

# To Scheduling Quality of Sets of Precise Form Which Consist of Tasks of Circular and Hyperbolic Type in Grid Systems

Andrey Saak, Vladimir Kureichik and Yury Kravchenko

**Abstract** Grid systems with centralized structure of the scheduling system and resource co-allocation are modeled by resource quadrant. A resource rectangle presents user's task. Quality of scheduling with heuristic algorithms is estimated by a Non-Euclidean heuristic measure which takes into consideration both the area and the form of an occupied resource region. One of a study problem is resource rectangle sets, denoted as sets of precise form, which have the square resource enclosure with no hollow space. The question that is posed concerns level polynomial algorithms adaptivity for the sets of precise form that consist of tasks of the circular and hyperbolic type.

**Keywords** Grid system · Centralized structure of the scheduling system · Resource rectangle · Set of precise form · Task of the circular type · Task of the hyperbolic type · Non-Euclidean heuristic measure · Level algorithm of scheduling of polynomial completeness

## 1 Introduction

Users' growing demand in computer power and rise of technology favour the transition to grid computing from meta computing [1, 2]. The effectiveness of Grid systems' performance depends on the quality of computer and time resources scheduling. Optimal resource scheduling is practically unreachable because of exponential completeness. In [3–7] an environment of resource rectangles, as

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A. Saak (✉) · V. Kureichik · Y. Kravchenko  
Southern Federal University, Rostov-on-Don, Russia  
e-mail: saak@tgn.sfedu.ru

V. Kureichik  
e-mail: vkur@sfedu.ru

Y. Kravchenko  
e-mail: krav-jura@yandex.ru

polynomial completeness scheduling theory tool, is developed for the purpose of computer and time resources distribution management. In the resource rectangles environment the operations on resource rectangles were introduced and the heuristic algorithms of resource distribution based on the presented operations were suggested. Polynomial completeness of such algorithms was showed. In [3–7] it is suggested and developed the quadratic classification of task sets. The polynomial algorithms, which were studied in [3–7], were adapted for respective quadratic type of a set of tasks. In [3] circular, hyperbolic and parabolic types were defined for the sets which consist of not less than two tasks. Quadratic type of one task was introduced in [8], where polynomial algorithms adaptivity for the sets consisted of tasks of circular type was researched.

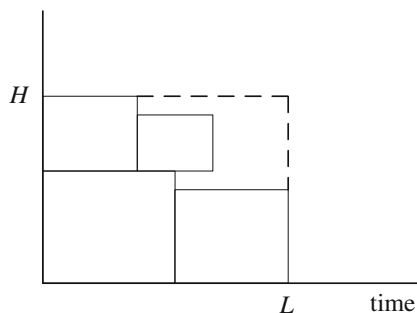
## 2 Problem Statement

Grid systems with centralized structure of the scheduling system and resource co-allocation are modeled by resource quadrant [3, 9]. User's task, which comes to be served by Grid system's scheduler, is presented as a resource rectangular with its horizontal and vertical dimensions, respectively, equaled to the number of time resource units and processors required to process the task [10]. Quality of scheduling with heuristic algorithms is estimated by the Non-Euclidean heuristic measure which takes into consideration both the area and the form of an occupied resource region

$$\frac{1}{2} \left( \frac{LH + (L - H)^2}{\sum_{j=0}^{k-1} a(j)b(j)} \right) \quad (1)$$

where L—length, H—vertical level of the resource enclosure (see Fig. 1) [3].

**Fig. 1** Users' task resource enclosure



Heuristic measure reaches its minimum of  $\frac{1}{2}$  in square packing with no empty space. In [11] a resource rectangle set was defined as the set of precise form, which has its square resource enclosure with no any empty spaces. Scheduling quality for a set of precise form which consists of the resource rectangles of circular type was the point of study in [11].

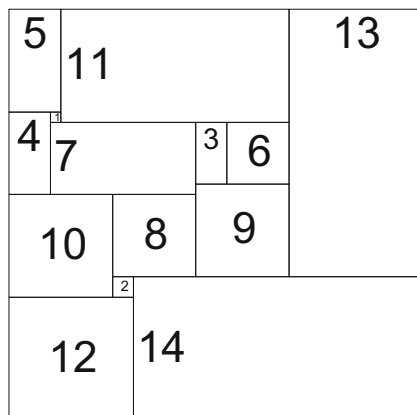
In this paper the question, that is posed, concerns polynomial algorithms adaptivity for sets of precise form which consist of resource rectangles of circular and hyperbolic type.

