SINR Maximization in Relay-Assisted Multi-user Wireless Networks

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Abstract This paper considers throughput maximization in relays based multi-user wireless networks by enhancing the worst signal-to-interference-plus-noise ratio (SINR) among multiple users. Unlike the existing approaches that use semidefinite relaxation coupled with Gaussian randomization (SDR-G), we utilize the d.c. (difference of two convex functions) structure of the resulting objective function to develop efficient iterative algorithms of low complexity. Numerical results demonstrate that the proposed algorithm locates solutions that are close to the upper bound by a few iterations, and hence, shows better performance than the other methods.

Keywords Beamforming · Wireless relay networks · Interference

1 Introduction

Relay-assisted wireless communication is a very active research topic (see e.g. [1, 2]). Using the spatial diversity of relays, it is possible to expand the range of communication [3]. For multi-user communication, wireless relay networks provide assistance in the communication between multiple sources and destination nodes [4]. To strengthen the signal of interest for a user at the destination, and suppress interferences and noise, a beamforming vector is designed at relay nodes [5].

Most of the previous works on relay beamforming in multiuser systems considered minimization of power consumed by relay nodes under constraints on individual

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signal-to-interference-plus-noise ratio (SINR). This nonconvex optimization problem is then transformed into a relaxed semi-definite program (SDP) [5, 6]. However, due to the individual relay power constraints, the relaxed solution no longer remains a rank-one, and thus remains sub-optimal in terms of performance. In order to further improve the solution, a technique known as randomization [5, 6] is utilized to search for an optimal beamformer vector candidate with appropriate scaling.

One of the key characteristics of the state-of-the-art wireless networks is that they must provide better QoS at a high data rate, expressed via high SINR, under limited power resources. Such requirements lead to a beamformer vector that when applied to the relay nodes provide maximin SINR among the given users. However, such an optimization is a highly non-convex problem. The existing approach is primarily based on using a matrix variable to reformulate the original problem as an SDP that rather increases the size of the problem by many folds.

The major contribution of this correspondence is to use d.c. programming [7] to express the objective function as the difference of two convex functions. Secondly, the concave part of the objective function is convexified at some given point which results into an iteratively decreasing program. Under some given convergence criteria, a suboptimal solution is obtained after a few iterations. In one of our previous works, we have considered a similar problem but in a robust channel environment [8].

The rest of the paper is structured as follows. Section 2 describes the system model and the beamforming problem. Section 3 reformulates the beamforming optimization as a d.c. objective function with convex constraints. Subsequently, with effective convexification an iterative algorithm is suggested to obtain the solutions. Simulation results that demonstrate the effectiveness od the algorithm are given in Section 4. Concluding remarks are described in Section 5.

Notations: Boldfaced uppercase and lowercase characters are used to denote matrices and column vectors. A positive semi-definite matrix (Hermitian) is represented as $\boldsymbol{A} \geq 0$. We define $\langle \boldsymbol{a}, \boldsymbol{b} \rangle := \boldsymbol{a}^T \boldsymbol{b}$ and $||\boldsymbol{a}||^2 := \langle \bar{\boldsymbol{a}}, \boldsymbol{a} \rangle$. Notation for the Hadamard is $\boldsymbol{a} \odot \boldsymbol{b}$.

2 System Model and Problem Formulation

Consider a multiuser system consisting of M source-destination pairs communicating with the help of N relay nodes as shown in Figure 1. While operating in half-duplex mode, two time-slots are used by the relay and user nodes for communication. In the first time-slot, the source send their signal to the relays. Let $\boldsymbol{s} \in \mathcal{C}^M$ be the signals transmitted by M source nodes. The signal is zero mean with variance $\sigma_s = \mathbb{E}[|s_i|^2]$. For $m = 1, 2, ..., M$, we define

$$\boldsymbol{h}_m = (h_{m1}, h_{m2}, ..., h_{mN})^T \in \mathcal{C}^N, \tag{1}$$

as the backward channel vector between the m^{th} source and relay nodes. Similarly, for the forward channel between the relay network and he i^{th} destination we set

