Geometry and Direct Kinematics of Six-DOF Three-Limbed Parallel Manipulator

Zh. Baigunchekov, M. Kalimoldaev, M. Utenov and T. Baigunchekov

Abstract In this paper the methods of structural synthesis and direct kinematics of six-DOF three-limbed parallel manipulator (PM) are presented. This PM is formed by connection of a mobile platform with a base by three dyads with cylindrical joints. Constant and variable parameters characterizing geometry of links and relative motions of elements of joints are defined. Direct kinematics of the PM is solved by iterative method.

Keywords Parallel manipulator · Limb · Geometry of link · Direct kinematics

1 Introduction

Most of the 6-DOF PM consist of six limbs (Merlet 2000; Gogu 2008–2014 and others). These PM possess the advantages of high stiffness, low inertia, and large payload capacity. However, such six–limbed fully PM have a limited workspace and complex kinematic singularities, which are their major drawbacks. Therefore in robotics literature (Yang et al. 2004; Mianovski 2007; Jin et al. 2009; Glazunov 2010 and other) a great interest to the PM with few number of limbs and larger workspace is observed.

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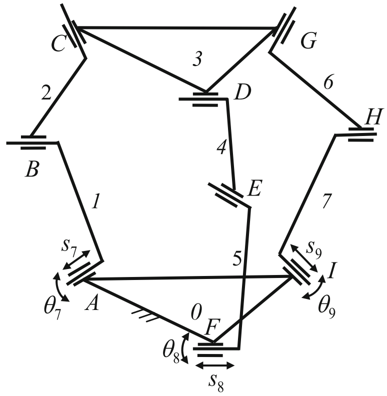


Fig. 1 PM with cylindrical joints

Following the above-mentioned trends in the development of PM, we proposed a novel structure of six-DOF three-limbed PM with cylindrical joints (PM 3CCC) (Baigunchekov et al. 2009), as shown in Fig. 1. This PM is formed by connection of a mobile platform 3 with a base 0 by three spatial dyads *ABC*, *DEF* and *GHI* of type CCC (C—cylindrical joint). Each of spatial dyads of type CCC do not impose restrictions on motion of the mobile platform, and six-DOF of the mobile platform are remained.

Each cylindrical joint has two-DOF: one rotation and one translation. In the considered PM the joints *A*, *F* and *I* are active joints, and the joints *B*, *C*, *D*, *E*, *G* and *H* are passive joints. Six variable parameters s_7, θ_7, s_8, θ_8, s_9, θ_9 of active joints *A*, *F* and *I* are the generalized coordinates. The results of singularity analysis of the PM 3CCC are presented (Baigunchekov et al. 2012). In this paper the geometry of this PM is described and its direct kinematics is solved.

2 Geometry of the PM 3CCC

To describe the geometry of the PM 3CCC two right-hand Cartesian coordinate systems *UVW* and *XYZ* are attached to each element of each joint. The *W* and *Z* axes of the coordinate systems *UVW* and *XYZ* are directed along the axes of rotation and translation of the cylindrical joints.

Transformation matrix $\mathbf{T}_{jk}$ between the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$, attached to the ends of the binary link with the *j*-th and *k*-th joints, has a form

$$\mathbf{T}_{jk} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & t_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & t_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & t_{44} \end{bmatrix}, \tag{1}$$

where $t_{11} = 1$, $t_{12} = t_{13} = t_{14} = 0$,

$$
\begin{aligned}
t_{21} &= a_{jk} \cdot \cos\gamma_{jk} + b_{jk} \cdot \sin\gamma_{jk} \cdot \sin\alpha_{jk},\\
t_{22} &= \cos\gamma_{jk} \cdot \cos\beta_{jk} - \sin\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \sin\beta_{jk},\\
t_{23} &= -\cos\gamma_{jk} \cdot \sin\beta_{jk} - \sin\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \cos\beta_{jk},\\
t_{24} &= \sin\gamma_{jk} \cdot \sin\alpha_{jk},\\
t_{31} &= a_{jk} \cdot \sin\gamma_{jk} - b_{jk} \cdot \cos\gamma_{jk} \cdot \sin\alpha_{jk},\\
t_{32} &= \sin\gamma_{jk} \cdot \cos\beta_{jk} + \cos\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \sin\beta_{jk},\\
t_{33} &= \cos\gamma_{jk} \cdot \cos\alpha_{jk} \cdot \cos\beta_{jk} - \sin\gamma_{jk} \cdot \sin\beta_{jk},\\
t_{34} &= -\cos\gamma_{jk} \cdot \sin\alpha_{jk},\ t_{41} = c_{jk} + b_{jk} \cdot \cos\alpha_{jk},\\
t_{42} &= \sin\alpha_{jk} \cdot \sin\beta_{jk},\ t_{43} = \sin\alpha_{jk} \cdot \cos\beta_{jk},
\end{aligned}
$$

