Chapter 13
Portfolio Optimization with Second-Order Stochastic Dominance Constraints and Portfolios Dominating Indices

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Abstract Portfolio optimization models are usually based on several distribution characteristics, such as mean, variance or Conditional Value-at-Risk (CVaR). For instance, the mean-variance approach uses mean and covariance matrix of return of instruments of a portfolio. However this conventional approach ignores tails of return distribution, which may be quite important for the portfolio evaluation. This chapter considers the portfolio optimization problems with the Stochastic Dominance constraints. As a distribution-free decision rule, Stochastic Dominance takes into account the entire distribution of return rather than some specific characteristic, such as variance. We implemented efficient numerical algorithms for solving the optimization problems with the Second-Order Stochastic Dominance (SSD) constraints and found portfolios of stocks dominating Dow Jones and DAX indices. We also compared portfolio optimization with SSD constraints with the Minimum Variance and Mean-Variance portfolio optimization.

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13.1 Introduction

Standard portfolio optimization problems are based on several distribution characteristics, such as mean, variance, and Conditional Value-at-Risk (CVaR) of return distribution. For instance, Markowitz [12] the mean-variance approach uses estimates of mean and covariance matrix of return distribution. Mean-variance portfolio theory works quite well when the return distributions re close to normal.

This chapter considers portfolio selection problem based on the stochastic dominance rule. Stochastic dominance takes into account the entire distribution of return, rather than some specific characteristics. Stochastic dominance produces a partial ordering of portfolio returns and identifies a portfolio dominating some other portfolios [11].

Hadar and Russell [8] demonstrated that a diversified portfolio can dominate a benchmark portfolio in the sense of the Second Order Stochastic Dominance (SSD). Several applications of stochastic dominance theory to portfolio selection are considered by Whitmore and Findlay [16]. Dentcheva and Ruszczynski [3] developed an efficient numerical approach for the portfolio optimization with SSD using partial moment constraints. Roman et al. [14] suggested a portfolio optimization algorithm for SSD efficient portfolios. They used SSD with a multi-objective representation of a problem with CVaR in objective. Kuosmanen [10] and Kopa and Chovanec [9] described SSD portfolio efficiency measure for diversification.

Rudolf and Ruszczynski [15] and Fabian et al. [5, 6] considered cutting plane method to solve optimization problem with SSD constraints. This chapter implemented an algorithm similar to the Rudolf and Ruszczynski [15]. We concentrated on numerical aspects of portfolio optimization with SSD constraints and conducted a case study showing that our algorithm works quite efficiently. We used Portfolio Safeguard (PSG) optimization package of AORDA.com, which has precoded functions for optimization with SSD constraints. We solved optimization problems for stocks in Dow Jones and DAX Indices and found portfolios which SSD dominate these indices. We also compared these portfolios with the Mean-Variance portfolios based on constant and time varying covariance matrices.

13.2 Second Order Stochastic Dominance (SSD)

Let denote by $F_X(t)$ the cumulative distribution function of a random variable X. For two integrable random variables X and Y, we say that X dominates Y in the second-order, if

$$\int_{-\infty}^{\eta} F_X(t)dt \le \int_{-\infty}^{\eta} F_Y(t)dt, \quad \forall \eta \in \mathbb{R} \tag{13.1}$$

In short we say that X dominates Y in SSD sense and denote it by $X \succcurlyeq_2 Y$ [7]. With the partial moment of a random variable X for a target value η, the SSD dominance is defined as follows

$$E([\eta - X]_+) \leq E([\eta - Y]_+), \quad \forall \eta \in \mathbb{R} \tag{13.2}$$

where, $[\eta - X]_+ = \max(0, \eta - X)$ [13].

13.2.1 SSD Constraints for a Discrete Set of Scenarios

Suppose that Y has a discrete distribution with outcomes, y_i, $i = 1, 2, \ldots, N$. Then the condition (13.2) can be reduced to the finite set of inequalities [13],

$$E([y_i - X]_+) \leq E([y_i - Y]_+), \quad i = 1, , 2, \ldots, N \tag{13.3}$$

We use inequalities (13.3) for obtaining a portfolio X dominating benchmark Y.

