An Efficient Procedure to Determine the Initial Basic Feasible Solution of Time Minimization Transportation Problem

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Abstract. To meet the challenge of delivering products to the customers in a minimum time, Time Minimization Transportation Problem (TMTP) provides a powerful framework to determine the better ways to deliver products to the customer. In this paper, we present a new procedure for determining an initial basic feasible solution of TMTP. Comparative study is accomplished between the presented procedure and the other existing procedures in virtue of sample examples which demonstrate that the presented procedure requires less number of iterations to reach the optimal transportation time.

Keywords: Time minimization transportation problem · Feasible solution · Pointer cost · Optimum solution

1 Introduction

Transportation problem (TP), one of the simplest combinatorial problems in Operations Research, deals with the circumstance in which a single identical vendible is transported from a number of sources to a number of destinations in such a way so that the total transportation cost is minimized while fulfilling all supply and demand limitations. The basic TP was originally developed by Hitchcock [1]. Efficient methods of solution derived from the simplex algorithm were flourished, primarily by Dantzig [2] and then by Charnes *et al.* [3]. This type of problem is known as cost minimizing TP, which has been deliberated since long and is famous by many researchers [4–26].

Again, the process of transporting exigent material, such as weapons used in military operations, salvage equipments, equipments used for dealing with emergency, people or medical treatment things and the fresh food with short storage period, where the speed of delivery is more important than the transportation cost, is known as TMTP. This problem was first addressed by Hammer [27]. An initial basic feasible solution for minimizing the time of transportation can be obtained by using any existing methods such as, North West Corner Method (NWCM) [11], Least Cost Method (LCM) [11], Vogel's Approximation Method (VAM) [11], Extremum

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Difference Method (EDM) [12], Highest Cost Difference Method (HCDM) [5, 6] and Average Cost Method (ACM) [4]. Extensive work has also been done on TMTPs by several researchers [17, 28–34]. They introduced various algorithms for solving TMTPs. Garfinkel and Rao [29] solved time minimization transportation problems. Hammer [27] and Szwarc [34] used labeling techniques to solve such kind of problems respectively. Ilijia Nikolic [30] presented an algorithm to minimize the total transportation time to TMTPs. Md. Main Uddin *et al*. [32] reduced the total transportation cost on the basis of time using VAM, M Sharif Uddin [31] used the HCDM whereas Mollah Mesbahuddin Ahmed used HCDM [17, 33] on Modified Transportation Cost Matrix (MTCM) to minimize the transportation time. Very recently Mollah Mesbahuddin Ahmed also introduced Allocation Table Method (ATM) [19] to do the same.

In this paper, we define and calculate the pointer cost by subtracting the time units of every cell of total opportunity time matrix from the sum of respective row and column highest and allocate to the cell corresponding to the maximum pointer cost. Finally we use the optimality test of time minimizing transportation problem.

2 Formulation of Time Minimization Transportation Problem

A general time minimization transportation problem is represented by the network in the following Fig. 1.

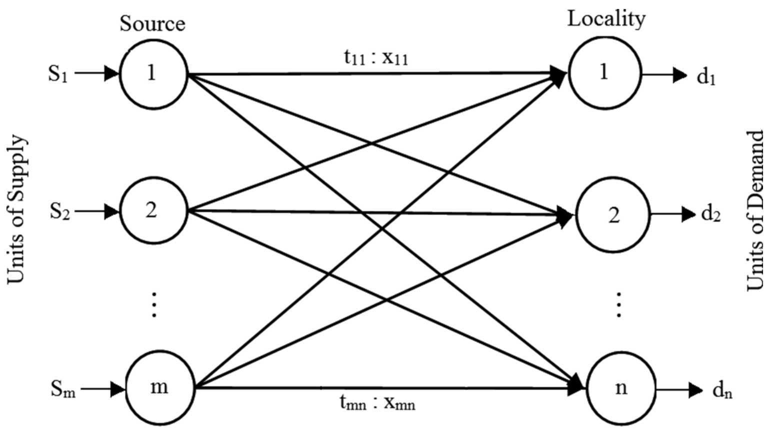

Fig. 1. Network representation of time minimization transportation problem

There are m sources and n destinations, each represented by a node. The arrows joining the sources and the localities represent the route through which the single identical vendible is transported. Suppose S_i is the availability of the same at ith (i = 1, 2, … …, m) source, d_j is the availability of the same at jth (j = 1, 2, … …, n) destination, t_{ij} denotes the unit transportation time from ith source to jth destination, x_{ij}

represents the amount is transported from *i*th source to *j*th destination. Then the LPP model of the balanced time minimization transportation problem is:

Minimize: Max $t_{ij} \mid x_{ij} > 0$

$$\begin{aligned} \text{s/t}, \sum\nolimits_{j=1}^{n} x_{ij} &= s_i \qquad i = 1, 2, \ldots\ldots, m \\ \sum\nolimits_{i=1}^{m} x_{ij} &= d_j \qquad j = 1, 2, \ldots\ldots, n \\ x_{ij} \geq 0 \text{ for all } i, j. \end{aligned} \tag{1}$$

3 Algorithm of Presented Method

The proposed algorithm for determining initial basic feasible solution consists of the following steps:

Step 1: Choose the smallest entry of every row and subtract it from each elements of the corresponding row of the TMTP and put them on the right-top of the corresponding elements.

$$t_{ij}^{t_{ij} - t_{ik}}, \text{ where } t_{ik} = \min(t_{i1}, t_{i2}, \ldots\ldots, t_{in}) \quad i = 1, 2, \ldots\ldots, m$$

Step 2: Apply the same process on each of the columns and put them on the left-bottom of the corresponding elements.

$${}_{t_{ij} - t_{kj}} t_{ij}, \text{ where } t_{kj} = \min(t_{1j}, t_{2j}, \ldots\ldots, t_{mj}) \quad j = 1, 2, \ldots\ldots, n$$

Step 3: Form the TOTM whose entries are the summation of right-top and left-bottom elements of Steps 1 and 2.

$$t_{ij} = (t_{ij} - t_{ik}) + (t_{ij} - t_{kj})$$

Step 4: Select the largest time element of every row, $\bar{u}_i$ and place them on the right besides corresponding row.

Step 5: Select the largest time element of every column, $\bar{e}_j$ and place them below the corresponding column.

Step 6: Determine the pointer cost, Δ_{ij} for each cell by subtracting the unit time of each cell from the sum of corresponding row and column highest time.

Step 7: Allocate maximum possible amount to the cell having largest value of the pointer cost, Δ_{ij}. If tie occurs, select the cell where the allocation is maximum.

Step 8: No further consideration is required for the row or column which is fulfilled. If both the row and column are fulfilled at a time, delete a row or a column randomly.

Step 9: Continue Step 4 to Step 8 for the remaining sub-matrix until all rows and columns are satisfied.

Step 10: Shift all the allocations to the original transportation matrix.

Step 11: Determine the largest time T_k corresponding to basic cells.

Step 12: Cross off all the non-allocated cells for which $t_{ij} \geq T_{k.}$

Step 13: Construct a loop associate with largest time T_k including a non-allocated cell in such a way that the allotment in the cell with T_k is shifted to the non-allocated cell in the loop. If no such loop can be formed, the solution under test is optimum. Otherwise move to the next step.

Step 14: Repeat Step 12 and 13 until an optimum basic feasible solution is obtained.

4 The Novelty of Our Algorithm

Here we have used TOTM proposed by Kirca and Satir in our presented algorithm, and we define and calculate the pointer cost (in step 6) by subtracting the time units of every cell of TOTM from the sum of respective row and column highest. We allocate maximum possible amount to the cell corresponding to the highest pointer cost.

5 Numerical Illustration

The proposed algorithm for finding an initial basic feasible solution of time minimizing transportation problem is illustrated by the following two randomly selected examples.

5.1 Example 1 (Table 1)

Iteration 1: 3 is the minimum element of the first row, so we subtract 3 from each elements of the first row. In a similar fashion, we subtract 6 and 5 from each elements of the 2nd and 3rd row respectively and place all the differences on the right-top of the corresponding elements in Table 2.

Iteration 2: In the same way, we subtract 4, 15 and 3 from each elements of the 1st, 2nd and 3rd column respectively and place the result on the left-bottom of the corresponding elements in Table 2.

