Chapter 14
Robust DEA Approaches to Performance Evaluation of Olive Oil Production Under Uncertainty

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Abstract In this chapter, we are concerned with performance evaluation of olive oil production using Data Envelopment Analysis (DEA) under uncertainty. In order to measure production efficiency of olive-growing farms, we first apply an imprecise DEA approach by taking into account optimistic and pessimistic perspectives on uncertainty realized in olive oil production yield. We then consider robust optimization based DEA under an uncertainty set where the random data belong. The robust DEA model enables to adjust level of conservatism that is defined by the price of robustness of the uncertainty set. The performance of imprecise and robust DEA models is illustrated via a case study of olive-growing farms located in the Aegean Region of Turkey. The numerical experiments reveal that the efficiency scores and efficiency discriminations dramatically depend on how the uncertainty is treated both in imprecise and robust DEA modeling. There exists a trade-off between the protection level and conservatism of the efficiency scores.

14.1 Introduction

Data Envelopment Analysis (DEA) is a well-established non-parametric approach for identifying relative efficiency of organizations or organizational units that are producing multiple outputs through the use of multiple inputs [13, 23]. The DEA approach has been widely applied for the performance evaluation of different

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aspects of business practices in various industries. Together with banking, healthcare, transportation and education, agriculture is one of the top-five industries that DEA has been applied [18]. In particular, DEA and related methodologies have been used for identifying relative technical efficiency of various types of agricultural establishments (for recent examples see [1, 2, 9, 15]).

The standard DEA methodology requires perfect information about data. In other words, multiple input and output parameters for each decision making unit are assumed to be known exactly. However, in various real world applications, some or all parameters involve uncertainty. Often little is known about the specific distributions of future uncertainties, and little data are available for estimating the probability distributions of these uncertainties. In many cases, it may be preferable to provide general information about the uncertainties, such as means, ranges, and directional deviations, rather than generating specific scenarios. In this case, they may be represented in the forms of ordinal or bound data.

The standard (deterministic) DEA approach was extended to Imprecise Data Envelopment Analysis (IDEA) to handle data uncertainty by Cooper et al. [11, 12]. The production efficiency of decision-making units is determined in view of such uncertain parameters that are assumed to take either optimistic or pessimistic perspectives. The reader is referred to Zhu [24, 25] and Park [19] for various applications of IDEA.

As we will discuss in more detail later, robust optimization is a technique for decision making under uncertainty that is concerned with finding the optimal solution when uncertain parameters in the problem take their worst-case values in prespecified uncertainty sets. Robust optimization was independently developed by Ben-Tal and Nemirovski [4] and Ghaoui and Lebret [16], and has experienced tremendous growth in the last decade mainly because of computational tractability and practical implementation (for example, see [5, 7, 8]).

Robust optimization has also been adopted to DEA for handling data uncertainty arising in input and output parameters. However, the robust DEA framework has not been yet widely applied in practice. Sadjadi and Omrani [20] considered robustifying uncertainty on output parameters for the performance assessment of electricity distribution companies. Shokouhi et al. [21] proposed a tractable robust approach for imprecise DEA where both input and output parameters are constrained within an uncertainty set. They applied a Monte-Carlo simulation to illustrate performance of the robust DEA model using a small example.

This chapter focuses on an agricultural performance evaluation problem under uncertainty using DEA. More precisely, the imprecise and robust DEA models are developed to measure technical efficiency of olive growing farms. To the best of our knowledge, this research is the first attempt to model olive oil production problem under uncertainty using the IDEA and robust DEA approaches. It is worthwhile to mention that imprecise DEA is applied to the olive oil production problem rather than standard (deterministic) DEA due to stochastic nature of output parameters associated with farms' olive oil yields. Uncertainty is represented as in the form of bound data varying dependently on the olive production. We apply both optimistic [11] and pessimistic [19] perspectives for data uncertainty within the imprecise DEA framework.

We also apply the robust DEA approach to find the worst-case production efficiency of olive-growing farms in view of uncertain olive oil production level varying within a pre-specified uncertainty set. A tractable robust DEA model is derived using an uncertainty set, introduced by Bertsimas and Sim [7], for random output parameters. This model enables to adjust the level of conservatism that is defined by the price of robustness of the uncertainty set.

We consider a real world case study of olive oil producing farms located in the Aegean Region of Turkey in order to illustrate performance of imprecise and robust DEA models. The production performance of those farms is measured in terms of efficiency scores under data uncertainty. We study how efficiency scores change between imprecise and robust DEA modeling. In addition to the comparison of efficiency scores in robust and imprecise DEA models, we also investigate the impact of the size of uncertainty sets and model parameters on the robust and imprecise DEA scores of smaller groups of farms via simulating the estimated olive oil production and its deviations.

The rest of this chapter is organized as follows. Section 14.2 provides an insight on imprecise DEA models. In Sect. 14.3, we present a brief introduction to robust optimization modeling of DEA and derive mathematical formulations of robust DEA models. Section 14.4 focuses on the case study and describes the data set in terms of input and output variables. Section 14.5 presents the numerical results of relative production efficiency obtained through imprecise and robust DEA approaches. Finally, Sect. 14.6 summarizes our findings.

14.2 DEA Modeling

This section is a brief introduction to deterministic and imprecise DEA modeling. We consider imprecise DEA as a benchmark approach for the olive oil production problem under uncertainty. Before formulating the imprecise DEA model, let us describe a standard DEA linear program since it is a fundamental model for both imprecise and robust DEA approaches.

14.2.1 Deterministic DEA Model

As mentioned before, DEA is used to measure relative efficiency of a decision making unit with respect to other units producing multiple outputs through the use of multiple inputs. The fundamental model (referred to the CCR DEA model) was introduced by the original work of Charnes et al. [10]. The CCR DEA model basically builds on the idea of maximizing the ratio of weighted combination of outputs to weighted combination of inputs.