$t_{44} = \cos\alpha_{jk}$, a_{jk}—a distance from the W_j axis to the Z_k axis measured along the direction of the common perpendicular t_{jk} between the W_j and Z_k axes; α_{jk}—an angle between positive directions of the W_j and Z_k axes measured counterclockwise about positive direction of t_{jk}; b_{jk}—a distance from direction of t_{jk} to direction of the X_k axis measured along the positive direction of the Z_k axis; β_{jk}—an angle between positive directions of t_{jk} and X_k axis measured counterclockwise about the positive direction of the Z_k axis; c_{jk}—a distance from direction of U_j axis to direction of t_{jk} measured along the positive direction of the W_j axis; γ_{jk}—an angle between positive directions of the U_j axis and t_{jk} measured counterclockwise about the positive direction of the W_j axis.

In comparision with the Denavit–Hartenberg transformation matrix, having four parameters, the transformation matrix (1) has six parameters fully characterizing the relative locations of the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$, because a free rigid body in space has six generalized coordinates.

A binary link jk of type CC is shown in Fig. 2. Axes of the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$, attached to the ends of this binary link, are chosen as follows: the W_j and Z_k axes are located along the axes of rotation and translation of the cylindrical joints j and k; the origins O_j and O_k of the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$ are located in points of intersection of the W_j and Z_k axes with the common perpendicular t_{jk} between these axes; the U_j and X_k axes are located along the common perpendicular t_{jk}; the V_j and Y_k axes are completed the right-hand Cartesian coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$.

At such choice of the coordinate systems $U_jV_jW_j$ and $X_kY_kZ_k$ nonzero parameters of the matrix $\mathbf{T}_{jk}$ are a_{jk} and α_{jk}. Then from the matrix (1) we obtain a matrix of the binary link jk of type CC

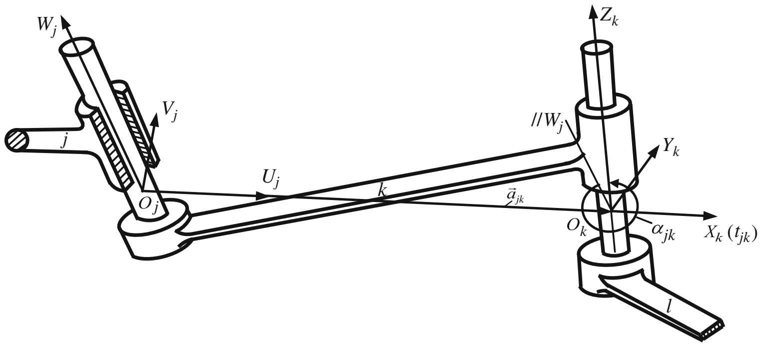


Fig. 2 Binary link *jk* of type CC

$$\mathbf{G}_{jk}^{\mathrm{CC}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_{jk} & 1 & 0 & 0 \\ 0 & 0 & \cos\alpha_{jk} & -\sin\alpha_{jk} \\ 0 & 0 & \sin\alpha_{jk} & \cos\alpha_{jk} \end{bmatrix}, \tag{2}$$

where parameters a_{jk} and α_{jk} are constant, and they characterize the geometry of the binary link *jk* of type CC.

