Chapter 7
Developing Virtual Mathematics Manipulatives: The SAMAP Project

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Abstract Online educational activities providing interactive environments where users can investigate the properties of abstract concepts and reflect on them are in great demand for all subjects in primary and secondary schools. However, the ubiquitous nature of these activities does not always guarantee students' conceptual development if enough consideration was not given to the design and implementation of the system and an appropriate role was not defined for the technology used. A computer system could play a wide range of roles changing from a 'tutor' acting as "a decision-making" subject to a 'tool' acting as an "auxiliary" object. One can also interpret this classification of roles as a system having total control of flow or a system allowing free explorations. A computer system is regarded as suitable to be used in education when it provides facilities that promote the student's conceptual development through engaging him/her in meaningful and authentic tasks. The new Turkish mathematics curriculum is based on constructivist educational approaches and advocates the wide usage of educational activities that help to make mathematical concepts and relations meaningful. The purpose of this chapter is to report the findings of a research project, SAMAP, funded by the Turkish National Science Foundation (TUBITAK), which aimed to develop virtual mathematics manipulatives in Turkish for the primary and secondary school curriculum.

7.1 Introduction

Recent developments in Information Communication Technologies (ICT) offer many new possibilities to enhance students' comprehension during the learning-teaching process. Hence, online educational activities providing interactive environments where users can investigate the properties of concepts and reflect on them are in great demand for all subjects in primary and secondary schools.

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P.S. Moyer-Packenham (ed.), *International Perspectives on Teaching and Learning Mathematics with Virtual Manipulatives*, Mathematics Education in the Digital Era 7,
DOI 10.1007/978-3-319-32718-1_7

However, the ubiquitous nature of these activities does not always guarantee students' conceptual development if enough consideration was not given to the design and implementation of the system and an appropriate role was not defined for the technology used (Durmuş and Karakırık 2006). A computer system could play a wide range of roles changing from a '*tutor*' acting as "*a decision-making*" subject to a 'tool' acting as an "*auxiliary*" object (O'Shea and Self 1983; Crook 1994). One can also interpret this classification as a system having total control of flow or a system allowing free explorations. In Jonassen's (1996) cognitive tool metaphor, an appropriate role for computers in line with the constructivist approaches is defined. A computer system is regarded as suitable to be used in education when it provides facilities that promote a student's conceptual development through engaging him/her in meaningful and authentic tasks.

Mathematics is rightly regarded as one of the most important subjects for the primary school primary school curriculum. Students at the concrete operational stage are introduced to fundamental abstract mathematical concepts and relations for the first time at this stage. Hence, it is vital to make abstract mathematical concepts and relations concrete with different models in order for students to be able to grasp them. Lack of such models leads students to focus on arithmetic and procedural skills rather than mathematical concepts and relations. Hence, there are many projects in this regard, such as National Library of Virtual Manipulatives (NLVM) (http://nlvm.usu.edu), WisWeb (http://www.fi.uu.nl/wisweb/en/) and the National Council of Teachers of Mathematics (NCTM) Illuminations (http://illuminations.nctm.org), to provide comprehensive sets of mathematics manipulatives to be used from kindergartens to the graduate studies to promote students' mathematical skills and understandings. The new Turkish mathematics curriculum, updated in 2005 (MEB 2005), is based on constructivist educational approaches and advocates the wide usage of educational activities that help to make mathematical concepts and relations meaningful. It also aims to promote the use of ICT and the Internet and to remove the digital gap among primary and secondary school students through the FATİH project (fatihprojesi.meb.gov.tr), which employs physical and virtual manipulatives.

Manipulatives are physical objects or concrete models that can make abstract ideas and symbols more meaningful and understandable to students (e.g., base-ten blocks and algebra tiles). A virtual manipulative is "an interactive, Web-based, visual representation visual representation of a dynamic object that provides opportunities for constructing mathematical knowledge" (Moyer et al. 2002). Virtual manipulatives are distinguished from other digital resources used for learning in their dynamic nature and provision of interactive experiences. The importance of using play and manipulation to grasp abstract mathematical concepts or using tools or concrete objects to mediate learning is emphasized by many educators for constructivist learning environments (Bruner 2003; Dienes 1971; Duffy and Cunningham 1996; Piaget 1952; Pea 1985; Vygotsky 1978). Many studies also confirm virtual and physical manipulatives physical manipulatives as effective tools of instruction (Butler et al. 2003; Sowell 1989; Suh and Moyer 2007). However, the provision of tools alone is not sufficient without adequately clarifying their place and their usage in the teaching-learning process. Hence, it is necessary to develop virtual manipulative sets that specifically highlight certain mathematical concepts and relations in the mathematics curriculum.

7.2 The SAMAP Project and Manipulatives Development Process

SAMAP is a Turkish acronym composed by the initial letters of the Turkish phrase “virtual mathematics manipulative project”. The SAMAP project was launched to develop an interactive, comprehensive and multi-lingual mathematical manipulative set, primarily focusing on the Turkish audience, for the primary and secondary school curriculum (Grades 1–8) in five strands of mathematics (numbers, geometry, measurement, data analysis and algebra). It was implemented by the author at Abant İzzet Baysal University, Bolu, and sponsored by the Turkish National Science Foundation, TUBITAK (Karakırık 2008, 2010). The SAMAP project included a graphic designer who was responsible for designing graphical elements (e.g., icons and images) displayed on the manipulatives and the website. The design and coding of the manipulatives and instructions and explanations provided in the manipulatives and on website were all managed by the author. SAMAP could be regarded as the first attempt to produce the Turkish version of the National Library of Virtual Manipulatives (NLVM) (NLVM) (Nlvm.usu.edu 1999). The general outline of the NLVM was adopted for SAMAP’s implementation. Many novel manipulatives as well as modified versions of available manipulatives were implemented in SAMAP. Most SAMAP manipulatives were designed with a direct reference to a mathematical objective from the Turkish mathematics curriculum.