Let us consider N decision making units. We assume that each decision making unit j (for $j = 1, \cdots, N$) uses M different inputs x_{ij} (for $i = 1, \cdots, M$) and produces S different outputs y_{rj} (for $r = 1, \cdots, S$). Let μ_r and w_i denote the weights (or decision variables) corresponding to output r and input i, respectively. The CCR DEA model calculates the efficiency score for the decision making unit o under consideration by solving the following linear problem:

$$\begin{aligned}\max\ & \sum_{r=1}^{S} \mu_r y_{ro} \\ \text{s.t.}\ & \sum_{i=1}^{M} w_i x_{io} = 1 \\ & \sum_{r=1}^{S} \mu_r y_{rj} - \sum_{i=1}^{M} w_i x_{ij} \leq 0,\ j = 1,\cdots,N \\ & \mu_r, w_i \geq 0, \qquad i = 1,\cdots,M,\ \ r = 1,\cdots,S.\end{aligned} \tag{14.1}$$

Notice that the standard DEA model (14.1) is constructed by using exact (deterministic) data values for input and output parameters for each decision making unit. However, it is not always possible to have perfect information about the data related to input and/or output values of decision making units. The gathered data may involve inaccuracy due to estimation error, and even data uncertainty may exist due to the nature of the underlying problem.

For these cases, Cooper et al. [11] first introduced the concept of imprecise data into the DEA framework. The term "imprecise data" reflects the situation where some of the input and output data are only known to lie within bounded intervals [14]. Thus, Imprecise Data Envelopment Analysis (IDEA) permits the incorporation of bounded or ranked data into the DEA models.

14.2.2 Imprecise DEA Model

Let D_r^+ and D_i^- denote sets for the input and output parameters including both imprecise and exact data. The values of y_r and/or x_i are not known exactly, but need to be determined in sets D_r^+ and D_i^-. Then the IDEA model based on the CCR DEA formulation is stated as follows.

$$\begin{aligned}\max\ & \sum_{r=1}^{S} \mu_r y_{ro} \\ \text{s.t.}\ & \sum_{i=1}^{M} w_i x_{io} = 1 \\ & \sum_{r=1}^{S} \mu_r y_{rj} - \sum_{i=1}^{M} w_i x_{ij} \leq 0,\ j = 1,\cdots,N \\ & y_r = (y_{rj}) \in D_r^+ \qquad r = 1,\cdots,S \\ & x_i = (x_{ij}) \in D_i^- \qquad i = 1,\cdots,M \\ & \mu_r, w_i \geq 0, \qquad i = 1,\cdots,M,\ \ r = 1,\cdots,S,\end{aligned} \tag{14.2}$$

where the decision variables μ_r and w_i represent weights corresponding to output and input data as defined before.

For the deterministic DEA model, the exact values of y_r and/or x_i taken from sets D_r^+ and D_i^- are simply substituted in (14.2). This leads to a linear program as stated by the standard DEA model (14.1). However, the IDEA model (14.2) becomes a nonlinear optimization problem. In order to solve the nonlinear programming problem, Cooper et al. [11] and Kim et al. [17] converted the nonlinear model into a linear program via scale transformations and variable alterations. Cooper et al. [12] then applied the IDEA model (where an imprecise output parameter for all units is defined in the form of intervals) to measure performance efficiency of a mobile telecommunication company in Korea. Zhu [25] also considered the IDEA model for the same telecommunication problem, but solved it as a standard DEA problem. He showed that the same efficiency scores as in Cooper et al. [12] can be obtained by simply substituting the output parameter of the unit under evaluation to its upper bound while fixing the output parameters of the remaining units to their lower bounds of the corresponding intervals. Therefore, the unit under evaluation is assumed to perform at its best (as fixed at the upper bound of the corresponding interval of the output parameters) while the other units are assumed to perform at the worst-case (as fixed at the lower bounds of the interval of output parameters).

Park [19] proved that the IDEA formulation (introduced by Cooper et al. [11]) in fact produces an "*optimistic*" strategy since the efficient score is evaluated at the best scenario (selected within pre-specified imprecise data interval) available for the underlying decision making unit. Therefore, the objective function for the IDEA model to achieve the optimistic strategy can be formulated as follows:

$$\max_{y_r \in D_r^+, x_i \in D_i^-} \max_{\mu, w} \sum_{r=1}^{S} \mu_r y_{ro}$$

Similarly, a "*pessimistic*" strategy within the IDEA context is achieved by the following min-max objective function

$$\min_{y_r \in D_r^+, x_i \in D_i^-} \max_{\mu, w} \sum_{r=1}^{S} \mu_r y_{ro}$$

The IDEA model with the min-max objective transforms the bounded data to exact data so that the model seeks to evaluate the unit under evaluation in the worst scenario possible. In other words, the unit under consideration is evaluated by the worst scenario possible (specified at the lower bound of the interval of the imprecise parameter) while the other units perform at their best scenario (specified at the upper bound of the interval of the imprecise parameter). Therefore, the solution of the-min-max optimization problem provides a conservative (worst-case) efficient score for the underlying unit.

If any unit is determined as efficient by both optimistic and pessimistic perspectives within IDEA approach, then it is declared as *"perfectly efficient"*. On the other hand, it is called *"potentially efficient"* when it is efficient under the optimistic strategy and inefficient under the pessimistic strategy [19].

Following the transformations introduced by Soyster [22], the IDEA model in view of the pessimistic perspective is formulated in a general form as follows;

$$
\begin{aligned}
\max\ & \textstyle\sum_{r=1}^{S} \mu_r\, inf\{y_{ro} \mid y_r \in D_r^+\} \\
s.t.\ & \sum_{i=1}^{M} w_i\, sup\left\{x_{io} \mid x_i \in D_i^-\right\} = 1 \\
& \sum_{r=1}^{S} \mu_r\, inf\left\{y_{ro} \mid y_r \in D_r^+\right\} - \sum_{i=1}^{M} w_i\, sup\left\{x_{io} \mid x_i \in D_i^-\right\} \le 0, \\
& \sum_{r=1}^{S} \mu_r\, inf\left\{y_{rj} \mid y_r \in D_r^+\right\} - \sum_{i=1}^{M} w_i\, sup\left\{x_{ij} \mid x_i \in D_i^-\right\} \le 0,\ j = 1,\cdots,N,\ j \ne o \\
& \mu_r, w_i \ge 0, \qquad i = 1,\cdots,M,\ \ r = 1,\cdots,S
\end{aligned}
\tag{14.3}
$$

where the 'sup' and 'inf' are replaced by 'max' and 'min', respectively, when D_r^+ and D_i^- are closed and bounded sets. The reader is referred to Park [19] for further information on the generalized linear program.

14.3 Robust DEA Approach

As mentioned in the introduction, the robust optimization approach to solving an optimization problem with uncertain data involves specifying appropriate uncertainty sets for the uncertain coefficients, and finding a solution that guarantees feasibility even if the uncertain coefficients take their worst-case values within the uncertainty sets. A brief introduction to the main ideas of robust linear optimization (the type of problem with which we are dealing in this chapter) is provided next; for further information, the reader is referred to Ben-Tal and Nemirovski [4, 5] as well as Ben-Tal et al. [6]. We then derive the robust DEA model that is to be applied for the case study of olive oil production problem described in Sect. 14.4.