13.2.2 Portfolio Optimization Problem with SSD Constraints

Let us denote:

w_j = portfolio weight of the instrument j, $j = 1, \ldots, n$.
p_i = probability of scenario i, $i = 1, \ldots, N$,
r_{ji} = return of instrument j on scenario i,
c_j = cost of investing in instrument $j = 1, \ldots, n$ (estimated return of an instrument is interpreted as negative cost $-c_j$),
$\mathbf{w}$ = vector of portfolio weights, $\mathbf{w} = (w_1, w_2, \ldots, w_n)^\top$,
$\mathbf{r}(\mathbf{w})$ = portfolio return as a function of portfolio weights $\mathbf{w}$,
$c(\mathbf{w})$ = portfolio cost as a function of portfolio weights $\mathbf{w}$.

Portfolio return on scenario i equals:

$$r_i(\mathbf{w}) = \sum_{j=1}^{n} w_j r_{ji}, \quad i = 1, 2, \ldots, N$$

Portfolio cost equals:

$$c(\mathbf{w}) = \sum_{j=1}^{n} c_j w_j$$

The benchmark portfolio Y has a discrete distribution with scenarios y_i, $i = 1, 2, \ldots, N$. We want to find a portfolio SSD dominating the benchmark portfolio Y and having minimum cost $c(\mathbf{w})$. We do not allow for shorting of instruments. Let us denote by W the set of feasible portfolios:

$$W = \{\mathbf{w} \in \mathbb{R}^n : w_j \geq 0, \; j = 1, 2, \ldots, n\}$$

The optimization problem is formulated as follows:

$$\begin{array}{ll} \text{minimize}_{\mathbf{w}} & c(\mathbf{w}) \\ \text{subject to:} & \mathbf{r}(\mathbf{w}) \succeq_2 Y \\ & \mathbf{w} \in W \end{array} \tag{13.4}$$

Since the benchmark portfolio has a discrete distribution, with (13.3) we reduce the portfolio optimization problem (13.4) to:

$$\begin{array}{lll} \text{minimize}_{\mathbf{w}} & \sum_{j=1}^{n} c_j w_j & \\ \text{subject to:} & E([y_i - r(\mathbf{w})]_+) \leq E([y_i - Y]_+), & i = 1,\ldots,N \\ & w_j \geq 0, & j = 1,\ldots,n \end{array} \tag{13.5}$$

A solution of the optimization problem (13.5) yields a portfolio dominating Y. The number of scenarios (which can be quite large) determines the number of SSD constraints in this optimization problem. Further, we suggest a procedure for elimination redundant constraints in (13.5).

13.3 Algorithm for Portfolio Optimization Problem with SSD Constraints

This section describes cutting plane algorithm for solving problems with SSD constraints in this study. An overview of the cutting-plane methods for SSD problems can be found in [5, 6, 15]. We start the description of the algorithm with the procedure for removing redundant constraints.

13.3.1 Removing Redundant Constraints

Let us consider benchmark scenarios y_{i_1} and y_{i_2} with indices i_1 and i_2 and denote the right hand side values of the constraints for scenarios in problem (13.5) by $C_{i_1} = E([y_{i_1} - Y]_+)$ and $C_{i_2} = E([y_{i_2} - Y]_+)$. If $y_{i_1} \leq y_{i_2}$ and $C_{i_1} \geq C_{i_2}$, for scenarios i_1 and i_2, then constraint i_1 is redundant and it can be removed from the constraint set. This procedure dramatically reduces the number of constraints in the optimization problem (13.5).

13.3.2 Cutting Plane Algorithm

Here are the steps of the algorithm for solving optimization problem (13.5). We denote by s the iteration number.