SAMAP manipulatives were coded by the author in an object-oriented manner using JAVA programming language. All SAMAP manipulatives were derived from the same JAVA code, which allowed for the creation of both an *applet* version, running on a webpage, and a stand-alone *application* version which could be downloaded. The SAMAP project initially employed the applet versions of the manipulatives on a website and later a SAMAP CD was produced with the application versions of the manipulatives. Figure 7.1 shows the outline of a typical SAMAP manipulative. Each SAMAP manipulative screen was divided into certain areas to provide a consistent and user-friendly environment: the *main working area* holds the actual implementation of the manipulative, *title bar* displays the manipulative’s title, *information panel* gives specific information about the manipulative or shows the latest feedback based on a user action and the *command panel* provides an interactive area where all graphical items providing user interaction (such as buttons, textboxes etc.) are placed. Since communication among panels is achieved through a special messaging service, many components seen on the screen are independent of each other and can be reused in the design of different manipulatives.

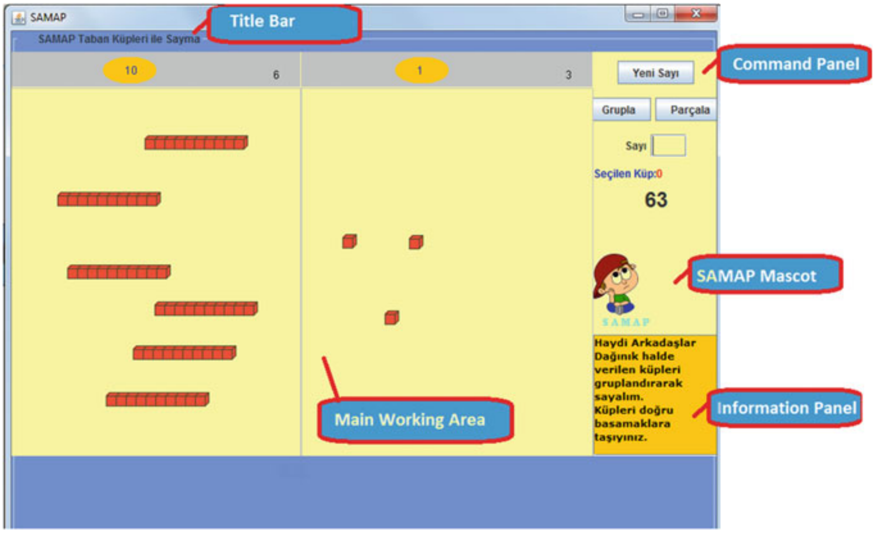


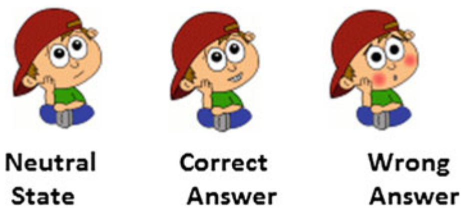
Fig. 7.1 An example SAMAP manipulative layout

All panels are optimized to 800 × 600 screen resolution and are adjusted automatically for lower resolutions and provide scrollbars if necessary. The command panel and the information panel are automatically placed at the bottom of the screen whenever a manipulative requires a larger horizontal working area. SAMAP's default language is Turkish but can detect and adapt itself to the user's local language. However, a property file holding language specific data needs to be updated to use SAMAP in another language. Manipulatives could interact with the website through JavaScript and display instructions and help pages specific to a manipulative.

The information panel also includes the SAMAP mascot, which provides instant graphical and audio feedback about the latest user action as portrayed in Fig. 7.2. The SAMAP mascot smiles for a correct action and it blushes for a wrong one. It is interesting to note that this mascot is known as SAMAP especially among young users.

Educational technologists need to determine the educational and technical specifications of a manipulative before implementation. Objectives from the Turkish

Fig. 7.2 States of a SAMAP mascot



primary mathematical curriculum were thoroughly investigated and activities that could be adapted to the computer were selected by the author and educational aims and specifications were determined. Technical specifications of the manipulatives were determined with both the requirements of the task at hand and the restrictions of the programming environment or the expertise of the programmers.

Virtual manipulatives, in essence, provide novel virtual artifacts (objects or tools) to be used to reflect or play with certain mathematics concepts and relations. The design of any artifact is determined by the specifications of its affordances. An affordance defines the relationship between the properties of an object and the capabilities of the agent that determine just how the object could possibly be used (Norman 2013). Hence, unique affordances for virtual manipulatives need to be identified to help students to learn specific mathematics concepts and relations. Moyer-Packenham and Westenskow's (2013) meta-analysis of virtual manipulatives suggest 5 affordance categories of virtual manipulatives, namely focused constraint, creative variation, simultaneous linking, efficient precision and motivation. The SAMAP manipulatives were designed in a way to support these pedagogical affordances to impact student learning.

The SAMAP manipulatives were designed to be very flexible but constrain student actions and restrict input when necessary and have *focused constraints*. For instance, drop down boxes and radio-buttons were widely employed to limit the user inputs to correct values. If a textbox was used to prompt user input, validity of the input was confirmed by necessary checks before the input was accepted. Students were allowed free explorations and solved specific problems with restricted actions depending on the running mode of the manipulatives.

The SAMAP manipulatives were designed to support *simultaneous linking* by using multiple representations of mathematics concepts in symbolic, graphical, textual or other forms. For instance, the set manipulations dynamically supports textual, pictorial, and symbolic forms for set operations.

The SAMAP manipulatives were designed to be attractive and entertaining and afford *motivation*. A specific icon was designed for each manipulative and a special title was chosen for each activity to attract students' attention. The SAMAP mascot was very motivating for younger students. Each manipulative was designed to have a specific meaningful task. The tasks were also designed to be challenging enough to increase motivation.

The SAMAP manipulatives were designed in a way to support *efficient precision*, by employing precise representations and simulating real behaviors. For instance, the *hit the target* manipulative uses an aircraft and cannon to demonstrate the concept of angle. The *finding symmetry* manipulative allows users to split an image flexibly and test whether they are comparable by dragging one piece to another and rotating them. However, efficient precision is sometimes disregarded on purpose to make students focus on the concept at hand. For instance, the right angle symbol is not used when the angle is 90° to emphasize their interchangeable usage in the *hit the target* manipulative. However, most students regarded this as an error and requested the regular right angle graphical symbol be used to denote that an angle having a value of 90° is a right angle.