Iteration 3: We add the right-top and left-bottom entry of each element of the transportation table obtained in Iteration 1 and Iteration 2 and formed the TOTM as following (Table 3)

Table 1. Time minimizing transportation problem for the numerical example

		Destination			
		1	2	3	Supply
Factory	1	4	15	3	80
	2	27	23	6	120
	3	7	26	5	300
Demand		140	90	270	

Table 2. Formation of total opportunity time matrix

		Destination				
		1	2	3	t_{ik}	Supply
Factory	1	${}_{0}4^{1}$	${}_{0}15^{12}$	${}_{0}3^{0}$	3	80
	2	${}_{23}27^{21}$	${}_{8}23^{17}$	${}_{3}6^{0}$	6	120
	3	${}_{3}7^{2}$	${}_{11}26^{21}$	${}_{2}5^{0}$	5	300
t_{kj}		4	15	3		
Demand		140	90	270		

Table 3. Total opportunity time matrix

		Destination			
		1	2	3	Supply
Factory	1	1	12	0	80
	2	44	25	3	120
	3	5	32	2	300
Demand		140	90	270	

Iteration 4: Select the largest time element of every row, $\bar{u}_i$ and place them on the right besides corresponding row. Also select the largest time element of every column, $\bar{e}_j$ and place them below the respective column (Table 4).

Iteration 5: Here, c_{11} is 1, largest unit time in the first row, $\bar{u}_i$ is 12 and in the first column, $\bar{e}_1$ is 44, so $\Delta_{11} = \bar{u}_1 + \bar{e}_1 - c_{11} = 12 + 44\text{-}1 = 55$. In a Similar fashion, we

Table 4. Largest time unit of every row and column

		Destination				
		1	2	3	$\bar{u}_i$	Supply
Factory	1	1	12	0	12	80
	2	44	25	3	44	120
	3	5	32	2	32	300
$\bar{e}_j$		44	32	3		
Demand		140	90	270		

calculate all the values of Δ_{ij}. Since the cell (3, 1) contains the largest value of Δ_{ij}, we allocate 140 units (minimum of 300 and 140) to this cell (Table 5).

Table 5. Initial basic feasible solution after Iteration 5

		Destination				
		1	2	3	$\bar{u}_i$	Supply
Factory	1	55 1	32 12	15 0	12	80
	2	44 44	51 25	44 3	44	120
	3	140 71 5	32 32	33 2	32	300
$\bar{e}_j$		44	32	3		
Demand		140	90	270		

Iteration 6: We adjust the supply and demand requirements corresponding to the cell (3, 1). Since the demand for this cell is satisfied, we delete the first column and calculate all the values of Δ_{ij} again for the resulting reduced transportation table. Since the cell (3, 3) contains the largest value of Δ_{ij}, we allocate 160 units (minimum of 160 and 270) to this cell (Table 6).

Iteration 7: We adjust the supply and demand requirements again corresponding to the cell (3, 3). Since the supply for this cell is satisfied, we delete the third row and

Table 6. Initial basic feasible solution after Iteration 6

		Destination				
		1	2	3	$\bar{u}_i$	Supply
Factory	1		32 12	15 0	12	80
	2		32 25	25 3	25	120
	3	140	32 32	160 33 2	32	160
$\bar{e}_j$			32	3		
Demand			90	270		

calculate all the values of Δ_{ij} again for the resulting reduced transportation table. Since the cell (1, 2), (2, 2) and (2, 3) contains largest value of Δ_{ij}, we allocate 110 units (minimum of 120 and 110) to the cell (2, 3) because the allocation in this cell is maximum corresponding to other cell (Table 7).

Table 7. Initial basic feasible solution after Iteration 7

		Destination				
		1	2	3	$\bar{u}_i$	Supply
Factory	1		25 12	15 0	12	80
	2		25 25	110 25 3	25	120
	3	140		160		
$\bar{e}_j$			25	3		
Demand			90	110		

Iteration 8: We adjust the supply and demand requirements again corresponding to the cell (2, 3). Since the demand for this cell is satisfied, we delete the third column and calculate all the values of Δ_{ij} again for the resulting reduced transportation table. Since

Table 8. Initial basic feasible solution after Iteration 8

		Destination				
		1	2	3	$\bar{u}_i$	Supply
Factory	1		80 25 12		12	80
	2		10 25 25	110	25	10
	3	140		160		
$\bar{e}_j$			25			
Demand			90			

only the second column is remaining with two unallocated cell in this case, we allocate 80 units (minimum of 80 and 90) to the cell (1, 2) and 10 units (minimum of 10 and 10) to the cell (2, 2) (Table 8).