Step 1. Initialization: $s = 0$. Assign an initial feasible set

$$W_0 = \{\mathbf{w} \in \mathbb{R}^n : w_j \geq 0,\ j = 1, 2, \ldots, n\}$$

Assign an initial verification set

$$V_0 = \{\mathbf{w} \in \mathbb{R}^n : E([y_i - r(\mathbf{w})]_+) \leq E([y_i - Y]_+),\ i = 1, 2, \ldots, N\}$$

Step 2. Solve the optimization problem

$$\begin{aligned} &\text{minimize}_{\mathbf{w}} \ \sum_{j=1}^{n} c_j w_j \\ &\text{subject to:} \ \ \mathbf{w} \in W_s \end{aligned} \tag{13.6}$$

If all constraints defining the set V_s are satisfied, then the obtained point is optimal to problem (13.5). Otherwise, go to Step 3.
Step 3. Find constraint in V_s with the largest violation and remove it from V_s. Denote this new set of constraints by V_{s+1} (after removing the constraint with the largest violation). Add removed constraint to the constraints defining set W_s and denote it by W_{S+1}. Increase the iteration counter $s = s + 1$ and go back to Step 2.

13.3.3 PSG Code for Optimization with SSD Constraints

The problem (13.5) can be directly solved with the Portfolio Safeguard (PSG) without any additional coding. Here is the code, which can be downloaded from this link.[1]

```
maximize
    avg_g(matrix_sde)
Constraint:= 1
    linear(matrix_budget)
MultiConstraint: <= vector_ubound_sd
    pm_pen (vector_benchmark_sd, matrix_sde)
Box: >= 0, <= 1
```

We have done the case study in PSG MATLAB Environment running many optimizations iteratively. However, here we provided just one code in PSG Run-File format to show that the SSD constrained problems can be easily coded and solved.

[1] Three example problems containing input data and solutions in PSG format are at the following link (see, Problem 1, Dataset 1, 2, 3): http://goo.gl/Fooals.

13.4 Case Study

We solved problem (13.5) with two data sets. The first dataset includes stocks from the Dow Jones (DJ) index and DJ index is considered as a benchmark. Similar, the second dataset includes stocks from the DAX index and the DAX index is used as a benchmark. The data were downloaded from the Yahoo Finance (http://finance.yahoo.com) and include 2500 historical daily returns of stocks from March 24, 2005 to Feb 27, 2015 for DJ index and from April 25, 2005 to Feb 27, 2015 for DAX index. The lists of stocks in indices are taken on March 2, 2015. Therefore, we considered only 29 stocks from the DJ index and 26 stocks from the DAX Index (the appendix contains the list of the stocks selected for this chapter). The stock returns on daily basis (r_{ji}) were calculated using logarithm of ratio of the stock adjusted closing prices (f_i),

$$r_{ji} = \ln(f_i/f_{i-1})$$

We adjusted the stocks prices of four companies from DAX Index.[2] Daily returns are considered as equally probable scenarios in the study.

The optimization problem with SSD constraint (in this case study) finds a portfolio SSD dominating the benchmark and having maximum expected portfolio return. Shorting is not allowed. The sum of portfolio weights is equal to 1,

$$\sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0, \quad j = 1,2,\ldots,n$$

We compared performance of the SSD based portfolios with Equally Weighted, Minimum Variance and Mean-Variance portfolios with the constant and time-varying covariance matrices. Here is a brief description of portfolios:

1. Equally Weighted (EW): All stocks in the portfolio are equally weighted. Every stock has same weight $(1/n)$, where n is the number of stocks in the portfolio.
2. Minimum Variance (MinVar): Minimum Variance portfolio has minimum variance without any constraint on portfolio return. Shorting is not allowed and the sum of the portfolio weights is equal to 1.
3. Mean-Variance (Mean-Var): Mean-variance portfolio [12] uses mean return and the variance of the stock returns. The approach finds efficient portfolios having minimum variance for a desired level of portfolio return or equivalently having maximum portfolio return for a given variance. We considered Mean-Var problems having variance in the objective function and the expected portfolio return 12 % per year in the constraint, and 0.2 upper bound constraint on the positions. Shorting is not allowed and the sum of the portfolio weights is equal to 1.

The classical Mean-Variance model considers the constant covariance matrix. We also considered the time dependent covariance matrix using DCC-GARCH model in MinVar and Mean-Var approaches. Further, we briefly describe the estimation procedure for the time-dependent covariance matrix.

[2] DB1.DE, FRE.DE, IFX.DE and MRK.DE stock prices are adjusted for splits.