### 3 Scheduling of a Set of Precise Form with the Tasks of Circular and Hyperbolic Type by Level Algorithms

A level algorithm by height with not-to-reach level was suggested in [7], an exceeding level algorithm by height and level algorithm by height with minimal deviation were introduced in [12]. For the sets of resource rectangles which don't have the property of its horizontal dimensions monotony, it is necessary on each step to define the right side of a resource enclosure as a sum of the value of the right side of derived resource enclosure and the value of maximal horizontal dimension of the elements in a vertical layer. Level algorithms by length are defined in the same way. For the sets of resource rectangles which don't have the property of its vertical dimensions monotony, it is necessary on each step to define the upper side of a resource enclosure as a sum of the value of the upper side of derived resource enclosure and the value of maximal vertical dimension of the elements in a horizontal layer.

We use the rectangle sets, which are induced by the elements of diverse square tiling [13], as a test example. At the same time diverse square tiling is square packing of consecutive squares with its sides equaled to consecutive natural numbers which begins from one (of sizes  $1 \times 1$  up to  $k \times k$ ), with possible two-times duplication of each square, with no empty spaces [13]. In the examples of diverse square tiling, which were given in [13], some equal square pairs have the common side and located horizontally or vertically. This allows considering such pair of squares as a rectangle with its sides ratio equaled to 1:2. In accordance with the definitions [8], a horizontally oriented rectangle could be considered as a rectangle of the circular type and vertically oriented rectangle would be the one of the hyperbolic type. As in [8] said, a square relates to the circular type. Thus the rectangle set, which is induced by the elements of diverse square tiling, contains rectangles of the circular and hyperbolic type. In [13] the examples of diverse square tiling for  $k = 9, 10, 11, 12, 13, 14$  (see Fig. 2) are given. Thereby, to produce test examples the sets with the maximal side of the enclosing square for corresponding  $k$  were used. The number on a rectangle shows the small side.

**Fig. 2** Diverse square tiling for  $k = 14$  [13]

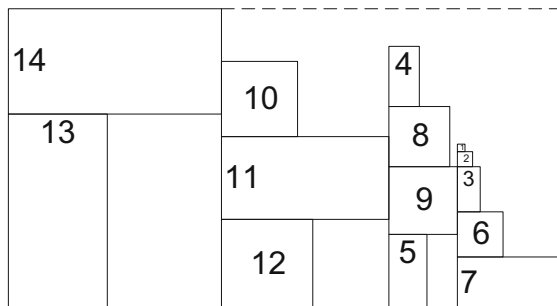


Let's denote the sets of the resource rectangles, which are induced by diverse square tiling and ordered by decrease of their heights, by the following way: set I for  $k = 9$ , set II for  $k = 10$ , set III for  $k = 11$ , set IV for  $k = 12$ , set V for  $k = 13$ , set VI for  $k = 14$ .

The results of set IV packing for the level algorithm by height with not-to-reach level are presented on Fig. 3.

The heuristic measure values of the resource enclosures of the level algorithm by height with not-to-reach level for the set which consists of the tasks of the circular and hyperbolic quadratic type are presented in Table 1.

**Fig. 3** Set VI packing by the level algorithm by height with not-to-reach level



**Table 1** The resource enclosures' heuristic measure values of the level algorithm by height with not-to-reach level

Set's number	Heuristic measure	Set's number	Heuristic measure
I	1.09	IV	0.95
II	1.00	V	1.16
III	0.86	VI	1.25

We could see that resource enclosures' heuristic measure values of the level algorithm by height with not-to-reach level don't exceed the value of

$$\frac{1}{2} + 0.75 \tag{2}$$

The results of set IV packing for the exceeding level algorithm by height are presented on Fig. 4.

The heuristic measure values of the resource enclosures of the exceeding level algorithm by height for the set which consists of the tasks of the circular and hyperbolic quadratic type are presented in Table 2.