14.3.1 Robust Linear Optimization

Consider, for example, a linear program

$$
\max\left\{\mathbf{c}'\mathbf{x} \mid \sum_{j=1}^{n} \tilde{a}_j x_j \le b,\ \ \mathbf{x} \in V\right\}
$$

where $\mathbf{c} \in R^{n\times 1}$, and V consists of all constraints whose parameters are certain. $\mathbf{x} \in R^{n\times 1}$ represents a vector of decision variables and $\tilde{\mathbf{a}} \in R^{n\times 1}$ is a vector of uncertain parameters. Let $\mathscr{U}_a$ denote an uncertainty set specified by the modeler. Robust optimization solves an optimization problem assuming that the uncertain data belong to an uncertainty set, $\tilde{\mathbf{a}} \in \mathscr{U}_a$. It looks for an optimal solution that remains feasible if the uncertainties take any values within that uncertainty set. This reformulation of the problem is referred to as the robust counterpart of the original optimization problem. In some special cases, the robust counterpart of the original problem involves the worst-case outcome of the stochastic data within the uncertainty set, and is a tractable optimization problem with no random parameters.

The robust counterpart of the underlying linear program is formulated as

$$\max_{\mathbf{x}} \min_{\tilde{\mathbf{a}}} \left\{ \mathbf{c}'\mathbf{x} \mid \sum_{j=1}^{n} \tilde{a}_j x_j \leq b, \;\; \tilde{\mathbf{a}} \in \mathscr{U}_a, \;\; \mathbf{x} \in V \right\}.$$

The size of the uncertainty set is often related to guarantees on the probability that the constraint involving uncertain coefficients will not be violated. There is a trade-off between the amount of protection against uncertainty that is desired and optimality—the smaller the probability that the constraint will be violated, the more the modeler gives up in terms of optimality of the robust solution relative to the solution to the original optimization problem.

Ellipsoidal, box and polyhedral are the most commonly used uncertainty sets, but more recently, asymmetric uncertainty sets have been used as well in order to capture the probability distribution characteristics of the uncertainties better. In practice, the shape is selected to reflect the modeler's knowledge of the probability distributions of the uncertain parameters, while at the same time making the robust counterpart problem efficiently solvable. Further results on probability bounds related to the size and the shape of uncertainty sets can be found, for example, in Bertsimas and Sim [7] and Bertsimas et al. [8].

For the robust DEA model, we apply an uncertainty set introduced by Bertsimas and Sim [7]. A brief description to this uncertainty set and its adaptation to the DEA modeling is presented next. Let's consider the constraint $\sum_{j=1}^{n} \tilde{a}_j x_j \leq b$ where the uncertain parameter $\tilde{a}_j$ will be robustified. Assume that each entry $\tilde{a}_j$ is modeled by a symmetric and bounded random variable that takes values in $[a_j - \hat{a}_j,\, a_j + \hat{a}_j]$. The random variable $\eta_j = \frac{\tilde{a}_j - \hat{a}_j}{\hat{a}_j}$ which obeys an unknown but symmetric distribution and takes values from an interval $[-1,1]$. Then the robust counterpart of the linear constraint is derived by a set of the following constraints

$$\left\{ \sum_{j=1}^{n} \hat{a}_j x_j + z\Gamma + \sum_{j=1}^{n} p_j \leq b, z + p_j \geq \hat{a}_j t_j, \right.$$
$$\left. -t_j \leq x_j \leq t_j, t_j \geq 0, p_j \geq 0, z \geq 0, \; j = 1, \cdots, n \right\}$$

where the parameter Γ adjusts the robustness of the model against the level of conservatism of the solution. It takes values in the interval $[0,n]$, not necessarily integer. It is crucial to decide the sufficient level where the some parameters are protected to get their worst-case values. When Γ is selected as 0, there is no protection against uncertainty (i.e. uncertainty is ignored). If $\Gamma = n$, then the constraint is completely protected against uncertainty.

14.3.2 Robust DEA Model

Assume that uncertain output parameters, $\tilde{y}_{rj}$ for $r = 1, \cdots, S$, and $j = 1, \cdots, N$, belong to an uncertainty set $\mathscr{U}_y$. The robust counterpart of the DEA model can be formulated as follows;

$$
\begin{aligned}
\max_{\mu, w} \; & \min_{\tilde{y}_{ro} \in \mathscr{U}_y} \sum_{r=1}^{S} \mu_r \tilde{y}_{ro} \\
\text{s.t.} \; & \sum_{i=1}^{M} w_i x_{io} = 1 \\
& \min_{\tilde{y}_{rj} \in \mathscr{U}_y} \sum_{r=1}^{S} \mu_r \tilde{y}_{rj} - \sum_{i=1}^{M} w_i x_{ij} \leq 0, \; j = 1, \cdots, N, \;\; j \neq o \\
& \mu_r \geq 0, \;\; w_i \geq 0, \qquad\qquad i = 1, \cdots, M, \;\; r = 1, \cdots, S,
\end{aligned}
\tag{14.3}
$$

Let θ be a free variable representing the inner minimization problem in the objective function. Then the objective function can be transformed into a constraint

$$
\min_{\tilde{y}_{ro} \in \mathscr{U}_y} \sum_{r=1}^{S} \mu_r \tilde{y}_{ro} - \theta \geq 0.
$$

