear functions

& Carpenter, 1993). However, the proliferation of reports that conclude the weak impact of digital technology on students' learning outcomes (For example, see Organisation for Economic Co-operation and Development, 2015) would suggest that it is the choice of technology and the ways it is used with students that is key to replicating the positive findings of the research settings.

Within mathematics education, academics in the field of educational technology have shifted their research lenses onto teachers in an attempt to bridge the gap between research and practice and to deepen the understanding of teachers' trajectories in knowledge and practice as they learn to implement mathematical technologies such as those described previously (Clark-Wilson, Aldon, et al., 2014; Clark-Wilson, Robutti, & Sinclair, 2014; Zehetmeier, 2015). Such understandings could ensure more research-informed approaches to the design, implementation and evaluation of professional development that aims to develop knowledge and associated teaching practices.

The Development of Teachers' Knowledge and Practice Concerning Dynamic Mathematical Technologies

Early studies explored how students and teachers of high school mathematics learned to use mathematical technological tools both for themselves (instrumentation) and subsequently in their role as designers/implementers/users of classroom tasks (instrumentalisation). These drew from Vygotsky's activity theory and led to the "instrumental approach" (Artigue, 2002; Guin & Trouche, 1999; Haspekian, 2005; Verillon & Rabardel, 1995). More recent research has focused the lens onto

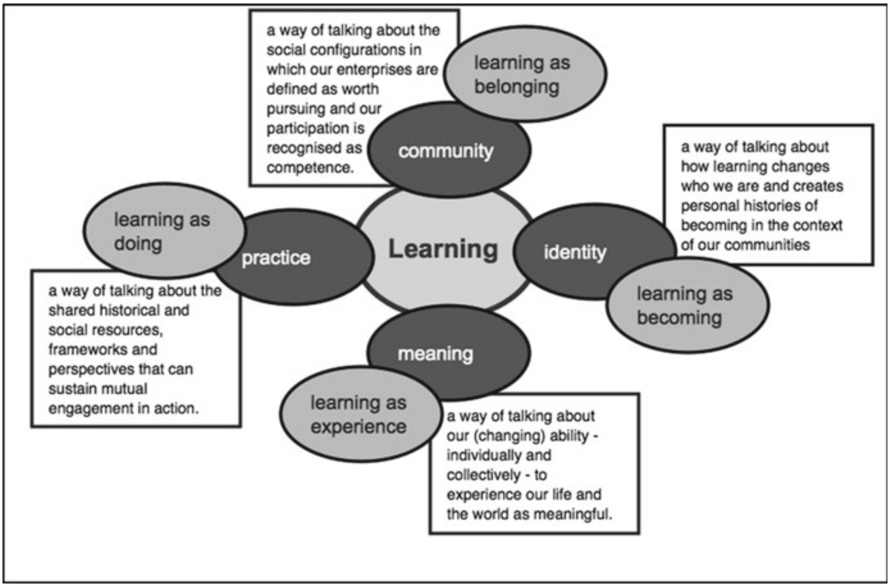

Fig. 4.2 Components of a social theory of learning (Wenger, 1998a, 1998b, p. 5)

teachers, resulting in the notions of epistemological "hiccups" (Clark-Wilson, 2010; Clark-Wilson & Noss, 2015) and "critical incidents" (Aldon, 2011) that occur during classroom practice as key triggers for teachers' cognitive learning. Consequently, the design of the CM teachers' professional development programme involved tasks for teachers that attempted to replicate these triggers, albeit in the less risky environment of a face-to-face PD session.

Designing Professional Development: A Community of Practice Perspective

According to Etienne Wenger's seminal work we all belong to multiple Communities of Practice (CoP) throughout our lives with varying levels of participation that impact differently on our learning (Wenger, 1998b). Wenger articulates how, in these communities, learning can be observed as the social construction of meanings within a community of practice, extending this notion and that of *situated learning* first described in the work of Lave (1988) and Lave and Wenger (1991). The components of Wenger's social theory of learning are shown in Fig. 4.2.