We could see that resource enclosures' heuristic measure values of the exceeding level algorithm by height don't exceed the value of

$$\frac{1}{2} + 0.56 \tag{3}$$

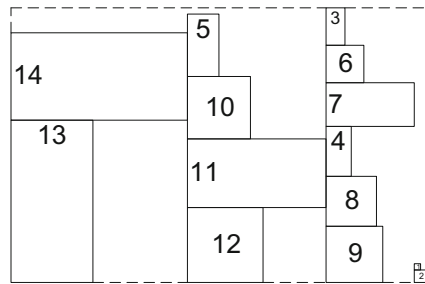
The results of set IV packing for the level algorithm by height with minimal deviation are presented on Fig. 5.

The heuristic measure values of the resource enclosures of the level algorithm by height with minimal deviation for the set which consists of the tasks of the circular and hyperbolic quadratic type are presented in Table 3.

We could see that resource enclosures' heuristic measure values of the level algorithm by height with minimal deviation don't exceed the value of

$$\frac{1}{2} + 0.58 \tag{4}$$

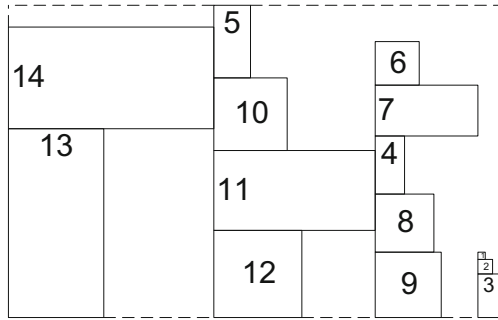
**Fig. 4** Set VI packing by the exceeding level algorithm by height



**Table 2** The resource enclosures' heuristic measure values of the exceeding level algorithm by height

Set's number	Heuristic measure	Set's number	Heuristic measure
I	1.06	IV	0.83
II	0.91	V	0.85
III	0.75	VI	1.06

**Fig. 5** Set VI packing by the level algorithm by height with minimal deviation



**Table 3** The resource enclosures' heuristic measure values of the level algorithm by height with minimal deviation

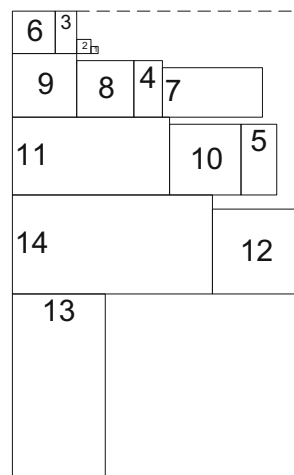
Set's number	Heuristic measure	Set's number	Heuristic measure
I	1.06	IV	0.73
II	0.93	V	0.85
III	0.86	VI	1.08

The results of set IV packing for the level algorithm by length with not-to-reach level are presented on Fig. 6.

The heuristic measure values of the resource enclosures of the level algorithm by length with not-to-reach level for the set which consists of the tasks of the circular and hyperbolic quadratic type are presented in Table 4.

We could see that resource enclosures' heuristic measure values of the level algorithm by length with not-to-reach level don't exceed the value of

**Fig. 6** Set VI packing by the level algorithm by length with not-to-reach level



**Table 4** The resource enclosures' heuristic measure values of the level algorithm by length with not-to-reach level

Set's number	Heuristic measure	Set's number	Heuristic measure
I	1.19	IV	0.80
II	0.73	V	0.94
III	0.82	VI	1.04

$$\frac{1}{2} + 0.69. \tag{5}$$

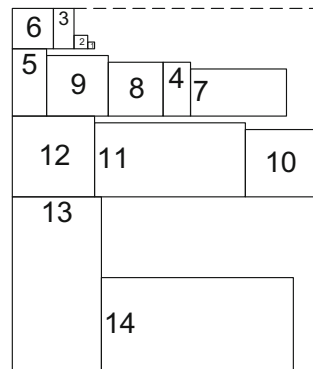
The results of set IV packing for the exceeding level algorithm by length are presented on Fig. 7.

The heuristic measure values of the resource enclosures of the exceeding level algorithm by length for the set which consists of the tasks of the circular and hyperbolic quadratic type are presented in Table 5.

