Scaling from Circuit Experiment to Real Traffic Based on Optimal Velocity Model

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Abstract The optimal velocity (OV) model was proposed to explain the physical mechanism of jam formation. The emergence of a traffic jam can be understood as a kind of dynamical phase transition. We confirmed the physical mechanism by two experiments. In this study, we investigate the relation between experimental results and observations of real traffic based on the OV model. In the OV model, the critical density at which a traffic jam occurs is determined by the OV function. The OV function is estimated from data of headway and velocity obtained by the experiments. Then, we propose a scaling rule of the OV function from the experiments to real traffic. Using this rule, we obtain critical density as a function of a single parameter. The obtained critical density is consistent with the observed values for highway traffic. From this result, we conclude that the jam formation in real traffic is explained by the same mechanism as the circuit experiments.

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1 Introduction

The optimal velocity (OV) model was proposed to explain why a traffic jam occurs [4]. The occurrence of a traffic jam is considered to be a kind of dynamical phase transition. If the car density is low, homogeneous flow, which corresponds to free flow in real traffic, is realised. If the car density exceeds a certain critical value, the homogeneous flow becomes unstable and transits to jammed flow. In order to confirm this physical mechanism of traffic jam, we carried out two circuit experiments. In the first experiment [12], we confirmed that the traffic jam occurs without bottlenecks, that is, without any causes which can be identified. The second experiment consisted of many sessions with various car density. From the experiment, we estimated the critical density [14]. This result shows that the density is the control parameter of jam formation. As a result of two experiments, the physical mechanism of traffic jam is confirmed. However, there is a criticism that circuit experiments are unrealistic situations and the results obtained by those experiments cannot be applied to real traffic.

In this study, we investigate the relation between circuit experiments and real traffic. We first determine the parameters of the OV model in the two experiments. The experimental values are different from those for real traffic, because the maximum velocities in the circuit experiments are smaller than those in real traffic. Next, we find a relation between the parameters in the circuit experiments and real traffic, and define a scaling rule for the parameters. If the relation is established, we can predict the critical density in real traffic without additional estimation of parameters. In our method, the critical density is given by a function of a single parameter. The predicted critical density is tested against observations of real traffic.

This paper is organised as follows. In Sect. 2, we briefly review the OV model. The estimation of the model parameters is shown in Sect. 3, and the scaling relation between the experiments and real traffic is shown in Sect. 4. A summary is given in Sect. 5.

2 Review of Model

The OV model is expressed by the equations of motion

$$\frac{d^2x_i}{dt^2} = a\left[V(x_{i+1} - x_i) - \frac{dx_i}{dt}\right], \tag{1}$$

where x_i is the position of the ith car. The parameter a is called sensitivity. The OV function $V(h)$ expresses the optimal velocity as a function of headway h. Typically, we adopt a hyperbolic tangent function as the OV function

$$V(h) = \alpha \tanh[\beta(h - h_0)] + v_0. \tag{2}$$

Sensitivity a and the OV function V are assumed to be common to all cars.

The OV model predicts that a homogeneous flow becomes unstable and transits to a jammed flow if

$$\left.\frac{dV(h)}{dh}\right|_{h=h_{\text{mean}}} > \frac{1}{2}a\,, \tag{3}$$

where h_{mean} is the mean headway. Then, the critical density $\rho_c = 1/h_{\text{critical}}$ can be analytically calculated from Eq. (3).

$$\rho_c = \left[\frac{1}{\beta}\cosh^{-1}\sqrt{\frac{2\alpha\beta}{a}} + h_0\right]^{-1}. \tag{4}$$

Therefore, the difference of critical density between the experiments and real traffic is reduced to the difference in the OV function.