Iteration 9: We adjust the supply and demand requirements again and we see that all supply and demand values are exhausted. Since all the rim conditions are satisfied and total number of allocation is 5.

Therefore, the initial basic feasible solution for the given problem is

$$x_{12} = 80,\ x_{22} = 10,\ x_{23} = 110,\ x_{31} = 140 \text{ and } x_{33} = 160.$$

Iteration 10: We shift all the allocations to the original matrix and see the time of basic cells is

$$t_{12} = 15,\ t_{22} = 23,\ t_{23} = 6,\ t_{31} = 7,\ \text{and } t_{33} = 5.$$

Therefore, the total transportation time required

$$\begin{aligned} T_0 &= \max\{t_{12}, t_{22}, t_{23}, t_{31}, t_{33}\} \\ &= \max\{15, 23, 6, 7, 5\} \\ &= 23 \end{aligned}$$

Optimality Test: Now largest time is $T_0 = 23$ in the cell (2, 2), therefore we cross off the non-basic cells (2, 1) and (3, 2) which contain larger time units than T_0 (Table 9).

Table 9. Optimality test

		Destination			
		1	2	3	Supply
Factory	1	4	$\boxed{80}$ 5	3	80
	2	~~27~~	$\boxed{10}$ 23	$\boxed{110}$ 6	120
	3	$\boxed{140}$ 7	~~26~~	$\boxed{160}$ 5	300
Demand		140	90	270	

Now we cannot form any loop originating from the cell (2, 2). Thus the obtained solution $x_{12} = 80$, $x_{22} = 10$, $x_{23} = 110$, $x_{31} = 140$ and $x_{33} = 160$ is optimum and the optimum time of shipment is max $\{15, 23, 6, 7, 5\} = 23$ time units and iteration no. is 0.

5.2 Example 2

Table 10. Time minimizing transportation problem for the numerical example

		Destination				
		1	2	3	4	Supply
Factory	1	3	6	8	4	20
	2	6	1	2	5	28
	3	7	8	3	9	17
Demand		15	19	13	18	

6 Result

Table 11 shows a comparison for minimum time and no. of iteration required among the solutions obtained by our proposed method and the other existing methods and also with the optimum solution by means of the above sample examples and it is seen that our proposed procedure requires less number of iterations to reach the optimal transportation time.

Table 11. A comparative study of different solutions

No.	Methods Name	Time		No. of iteration	
		Ex. 1	Ex. 2	Ex. 1	Ex. 2
1	Proposed Method	23	7	0	0
2	North West Corner Method	27	9	2	3
3	Row Minimum Method	26	9	2	2
4	Column Minimum Method	23	9	0	2
5	Least Cost Method	27	9	2	2
6	Vogel's Approximation Method	26	9	1	1
7	Extremum Difference Method	26	7	1	0
8	Highest Cost Difference Method	27	9	2	2
9	Average Cost Method	26	8	1	1
10	TOTM-MMM	27	9	2	1
11	TOTM-VAM	26	9	1	1
12	TOTM-EDM	26	9	1	1
13	TOTM-HCDM	27	9	1	1
14	TOTM-SUM	26	7	1	0
15	MTCM-HCDM	27	9	3	3
16	Allocation Table Method	27	9	1	1
	Optimum Solution	23	7	0	0

7 Conclusion

A new procedure for finding an initial basic feasible solution of time minimization transportation problem is presented and also illustrated numerically to test the efficiency of the method. Comparative study among the solution obtained by presented method and the other existing methods is also carried out by means of sample examples and it is seen that our proposed procedure requires less number of iterations to reach the optimal transportation time. Therefore, the algorithm claims its wide application in the field of optimization in solving the time minimization transportation problem.

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