The SAMAP manipulatives were designed to have open-ended tasks that encourage *creativity* and enable students' multiple solutions. For instance, the *logo* manipulative allows users to produce their own drawings (e.g., cars and houses). Many manipulatives require subject-specific problem-solving knowledge to solve the presented problem in a task and allow multiple solutions for the problem. For instance, the *set* manipulative could interpret any set operations and allow different symbolic representations for any area on the Venn diagram beside symbolic expressions provided in the drop down boxes.

The SAMAP project included different types of manipulatives such as manipulatives for solving certain mathematics problems, exemplifying or simulating certain mathematical concepts or relations, doing certain mathematical calculations and procedures, and innovative applications of mathematical concepts and activities that could provoke discussions in the class. Manipulatives that could be employed for both individualized instruction and collaborative work were implemented. Each manipulative is automatically adapted to different class levels through certain parameters. For instance, base ten blocks were adapted to employ numbers up to 20, 99, 999 and 9999 for first, second, third and fourth grade students respectively, thereby employing the focused constraint affordance. Furthermore, setting the base to a number other than 10 is also permitted for grades 5–8.

Around 75 distinct (100 with variations) mathematical manipulativewere implemented by the author at Abant İzzet Baysal University during the three years of the project between the years of 2005 and 2008. The number of manipulatives can be roughly categorized with respect to the five strands of mathematics as follows:

- numbers (28),
- geometry (20),
- data analysis and probability (14),
- measurement (8),
- algebra (5).

These numbers roughly correspond to the weights each mathematical strand occupies in the primary level. These numbers are regarded as in line with the project aims since numerical activities focusing on only arithmetic and numerical operations without any problem solving tasks were generally avoided. For instance, activities requiring arithmetic operations but not attaching any specific meaning to the numbers used were avoided. In fact, activities of this type were designed only after specific requests by in service teachers (e.g., one such example is the *number pyramid* manipulative in number strand). The relevant files for each manipulative were brought together in a JAR file. Then deployment, instruction and web pages for the manipulatives were designed. Figure 7.3 shows the sequence of web pages to reach specific manipulatives in a strand.

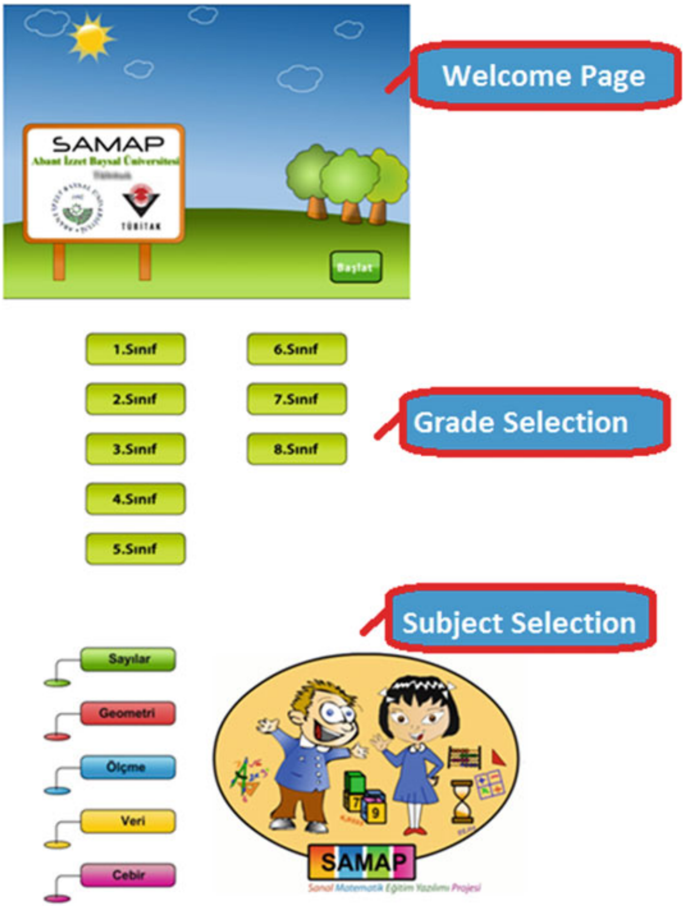


Fig. 7.3 Web pages to reach specific manipulatives in a strand

7.3 Examples of SAMAP Manipulatives

Here are two examples of SAMAP manipulatives exemplifying individualized and collaborative usage in the class to demonstrate the functionality and nature of the manipulatives.

Figure 7.4 shows a SAMAP manipulative for operations on sets and shows its results dynamically in multiple representations namely symbolic, verbal, and graphical forms. The manipulative has many options in the form of radio-buttons on the bottom-left of the screen to change the appearance and the functionality of the set operations such as:

- number of sets available (one, two, or three),
- types of the sets (intersected sets, disjoint sets and subsets), and

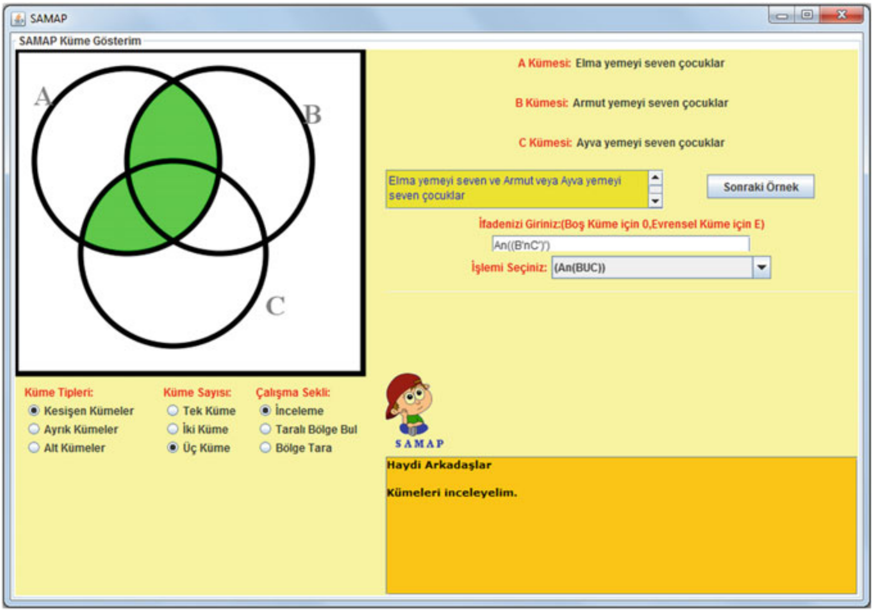


Fig. 7.4 The set manipulation of SAMAP

- running mode (free investigation, finding the shaded region, and shading a specified region).