Next, to derive the robust counterpart of the DEA model (so-called as the robust DEA model) using the uncertainty set introduced by Bertsimas and Sim [7], both inner minimization problems in the constraints are first solved using dual linear programs. Then the corresponding robust counterparts are reinjected into the corresponding constraints. The robust DEA model can be stated as

$$
\begin{aligned}
\max \; & \theta \\
\text{s.t.} \; & \sum_{i=1}^{M} w_i x_{io} = 1 \\
& \sum_{r=1}^{S} \mu_r \hat{y}_{ro} - \theta - z_o \Gamma_o - \sum_{r=1}^{S} p_{ro} \geq 0 \\
& \sum_{i=1}^{M} w_i x_{ij} - \sum_{r=1}^{S} \mu_r \hat{y}_{rj} - \theta - z_j \Gamma_j - \sum_{r=1}^{S} p_{rj} \geq 0, \; j = 1, \cdots, N, \;\; j \neq o \\
& z_j + p_{rj} \geq \hat{y}_{rj} t_r, \;\; p_{rj} \geq 0, \;\; z_j \geq 0 \qquad j = 1, \cdots, N, \; r = 1, \cdots, S \\
& -t_r \leq \mu_r \leq t_r, \;\; t_r \geq 0, \qquad r = 1, \cdots, S \\
& \mu_r \geq 0, \;\; w_i \geq 0, \qquad i = 1, \cdots, M, \; r = 1, \cdots, S.
\end{aligned}
\tag{14.4}
$$

where Γ_o and Γ_j represent are the price of robustness of the uncertainty sets defined for the uncertain parameters in the objective function and the constraints, respectively. As explained in detail by Bertsimas and Sim [7] and Sadjadi and Omrani [20], the sufficient level for Γ_j parameter is determined as $\Gamma_j = 1 + \phi^{(-1)}(1 - e_j)\sqrt{N}$ where e_j represents the probability that the constraint j is to be violated, ϕ is the cumulative distribution of standard Gaussian variable. It is also worthwhile to note that the robust DEA model has more variables and constraints than the IDEA model has. On the other hand, it still remains as a tractable linear program.

14.4 Case Study: Performance of Olive Oil Growing Farms

In this study, we consider a real case of olive oil production problem to apply the imprecise and robust DEA models introduced in Sects. 14.2 and 14.3. A sample that consists of 89 olive oil growing farms (labeled as 1–89) in the Aegean region of Turkey is selected to perform an efficiency analysis. The farms are located in the same agricultural area in Izmir; therefore, possible effects of geographical and weather conditions on the oil production are eliminated.

The data were gathered from Taris Olive Oil company, which was established in 2001 by the Union of Taris Olive and Olive Oil Cooperatives. Currently, 33 cooperatives are affiliated with the Union. The company is responsible for the trading of olives cultivated by the olive producing farms located in the Aegean region of Turkey. According to International Olive Council, Turkey produces 4.9 % of world production of olive oil and takes the sixth place in the world olive oil production league (http://www.internationaloliveoil.org/).

Taris has recently started gathering data in order to keep record of the suppliers' performance. The farmers are requested to fill a questionnaire about specifications of farms as well as their performance during the year. For the computational experiments, we use the raw data (relevant to specifications of the farms) that were collected in 2011. Thus, the data set basically reflects the performance of olive oil (not table olives) producers in 2010.

Following to most studies in the literature [3], we also consider total olive land utilized by each farm, cultivation cost, labor as inputs to the DEA models. *Land* is measured by decares (1 *decare* $= 1000\,\mathrm{m}^2$). *Cultivation cost* represents the aggregated monetary value in Turkish Liras for cultivation and miscellaneous costs (such as fertilizer, pesticides or fuel costs) spent by the farm in order to operate during a year. We determine *labor* as the number of workers employed to process the harvesting rather than the monetary terms. Apart from land, cost and labor, we also consider *number of olive trees* as the fourth input parameter [2]. Although Land is an input factor, because of the different densities of trees in the given land areas, we also consider number of trees as an input parameter.

As an output parameter, *olive oil yield* is chosen. The olive oil production for each farm depends on the total olive production and is assumed to be uncertain. The other factors such as weather and age of olive trees that might affect the olive production are not taken into account in this study. Since there is no exact measurement for the olive oil production, we consider an expert knowledge in designing the output parameter as the projection of olive production of each farm. Recall that Taris is interested in the olive oil yield rather than the olive production itself.

In current practice, nominal value of olive oil yield for each farm is estimated as 20 % of the olive production. For instance, for a farm that is producing 15,000 kg olives in a year, the nominal olive oil production is expected to be 3000 kg. However, as confirmed by the experts, this value in reality fluctuates within the range of 25 % of the oil production. In this case, the annual olive oil yield varies between 2250 and 3750 for the farm with 3000 kg of nominal olive oil production. Therefore, we develop the imprecise and robust DEA models to evaluate relative production performance of those olive-growing farms with multiple (deterministic) input parameters (land, cost, labour and number of olive trees) and uncertain output parameter (olive oil production). The deterministic DEA model is used as a benchmark to compare relative performance of those farms using the imprecise and robust DEA models. The DEA models (described in Sects. 14.2 and 14.3) are implemented using General Algebraic Modeling System (GAMS) and solved by a linear programming algorithm.

14.5 Computational Results

We conducted a series of computational experiments to investigate the performance of the imprecise and robust DEA models. Specifically, the experiments aim to answer the following questions:

- How do deterministic, imprecise and robust DEA models perform for the olive oil production problem?
- What are the impact of size and shape of the symmetric uncertainty sets on the robust relative efficiency of olive growing farms?
- How do the robust and imprecise DEA models respond to the changes in size of uncertainty ranges and sample sizes?

In order to measure the relative performance of olive growing farms, we apply the DEA approach. The efficiency scores of farms are computed by solving the linear programs corresponding to the DEA models presented in Sects. 14.2 and 14.3. There is no consensus in the farm efficiency literature on deciding the returns-to-scale assumption. Since all of our farms are located in a small specific region and operate in a similar scale size, constant returns-to-scale is assumed. In summary, we consider the following DEA models;

- *Nominal (deterministic) model* applies the standard DEA approach and uses certain data values of input and output variables. The olive oil production of each farm is calculated as 20 % of the actual olive production.
- *Imprecise DEA approach* considers optimistic and pessimistic views by fixing the upper and lower bounds of the corresponding intervals for the output values in the IDEA model.
- *The robust DEA models* require another input parameter that measures the level of robustness (the price of robustness to use the term from Bertsimas and Sim [7]). The level of robustness varies from 0 to 1.0 and the corresponding DEA models are labelled as $R(0.0), R(0.1), \cdots, R(1.0)$, respectively. In particular, the robust DEA model at zero price of robustness, $R(0.0)$, corresponds to the deterministic (nominal) DEA model.

As mentioned in Sect. 14.3, the robust DEA model requires to specify parameter Γ. In the olive oil production problem, there exists only one uncertain data point at each constraint. The protection level against uncertainty Γ is defined to vary within interval [0,1]. Since Γ is not necessarily integer, we test the robust DEA model at different protection levels by fixing it at any value within the range [0, 1]. The state $\Gamma = 0$ represents no protection for uncertainty, which corresponds to the nominal model. On the other hand, $\Gamma = 1$ describes a full protection against uncertainty.