Properties of traffic jams in the OV model are summarised as follows. When the jammed flow becomes stationary, the trajectories of all cars in the headway-velocity space are expressed by a hysteresis-like loop shown in Fig. 1 [3]. In other words, the motion of all cars becomes periodic. In most of the period, however, cars stay in the states represented by the two cusps of the loop. The lower cusp represents the state of cars inside jam clusters, and indicates the minimum headway at which cars stop. The upper cusp represents the state in which cars are running almost freely in the regions outside the jam clusters. The backward velocity of a jam cluster is given by the velocity-axis intercept of the line connecting the upper and lower cusps (Fig. 1). Here, we note that the inflection point of the hyperbolic tangent function also lies on this line.

Because the motion of all cars is periodic, each car retraces the motion of the preceding car with a certain time delay. The time delay T is equal to the time interval at which cars depart from a jam cluster one after another. Therefore, T is given by

$$T = \frac{h_{\min}}{v_{\text{back}}}, \tag{5}$$

where $h_{\min}$ is the minimum headway and v_{back} is the backward velocity of jam clusters.

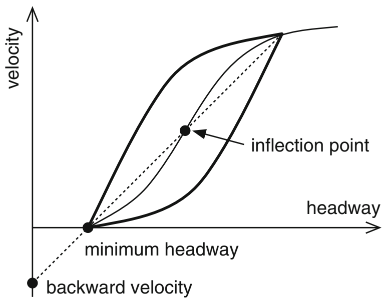

Fig. 1 Typical hysteresis-like loop. The OV function is represented by a *thin solid curve*. The *thick solid* loop represents the periodic motion of the cars. The *dashed line* connects two cusps. *Black dots* on the *line* represent the inflection point, the minimum headway and the backward velocity, respectively

It is known that a relation exists between sensitivity a and the time delay T in the framework of the OV models [1, 2, 8, 13]. The value of aT is known as $1.6 \sim 1.8$, and is insensitive to changes in the model parameters. Here, we set

$$aT = 1.8. \tag{6}$$

Then, the sensitivity is not a free parameter, and is essentially determined by the OV function through T.

3 Estimation of OV Function

In this section, we estimate the parameters in the OV functions from the experimental data. OV functions express the relation between headway and velocity. In the experiments, three types of flow, free, jammed, and stop-and-go flow are realised. Figure 2 shows relations between headway and velocity for the three types of flow. Obviously, data points cover only a part of the OV function in the cases of free and jammed flows. We can estimate the OV function in the case of stop-and-go flows.

In the estimation, we first choose five representative points to determine the OV function, and next fit a function to these points.

Two of the five points are two cusps of the loop shown in Fig. 1. The lower cusp is given by the minimum headway, which is the headway in jam clusters. To determine the minimum headway, we select data of stopped cars and average their headway. The upper cusp is found in the data sequence at the moment that stopped cars exist.

Three of the five points are determined by the distribution of data points of headway and velocity. We first obtain smooth distribution by Parzen window density estimation. In this method, we assign a Gaussian distribution for each data point and sum them over all data points. Two peaks and one saddle point of the smoothed distribution are found. Then, we can determine five representative points. Figure 3a shows the smoothed distribution and the five points. The OV function fitted to these points is obtained by the standard least square method. The estimated OV function is also shown in Fig. 3a. We observed the stop-and-go flow in four cases in the two experiments. Then, four OV functions are obtained for these cases. Figure 3b shows the OV functions for four cases.

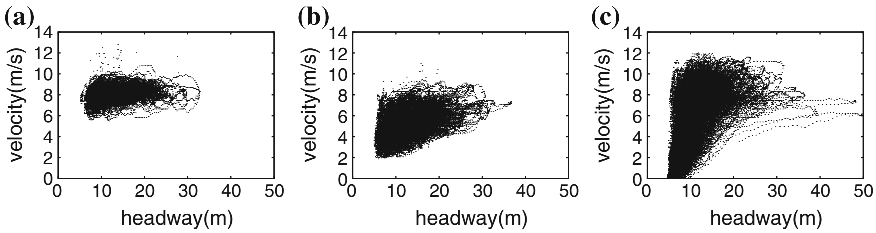

Fig. 2 Headway-velocity relations for the three types of flow: free flow (a); jammed flow (b); stop-and-go flow (c) *Dots* represent headways and velocities for all cars