Furthermore, the manipulative has the following components to provide further display options:

- interactive graphical display of the results of the set operations on the top right of the screen,
- a drop down box showing all of the available results of set operations in symbolic form $(A \cap (B \cup C))$ on the bottom-right of the screen,
- a textbox to enter any set operations in symbolic form,
- a button to change the text examples resembling the current set operations (there are 7 different text examples for three intersected sets such as children eating apples, pears, and quinces to show the sets A, B, and C respectively for the current selection), and
- a text area where the results of the current set operations are displayed with respect to the selected text example.

As Fig. 7.4 shows, this manipulative requires subject-specific problem-solving knowledge for solving all relevant problems for the sets. Some of the capabilities of the system, such as finding all different symbolic representations for the current selection, such as $A \cap ((B' \cap C'))'$ or $\left(A' \cup \left(B' \cap C'\right)\right)'$ which could be found by applying De Morgan's Laws for the current selection, were hidden to reduce the complexity of the manipulative. In fact, one could argue that this manipulative has the potential to be turned into more distinct manipulatives by just focusing on one of the representations, on one of the set operations, or on one of the running modes. Hence, in an individualized or collaborative teaching scenario, this manipulative has the functionality to display the results of set operations dynamically in different forms.

Although this manipulative resembles the *Venn Diagrams* manipulative available at the NLVM, it has novel and flexible features to distinguish it as a completely different manipulative. For instance, one could use various real life situations such as reading (newspapers, magazines, and books), attending lectures (math, physics, and chemistry) and learning languages (English, French, and German) as well as eating various foods to concretize the meaning of the set operations in the example while the NLVM Venn Diagrams does not have this capability.

Figure 7.5 shows another SAMAP manipulative, which focuses on the concept of arithmetic mean that dynamically computes and displays the arithmetic mean of a set of random or user-defined data between 0 and 100. Unlike the previous example, this manipulative has neither a comprehensive problem-solving capability nor range of options for user-interaction. It only accepts random or user-defined data and displays the data points and their arithmetic mean on the screen. In other words, it enables users to see how the arithmetic mean changes when data set

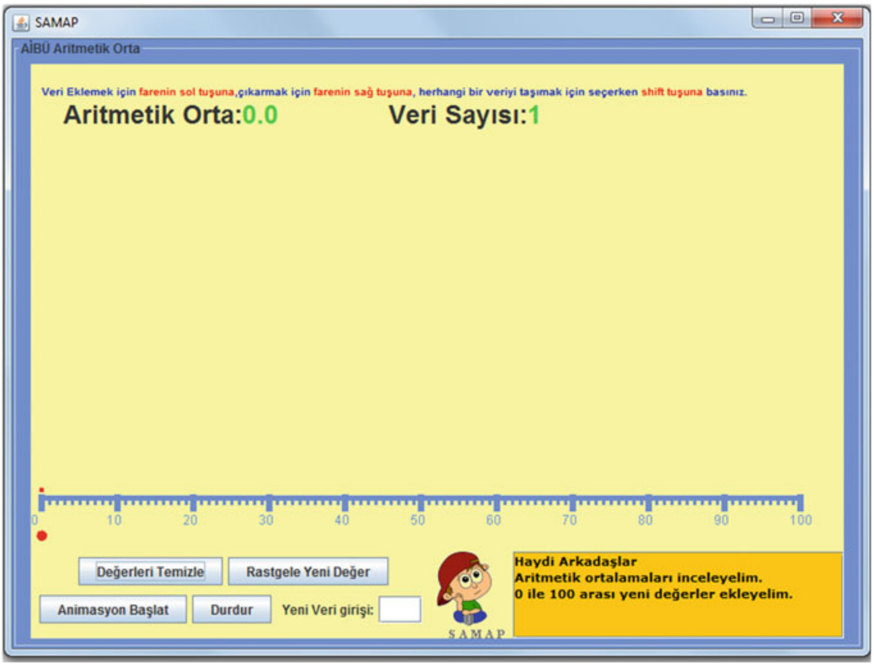


Fig. 7.5 The arithmetic mean manipulative of SAMAP

changes. Many users, including teachers, who seem to be inclined to press buttons randomly or change some parameters in a manipulative to see what it leads to had difficulty seeing or predicting the rationale for this manipulative since it has neither a specific problem-solving tool nor a didactic style to teach something.

This does not make this manipulative less effective since it is purportedly designed to be used with the guidance of an instructor in a teaching setting to provoke discussions about the arithmetic mean. The manipulative enables users to enter new data or remove or change existing data to resolve some of their hypotheses about the arithmetic mean. However, the instructor needs to ask leading questions to help users to grasp the subtleties of the concept of arithmetic mean. For instance, one could ask the following leading questions that could be answered with the help of this manipulative to demonstrate different aspects of arithmetic mean:

- state an observation about how arithmetic mean changes when new random data are inserted;
- enter new data in order to increase/decrease the current arithmetic mean;
- remove any data on the screen in order to increase/decrease the current arithmetic mean; and,
- change any data on the screen in order to increase/decrease the current arithmetic mean.

All of the above questions point to some aspect of the concept of the arithmetic mean. In fact, this manipulative never requires users to compute the arithmetic mean by hand. Rather it forces users to think and comment on the influence of data on the arithmetic mean.

7.4 Evaluation of SAMAP Manipulatives

The main aim of the SAMAP project was to be able to develop a comprehensive virtual manipulative set for mathematics. The technical side of the project was overwhelming and the project was understaffed for the evaluation part since one of the team members responsible for evaluating the manipulatives had to leave at the beginning of the project. Hence, the effectiveness of the manipulatives was mainly evaluated through online questionnaires (e.g., the SAMAP assessment scale surveys) and users' comments on the technical and educational aspects of the manipulatives. Suggestions for improvements were collected through the website during the project. There were no quantitative or qualitative studies conducted to make comparisons of SAMAP manipulatives with concrete manipulatives or other instructional treatments because of the author's belief that virtual manipulatives need to be developed to promote students' conceptual understanding rather than to improve students' performance on tasks and tests. However, SAMAP manipulatives were widely used in primary schools and were introduced to pre-service teachers in educational technology courses in several Turkish Universities. The next section includes findings from implementation efforts, online evaluation, and reports from pre-service teachers.