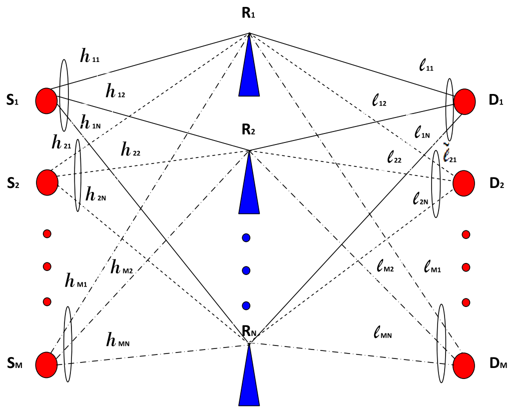

Fig. 1 A multi-user wireless relay network.

$$\boldsymbol{\ell}_i = (\ell_{i1}, \ell_{i2}, ..., \ell_{iN})^T \in \mathcal{C}^N, \ i = 1, 2, ..., M \tag{2}$$

The relays receive the following signal

$$\boldsymbol{y}_{up} = \sum_{m=1}^{M} \boldsymbol{h}_m s_m + \boldsymbol{n}_r, \tag{3}$$

where $\boldsymbol{n}_r \sim \mathcal{N}(0, \sigma_r^2 \boldsymbol{I}_N)$ denotes the additive white Gaussian noises at the relay receivers. Suppose $\boldsymbol{x} = (x_1, x_2, ..., x_N)^T$ are the beamforming weights. After multiplying with these weights, the signals forwarded by the relays to the destinations take the form

$$\boldsymbol{y}_{amp} = \boldsymbol{x} \odot \boldsymbol{y}_{up} = \sum_{m=1}^{M} \boldsymbol{x} \odot \boldsymbol{h}_m s_m + \boldsymbol{x} \odot \boldsymbol{n}_r. \tag{4}$$

Subsequently, the signal received by the destination node i is

$$\begin{aligned} \boldsymbol{y}_{di} &= \langle \boldsymbol{\ell}_i, \boldsymbol{y}_{amp} \rangle + n_{di} \\ &= \sum_{m=1}^{M} \langle \boldsymbol{c}_{mi}, \boldsymbol{x} \rangle s_m + \langle \boldsymbol{\ell}_i \odot \boldsymbol{n}_r, \boldsymbol{x} \rangle + n_{di}, \end{aligned} \tag{5}$$

where $n_{di} \sim \mathcal{N}(0, \sigma_d^2)$, and $\boldsymbol{c}_{mi} = \boldsymbol{\ell}_i \odot \boldsymbol{h}_m$ is the compounded channel vector between source m and destination i.

After defining system parameters, we can now express the signal-to-interference-plus-noise-ratio (SINR) at the i^{th} destination as

$$\boldsymbol{SINR}_i(\boldsymbol{x}) = \frac{\sigma_s^2 |\langle \boldsymbol{x}, \boldsymbol{c}_{ii} \rangle|^2}{\sigma_s^2 \sum_{m \neq i} |\langle \boldsymbol{x}, \boldsymbol{c}_{mi} \rangle|^2 + \boldsymbol{x}^H \boldsymbol{L}_i \boldsymbol{x} + \sigma_d^2}. \tag{6}$$

where $\boldsymbol{L}_i = \sigma_r^2 \text{diag}([|\ell_{i,1}|^2, \ldots, |\ell_{i,N}|^2])$. The beamforming power consumed by the n^{th} relay node is $\boldsymbol{P}_n(x_n) = r_n|x_n|^2$ where $r_n = \sigma_s^2 \sum_{m=1}^{M} |h_{mn}|^2 + \sigma_r^2, n = 1, 2, ..., N$.

Hence, SINR's threshold maximization is given as,

$$\max_{\boldsymbol{x}\in\mathcal{C}^N} \min_{i=1,2,...,M} \boldsymbol{SINR}_i(\boldsymbol{x})$$
$$\text{s.t. } r_n|x_n|^2 \leq \gamma_n, \; n = 1, 2, ..., N, \tag{7}$$

which has a nonconvex objective function, and thus difficult to solve.