Nonzero parameters of the cylindrical joint *j* shown in Fig. 3 are θ_j and s_j. Then from the matrix (1) we obtain a matrix of the cylindrical joint *j*

Fig. 3 Cylindrical joint *j*

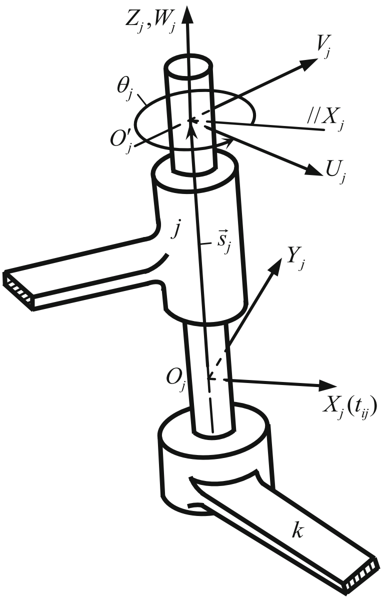
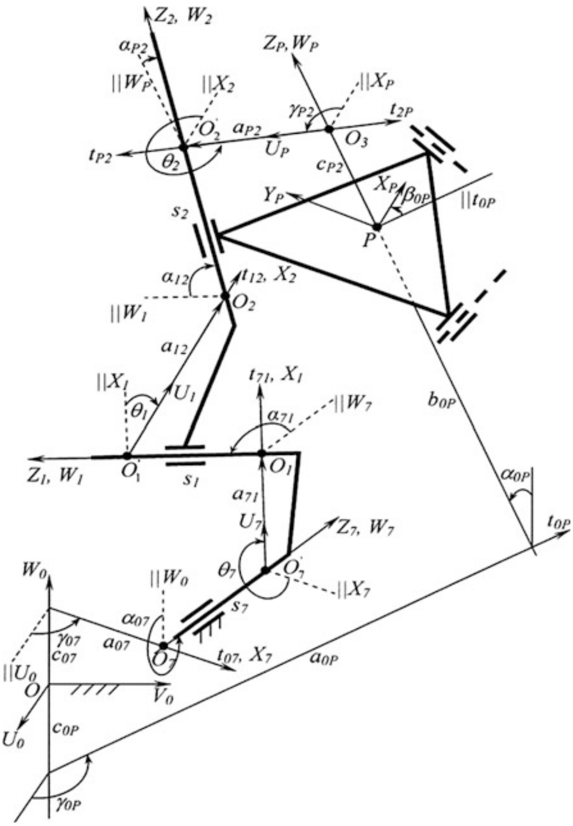


Fig. 4 The first limb



$$
\mathbf{P}_j^{\mathrm{C}}(\theta_j, s_j) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_{jk} & -\sin\theta_j & 0 \\ 0 & \sin\theta_j & \cos\theta_j & 0 \\ s_j & 0 & 0 & 1 \end{bmatrix}, \tag{3}
$$

where s_j—a distance from the X_j axis to the U_j axis measured along the directions of the Z_j and W_j axes; θ_j—an angle between the positive directions of the X_j and U_j axes measured counterclockwise about the positive directions of the Z_j and W_j axes. Parameters s_j and θ_j are variable, and they characterize relative translation and rotation motions of the j-th cylindrical joint elements.

Choosing the coordinate systems *UVW* and *XYZ*, as shown in Figs. 2 and 3, the constant and variable parameters of the PM 3CCC have been obtained. Constant and variable parameters of the first limb *ABC* of the PM 3CCC are shown in Fig. 4, where θ_7 and s_7 are the generalized coordinates of the active joint *A*; θ_2, s_2 and θ_3, s_3 are the variable parameters of the passive joints *B* and *C*; all other parameters are the constant parameters characterizing the geometry of links. Constant and variable parameters of two other symmetrical legs are defined similarly.

3 Direct Kinematics

In direct kinematics of the PM 3CCC position of coordinate system $X_P Y_P Z_P$ attached to the mobile platform 3 are defined with respect to the base frame $U_o V_o W_o$ by known constant geometrical parameters of the links and the generalized coordinates s_i, θ_i, (i = 7, 8, 9).

For automation of calculation of the direct kinematics we use following designations: n—number of mobile links, m—number of input links, L—number of the closed loops, n_l—number of links in the l-th loop (l = 1, 2, …, L), (l, a)—index of the a-th link in the l-th loop. Dependent variable parameters of the passive joints $h_{(l,a)}^{(\omega)}$ are numerated 1, 2,…, $n - m$, independent variable parameters of the active joints $q_{(l,a)}^{(\omega)}$ are numerated $n - m + 1$, $n - m + 2$, …, n, where ω—number of DOF of the joint (l, a). For the considered PM 3CCC $n = 7$, $m = 3$, $L = 2$, $n_1 = n_2 = 6$, $\omega = 2$. PM 3CCC has two closed loops $ABCDEFA$ and $ABCGHIA$.