7.4.1 Online Evaluation of SAMAP Manipulatives

A project website was launched at the beginning of the project and all SAMAP manipulatives were gradually integrated to the website. Users were required to register to be able to use the site and participate in the study on voluntarily basis. Hence, all numbers reported in the study refer to unique users. The study was conducted between years 2006 and 2009. The online users of the SAMAP website were classified into 9 different categories with respect to different class levels and experience as shown in Table 7.1. Users that did not fall into one of these categories, such as parents and non-students, were classified as other and not included.

Nearly ten thousand unique users accessed the website during the project. A special web page was prepared for each manipulative containing the instructions on how to use the manipulative and online questionnaires depending on a particular user's classification. Various bits of statistical information about the users, such as their locations and their computer screen resolutions, were collected using the Google analytics tool. The number of users and completed questionnaires for different numbers of manipulatives in each strand are shown in Table 7.2. Five strands of mathematics, namely numbers, geometry, measurement, data analysis and algebra, are decoded using the abbreviation ST1 to ST5, respectively, in all the results that follow.

Online questionnaires using Likert scale responses were administered to every group for each manipulative. Different questionnaires having different numbers of items were prepared with a measurement expert. Questions about technical and educational aspects of the manipulatives, as well as users' attitudes, were asked in the questionnaires. Table 7.3 shows the number of questionnaire items and the aspects of questions for each group. Participants were asked to complete the

Table 7.1 Classification of online users of SAMAP website

Group No	Group members
Group 1	1st grade, 2nd grade and 3rd grade primary school students
Group 2	4th grade and 5th grade primary school students
Group 3	6th grade, 7th grade and 8th grade secondary school students
Group 4	First and second year pre-service teachers
Group 5	Third year pre-service teachers
Group 6	Fourth year pre-service teachers
Group 7	Graduate students
Group 8	Teachers
Group 9	Academicians

Table 7.2 The number of questionnaires completed for each strand of mathematics

	Size (N)	Number of manipulatives	ST1	ST2	ST3	ST4	ST5	Total
Group 1	43	34	32	23	14	4	–	73
Group 2	20	23	10	5	8	3	–	26
Group 3	16	23	5	10	2	1	2	20
Group 4	44	43	63	23	9	9	2	106
Group 5	51	62	116	74	54	42	1	287
Group 6	33	36	22	16	4	4	12	58
Group 7	16	64	57	44	31	38	34	204
Group 8	45	55	57	22	16	4	3	102
Group 9	12	30	14	8	7	5	8	42
Total	280	370	376	225	145	110	62	918

Table 7.3 The number of questionnaire items for each Group

	Attitude		Educational		Technical		Total
	#	%	#	%	#	%	Total
Group 1	3	60	1	20	1	20	5
Group 2	3	27	2	18	6	55	11
Group 3	4	27	4	27	7	47	15
Group 4	4	20	5	25	11	55	20
Group 5	4	15	8	31	14	54	26
Group 6–7	4	10	20	49	17	41	41
Group 8–9	4	8	28	55	19	37	51

questionnaires for any manipulative they preferred. User comments were also collected through a text area. The full analysis of the questionnaires (Karakırık and Cakmak 2009) is beyond the scope of this chapter. We will just summarize the results of each group and focus on primary school students in Group 1 and Group 2.

7.4.2 Group 1

Group 1 included 1st to 3rd grade students. A simple questionnaire consisting of five three-level Likert items was prepared and pilot-tested with 10 children of this level to ensure comprehensibility of questionnaire items. Students were required to choose one of the three emoticons, ☺, 😐 and ☹ to answer an item to denote "agree" (A), "no opinion" (N) and "do not agree" (D). The results of questionnaires for Group 1 are displayed in Table 7.4.

Table 7.4 The results of questionnaires from Group 1

Items	Opinion	ST1	ST2	ST3	ST4	Total	%
1. I liked this game	A	29	20	11	2	62	0.85
	N	2	2	1	0	5	0.07
	D	1	1	2	2	6	0.08
2. I like to play this game again	A	26	17	10	3	56	0.77
	N	5	4	2	1	12	0.16
	D	1	2	2	0	5	0.07
3. It is easy to play this game	A	28	18	12	1	59	0.81
	N	4	4	1	3	12	0.16
	D	0	1	1	0	2	0.03
4. I can play this game on my own	A	28	20	13	3	64	0.88
	N	3	1	0	1	5	0.07
	D	1	2	1	0	4	0.05
5. I learned something new in this game	A	23	18	10	2	53	0.73
	N	6	4	0	2	12	0.16
	D	3	1	4	0	8	0.11

Note A agree; *N* no opinion; *D* do not agree

Forty-three students completed 73 questionnaires for 34 different manipulatives in Group 1. Students in this group stated that they loved the manipulatives (85 %), wanted to play again (77 %), found them easy (81 %), were able to play the games themselves (88 %), and learned something new (73 %). Students' comments also confirmed that they liked SAMAP manipulatives and found them easy to use.

7.4.3 Group 2

Group 2 included 4th and 5th grade students. A pilot study was performed on 5th-grade students to determine the comprehensibility of 29 questionnaire items. Based on the pilot study, a final questionnaire, consisting of eleven three-level Likert items, was prepared like Group 1 but written expressions were used instead of emoticons. The questionnaire administered to Group 2 and the results are displayed in Tables 7.5 and 7.6 respectively.