Central to Wenger's definition of a CoP is that it is a self-organising system that develops around things that matter to the members, even if the "raison d'être" for the CoP has been externally mandated. In such cases the members develop practices that respond to such mandates through their participation in the CoP. According to

Wenger, "a community of practice exists because it produces a shared practice as members engage in a collective process of learning." (Wenger, 1998a, p. 4). For the Cornerstone Maths project, the existence of the CoP is "legitimised" through the formal process whereby Headteachers register their school's involvement and commit to actions that seek to maximise the impact of the teachers' participation on students' learning outcomes. This legitimised relationship can bring the possibility that the participating teachers' actions might be scrutinised, over-managed or lead to new demands being made of them, for example, by being asked to "roll-out" CM in the school or to lead the professional development about CM to other colleagues within and even beyond the school.

In Wenger's terminology, the "joint enterprise" of the CM project CoP concerns:

- A common understanding of the work of the CoP, which is continually renegotiated by the members, i.e. the fundamental aim to provide opportunities for students to engage in mathematical activity that is mediated by the CM digital technology.
- Relationships of mutual engagement that bind the group together.
- The products of the CoP in the form of routines, ways of thinking, artefacts, vocabulary and ultimately, pedagogic styles.

Crucial to the design of the CM PD is that the members "develop among themselves their own understanding of what their practice is about" within the context of the CM approach to teaching and learning mathematics (Wenger, 1998a, p. 4).

Wenger describes the practices associated with his social theory of learning in relation to the participants' modes of belonging to the community of practice through their *engagement*, *imagination* and *alignment*. These are articulated further in Appendix and are used later in the paper to make sense of the findings of a particular group of project teachers.

A Methodology for Eliciting Teachers' Trajectories of Knowledge and Practice

The project recruited 72 teachers from 31 schools for the first PD cycle, which involved the following activities:

- Completion of an on-line questionnaire that collected contextual data and probed teachers' mathematical knowledge of algebraic variables and their prior use of dynamic technology in mathematics.
- Participation in an initial one-day face-to-face PD meeting, which included familiarisation with the CM curriculum unit, hands-on PD tasks with the CM software and collaborative lesson planning in school pairs within a shared space in an online project community.
- Participation in asynchronous follow-up support through the online project community and by email.

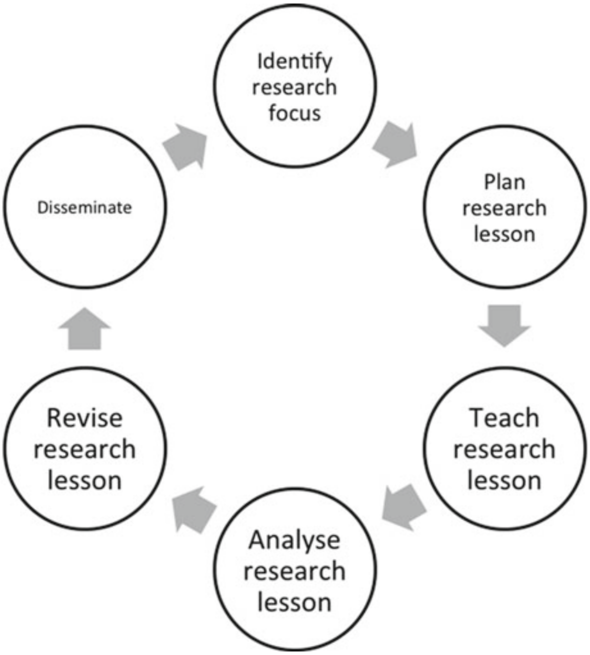

Fig. 4.3 The lesson study approach (Adapted from Foster et al., 2014)

- Participation in synchronous follow-up support provided by online meetings.
- [for a sample of teachers] Classroom observation of a CM lesson by the researcher, with pre- and post-lesson discussions.
- [for a sample of schools] Group observations of a CM lesson by the researcher and/or other members of the department, with pre- and post-lesson discussions.
- Participation in a final half-day face-to-face PD meeting.

We adapted a version of lesson study that had been developed for another Nuffield-funded research project in England, *Lessons for Mathematical Problem Solving* (Foster, Swan, & Wake, 2014) (Fig. 4.3).