13.4.1 Estimation of Time-Varying Covariance Matrix

We considered a dynamic conditional correlation DCC-GARCH (DCC) model for the estimation of large time-dependent covariance matrices [4]. We estimated the time dependent covariance matrix using DCC-GARCH model (assuming that correlations may change over time). The time-dependent covariance matrix H_t is extracted from the DCC-GARCH model, where $H_t = D_t R_t D_t$. Here, D_t is the diagonal matrix from a univariate GARCH model and R_t is the time dependent correlation matrix. This chapter assumes the simplest conditional mean return equation where $\bar{r}_j = N^{-1} \sum_{i=1}^{N} r_{ji}$ is the sample mean and the deviation of returns $(r_t - \bar{r})$ is conditionally normal with zero mean and time-dependent covariance matrix H_t [2]. We consider the time-dependent covariance matrix H_t in a simple DCC(1,1)-GARCH model. We used H_t in MinVar and Mean-Var problems.

The next Sect. 13.4.2 compares SSD constrained optimization with the MinVar and the Mean-Var approaches for all available historical data in a static setting. The code was implemented with MATLAB R2012b. We have used PSG riskprog function in MATLAB environment to solve MinVar and Mean-Variance portfolio problems. For the estimation of the time-dependent covariance matrix we have used MFE Toolbox[3]. The Sect. 13.4.3 we compares out-of-sample performance of portfolios in time series framework. The calculations were performed on a computer having 3.4 GHz CPU and 8 GB of RAM.

13.4.2 Comparing Numerical Performance of Various Portfolio Settings

We benchmarked the cutting plane algorithm described in Sect. 13.3.2 with the direct PSG code described in Sect. 13.3.3. We got the same results with both approaches. Further in tables we report performance of the direct PSG code. The dataset includes 2500 historical daily stock returns. Firstly we optimized portfolios with all approaches using available 2500 historical daily returns.

Table 13.1 shows the expected yearly returns of portfolios for all considered approaches.

The SSD dominating portfolios can be used for actual investments. At least in the past, these portfolios SSD dominated the corresponding indices. Moreover, the expected yearly return of the portfolio SSD dominating the DJ index equals 0.10029 and significantly exceeds the DJ index return in this period. Similar observations are valid for the portfolio of DAX index; the expected yearly return of portfolio SSD dominating the benchmark equals 0.14894.

We compared solving times of SSD constraint optimization problem (using direct PSG optimization) with the MinVar and Mean-Var approaches (using PSG *riskprog*

[3] DCC-GARCH models are estimated with Kevin Sheppard's (Multivariate GARCH) MFE Toolbox. http://www.kevinsheppard.com/MFE_Toolbox

Table 13.1: Expected yearly returns of portfolios

Portfolios	DJI	DAX
EW	0.09617	0.10179
MinVar	0.08668	0.14135
Mean-Var	0.12693	0.12693
DCC MinVar	0.09034	0.13466
DCC Mean-Var	0.12693	0.12693
SSD	0.10029	0.14894
Benchmark	0.05682	0.10484

subroutine). Data loading and solving times are given in Table 13.2. The optimization is done almost instantaneously and data loading takes some fraction of a second. The time-dependent covariance matrix estimation with MFE Toolbox additionally takes about 30 s (for MinVar and Mean-Var optimization).

Table 13.2: Loading and solving times (in seconds) with PSG in MATLAB Environment

	DJ		DAX	
Problem	Loading	Solving	Loading	Solving
SSD constrained (PSG code)	0.24	0.01	0.23	0.01
MinVar (PSG riskprog)	0.22	0.01	0.23	0.01
Mean-Var (PSG riskprog)	0.31	0.01	0.32	0.01

13.4.3 Out-of-Sample Simulation

Secondly, we have evaluated the out-of-sample performance of considered approaches. We considered a time series framework where the estimation period (750 and 1000 days) is rolled over time. Portfolios are re-optimized on every first business day of the month using the recent historical daily returns (750 or 1000). We kept constant positions during the month. Regarding the return constraint in the Mean-Var problem, if the expected return 12 % per year is not feasible (in the beginning of the month), than we set 6 % expected return constraint and if we still do not have feasibility, we reduce the expected return to 3 %, and then to 0 %. A difficulty in estimation of the covariance matrices with DCC model is that the time-dependent conditional correlation matrix has to be positive definite for all time moments [1]. We observe that with a small in-sample time intervals (such as 250 days) the variance-covariance matrix may not be positive-definite. Therefore, we have used 750 and 1000 days in-sample periods.