Twenty students completed 26 questionnaires for 23 different manipulatives in Group 2. Students in Group 2 loved the manipulatives (88 %), found them easy (96 %), thought the descriptions were clear (92 %), liked the screen layouts (77 %), wanted to play again (85 %), thought they drew attention to the mathematics (73 %), and reported that they learned something new (69 %). Many students stated

Table 7.5 The questionnaire administered to Group 2

Q1. I like this activity
Q2. I want to use this activity again
Q3. It is easy to use this activity
Q4. I can use this activity on my own
Q5. This activity has increased my interest in mathematics
Q6. The screen layout of this activity is nice
Q7. This activity has the opportunity to correct my mistakes
Q8. This activity gives adequate warning and information
Q9. This activity has the opportunity to test myself
Q10. I learned new topics in this activity
Q11. The description of this activity is clear

Table 7.6 The results of questionnaires from Group 2

	ST1			ST2			ST3			ST4			Total			%		
	A	N	D	A	N	D	A	N	D	A	N	D	A	N	D	A	N	D
Q1	10	0	0	4	0	1	6	0	2	3	0	0	23	0	3	0.9	0	0.1
Q2	10	0	0	4	1	0	6	1	1	2	1	0	22	3	1	0.9	0.1	0
Q3	10	0	0	5	0	0	7	0	1	3	0	0	25	0	1	1	0	0
Q4	9	1	0	5	0	0	8	0	0	3	0	0	25	1	0	1	0	0
Q5	9	1	0	3	2	0	5	3	0	2	1	0	19	7	0	0.7	0.3	0
Q6	8	1	1	4	0	1	5	2	1	3	0	0	20	3	3	0.8	0.1	0.1
Q7	9	1	0	3	1	1	6	0	2	3	0	0	21	2	3	0.8	0.1	0.1
Q8	8	1	1	4	1	0	4	1	3	2	1	0	18	4	4	0.7	0.2	0.2
Q9	9	1	0	4	0	1	5	3	0	3	0	0	21	4	1	0.8	0.2	0
Q10	5	5	0	4	1	0	6	1	1	3	0	0	18	7	1	0.7	0.3	0
Q11	9	1	0	5	0	0	7	0	1	3	0	0	24	1	1	0.9	0	0

that they found SAMAP manipulatives useful and entertaining. Some wanted more challenging activities with time restrictions. Furthermore, manipulatives related to numbers and geometry areas were used more frequently by students in Group 1 and Group 2 than measurement and data analysis since they are more likely to include arithmetic operations. Students preferred to employ manipulatives involving certain procedural skills and mathematical calculations such as the number pyramid manipulative in Fig. 7.6.

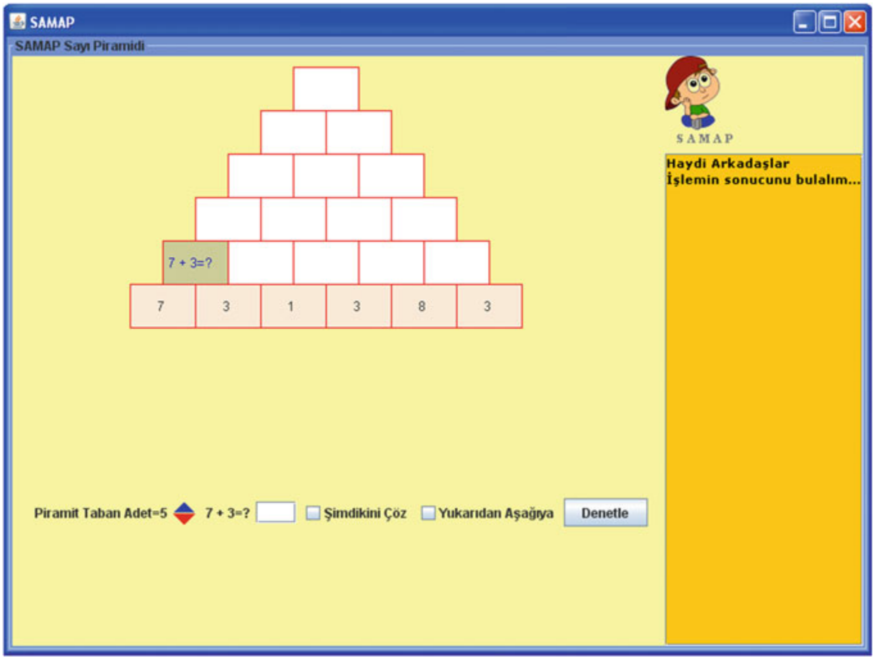


Fig. 7.6 The number pyramid manipulative of SAMAP

7.4.4 Group 3

Group 3 included secondary school students. Sixteen students completed 20 questionnaires for 23 manipulatives. There were positive and negative opinions in this group about the manipulatives. Some stated that manipulatives were not interesting enough for them to use since they found the activities simple. Others stated that activities were nice and useful. The analysis of the questionnaire items revealed that students wanted to use the activities individually but did not have much desire to re-use the activities.

7.4.5 Groups 4–7

Groups 4–7 included undergraduate and graduate students. The highest numbers of questionnaires were completed by these groups. One-hundred forty-four students completed 655 questionnaires for 205 manipulatives. These groups completed similar questionnaires, except first and second year undergraduate students were asked fewer questions on educational aspects. Participants in these groups complained about the number of questions in the questionnaires and were careless about completing test items since it was mainly introduced in a course. However, many useful comments were gathered from these groups especially about errors found in the manipulatives and suggestions for modifications for better interactivity. Furthermore, most of the participants stated that they liked the manipulatives and were optimistic about their possible usage in classes to improve mathematics education.

7.4.6 Group 8

Group 8 included primary school teachers. A comprehensive questionnaire, consisting of 51 five-level Likert items, was used with the range of options from “strongly agree” to “strongly disagree.” Teachers were asked detailed questions about the technical and educational aspects of the manipulatives. Forty-five teachers completed 102 questionnaires for 55 manipulatives. Analysis of the questionnaires revealed that most teachers thought that students would like the manipulatives but they would be unable to use them on their own. Teachers found the manipulatives in-line with the curriculum with specific learning objectives but they were generally pessimistic about their possible contributions to students’ mathematics achievement.

Teachers described manipulatives working as desired, but found some manipulatives were inflexible and did not enable them to construct their own problems, objects or movements. For instance, they wanted to change the analog clock by dragging the minute hand using the mouse rather than using the keyboard or the digital clock. One teacher commented that she used the manipulatives in the class and students liked the manipulatives. In fact, many teachers sent emails to the author to express their positive feelings about the manipulatives during the evaluation process. Some teachers demanded more explanations and instructions to be provided to direct students and teachers on how to use the manipulatives.

7.4.7 Group 9

Group 9 included academicians working actively in universities. Twelve academicians completed 42 questionnaires for 30 manipulatives. Academicians also found the manipulatives promising and useful. Some stated that manipulatives could be used as complementary materials in the class and may help students to reflect on mathematics concepts. Some demanded more feedback be provided and suggested improvements in manipulative design.