The common research question that provided the focus for all of the teachers and researchers in the project as they created lesson plans to teach the research lesson was "to develop students' appreciation of an algebraic variable as a dynamic concept".

Our prior work had established the notion of "landmark" activities within CM, defined as those which

> indicate a rethinking of the mathematics or an extension of previously held ideas—the 'aha' moments that show surprise—and provide evidence of students' developing appreciation of the underlying concept (Clark-Wilson, Hoyles, & Noss, 2015).

Hence, all teachers planned to teach the same CM lesson and, although the CM curriculum unit does include outline lesson plans, we worked with the teachers to (re-)design the lesson to take account of their particular classroom contexts (student prior attainment, chosen technology etc.). The visibility of these "re-designs" was an important methodological tool that provided an insight into the aspects of the lesson that the teachers considered to need a greater or lesser emphasis and, in doing so, aspects of their knowledge and intended pedagogy. The subsequent sample of lesson observations, which were selected to give a diversity of teachers' prior mathematical and pedagogical knowledge/experience with dynamic technology in lower secondary classrooms, provided opportunities to probe teachers' developing knowledge and practices.

One Task: Four Lessons—Sixteen Stories

The case study of a group of four participating teachers from one school has been selected as an illustrative example of how their engagement with the project has impacted on their developing knowledge and practice within the very specific domain of the study. They all began with a plan to teach the same research lesson to a chosen class of 11–14-year-olds. All four teachers (Sasha, Darren, Nitesh and Cheryl) taught the lesson to their class, which was observed by the remaining three teachers.

The school, Greenfields High School, is a larger than average 11–18 secondary school in a relatively affluent area of Greater London that achieved examination outcomes in 2014 that are consistent with the national average. The mathematics department had 17 members and it was notable that the Head of Department chose to give four of the department the opportunity to participate in the project. One of the group, Sasha, was the co-ordinator of the 11–14 mathematics scheme for the department and all of the teachers were between 20 and 29 years of age with less than 5 years teaching experience. They all held first degrees in mathematical sciences and had completed post-graduate certificates in education. In their responses to the initial on-line survey, half of the group reported only occasional use of dynamic mathematical technologies by their lower secondary classes and the other two teachers reported no prior use. The teachers indicated that their barriers to such use were: a lack of knowledge of suitable technologies; a lack of time to explore possible technologies (either individually or with colleagues). Notably, a lack of access to suitable technology was not reported as a barrier.

In their research lesson plan, which was developed collaboratively during the initial face-to-face PD meeting, the teachers' "re-design" included the following aspects:

- Organisation of the technology and how students would be grouped.
- Key learning outcomes for students, which focused on an understanding that, within the dynamic representation, algebraic variables with the same name, behave in the same way.
- An opportunity to check that students were "instrumented" in their use of the software to enable them to achieve the mathematical outcomes of the lesson.

- Specific questions for the teacher to pose whilst demonstrating a particular counter-example.
- Consideration of how the students might respond to the lesson tasks—and some possible teacher reactions.

Supported by their Head of Department, whose authority enabled the teachers to be released from their own classroom teaching to each observe their three colleagues' research lessons, the group came together for a one hour meeting in their school to discuss the lesson outcomes. I observed and audio-recorded this meeting, in which each teacher began by giving their own recollection of their lesson in relation to the common research focus and, following this, the remaining teachers were invited to recount their observations. I intervened on occasion to clarify their descriptions and to probe the teachers' actions in more detail. As I was not present in any of the classrooms for the lessons, my questions were genuine as I sought to create a picture of the lesson.

Cheryl had been the first to teach the research lesson to a more-able set of 12–13-year-olds. Her overall reflection was that, although she concluded that the students had all achieved the desired learning outcome—that students could appreciate that when two mathematical variables have the same name (or are *linked*, using the terminology of the software), then they behave dynamically in the same way—she had over-structured the lesson, insisting on leading them through the software steps (the instrumentation phase) rather than allowing the students "enough freedom to explore it for themselves".