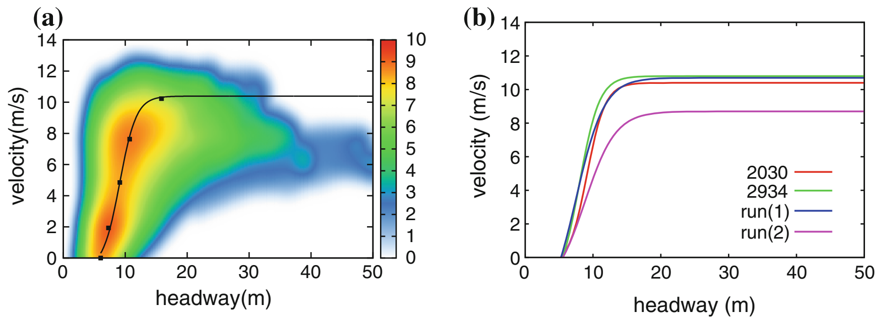

Fig. 3 OV functions. Smoothed distribution of headway and velocity. *Colours* represent the logarithm of the summation of Gaussian distributions. *Black dots* represent two peaks and a saddle point of the distribution. Two cusps are also shown by *black dots*. *Solid curve* represents the fitted OV function (**a**), OV functions are determined for four cases of stop-and-go flow. Two legends 2030 and 2934 represent session IDs in the second experiment, and run (I) and (II) represent two sessions in the first experiment (**b**)

4 Scaling Relation

In this section, we propose a scaling rule for the OV function. The OV function (Eq. 2) has four parameters, α, β, h_0, and v_0. The scaling rule should be defined by a single scaling parameter, and therefore three relations are necessary to reduce free parameters. For this purpose, we use two observational facts.

One is a relation among inflection points for experiments and real traffic. The inflection points for real traffic can be easily identified from car following experiments on real highways [11, 16]. Figure 4 shows examples observed on Chuo, Tomei, and Tokyo metropolitan highways [11]. The inflection point is considered to be the most unstable point in the OV model, and therefore is expected to exist at the place

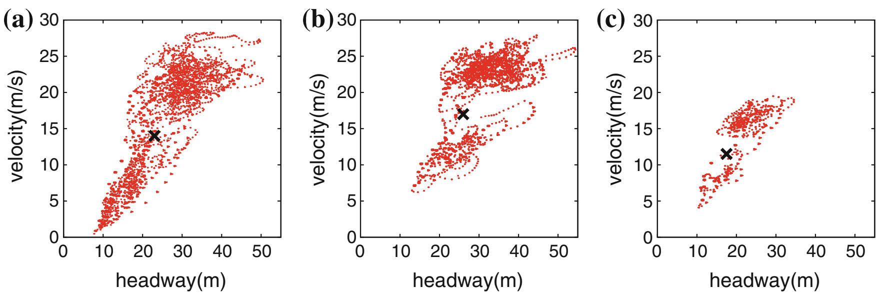

Fig. 4 An example of the car following experiment: Chuo (a); Tomei (b); Tokyo (c) *Red dots* represent the position of data points of headway and velocity. The *black cross* represents guessed inflection points

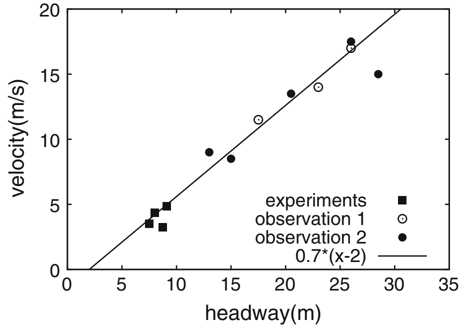

Fig. 5 *Black squares* represent inflection points from our experiments. *White and black circles* represent inflection points from Japanese highways reported in [11] and [16], respectively. *Solid line* represents the line fitted to the data