7.4.8 Overall

In summary, the results of the online study reveal that participants liked and enjoyed SAMAP manipulatives. Manipulatives were generally regarded as having potential to be useful for teaching mathematics concepts. The level of interactivity provided in SAMAP manipulatives was generally found satisfactory, despite the many suggestions for modifications that were put forward. Furthermore, many schools utilized SAMAP manipulatives in classroom settings for teaching mathematics all over Turkey.

7.5 Analysis of Pre-service Teachers' Reports of SAMAP Manipulatives

It is not easy to evaluate the effectiveness of manipulatives through only questionnaires or statistical means since ICT applications may not always lead to significant changes as measured by classical evaluation methods. Hence, SAMAP manipulatives were incorporated into educational technology courses in several Turkish Universities between 2009 and 2014, including Abant İzzet Baysal and Selcuk Universities. Pre-service primary school and mathematics teachers and graduate students were required to submit detailed reports on the technical and educational aspects of SAMAP manipulatives.

SAMAP manipulatives were introduced to the senior pre-service primary school mathematics teachers and classroom teachers within a mathematics methods course that met three hours per week. Some of the manipulatives were implemented in a classroom setting and pre-service teachers were required to use and investigate the manipulatives for two weeks and prepare a report on their effectiveness and state their opinions for improvement. More than a thousand pre-service teachers' reports were collected in five years in two universities containing pre-service teachers' own words and reflections about many SAMAP manipulatives. Many encouraging comments and feedback were received from these reports. These comments were taken into account to revise some of the manipulatives. Most of the participants had optimistic opinions about the effectiveness of the manipulatives. Major issues highlighted in the reports about virtual manipulatives could be summarized as follows:

SAMAP manipulatives were seen as entertaining, easy to learn and useful activities by most of the pre-service teachers. Many reports talk about the motivational aspect of many manipulatives by referring their entertaining nature. The manipulatives were thought to be attractive for children and they were thought to be appropriate to be implemented in class. Mathematics was said to be more enjoyable through these manipulatives. They were also generally found to be appropriate for the Turkish curriculum and class levels. Furthermore, manipulatives were said to have the potential to change students' perception of mathematics after seeing mathematical models of some real life problems.

SAMAP manipulatives are successful in modeling mathematical concepts for primary school, which may contribute to the persistence of learning mathematics. For instance, the *function machine* manipulative in which users were required to guess a function definition is said to make the function concept very concrete. The input and output areas of the manipulative that models a function were thought to be very explanatory.

Likewise, the *counting scales* manipulative is said to be useful in making addition and subtraction operations concrete. It was found very helpful to teach the properties of positive and negative numbers by showing different colored scales and requiring users to hide scales until they have scales of just one color. A scale was hidden by dragging a different colored scale over it. Users quickly get the idea that one cannot hide a positive or a negative scale by another positive or negative scale respectively.

Virtual manipulatives have been viewed as helping to develop students' thinking skills, making connections among mathematical concepts and improving their estimation skills. For instance, it was stated that the *fill and pour* manipulative helped students to make connections between different numbers and improved their estimation skills in an entertaining way.

Virtual manipulatives may contribute students' problem-solving skills. For instance, several measurement activities involving real-life situations were found useful to teach practical problem solving skills and helpful for students to realize how to apply mathematical concepts to real life.

SAMAP manipulatives were generally found to be very useful. For instance, the "*tossing coins*" manipulative was seen as useful in exemplifying the probability concept despite its simple design and functionality. It was especially regarded helpful for showing the difference between theoretical and empirical probability. It helps users to discover that this difference gets smaller when the number of tossed coins increases.

SAMAP manipulatives reinforce what was learned in class. For instance, the *number decipher* manipulative was found very useful in consolidating arithmetic operations on natural numbers. Students need to make use of higher-order thinking skills in this activity and take into consideration the next operation to apply. This kind of manipulative involving a puzzle may encourage students to enjoy solving novel problems.

The manipulatives have a pedagogically appropriate design. For instance, the "*set manipulations*" manipulative covers all set operations in a manner from simple to difficult.

The manipulatives were seen as appropriate for individualized learning since the design takes student differences into account. One can set several parameters in the manipulatives in accordance with his/her speed of learning. This encourages students' active participation and use of technology in the class. For instance, the *taking symmetry* manipulative exemplifies the symmetries of figures or lines and provides a good pretext for using technology in mathematics classes.

The manipulatives provide valuable feedback and have potential to remove students' misconceptions. For instance, it was reported that some students make mistakes in multiplication procedures even at the secondary school level. Virtual manipulatives are useful in correcting students' misconceptions.

The dynamic nature and interactivity of SAMAP manipulatives were appreciated by most pre-service teachers. For instance, the *hit the target* and *finding symmetries* manipulatives were found very interactive and visually demonstrated the angle and symmetry concepts respectively.

Some SAMAP manipulatives were found very ordinary and not interesting enough to motivate students (e.g., *factor tree).* It was criticized for employing big numbers in the activity for 6th grade and for the provision of all factors when one is entered by the user. Similarly, the *finding balance* manipulative was found simple and inflexible since it contains a limited number of weights.

It was concluded from the reports that manipulatives, including animations and multiple representations to illustrate the properties of certain mathematical concepts, were more popular and seen as educationally effective by the participants. It was interesting to note that many preferred manipulatives to be used on an individual basis rather than collaboratively. This reflects the perception of the participants about the place of technology in classes. Many aim to employ manipulatives to concretize some mathematical concepts, which may be difficult to achieve by physical manipulatives or other means. Manipulatives requiring teachers' guidance and whose aims and functionalities are not so apparent though operational instructions were given, such as the *Arithmetic mean* manipulative in Fig. 7.5, are found to be troublesome by most participants. This may reflect teachers' expectations to find ready-made activities and their unwillingness to invest time and energy to prepare their own activities through flexible manipulatives and problem-solving tools.