14.5.1 Performance Comparison of Imprecise and Robust DEA Approaches

We are first concerned with performance comparisons of all DEA models in terms of relative production efficiency of the farms. Table 14.1 presents the optimal efficiency scores obtained by the deterministic DEA and imprecise DEA approaches in view of optimistic and pessimistic perspectives. In Table 14.2, the results of the robust DEA approach with various level of robustness (at 0.2, 0.4, 0.5, 0.6, 0.8 and 1.0) are summarized in terms of worst-case efficiency scores of farms. The farms taking place on the efficient frontier possess an efficiency score of 1.0 (and highlighted in bold). Notice that the inefficient farms have efficiency scores less than 1.0. Table 14.3 summarizes the statistics of efficiency scores obtained by deterministic, imprecise and robust DEA models in terms of average, minimum and maximum as well as the number of efficient and inefficient farms.

From the results presented in Tables 14.1, 14.2 and 14.3, we can make the following observations;

- The DEA models show different characteristics in terms of the number of efficient farms. The optimistic (pessimistic) strategy obtained by IDEA provides 32 farms efficient while the deterministic DEA produces only 8 efficient farms. On the other hand, no farm is declared as efficient according to the robust DEA strategy. Only farm 42 (see Table 14.2) is determined as perfectly efficient since it is declared as efficient by all nominal, optimistic and pessimistic imprecise DEA models.
- The IDEA model with optimistic approach seems the least conservative way of evaluation in the presence of uncertain data in the form of bounds. The efficiency scores obtained by the optimistic model are consistently greater than or equal to the efficiency scores produced by the nominal DEA and pessimistic IDEA models. As mentioned in Sect. 14.2, the IDEA in view of an optimistic perspective assumes that a farm under consideration performs at its best production whereas the rest of farms perform at their worst production efficiency. The highest average efficiency score is achieved by the optimistic IDEA as 74.4 %. As expected, it outperforms the deterministic DEA strategy that, in average, provides 52.4 % of overall scores.
- On the other hand, the IDEA model with pessimistic perspective is seen the most conservative way of finding efficiency scores of olive growing farms as it produces the lowest efficiency scores comparing with the nominal DEA and the

Table 14.1: Efficiency scores obtained by the deterministic and imprecise DEA models

Farms	Deterministic DEA	IDEA Optimistic	IDEA Pessimistic	Farms	Deterministic DEA	IDEA Optimistic	IDEA Pessimistic
1	**1.00**	**1.00**	0.63	46	0.25	0.41	0.15
2	0.22	0.36	0.13	47	0.27	0.44	0.16
3	0.42	0.70	0.25	48	0.27	0.45	0.16
4	0.07	0.11	0.04	49	0.16	0.27	0.10
5	**1.00**	**1.00**	0.77	50	0.26	0.43	0.16
6	0.34	0.57	0.20	51	0.21	0.35	0.13
7	0.45	0.75	0.27	52	0.48	0.80	0.29
8	0.20	0.34	0.12	53	0.38	0.64	0.23
9	0.24	0.41	0.15	54	0.35	0.58	0.21
10	0.10	0.16	0.06	55	0.59	0.99	0.36
11	0.50	0.83	0.30	56	0.21	0.36	0.13
12	0.56	0.93	0.33	57	0.79	**1.00**	0.48
13	0.52	0.87	0.31	58	0.37	0.62	0.22
14	0.34	0.57	0.21	59	0.42	0.70	0.25
15	0.12	0.20	0.07	60	0.66	**1.00**	0.39
16	**1.00**	**1.00**	0.91	61	0.37	0.62	0.22
17	0.72	**1.00**	0.43	62	0.38	0.64	0.23
18	0.67	**1.00**	0.40	63	0.47	0.79	0.28
19	**1.00**	**1.00**	0.99	64	0.90	**1.00**	0.54
20	0.73	**1.00**	0.44	65	0.76	**1.00**	0.46
21	0.65	**1.00**	0.39	66	0.50	0.83	0.30
22	0.67	**1.00**	0.40	67	0.71	**1.00**	0.43
23	0.50	0.83	0.30	68	0.58	0.97	0.35
24	**1.00**	**1.00**	0.75	69	0.81	**1.00**	0.49
25	0.33	0.54	0.20	70	**1.00**	**1.00**	0.85
26	0.56	0.94	0.34	71	0.76	**1.00**	0.46
27	0.97	**1.00**	0.58	72	0.51	0.86	0.31
28	0.10	0.16	0.06	73	0.59	0.98	0.35
29	0.58	0.96	0.35	74	0.75	**1.00**	0.45
30	0.74	**1.00**	0.45	75	0.55	0.91	0.33
31	0.19	0.31	0.11	76	0.31	0.52	0.19
32	0.34	0.56	0.20	77	0.37	0.62	0.22
33	0.99	**1.00**	0.59	78	0.95	**1.00**	0.57
34	**1.00**	**1.00**	0.82	79	0.44	0.74	0.27
35	0.19	0.32	0.11	80	0.31	0.51	0.18
36	0.19	0.31	0.11	81	0.66	**1.00**	0.40
37	0.71	**1.00**	0.43	82	0.82	**1.00**	0.49
38	0.08	0.13	0.05	83	0.50	0.83	0.30
39	0.42	0.69	0.25	84	0.30	0.49	0.18
40	0.56	0.93	0.34	85	0.58	0.97	0.35
41	0.51	0.86	0.31	86	0.47	0.79	0.28
42	**1.00**	**1.00**	**1.00**	87	0.80	**1.00**	0.48
43	0.14	0.23	0.08	88	0.60	**1.00**	0.36
44	0.64	**1.00**	0.38	89	0.69	**1.00**	0.42
45	0.32	0.53	0.19				

Table 14.2: Efficiency scores obtained by the robust DEA model at various level of robustness