Cheryl continued to say,

> thinking about the linking especially, it didn't actually take too much nudging, if anything, I let too much slip on it, and they would have been able to do that on their own …
>
> … I thought they were going to find it a lot harder than they did, which I think is why I over-structured it—but if I was going to go back and do it again—it didn't need as much structures that, it could have been a lot more free.

Sasha agreed with Cheryl's evaluation, adding,

> I think we generally quite agreed as well after Cheryl's lesson that it was really good and that they'd all got to the place that we wanted to get to but that the main point was that they needed a bit more freedom, as Cheryl said, to kind of actually discover things for themselves, rather than being led.

I probed the teachers to try to find out what it was that the group felt that the students should have discovered for themselves and why this might be a more desirable outcome, to which Darren (addressing Cheryl) added his own observation,

> I think that you scaffolded it very well for them to have success and it wasn't just success but it was really meaningful success. So when they discovered linking they were really … they felt like they'd accomplished something and then that kind of slowly filtered down. I know I saw one pair who discovered it [linking algebraic variables] for themselves, 'oh if we name them the same it comes up with linking'—the pair next to them looked and said 'oh what have you done' and then they said 'oh what you do is you name them the same' …

The group reflected on the challenges of trying to remain a passive observer during the lesson observations but also appreciated the value of the knowledge that was gained when in this role. Nitesh commented,

> I was trying not to get involved too much and I think it was the hardest thing to see someone struggle and you just want to jump in to help … There were lots of conversations happening without Cheryl actually being there, which was nice, like in pairs and stuff. So it was nice to see. It was more the fact that, you didn't need to do anything—they figured it out for themselves.

Darren taught the lesson next. His experience of observing in Cheryl's classroom directly impacted on his own lesson plan as he gave his class much more time during his lesson to explore the software for themselves.

> I gave them too much freedom—towards around say the 35 to 45 minute mark I was starting to lose them because they had struggled for too long … On the first question, I didn't intervene early enough, I think I gave them too much freedom because there's two or three groups that were doing really really well and there was a couple of groups that were plodding on quite nicely, but there were three or four groups that were getting a bit frustrated with it and they were sort of quite hard to get back on side towards the end of the lesson … So when I saw Cheryl's lesson I gave them more freedom but I pushed it too far the other way. But from that we got a scale …

This observation was reiterated by the other teachers, who were highly supportive of Darren as his class, although slightly older (13–14 years), had lower levels of prior mathematical attainment and were less motivated than Cheryl's group. The general feeling was that due to the impending end of the lesson, Darren had rushed his final plenary, which was when he intended to discuss with the class why and how algebraic variables might need to be linked within the dynamic software—and in mathematics more generally.

The third lesson to be taught was by Sasha, who chose a class of 12–13-year-olds who were a lower attaining mathematics group, which was acknowledged by the other teachers to include a number of students with classroom behaviours that were challenging to manage. However, it was notable that two of the teachers had observed how two of these students achieved success in the lesson—and the role that Sasha had played in their achievement. Jason was particularly impressed by the way that Sasha had maintained the focus of the Pupil workbook, which contained the task instructions, during the lesson.

As the final teacher to teach the lesson, Nitesh, acknowledged that he had been at a distinct advantage as he benefited from the cumulative knowledge and experience of the group. He taught the research lesson to his class of 12–13-year-old students, who were of a slightly lower level of attainment than Sasha's class.

Cheryl commented that the lesson was well-structured, especially in the way that Nitesh integrated the opportunities for the students to record their findings in the Pupil workbook alongside their explorations with the dynamic software. Darren commented that the students in Nitesh's class seemed to value their work in their booklets more than his own class but more importantly, both Cheryl and Darren had acknowledged how it was Nitesh's actions in the classroom that had supported this particular outcome. Nitesh himself was impressed by the mathematical outcomes of his class, although he still felt that he could have had clearer expectations with respect to their written recordings.

Nitesh emphasised the use of the dynamic slider with his students as they checked whether the algebraic expressions they had created matched with the pattern and questioned how well his students had fully made sense of the expressions they had created, saying,

> Next time I do this, I'll focus on more about algebraic expressions and what they mean, as opposed to only creating the linked pattern.