7.6 Discussion

A recent meta-analysis on virtual manipulatives reported that virtual manipulatives have produced overall moderate effects in favor of the virtual manipulatives when compared with other instructional treatments (Moyer-Packenham et al. 2014). Moyer-Packenham and Suh (2012) reported that virtual manipulatives have significantly different effects on different achievement groups because of students' different types of experiences with the virtual manipulative. All virtual manipulatives cannot provide similar learning performance and efficiency and children in different age groups might respond in different ways to virtual manipulatives (Moyer-Packenham et al. 2015). Virtual manipulatives in different mathematical strands also produce varying results. The quality of interactions provided or the number of different representations employed in these manipulatives greatly affect these results. For instance, virtual manipulatives prepared for the geometry strand are thought to produce the biggest effect for conceptual understanding since the enigmatic nature of geometry allows easy investigation of the concepts through more representations.

It is also well-documented that adequately designed virtual manipulatives could be used interchangeably with physical ones (Burns and Hamm 2011; Ozgün-Koca and Edwards 2011) as in our earlier prediction (Durmuş and Karakırık 2006) and even support further affordances. These affordances might include bringing out emotional connections to learning and self-regulatory behaviors (McLeod et al. 2012) and highlighting the visual and kinesthetic senses (Namukasa et al. 2009) and mathematical discourse (Anderson-Pence and Moyer-Packenham 2015). Özgün-Koca et al. (2013) advocates implementing activities that make use of both physical and virtual manipulatives. Using virtual manipulatives has also been reported to minimize the impact of extraneous demographic variables on learning (Moyer-Packenham et al. 2013). The focus of the recent studies that compare the effectiveness of physical and virtual manipulatives seems to shift from comparing students' performances to evaluating manipulatives from the perspective of the students (McLeod et al. 2012) by taking into account the quality of interaction provided to improve conceptual knowledge. Virtual manipulatives are far more effective in demonstrating and teaching conceptual knowledge (Suh and Moyer 2007), which is difficult to measure by classical standardized tests that compare students' performances. They are also used as an effective practice to teach mathematical concepts to students with learning disabilities (Satsangi and Bouck, 2015).

The aim of the SAMAP manipulatives was to promote higher order thinking skills by providing an environment for investigating mathematical concepts and relations rather than focusing on simple calculations and mathematical operations. However, many teachers and students were uncertain how to make use of the manipulatives since they were used to repetitive calculations instead of investigating mathematical concepts. Furthermore, many teachers only make use of the SAMAP manipulatives as extracurricular activities since they lack the experience to structure a mathematics lesson with virtual manipulatives (Reimer and Moyer 2005). The author argues that SAMAP manipulatives could be used to promote discussions, to increase students' participation and enhance their conceptual understanding in mathematics classrooms (Karakırık 2011). It was reported that linked virtual manipulatives enhance students' collaboration and mathematical discourse (Anderson-Pence and Moyer-Packenham 2015). As Wegerif and Dawes (2004) rightly points out a dialogical approach is required rather than dialectic one and dialogue among students and between students and the teacher is important in creating learning environments that promote thinking and productive learning.

SAMAP manipulatives were designed both for individual use and collaborative use in the classroom with the guidance of teachers. Most of the SAMAP manipulatives are related to the embodiment or concretization of some mathematical concepts and the creation of meaningful problem-solving scenarios, and avoid activities focused solely on procedural skills (Kaput 1992). A flexible design that enabled the most user interaction through expert systems was preferred. SAMAP manipulatives attach great importance to student interaction and the active participation of students are taken into account in designing manipulatives by adopting a constructivist approach. Repetitive tasks or animations are mostly avoided and open-ended tasks are preferred. A virtual manipulative may provide direct or indirect feedback either in structurally didactic manner or without providing guidance for any solution path (Anderson-Pence and Moyer-Packenham 2015). Implementing classroom activities highlighting the mathematical concepts is argued to be the most outstanding characteristics of a constructivist classroom. In this respect, manipulatives could be regarded as effective and successful to the extent they foster reflective and deep thought among students and teachers and introduce them new mathematical knowledge. This could be regarded as guided discovery activities that both involve discovery and guidance as suggested by Kuhn (2007).

Two main types of manipulatives emerged in this project for the purpose of demonstrating mathematical concepts of an abstract nature which implements both 'learning with models' and 'learning to model' approaches (Durmuş and Karakırık 2006). The first type illustrates different aspects of a mathematical concept through multiple representations. These manipulatives include more exploration activities and help students to see and investigate the mathematical relations and concepts. The second type provides problem-solving or modeling tools by which students were able to express themselves and devise their own models or problem solutions. These manipulatives included meaningful problem situations where students can internalize the mathematical concepts and problem solving strategies. Each SAMAP manipulative is designed by taking both types into consideration and having both demonstration and interaction modes where applicable.

Different representations of a concept have different inferential powers (Cox and Brna 1995) and may highlight different computational properties of a concept (Larkin and Simon 1987). The design parameters, unique functions supporting learning and the cognitive tasks to interact with multiple representations, needs to be taken into consideration to improve the effectiveness of multiple representations (Ainsworth 2006). The SAMAP manipulatives extensively use multiple representations of mathematics concepts such as graphical, algebraic, and written representations and allow users to change the representations dynamically. It is argued that virtual manipulatives are far more flexible and helpful for conceptual learning since they help students to visualize different interpretations and properties of a concept through dynamic representations.

7.7 Conclusion

SAMAP manipulatives covering objectives in the Turkish mathematics curriculum have been widely used by Turkish primary school teachers and students. In light of feedback from SAMAP users, it could be argued that many students are satisfied with the resources provided by the manipulatives. Therefore, the author argues that SAMAP manipulatives offer a favorable environment for discussion of mathematical concepts. Students accustomed to procedural operations in mathematics classes do not seem to fully understand the purpose of the manipulative at first, but can benefit from them over time with proper guidance of teachers. This emphasizes the leading roles of teachers in conceptual activities where it is not easy to see how to apply certain mathematical concepts in a problem situation. Considering the failure of early optimistic artificial intelligence studies that aimed to create a virtual teacher, no manipulative is expected to be smart enough to create a discussion platform for students. It is believed that manipulatives having special problem-solving expertise in well-defined scenarios should be developed to encourage students to investigate certain mathematical concepts: SAMAP activities aim to serve this purpose.

Acknowledgements The SAMAP project is supported by the Turkish Science Foundation TUBİTAK by project number SOBAG 106K140.

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