Table 13.3 shows out-of-sample total compounded returns of considered portfolios. In particular, we observe that the SSD constrained portfolio for DJ stocks with 750 days in-sample and DAX stocks with 1000 days in-sample, have highest compounded returns among all portfolios.

Table 13.3: Out-of-sample total compounded returns of the portfolios

	DJ		DAX	
Portfolios	750	1000	750	1000
EW	1.6698	2.7560	1.2120	1.9022
MinVar	1.4787	2.0035	1.6186	2.4410
Mean-Var	1.6715	2.1487	1.4912	2.6219
DCC MinVar	1.3876	2.0463	1.7763	2.2062
DCC Mean-Var	1.6499	2.2556	1.4344	2.3435
SSD	1.8817	2.1692	1.3987	2.8827
Benchmark	1.2729	2.2275	1.3399	1.8674

Figures 13.1, 13.2, 13.3, and 13.4 show the out-of-sample compounded daily returns of the portfolios.

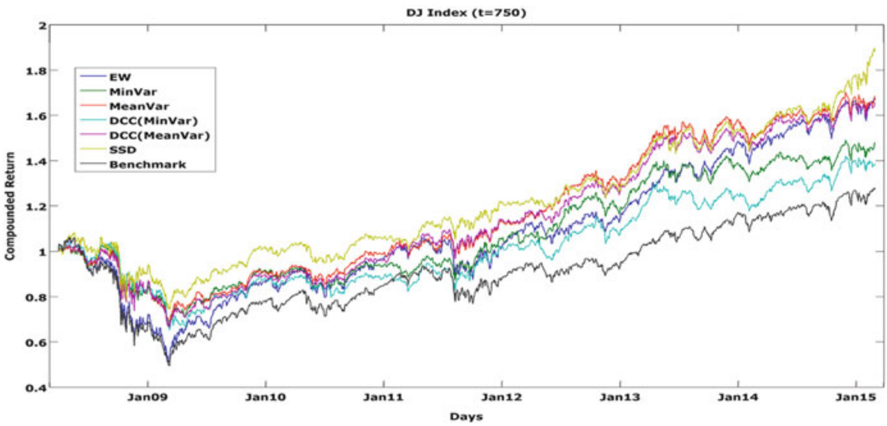

Fig. 13.1: Compounded (on daily basis) returns of portfolios including DJ stocks, $t = 750$

The out-of-sample performances of portfolios are represented in Tables 13.4, 13.5, 13.6, and 13.7. The tables include yearly compounded portfolio returns for 2009–2014 years, the total compounded portfolio return (T_R) and Sharpe Ratio (Sh_R).

- Table 13.4 (DJ stocks, $t = 750$). SSD constrained portfolio has the highest Total Compounded Return (1.8817) and Sharpe Ratio (0.7906).
- Table 13.5 (DJ stocks, $t = 1000$). SSD constrained portfolio has Sharpe Ratio (1.2771) higher than the all considered portfolios except Equally Weighted portfolio.