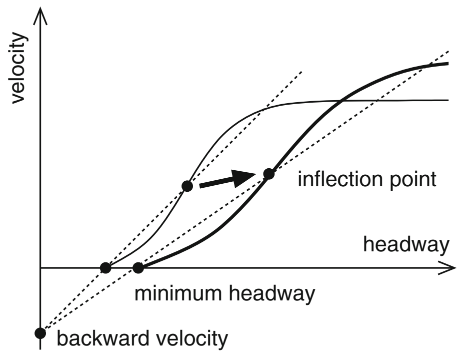

Fig. 6 Illustration of the scaling rule. *Solid curves* represent two OV functions related by the scaling rule. Each *dashed line* connects the inflection point and the point corresponding to the backward velocity

where there are no data points. The position of the inflection point for each case is estimated by eye.

Figure 5 shows inflection points observed in the experiments and on real highways. We suppose that there is a linear relation

$$v = 0.7(h - 2), \tag{7}$$

among inflection points.

The other observational fact is that the backward velocity of jam clusters is common for the experiments and real traffic. Observations on real highways show that the backward velocity is roughly 20 km/h [12, 15]. On the other hand, the backward velocity is roughly 6 m/s in our circuit experiment [14]. Obviously, both jam clusters have almost the same backward velocity.

Now, we can define a scaling rule by use of the above two facts and the property of jam in the OV model. The scaling rule is summarised as follows: (1) Inflection point lies on the line (Eq. 7), (2) Backward velocity is 6 m/s, (3) The OV function passes the point corresponding to minimum headway determined by the infection point and the backward velocity. Figure 6 shows an illustration which explains the scaling rule.

From this scaling rule, we can find relations among parameters of OV functions. Suppose two OV functions for experiments and for real traffic as

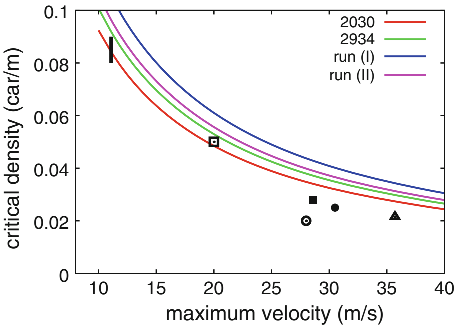

Fig. 7 Critical densities for sessions 2030, 2934, run (I), and run (II) shown in Fig. 3b. *Black solid bar* represents the range of critical density estimated in [14]. *White square*, *black square*, *white circle*, *black circle*, and *black triangle* represent the critical densities for real highways reported in [7], [5], [9], [6], and [10], respectively

$$V(h) = \alpha \tanh[\beta(h - h_0)] - v_0, \tag{8}$$
$$V'(h) = \alpha' \tanh[\beta'(h - h'_0)] - v'_0, \tag{9}$$

respectively. Then, the relations among parameters are given by

$$\alpha' = \frac{v'_0}{v_0}\alpha, \tag{10}$$
$$\beta' = \frac{h_0 - h_{\rm min}}{h'_0 - h'_{\rm min}}\beta, \tag{11}$$
$$h'_{\rm min} = \frac{v_{\rm back}}{v'_0 + v_{\rm back}} h'_0 \ . \tag{12}$$

and Eqs. (6) and (7). Any of parameters, α', β', etc., can be used as scaling parameter. For convenience, we adopt the maximum velocity $\alpha' + v'_0$ as the scaling parameter, because it corresponds to the speed limit of a road.

Then, the critical density (Eq. 4) can be expressed by a function of the maximum velocity. Because we found four OV functions as shown in Fig. 3b, we obtain four expressions for the critical density. Figure 7 shows the profiles of critical density in the four cases and observed values on real highways [5–7, 9, 10] and the experiments [14]. The estimated critical density roughly agrees with the observed values.