3 D.C. Programming Based Solution

This section reformulates the original beamforming design problem (7) as a d.c. optimization problem. It can be noticed that (7) is equivalent to

$$\max_{\boldsymbol{x}\in\mathcal{C}^N, \boldsymbol{y}\in R_+^M} \min_{i=1,2,...,M} \varphi_i(\boldsymbol{x}, y_i) := \frac{|\langle \boldsymbol{x}, \boldsymbol{c}_{ii}\rangle|^2}{y_i + \sigma_d^2/\sigma_s^2} \; : \tag{8a}$$

$$\sum_{m\neq i} |\langle \boldsymbol{x}, \boldsymbol{c}_{mi}\rangle|^2 + \frac{1}{\sigma_s^2}\boldsymbol{x}^H \boldsymbol{L}_i \boldsymbol{x} \leq y_i, \; i = 1, 2, ..., M \tag{8b}$$

$$r_n|x_n|^2 \leq \gamma_n, \; n = 1, 2, ..., N. \tag{8c}$$

It can be proved that each fractional function $|\langle \boldsymbol{x}, \boldsymbol{c}_{ii}\rangle|^2/(y_i + \sigma_D^2/\sigma_s^2)$ is convex (for proof see [9]). Now, although each $\varphi_i(\boldsymbol{x}, y_i)$ is convex, their minimum $\min_{i=1,2,...,M} \varphi(\boldsymbol{x}, y_i)$ is not concave and (8) is not a convex program. However, using decomposition [7]

$$\min_{i=1} \varphi_i(\boldsymbol{x}, y_i) = \sum_{i=1}^{M} \varphi_i(\boldsymbol{x}, y_i) - \max_{i=1,2,...,M} \sum_{j\neq i} \varphi_j(\boldsymbol{x}, y_j)$$

we see that (8) is the following d.c. program

$$-\min_{\boldsymbol{x}\in\mathcal{C}^N, \boldsymbol{y}\in R_+^M} [f_{01}(\boldsymbol{x}, \boldsymbol{y}) - f_{02}(\boldsymbol{x}, \boldsymbol{y})] \; : \; (8b) \tag{9}$$

with the convex functions

$$f_{01}(\boldsymbol{x}, \boldsymbol{y}) := \max_{i=1,2,...M} \sum_{j\neq i} \varphi_j(\boldsymbol{x}, y_j), \;\; f_{02}(\boldsymbol{x}, \boldsymbol{y}) := \sum_{i=1}^{M} \varphi_i(\boldsymbol{x}, y_i), \tag{10}$$

as maximum of convex function and as sum of convex function. Hence, we obtain the following iterative solution

$$\min_{\boldsymbol{x}\in\mathcal{C}^N, \boldsymbol{y}\in R_+^M} [f_{01}(\boldsymbol{x}, \boldsymbol{y}) - f_{02}(\boldsymbol{x}^{(\kappa)}, \boldsymbol{y}^{(\kappa)}) - \sum_{i=1}^{M} \langle \nabla\varphi_i(\boldsymbol{x}^{(\kappa)}, y_i^{(\kappa)}), (\boldsymbol{x}, y_i) - (\boldsymbol{x}^{(\kappa)}, y_i^{(\kappa)})\rangle] : (8b) \tag{11}$$

where

$$\langle \nabla\varphi_i(\boldsymbol{x}^{(\kappa)}, y_i^{(\kappa)}), (\boldsymbol{x}, y_i) - (\boldsymbol{x}^{(\kappa)}, y_i^{(\kappa)})\rangle = \frac{2\text{Re}(\overline{\langle \boldsymbol{x}^{(\kappa)}, \boldsymbol{c}_{ii}\rangle}.\langle \boldsymbol{c}_{ii}, \boldsymbol{x} - \boldsymbol{x}^{(\kappa)}\rangle)}{y_i^{(\kappa)} + \sigma_d^2/\sigma_s^2} - \frac{|\langle \boldsymbol{x}^{(\kappa)}, \boldsymbol{c}_{ii}\rangle|^2 (y_i - y_i^{(\kappa)})}{(y_i^{(\kappa)} + \sigma_d^2/\sigma_s^2)^2}.$$

The detailed procedure of the above algorithm has been described in [8].