	Robust DEA								Robust DEA						
Farms	R(0.0)	R(0.2)	R(0.4)	R(0.5)	R(0.6)	R(0.8)	R(1.0)	Farms	R(0.0)	R(0.2)	R(0.4)	R(0.5)	R(0.6)	R(0.8)	R(1.0)
1	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	46	0.25	0.22	0.20	0.19	0.18	0.17	0.15
2	0.22	0.20	0.18	0.17	0.16	0.14	0.13	47	0.27	0.24	0.22	0.21	0.20	0.18	0.16
3	0.42	0.38	0.34	0.33	0.31	0.28	0.25	48	0.27	0.24	0.22	0.21	0.20	0.18	0.16
4	0.07	0.06	0.06	0.05	0.05	0.05	0.04	49	0.16	0.15	0.13	0.13	0.12	0.11	0.10
5	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	50	0.26	0.23	0.21	0.20	0.19	0.17	0.16
6	0.34	0.31	0.28	0.27	0.25	0.23	0.20	51	0.21	0.19	0.17	0.16	0.16	0.14	0.13
7	0.45	0.41	0.37	0.35	0.33	0.30	0.27	52	0.48	0.44	0.40	0.38	0.36	0.32	0.29
8	0.20	0.19	0.17	0.16	0.15	0.14	0.12	53	0.38	0.35	0.31	0.30	0.28	0.26	0.23
9	0.24	0.22	0.20	0.19	0.18	0.16	0.15	54	0.35	0.32	0.29	0.27	0.26	0.23	0.21
10	0.10	0.09	0.08	0.07	0.07	0.06	0.06	55	0.59	0.54	0.48	0.46	0.44	0.39	0.36
11	0.50	0.45	0.41	0.39	0.37	0.33	0.30	56	0.21	0.19	0.18	0.17	0.16	0.14	0.13
12	0.56	0.50	0.46	0.43	0.41	0.37	0.33	57	0.79	0.72	0.65	0.62	0.59	0.53	0.48
13	0.52	0.47	0.43	0.41	0.39	0.35	0.31	58	0.37	0.34	0.31	0.29	0.28	0.25	0.22
14	0.34	0.31	0.28	0.27	0.25	0.23	0.21	59	0.42	0.38	0.34	0.33	0.31	0.28	0.25
15	0.12	0.11	0.10	0.09	0.09	0.08	0.07	60	0.66	0.59	0.54	0.51	0.49	0.44	0.39
16	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	61	0.37	0.34	0.31	0.29	0.28	0.25	0.22
17	0.72	0.65	0.59	0.56	0.53	0.48	0.43	62	0.38	0.35	0.31	0.30	0.28	0.26	0.23
18	0.67	0.61	0.55	0.52	0.50	0.45	0.40	63	0.47	0.43	0.39	0.37	0.35	0.31	0.28
19	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	64	0.90	0.82	0.74	0.70	0.67	0.60	0.54
20	0.73	0.66	0.60	0.57	0.54	0.49	0.44	65	0.76	0.69	0.63	0.59	0.56	0.51	0.46
21	0.65	0.59	0.53	0.51	0.48	0.43	0.39	66	0.50	0.45	0.41	0.39	0.37	0.33	0.30
22	0.67	0.61	0.55	0.52	0.50	0.45	0.40	67	0.71	0.64	0.58	0.55	0.53	0.47	0.43
23	0.50	0.45	0.41	0.39	0.37	0.33	0.30	68	0.58	0.52	0.47	0.45	0.43	0.39	0.35
24	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	69	0.81	0.73	0.66	0.63	0.60	0.54	0.49
25	0.33	0.29	0.27	0.25	0.24	0.22	0.20	70	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60
26	0.56	0.51	0.46	0.44	0.42	0.38	0.34	71	0.76	0.69	0.62	0.59	0.56	0.51	0.46
27	0.97	0.87	0.79	0.75	0.71	0.64	0.58	72	0.51	0.47	0.42	0.40	0.38	0.34	0.31
28	0.10	0.09	0.08	0.07	0.07	0.06	0.06	73	0.59	0.53	0.48	0.46	0.44	0.39	0.35
29	0.58	0.52	0.47	0.45	0.43	0.39	0.35	74	0.75	0.68	0.61	0.58	0.55	0.50	0.45

Table 14.2: (continued)

	Robust DEA								Robust DEA						
Farms	R(0.0)	R(0.2)	R(0.4)	R(0.5)	R(0.6)	R(0.8)	R(1.0)	Farms	R(0.0)	R(0.2)	R(0.4)	R(0.5)	R(0.6)	R(0.8)	R(1.0)
30	0.74	0.67	0.61	0.58	0.55	0.50	0.45	75	0.55	0.49	0.45	0.42	0.40	0.36	0.33
31	0.19	0.17	0.15	0.14	0.14	0.12	0.11	76	0.31	0.28	0.26	0.25	0.23	0.21	0.19
32	0.34	0.31	0.28	0.26	0.25	0.23	0.20	77	0.37	0.34	0.30	0.29	0.27	0.25	0.22
33	0.99	0.90	0.81	0.77	0.73	0.66	0.59	78	0.95	0.86	0.78	0.74	0.70	0.63	0.57
34	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	79	0.44	0.40	0.36	0.34	0.33	0.30	0.27
35	0.19	0.17	0.15	0.15	0.14	0.13	0.11	80	0.31	0.28	0.25	0.24	0.23	0.20	0.18
36	0.19	0.17	0.15	0.14	0.14	0.12	0.11	81	0.66	0.60	0.54	0.51	0.49	0.44	0.40
37	0.71	0.65	0.59	0.56	0.53	0.48	0.43	82	0.82	0.74	0.67	0.64	0.61	0.55	0.49
38	0.08	0.07	0.07	0.06	0.06	0.05	0.05	83	0.50	0.45	0.41	0.39	0.37	0.33	0.30
39	0.42	0.38	0.34	0.32	0.31	0.28	0.25	84	0.30	0.27	0.24	0.23	0.22	0.20	0.18
40	0.56	0.51	0.46	0.44	0.41	0.37	0.34	85	0.58	0.53	0.48	0.45	0.43	0.39	0.35
41	0.51	0.46	0.42	0.40	0.38	0.34	0.31	86	0.47	0.43	0.39	0.37	0.35	0.32	0.28
42	**1.00**	0.91	0.82	0.78	0.74	0.67	0.60	87	0.80	0.72	0.65	0.62	0.59	0.53	0.48
43	0.14	0.12	0.11	0.11	0.10	0.09	0.08	88	0.60	0.55	0.49	0.47	0.45	0.40	0.36
44	0.64	0.58	0.52	0.50	0.47	0.42	0.38	89	0.69	0.63	0.57	0.54	0.51	0.46	0.42
45	0.32	0.29	0.26	0.25	0.23	0.21	0.19								

optimistic IDEA models. This case also confirms the findings in [19]. The average efficiency score is 33.6 % for the pessimistic model and 88 farms are found to be inefficient. The discrimination of efficiency scores exhibits a noticeable change between optimistic to pessimistic modeling.