5 Summary

In this study, we investigated the relation between critical densities for the circuit experiments and real traffic based on the OV model. In the OV model, the difference of critical densities is essentially determined by the difference of OV functions. For the purpose, we first estimated the OV function from the data obtained by the circuit experiments. In order to find the relation between OV functions, we used two observational facts. One is the relation among inflection points of OV functions, and the other is the common backward velocity of jam clusters. These facts determined the scaling relations among the parameters of OV functions. As a result, we can express the critical density as a function of a scaling parameter. The agreement of estimated critical density with observed values is fair. Then, we can conclude that the jam formation in real traffic is explained by the same mechanism as in the circuit experiments.

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References

1. Bando, M., Hasebe, K., Nakanishi, K., Nakayama, A.: Analysis of optimal velocity model with explicit delay. Phys. Rev. E **58**(5), 5429 (1998)
2. Bando, M., Hasebe, K., Nakanishi, K., Nakayama, A.: Delay of vehicle motion in traffic dynamics. Jpn. J. Ind. Appl. Math. **17**(2), 275–294 (2000)
3. Bando, M., Hasebe, K., Nakayama, A., Shibata, A., Sugiyama, Y.: Structure stability of congestion in traffic dynamics. Jpn. J. Ind. Appl. Math. **11**(2), 203–223 (1994)
4. Bando, M., Hasebe, K., Nakayama, A., Shibata, A., Sugiyama, Y.: Dynamical model of traffic congestion and numerical simulation. Phys. Rev. E **51**(2), 1035 (1995)
5. Hall, F.L., Allen, B.L., Gunter, M.A.: Empirical analysis of freeway flow-density relationships. Trans. Res. Part A Gen. **20**(3), 197–210 (1986)
6. Kikuchi, M., Nakayama, A., Nishinari, K., Sugiyama, Y., Tadaki, S., Yukawa, S.: Long-term traffic data from Japanese expressway. In: Traffic and Granular Flow'01, pp. 257–262. Springer (2003)
7. Koshi, M., Iwasaki, M., Ohkura, I.: Some findings and an overview on vehicular flow characteristics. In: Proceedings of the 8th International Symposium on Transportation and Traffic Flow Theory, vol. 198, pp. 403–426. University of Toronto: Toronto, Ontario (1983)
8. Nakanishi, K., Itoh, K., Igarashi, Y., Bando, M.: Solvable optimal velocity models and asymptotic trajectory. Phys. Rev. E **55**(6), 6519 (1997)
9. Neubert, L., Santen, L., Schadschneider, A., Schreckenberg, M.: Single-vehicle data of highway traffic: a statistical analysis. Phys. Rev. E **60**(6), 6480 (1999)
10. Nishinari, K.: Jamology: physics of self-driven particles and toward solution of all jams. In: Distributed Autonomous Robotic Systems 8, pp. 175–184. Springer (2009)
11. Oba, T.: An experimental study on car-following behavior. Ph.D. thesis, Thesis of Master of Engineering, University of Tokyo (1988)
12. Sugiyama, Y., Fukui, M., Kikuchi, M., Hasebe, K., Nakayama, A., Nishinari, K., Tadaki, S.I., Yukawa, S.: Traffic jams without bottlenecks experimental evidence for the physical mechanism of the formation of a jam. New J. Phys. **10**(3), 033001 (2008)
13. Sugiyama, Y.B., Yamada, H.: Simple and exactly solvable model for queue dynamics. Phys. Rev. E **55**(6), 7749 (1997)
14. Tadaki, S.I., Kikuchi, M., Fukui, M., Nakayama, A., Nishinari, K., Shibata, A., Sugiyama, Y., Yosida, T., Yukawa, S.: Phase transition in traffic jam experiment on a circuit. New J. Phys. **15**(10), 103034 (2013)
15. Wardrop, J.: Experimental speed/flow relations in a single lane. In: Proceedings of the 2nd International Symposium on the Theory of Road Traffic Flow. Ed. J. Almond OECD (1965)
16. Xing, J.: A study on the bottleneck phenomenon and car-following behavior on motorways. Ph.D. thesis, Thesis of Doctor of Engineering, University of Tokyo (1992)