4 Numerical Results

For simulations, the powers of backward and forward noises are normalized to $\sigma_{\text{R}}^2 = \sigma_{\text{D}}^2 = 1$. On the other hand, all the sources have the same signal power $\sigma_s^2 = 100$. Generation of the forward and backward channels follows circularly symmetric complex Gaussian distribution in the simulation settings. In order to execute the proposed DCI method, first we solve the following convex program for $n \in \{1, 2, \ldots, N\}$ with any $10\log_{10}(\alpha_i) > 0$ dB

$$\begin{aligned} &\min_{\boldsymbol{X}\in\mathcal{C}^{N\times N}} \max \boldsymbol{X}_{n,n} \quad \text{s.t.} \quad \boldsymbol{X} \geq 0, \\ &\sigma_s^2\langle \boldsymbol{C}_{ii}, \boldsymbol{X}\rangle \geq \alpha_i \left(\sigma_s^2 \sum_{m\neq i}^{M} \langle \boldsymbol{C}_{mi}, \boldsymbol{X}\rangle + \sigma_{\text{R}}^2 \langle \boldsymbol{L}_i, \boldsymbol{X}\rangle + \sigma_{\text{D}}^2 \right) \end{aligned} \tag{12}$$

to obtain the initial $(\boldsymbol{x}^{(0)}, \boldsymbol{y}^{(0)})$ feasible point as explained in [8].

Total relay power is assumed to be equally divided among the relay nodes. We have plotted the minimum SINR among all users achieved by different approaches. The upper bound for the SINR performance can be obtained by solving the relaxed SDPs as explained in [5, 6]. In every figure, the DCI method performs well enough to almost overlap the upper bound of the SDR curve which shows the effectiveness of our proposed method.

Figure 2 shows that with our proposed DCI approach achieves an SINR of 6 for each user when $P_T = 10$ dB. On the other hand, the SINR obtained by SDP randomization only gives 3 dB for the same amount of total relay power. Moreover, the fair distribution of SINR among all users is also not achievable by the randomization technique.

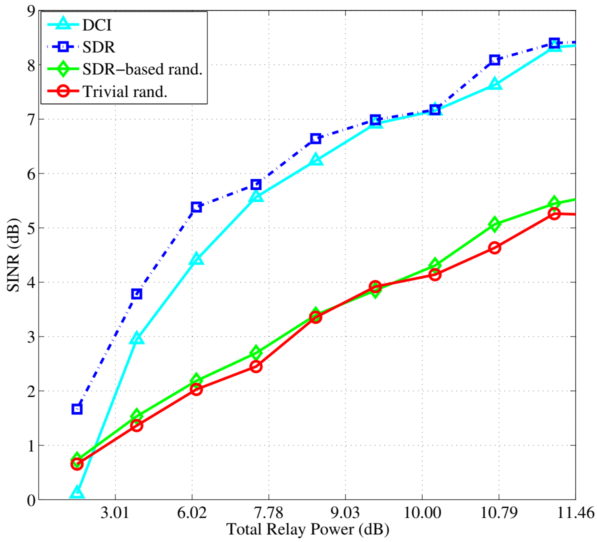

Fig. 2 Minimum SINR plotted against relay power when $M = 3$ and $N = 10$.

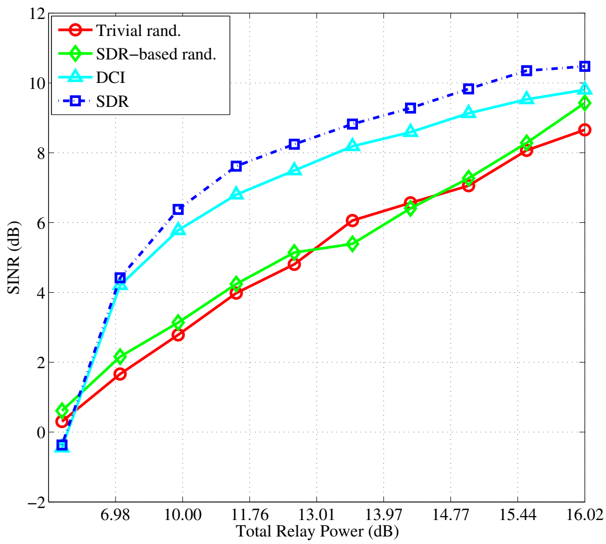

Fig. 3 Minimum SINR plotted against relay power when $M = 4$ and $N = 16$.