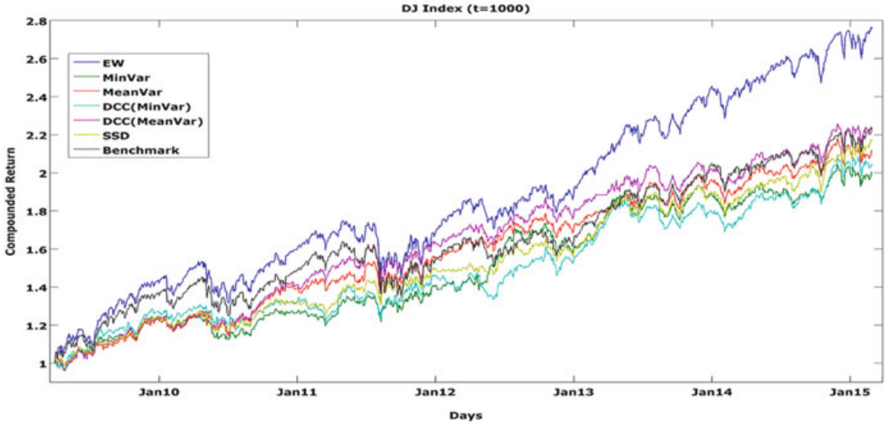

Fig. 13.2: Compounded (on daily basis) returns of portfolios including DJ stocks, $t = 1000$

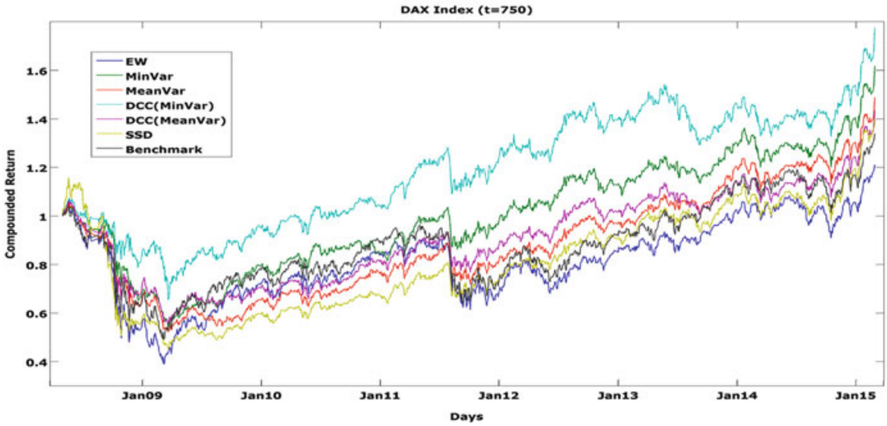

Fig. 13.3: Compounded (on daily basis) returns of portfolios including DAX stocks, $t = 750$

- Table 13.6 (DAX stocks, $t = 750$). SSD constrained portfolio has Sharpe Ratio (0.2732) and Total Compounded Return (1.3987) higher than the Benchmark and Equally Weighted portfolios.
- Table 13.7 (DAX stocks, $t = 1000$). SSD constrained portfolio has the highest Total Return (2.8827) and Sharpe Ratio (1.3523).

Table 13.8 shows weights of SSD constrained portfolios at the last month of the out-of-sample period. Also, the table shows SSD dominating portfolios over all in-sample 2500 days. The table shows only stocks with non-zero positions.

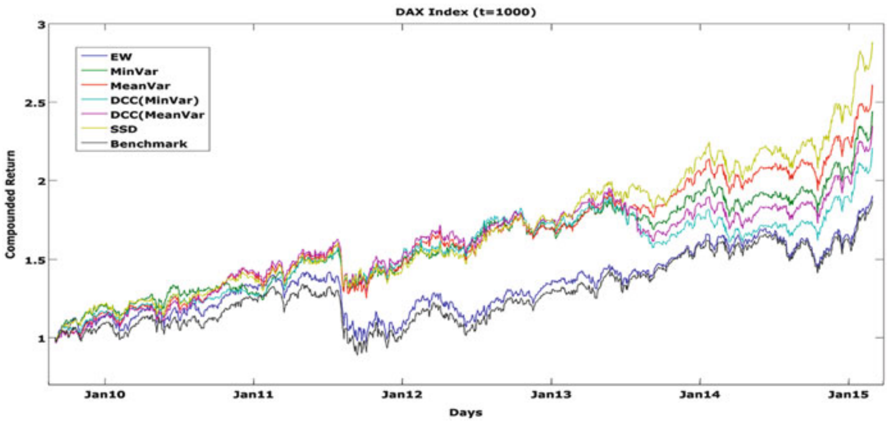

Fig. 13.4: Compounded (on daily basis) returns of portfolios including DAX stocks, $t = 1000$

Table 13.4: Yearly compounded returns, total compounded return (T_R), Sharpe ratio (Sh_R) for DJ stocks ($t = 750$)