- Under the full protection against uncertainty, the average efficiency score for the farms is obtained as 31.5 %. This is slightly lower than the average score for the pessimistic IDEA model, which yields 33.6 % average efficiency. Note that 81 out of 89 farms have protected their efficiency scores at the same level with the pessimistic IDEA model. On the other hand, in case of no protection against uncertainty, not surprisingly, the model produces the same efficiency scores with the nominal (deterministic) model (an average of 52.4 %). This verifies the efficiency scores obtained by the deterministic model.

Table 14.3: Statistics of efficiency scores

	Deterministic	Imprecise DEA		Robust DEA					
	DEA	Optimistic	Pessimistic	R(0.2)	R(0.4)	R(0.5)	R(0.6)	R(0.8)	R(1.0)
Efficiency scores (%)									
Average	52.4	74.4	33.6	47.4	42.9	40.8	38.8	35.0	31.5
Min	6.8	11.3	4.1	6.1	5.5	5.3	5.0	4.5	4.1
Max	100	100	100	90.5	81.8	77.8	73.9	66.7	60.0
Number of efficient and inefficient farms									
Efficient	8	32	1	0	0	0	0	0	0
Inefficient	81	57	88	89	89	89	89	89	89

- No farm is reported as efficient by the robust DEA approach. For instance, the robust DEA model with 50 % protection against the uncertainty produces the maximum efficiency score as 77.8 % and the average efficiency score is 40.8 %. Using this reference efficiency score obtained by robust DEA model (at 0.5 price of robustness for all constraints), we can compare the relative production performance of the DEA models. We observe that
  - The efficiency scores of all farms obtained by the robust DEA ($E_{R(0.50)}$) are always lower than those scores achieved by the nominal model (E_N) and the robust DEA model with no protection ($E_{R(0.0)}$).
  - Their efficiency scores ($E_{R(0.50)}$) are persistently larger than those achieved by the pessimistic imprecise DEA (E_{pes}) and the robust DEA model with full protection against uncertainty ($E_{R(1.0)}$).
  - On the other hand, the imprecise DEA scores under the optimistic view (E_{opt}) are grater than the scores of all other DEA approaches.

As a result, we can state the following relationship between the efficiency scores of all farms obtained by various DEA approaches as

$$E_{R(1.0)} \leq E_{pes} \leq E_{R(0.50)} \leq E_N = E_{R(0.0)} \leq E_{opt}.$$

- Finally, we can comment on the impact of level of conservatism on the average efficiency scores obtained by the robust DEA models at various price of robustness. As the level of conservatism increases from no-protection to full-protection against uncertainty (i.e. varying price of robustness within [0, 1]), the average efficiency scores for the sample decline from 52.4 to 31.5 %. Similarly, the lowest (highest) average efficiency scores decrease from 6.8 (100) to 4.1 % (60 %). This basically shows a trade-off between the level conservatism and the production efficiency level that a decision maker needs to take into account.

14.5.2 Impact of Model Parameters

We also investigate possible impact of the model parameters and the size of future uncertainty on imprecise and robust production performance of olive oil producing farms. We adopt the simulation framework suggested by Shokouhi et al. [21] for the case study. The olive-growing farms are clustered into smaller groups according to their efficiency scores obtained by the nominal DEA approach. In this chapter, due to the length restriction, we only present the results of three groups each of whom consists of eight farms with the same or similar nominal performance. The farms in *Group 1* are all declared as efficient whereas *Group 2* consists of the least efficient farms according to the nominal DEA approach. *Group 3* involves such farms showing medium level (around 50 %) of efficiency.

We design two experiments, labeled as *Experiment I* and *Experiment II*, with different size of uncertainty levels for the output parameters. More precisely, *Experiment I* assumes the initial range for the uncertain olive oil yield of a farm where the olive oil production of a farm deviates from its nominal production by 1/4 (25 %). On the other hand, *Experiment II* is designed to observe the impact of the interval size on results by considering narrower ranges for olive oil yields of the farms where the olive oil production of a farm is assumed to deviate from its nominal production by 1/6 (approximately 16 %) rather than 1/4.

A brief description of the simulation procedure is as follows. First, we calculate the optimal weights associated with each farm using the nominal values of input and the estimated output parameters within the imprecise and robust DEA models (given the price of robustness). Secondly, we randomly generated 1000 nominal values of olive oil production levels (using uniform distribution) for the uncertain output parameters. Thus, the corresponding intervals for each simulated point within the ranges of 25 and 16 % olive production are then determined. Finally, the optimal weights (obtained with the estimated oil production level) and oil olive production intervals at each generated random points of the output parameters are then used to find the robust efficiency scores by solving the robust DEA models. The same procedure is repeated with 1000 output intervals in the same manner. We then analyse the statistics of 1000 efficiency scores associated with each farm within three groups in terms of average and standard deviation of scores.

Table 14.4 illustrates the simulation results for Experiment I (top) and Experiment II (bottom) using three groups of farms in terms of average efficiency scores obtained at various price of robustness with robust DEA and IDEA with optimistic (IDEA-O) and pessimistic (IDEA-P) approaches. These results basically show, on average, how the olive oil production performance of each farm changes under imprecise and robust DEA when the range of uncertainty varies.

From the simulation results in Table 14.4, we observe that

- The average efficiency scores in Experiment II are always higher than those in Experiment I regardless the choice of models (imprecise and robust DEA at each price of robustness). This implies that as the size of intervals for the random parameters decreases (i.e. random parameter approaches to the estimated nominal value), the average efficiency score increases.
- As the simulation results confirm, the imprecise DEA with optimistic view produces the highest average efficiency scores (labelled as SE), obtained out of 1000 simulated points for all farms. In both Experiments I and II, as the price of robustness varies between 0 and 1, the average efficiency score decreases. As a result, we can state the following relationship between the efficiency scores of all farms obtained by various DEA approaches as