We could see that resource enclosures' heuristic measure values of the exceeding level algorithm by length don't exceed the value of

$$\frac{1}{2} + 0.36. \tag{6}$$

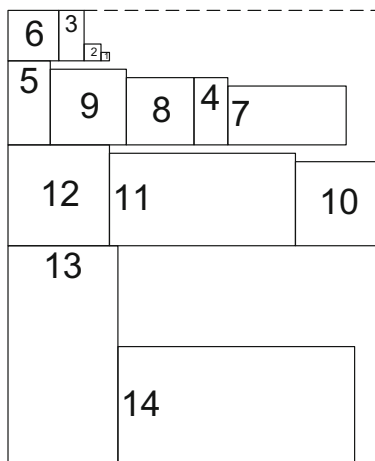
**Fig. 7** Set VI packing by the exceeding level algorithm by length



**Table 5** The resource enclosures' heuristic measure values of the exceeding level algorithm by length

Set's number	Heuristic measure	Set's number	Heuristic measure
I	0.74	IV	0.72
II	0.81	V	0.86
III	0.71	VI	0.77

**Fig. 8** Set VI packing by the level algorithm by length with minimal deviation



The results of set IV packing for the level algorithm by length with minimal deviation are presented on Fig. 8.

The heuristic measure values of the resource enclosures of the level algorithm by length with minimal deviation for the set which consists of the tasks of the circular and hyperbolic quadratic type are presented in Table 6.

We could see that resource enclosures' heuristic measure values of the level algorithm by length with minimal deviation don't exceed the value of

$$\frac{1}{2} + 0.30. \tag{7}$$

The graphs of the resource enclosures' heuristic measure values, which were obtained with the use of the level algorithms by height and length when scheduling sets I to IV, are presented on Fig. 9.

We could see that the level algorithm by length with minimal deviation has the smallest maximum value equaled to

$$\frac{1}{2} + 0.30 \tag{8}$$

of the heuristic measure values when considering tested sets of resource rectangles.

**Table 6** The resource enclosures' heuristic measure values of the level algorithm by length with minimal deviation

Set's number	Heuristic measure	Set's number	Heuristic measure
I	0.74	IV	0.78
II	0.73	V	0.80
III	0.77	VI	0.77



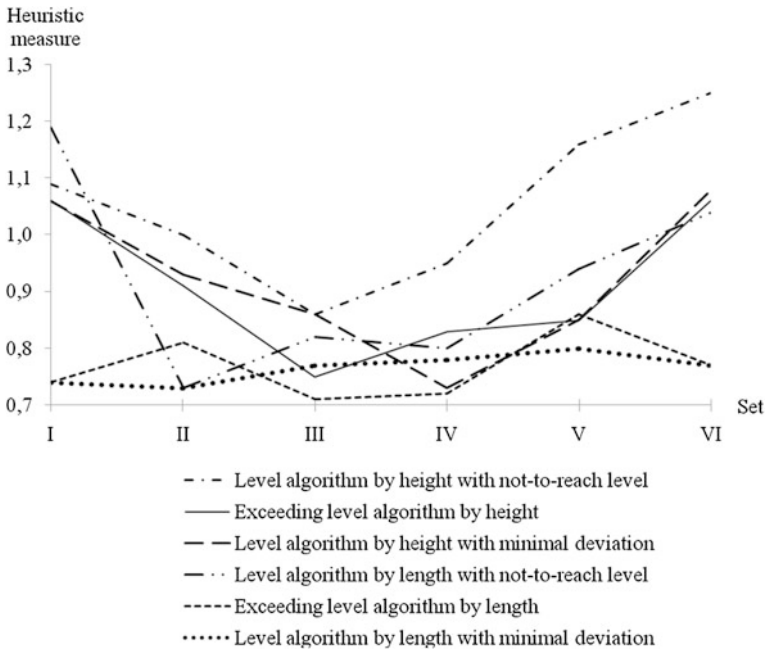


Fig. 9 The resource enclosures' heuristic measure values of level algorithms

The research allows recommending the polynomial algorithms, which were considered here, for implementation in Grid systems with centralized structure and resource co-allocation for serving sets which consist of tasks of the circular and hyperbolic quadratic type.

### 4 Conclusion

For scheduling by sets of precise form which consist of resource rectangles, which don't have the property of their dimensions monotony, in the resource rectangles environment the level algorithms by height and length were suggested. Having some sets of precise form consisted of tasks of the circular and hyperbolic quadratic type as an example, the resource enclosures' heuristic measure values were calculated. It was shown that the developed polynomial algorithms were suitable for mentioned class of sets of user's tasks in Grid systems.

**Acknowledgments** The study was performed by the grant from the Russian Science Foundation (project # 14-11-00242) in the Southern Federal University.

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