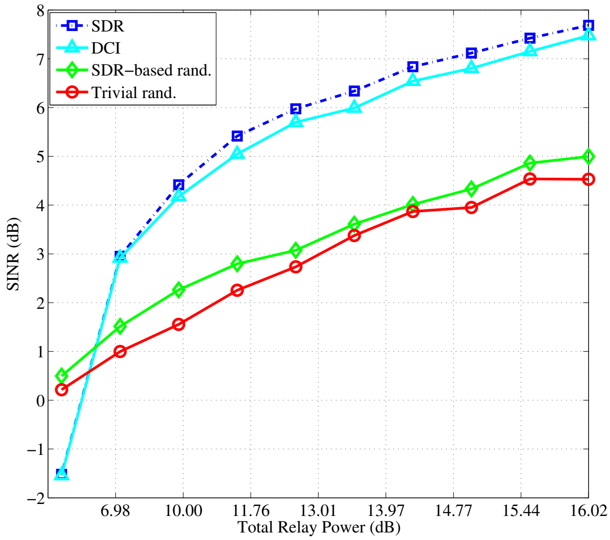

Fig. 4 Minimum SINR plotted against relay power when $M = 5$ and $N = 20$.

Table 1 Numbers of iterations for convergence

$(M, N) = (3, 10)$		$(M, N) = (4, 16)$		$(M, N) = (5, 20)$	
P_T (dB)	Iterations	P_T (dB)	Iterations	P_T (dB)	Iterations
0.04	13.57	0.04	21.03	-10	23.71
4.08	13.25	7.27	20.39	7.51	21.88
6.14	12.62	9.85	20.16	10.48	21.20
7.53	12.11	11.46	18.77	12.23	20.25
8.58	12.23	12.63	18.67	13.48	19.08
9.43	11.33	13.55	17.60	14.44	17.99
10.14	10.15	14.31	16.59	15.27	17.07
10.75	9.16	14.96	15.73	15.90	14.83
11.28	7.95	15.52	12.69	16.48	12.72
11.76	6.11	16.02	7.48	16.99	8.84

Figures 4 and 3 also analyze the impact of increasing number of relay nodes from $N = 12$ to $N = 16$ for a fixed number of users $M = 4$. As indicated, the individual SINR increases when more relays are utilized.

Table 1 presents the iterative convergence performance by the proposed DCI method. Table 2 gives the averaged value of the rank of the sub-optimal matrix-based solution $\boldsymbol{X}_{\text{opt}}$ of SDP [5, 6] at α_{opt}. As it can be seen that due to the high rank of the relaxed SDP based solution, it is not possible to guarantee the optimality. Furthermore, In terms of performance, this randomization is nearly same as a simple trivial randomization achieved by solving the SDP program for a feasible solution $\boldsymbol{X}$ of SDP [5, 6].

Table 2 Average rank i_{opt} of $\boldsymbol{X}_{opt}$ by SDP relaxation

$(M, N) = (3, 10)$		$(M, N) = (4, 16)$		$(M, N) = (5, 20)$	
P_T (dB)	Avg. i_{opt}	P_T (dB)	Avg. i_{opt}	P_T (dB)	Avg. i_{opt}
0.04	2.18	0.04	2.55	0.04	2.55
4.08	2.09	7.27	2.52	7.27	2.58
6.14	2.11	9.85	2.47	9.85	2.62
7.53	2.11	11.46	2.50	11.46	2.64
8.58	2.11	12.63	2.52	12.63	2.62
9.43	2.14	13.55	2.65	13.55	2.62
10.14	2.11	14.31	2.46	14.31	2.57
10.75	2.14	14.96	2.49	14.96	2.63
11.28	2.15	15.52	2.54	15.52	2.59
11.76	2.14	16.02	2.52	16.02	2.55

5 Conclusion

This paper has presented an efficient algorithm for the beamforming design problem while maximizing the worst SINR when multiple users communicate with the assistance of relay nodes. The proposed approach reformulates the original nonconvex problem as a low-dimension d.c. program and then develop an efficient iterative procedure to obtain a local optimal solution. Numerical results demonstrate that the developed algorithms perform much better than the existing relaxed SDP based solutions in terms of the throughput.

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