Let made a Table 1 of conformity between the numbers of links and joints of the PM 3CCC.

Using the Table 1 of conformity between the numbers of links and joints we can write the loop-closure equations of the loops $ABCDEFA$ and $ABCGHIA$

$$\left.\begin{aligned}\mathbf{M}_{71}\cdot\mathbf{M}_{12}\cdot\mathbf{M}_{23}\cdot\mathbf{M}_{34}\cdot\mathbf{M}_{48}\cdot\mathbf{M}_{87}=\mathbf{E}\\ \mathbf{M}_{71}\cdot\mathbf{M}_{12}\cdot\mathbf{M}_{25}\cdot\mathbf{M}_{56}\cdot\mathbf{M}_{69}\cdot\mathbf{M}_{97}=\mathbf{E}\end{aligned}\right\} \tag{4}$$

or

$$\mathbf{M}_{l,1}\cdot\mathbf{M}_{l,2}\cdot\ \ldots\ \cdot\mathbf{M}_{l,a}\cdot\ \ldots\ \cdot\mathbf{M}_{l,n_l}=\mathbf{E};\ \ l=1,\ 2, \tag{5}$$

where $\mathbf{E}$ is a unit matrix, $\mathbf{M}_{jk}=\mathbf{P}_{jk}^{\mathrm{C}}\cdot\mathbf{G}_{jk}^{\mathrm{CC}}$, $j,\ k$ =1, 2,…, 9.

For the direct kinematics of the PM 3CCC the iterative method (Uicker et al. 1964) is used. According to this method unknown dependent variable parameters $h_{(l,a)}^{(\omega)}$ are written through their initial approaches $h_{(l,a)}^{(\omega)*}$ and deviations $dh_{(l,a)}^{(\omega)}$ by expression

$$h_{(l,a)}^{(\omega)}=h_{(l,a)}^{(\omega)*}+dh_{(l,a)}^{(\omega)}, \tag{6}$$

where $h_{(l,a)}^{(\omega)}\in\theta_{l,a},\ s_{l,a}$.

Table 1 Conformity between the numbers of links and joints

l	1	1	1	1	1	1	2	2	2	2	2	2
a	1	2	3	4	5	6	1	2	3	4	5	6
$(l,\ a)$	7	1	2	3	4	8	7	1	2	5	6	9

The system of matrix Eq. (5) are transformed to the system of linear equations, from which the deviations $dh_{(l,a)}^{(\omega)}$ are determined. Adding the determined deviations to their previous values, more exact values of the passive joints variable parameters have been obtained.

Position of the coordinate system $X_P Y_P Z_P$ attached to the mobile platform 3 with respect to the base coordinate system $U_o V_o W_o$ can be defined by matrix $\mathbf{S}_P$

$$\mathbf{S}_P = \mathbf{G}_{07} \cdot \mathbf{M}_{71} \cdot \mathbf{M}_{12} \cdot \mathbf{M}_{23} \cdot \mathbf{G}_{3P}, \tag{7}$$

where $\mathbf{G}_{07}$ is a transformation matrix between the coordinate system $X_7 Y_7 Z_7$ and the absolute coordinate system $U_0 V_0 W_0$, $\mathbf{G}_{3P}$ is a transformation matrix between the coordinate systems $X_P Y_P Z_P$ and $X_3 Y_3 Z_3$ of the mobile platform 3.

4 Conclusions

A novel six-DOF three-limbed PM 3CCC is formed by connection of the mobile platform with the base by three spatial dyads with cylindrical joints. Constant and variable parameters of the PM 3CCC are defined on the basis of the transformation matrix of two systems of coordinates attached to each element of each joints. Constant parameters characterize geometry of links, and variable parameters characterize relative motions of the joint elements. The loop-closure matrix equations of the PM 3CCC are made up. The direct kinematics of the PM 3CCC is solved by iterative method of solution of the loop-closure matrix equations.

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