Portfolios	2009	2010	2011	2012	2013	2014	T_R	Sh_R
EW	1.2257	1.1256	1.0467	1.1249	1.3063	1.1029	1.6698	0.5533
MinVar	1.0903	1.0282	1.1259	1.1158	1.1814	1.0451	1.4787	0.4584
Mean-Var	1.0618	1.0993	1.1424	1.1399	1.2220	1.0473	1.6715	0.7034
DCC MinVar	1.0277	1.0119	1.1513	1.0612	1.1574	1.1090	1.3876	0.4025
DCC Mean-Var	1.0405	1.0865	1.1864	1.1041	1.2104	1.0678	1.6499	0.6835
SSD	1.0931	1.0493	1.1495	1.0443	1.2089	1.1498	1.8817	0.7906
Benchmark	1.154	1.0958	1.0321	1.0652	1.2584	1.0688	1.2729	0.2239

Table 13.5: Yearly compounded returns, total compounded return (T_R), Sharpe ratio (Sh_R) for DJ stocks ($t = 1000$)

Portfolios	2010	2011	2012	2013	2014	T_R	Sh_R
EW	1.1256	1.0467	1.1249	1.3063	1.1029	2.7560	1.2902
MinVar	1.0294	1.1109	1.1349	1.1852	1.0593	2.0035	0.9230
Mean-Var	1.1241	1.1129	1.0920	1.1993	1.0736	2.1487	1.1965
DCC MinVar	1.0437	1.1064	1.0530	1.1702	1.1321	2.0463	0.9803
DCC Mean-Var	1.1207	1.1548	1.0866	1.1822	1.0899	2.2556	1.2653
SSD	1.0721	1.1327	1.0525	1.2078	1.1153	2.1692	1.2771
Benchmark	1.0958	1.0321	1.0652	1.2584	1.0688	2.2275	1.0165

Table 13.6: Yearly compounded returns, total compounded return (T_R), Sharpe ratio (Sh_R) for DAX stocks ($t = 750$)

Portfolios	2009	2010	2011	2012	2013	2014	T_R	Sh_R
EW	1.2890	1.1677	0.8119	1.2526	1.2034	1.0217	1.2120	0.1245
MinVar	1.1609	1.0742	1.1485	1.1590	1.1537	1.0698	1.6186	0.5457
Mean-Var	0.9763	1.1691	1.0700	1.1896	1.2448	1.0740	1.4912	0.4635
DCC MinVar	1.1231	1.0966	1.1853	1.1346	1.0468	1.0436	1.7763	0.6348
DCC Mean-Var	0.9799	1.1565	1.0793	1.1612	1.1282	1.0789	1.4344	0.3939
SSD	1.0284	1.1304	1.0950	1.2322	1.2031	1.0885	1.3987	0.2732
Benchmark	1.1891	1.1411	0.8181	1.2671	1.2414	1.0123	1.3399	0.2279

Table 13.7: Yearly compounded returns, total compounded return (T_R), Sharpe ratio (Sh_R) for DAX stocks ($t = 100$)

Portfolios	2010	2011	2012	2013	2014	T_R	Sh_R
EW	1.1677	0.8119	1.2526	1.2034	1.0217	1.9022	0.6784
MinVar	1.0714	1.1479	1.1541	1.1371	1.0746	2.4410	1.1886
Mean-Var	1.2238	1.0407	1.1562	1.2211	1.0962	2.6219	1.3164
DCC MinVar	1.1033	1.1825	1.1501	1.0282	1.0606	2.2062	1.0335
DCC Mean-Var	1.2547	1.0542	1.1593	1.0626	1.0867	2.3435	1.0169
SSD	1.1452	1.0343	1.2235	1.2323	1.1425	2.8827	1.3523
Benchmark	1.1411	0.8181	1.2671	1.2414	1.0123	1.8674	0.6676

Table 13.8: SSD constrained portfolios (table shows only selected stocks)