The group was very positive about their overall experience within the cycle of planning, teaching and multiple observations and they all commented that they planned to teach the CM curriculum unit to another class.

Conclusions and Further Research

The mathematics department at Greenfields High School is already a CoP with established modes of belonging. The CM Project CoP began as a peripheral CoP to the four teachers as they began to engage in its activities and through their participation, assume aspects of its aims into their departmental practices. The teachers embraced the CM PD tasks, the collaborative research lesson planning task and most importantly, once they returned to school, the opportunity to engage in the lesson study cycle. In Wenger's terms, there was an appreciation of the *joint enterprise* of working to integrate student use of dynamic technology in their lower secondary lessons, the *mutual engagement* was noticeably established and, as the findings show, the emergence of a shared repertoire of dynamic technology use within the specified mathematical topic was beginning to emerge.

An important aspect of the teachers' development in their mathematical knowledge for teaching concerned their emerging mathematical vocabulary and the accompanying curriculum scripts that supported the classroom discourse alongside the dynamic technology. Although the software itself prompted the students to generate new language in the classroom as they "built" their algebraic patterns, "named" their algebraic variables and ultimately "linked" these variables, the teachers needed to think through what they would say as they made use of the software in both whole-class contexts and when supporting groups of students. By mutually observing each other it was very obvious that, by reflecting on their own approach, they could relate directly to the merits of another teacher's actions and their accompanying dialogue. The teachers also appreciated how, within these discourses, they needed to prioritise the language of the mathematics over that of the technology.

The teachers' engagement with the CM CoP was evidenced by their pursuit of the project's aims "in concert with others" through their "mutual engagement" in the project tasks. Their shared experiences, particularly within each others' classrooms, served to build their interpersonal relationships as well as open up peripheries of their own classroom experiences that had the potential to support them to develop new teaching practices.

A second facet to the development of teachers' mathematical knowledge for teaching concerns the way in which their imaginations enabled them to (re-)view their own practices alongside that of their colleagues and use their experiences to create their own visions for their own classroom practices with dynamic technology. The sharing of their stories of the classroom observations was fundamental to these processes as they imagined what their future versions of the research lesson might be. Much of their conversation was about seeing the students' mathematical behaviours in a new light. Darren spoke quite passionately about how seeing a particular student achieve highly during Sasha's lesson had prompted him to think about how he might adapt his teaching approach to engage more of his students. It was significant that all of the teachers planned to teach algebraic patterns and expressions using the CM curriculum unit in the future.

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Appendix: Learning Practices Within Wenger's Social Practice of Learning Model

Engagement

- definition of a common enterprise in the process of pursuing it in concert with others;
- mutual engagement in shared activities;
- the accumulation of a history of shared experiences;
- the production of a local regime of confidence;
- the development of interpersonal relationships;
- a sense of interacting trajectories that shape identities in relation to one another;
- the management of boundaries;
- the opening of peripheries that allow for various degrees of engagement.

Imagination

- recognising our experience in others, knowing what others are doing, being in someone else's shoes;
- defining a trajectory that connects what we are doing to an extended identity, seeing ourselves in new ways;

- locating our engagement in broader systems in time and space, conceiving
- sharing stories, explanations, descriptions;
- opening access to distant practices through excursions and fleeting contacts—visiting, talking, observing, meeting;
- assuming the meaningfulness of foreign artefacts and actions;
- creating models, reifying patterns, producing representational artefacts;
- documenting historical developments, events and transitions; reinterpreting histories and trajectories in new terms; using history to see the present as only one of many possibilities and the future as a number of possibilities;
- generating scenarios, exploring other ways of doing what we are doing, other possible worlds and other identities.

Alignment

- investing energy in a directed way and creating a focus to coordinate this investment of energy;
- negotiating perspectives, finding common ground;
- imposing one's view, using power and authority;
- convincing inspiring, uniting;
- defining broad visions and aspirations, proposing stories of identity;
- devising proceduralisation, quantification and control structures that are portable (i.e. usable across boundaries);
- walking boundaries, creating boundary practices, reconciling diverging perspectives.

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