$$SE_{R(1.0)} \leq \cdots \leq SE_{R(0.0)} \leq SE_{opt}.$$

- On the other hand, the IDEA approach with pessimistic view produces the lowest average efficiency scores (obtained by the simulation experiments) for most farms in three groups. The lowest scores are indicated in bold in Table 14.4. Therefore, $SE_{pes} \leq SE_{R(1.0)}$. Notice that the order (between the IDEA approach with pessimistic view and the most conservative robust optimization approach) that was already established from Tables 14.1 and 14.2 has changed. For these cases, IDEA apparently becomes more conservative than robust DEA. This result leads us to conclude that for such homogeneous smaller samples of farms (in groups 1, 2 and 3) with similar nominal performance, the pessimistic IDEA approach produces more conservative scores than the robust DEA model with full protection no matter which uncertainty range (25 or 16 %) is chosen. However, for more diversified sample of farms, the robust DEA strategy is more conservative than IDEA with pessimistic view as illustrated in Tables 14.2 and 14.3.
- In order to illustrate the overall performance (labeled as "AverGP" in Table 14.4) of each farm within the three different groups, we compute the average of the efficiency scores obtained by the robust DEA at eleven values of price of robustness $(0, 0.1, 0.2, \cdots, 1.0)$. One can easily see that all farms in both Groups 1 and 3 exhibit average performance around 80 and 85 % in Experiments I and II, respectively. However, farms in Group 2 show different average performance varying from 42 to 81 % in Experiment I (and similarly, 45–85 % in Experiment II). Recall that these farms are reported as the least efficient by the deterministic DEA model.

Table 14.4: Expected efficiency scores (%) obtained by the simulation experiments

	Group 1								Group 2								Group 3							
Farm ID	1	5	16	19	24	34	42	70	4	10	15	28	31	38	43	49	11	13	23	41	52	66	72	83
Approach	*Experiment I: Range of 25 % of the oil production*																							
IDEA-O	90	87	91	89	91	88	88	88	80	90	90	88	94	90	90	91	94	86	87	88	93	88	85	91
IDEA-P	**57**	78	79	88	**68**	73	88	80	**29**	**57**	**47**	**34**	88	**35**	**52**	77	**64**	74	78	**56**	**53**	70	**58**	**55**
R(0.0)	88	89	88	91	89	88	90	88	48	90	78	56	94	58	87	90	86	87	89	86	86	86	90	87
R(0.2)	84	85	85	86	86	84	86	84	46	86	74	53	86	55	81	88	84	84	85	83	83	82	86	83
R(0.4)	81	81	81	81	81	81	81	81	43	82	70	50	81	53	77	83	81	82	80	80	80	82	79	80
R(0.5)	79	78	79	79	79	79	80	78	43	79	68	49	79	51	76	79	80	78	78	78	78	78	80	78
R(0.6)	77	77	78	77	77	77	78	77	41	77	66	48	77	50	74	78	77	77	77	76	76	77	78	77
R(0.8)	73	73	74	74	73	73	73	74	39	73	63	46	73	48	69	74	73	73	74	74	73	73	73	74
R(1.0)	70	**70**	**70**	**70**	70	**69**	**70**	**71**	37	70	60	43	**70**	45	66	**71**	70	**70**	**70**	70	70	**69**	70	71
AverGP	79	79	80	80	79	79	80	79	42	80	68	49	80	51	76	81	79	79	79	78	78	79	79	78
Approach	*Experiment II: Range of 16 % of the oil production*																							
IDEA-O	97	95	96	96	96	96	96	95	77	96	96	88	96	91	96	95	98	95	95	95	95	96	95	95
IDEA-P	**67**	**67**	**67**	**73**	**67**	**68**	**67**	**67**	**35**	**67**	**56**	**40**	89	**41**	**62**	88	**75**	87	86	**68**	**64**	84	**69**	**67**
R(0.0)	92	93	92	94	92	92	93	93	50	93	80	57	96	60	88	93	91	91	93	90	91	93	93	92
R(0.2)	89	90	89	90	90	89	90	89	48	90	77	56	90	58	85	92	89	90	89	88	88	88	90	88
R(0.4)	86	87	87	87	86	87	87	87	46	87	74	53	87	57	82	89	86	87	86	86	86	88	87	86
R(0.5)	85	85	85	86	85	85	86	85	46	86	73	52	85	55	81	85	86	85	85	85	84	84	84	85
R(0.6)	84	84	85	84	84	84	84	84	45	84	72	52	84	55	80	84	84	83	84	83	83	84	83	84
R(0.8)	81	81	82	81	81	81	81	81	41	78	67	48	78	50	73	78	81	81	81	81	81	81	81	81
R(1.0)	78	79	79	78	79	78	78	79	39	73	62	45	**73**	47	69	**74**	78	**79**	**79**	78	79	**78**	78	79
AverGP	85	85	86	86	85	85	86	85	45	84	72	52	84	54	80	85	85	85	85	85	84	85	85	85

14.6 Conclusions

In this study, we are concerned with performance evaluation of olive oil production using DEA under uncertainty. In particular, we study the sensitivity of efficiency scores obtained through imprecise and robust optimization based DEA approaches in a real world agricultural problem. The olive oil production problem involves the efficiency assessment of a sample of farms located in a specific region in Turkey. The only output factor (olive oil yield of the farm) is uncertain varying between bounds that depend on the olive production. For computational experiments, we implement two basic approaches of imprecise DEA and robust optimization based DEA models.

The results indicate that the optimistic model yields higher levels of efficiency for the farms, whereas the pessimistic model scores are way below than the optimistic and nominal models as expected. The discrimination of the scores is considerably worse in the pessimistic model where only one farm remains efficient. In robust DEA modeling, as the level of conservatism increases from no-protection to full-protection against uncertainty, the average, minimum and maximum efficiency scores for the sample decline. This indicates a trade-off between the level conservatism and the efficiency levels.

We compare the efficiency scores of the DEA models in order to establish performance ranking of deterministic, imprecise and robust DEA approaches. The IDEA with optimistic view yields considerably higher levels of efficiency than any other DEA models considered in this study. When no robustness is assumed, the efficiency scores are exactly the same with those of the nominal model. Under full robustness, the efficiency scores of robust DEA are less than or equal to the scores of the IDEA with pessimistic view. Therefore, it can be stated that the most conservative robust DEA model can yield lower efficiency scores than the most pessimistic imprecise DEA model.

In order to measure sensitivity of different DEA approaches to changing uncertainty ranges and parameters, we perform simulation based experiments using homogeneous groups of farms. The simulation results reveal that when the uncertainty ranges are close to the estimated nominal values, the average efficiency scores increase. In addition, when the level of conservatism increases from no-protection to full-protection against uncertainty, the average efficiency scores for the sample decline. Therefore, we can conclude that the choice of price of robustness and size of intervals play an important role on the performance of the robust DEA models. As future research directions, one may investigate model behavior in cases where uncertainty is observed in both input and output parameters simultaneously. In particular, data driven uncertainty sets would be worthwhile to investigate.

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