DJ	Weights			DAX	Weights		
Code	750 days	1000 days	2500 days	Code	750 days	1000 days	2500 days
BA	0.06984	0.00808	0.00243	ADS	0.00000	0.00000	0.01287
DIS	0.34832	0.10397	0.00000	BAYN	0.00000	0.00000	0.02686
HD	0.19280	0.19317	0.00000	BEI	0.00000	0.02238	0.20000
IBM	0.00000	0.00000	0.01959	CON	0.33042	0.04653	0.00000
JNJ	0.00000	0.22105	0.20000	DPW	0.06556	0.12844	0.00000
KO	0.00000	0.00000	0.16569	DTE	0.02086	0.00000	0.00000
MCD	0.00000	0.00000	0.20000	FME	0.00000	0.00000	0.20000
MMM	0.08726	0.00000	0.00000	FRE	0.31189	0.43899	0.12149
MRK	0.07009	0.15284	0.00000	LIN	0.00000	0.00000	0.10719
NKE	0.05919	0.05844	0.01776	MRK	0.27127	0.36366	0.12265
PG	0.00000	0.02058	0.19368	MUV2	0.00000	0.00000	0.08076
T	0.00000	0.00000	0.01456	SAP	0.00000	0.00000	0.12819
TRV	0.08155	0.00000	0.00000				
UNH	0.09096	0.08374	0.00000				
VZ	0.00000	0.00000	0.05402				
WMT	0.00000	0.15814	0.13226				

13.5 Conclusions

In this chapter we tested algorithms for portfolio optimization with SSD constraints. The algorithms are very efficient and solve optimization problems nearly instantaneously (solution times less than 0.01 s for the considered cases).

We have done out-of-sample simulations and compared SSD constrained portfolios with the minimum variance and mean-variance portfolios. The portfolios were constructed from the stocks of DJ and DAX indices. SSD constrained portfolio demonstrated quite good out-of-sample performance and in some cases had highest compounded return and Sharpe ratio (among all considered portfolios).

Appendix: Company Codes and Names

	DAX code	Name	DJ code	Name
1	ADS	Adidas AG	AXP	American Express Company
2	ALV	Allianz SE	BA	The Boeing Company
3	BAS	BASF SE	CAT	Caterpillar Inc.
4	BAYN	Bayer AG	CSCO	Cisco Systems, Inc.
5	BEI	Beiersdorf AG	CVX	Chevron Corporation
6	BMW	Bayerische Mot. Werke Aktienges.	DD	E. I. du Pont de Nemours and Company
7	CBK	Commerzbank AG	DIS	The Walt Disney Company
8	CON	Continental Aktiengesellschaft	GE	General Electric Company
9	DAI	Daimler AG	GS	The Goldman Sachs Group, Inc.
10	DB1	Deutsche Boerse AG	HD	The Home Depot, Inc.
11	DBK	Deutsche Bank AG	IBM	Int. Business Machines Corporation
12	DPW	Deutsche Post AG	INTC	Intel Corporation
13	DTE	Deutsche Telekom AG	JNJ	Johnson & Johnson
14	EOAN	E.ON SE	JPM	JPMorgan Chase & Co.
15	FME	Fres. Med. Care AG & Co. KGAA	KO	The Coca-Cola Company
16	FRE	Fresenius SE & Co KGaA	MCD	McDonald’s Corp.
17	HEI	HeidelbergCement AG	MMM	3M Company
18	IFX	Infineon Technologies AG	MRK	Merck & Co. Inc.
19	LHA	Deutsche Luft. Aktiengesellschaft	MSFT	Microsoft Corporation
20	LIN	Linde Aktiengesellschaft	NKE	Nike, Inc.
21	MRK	Merck KGaA	PFE	Pfizer Inc.
22	MUV2	Münchener R. G. A.	PG	The Procter & Gamble Company
23	SAP	SAP SE	T	AT&T, Inc.
24	SDF	K+S Aktiengesellschaft	TRV	The Travelers Companies, Inc.
25	SIE	Siemens Aktiengesellschaft	UNH	UnitedHealth Group Incorporated
26	TKA	ThyssenKrupp AG	UTX	United Technologies Corporation
27			VZ	Verizon Communications Inc.
28			WMT	Wal-Mart Stores Inc.
29			XOM	Exxon Mobil Corporation

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