

Chapter 5

Centrifugal Effects as the Mechanism of the Solar System Creation from a Common Gaseous Cloud

Abstract The irresistible difficulty in cosmogony is the observed fact that the planets of the Solar System, having only $\sim 0.015\%$ of the system mass, possesses 98% of their orbital angular momentum. At the same time, $\sim 99.85\%$ of the Sun's mass produces no more than 2% of the angular momentum, which is accepted to be the conservative parameter. This fact was found in the framework of the hydrostatic equilibrium of the relevant bodies. It is shown in this chapter that, in the framework of Jacobi dynamics, based on new understanding of physics in the matter of gravitational interaction, creation of a new body occurs within the parental cloud as a result of its separation in density on shells, where the outer shell reaches the state of weightlessness. Here the orbital moment of momentum of a created secondary body represents the total kinetic moment of the parental body's cloud owing to the energy conservation law. It means that the orbital moment of momentum of each planet represents the kinetic momentum of the protosun at the time of planet separation and orbiting. The planet's orbital moment of momentum is formed by the total potential energy of the protosun. But the planet's angular moment of axial rotation is formed by the tangential component of the planet's own potential energy. So, the above fact appears to be a misunderstanding. Appearance of weightlessness of the upper shell during a body's matter differentiation, conditions of a body separation and orbiting, the structure of the potential end kinetic energies of a non-uniform body, conditions of dynamical equilibrium of oscillation and rotation of a body, equations of oscillation and rotation of a body and their solution, the nature and mechanism of body shells differentiation and physical meaning of the Archimedes and Coriolis forces are considered as particular tasks, which found here a mathematical solution. The problem of initial values of mean density and radius of a created body also has its own solution. The discussed physics and kinematics of creation and separation of Solar System bodies prove Huygens' law of motion on the semi-cubic parabola of his watch pendulum, which synchronously follows the Earth's motion. Relationship between the evolute and evolvent represents the relationship between function and its derivative or between function and its integral. In the case of the Huygens' oscillating pendulum, the suspension filament starts unrolling in a fixed point. In the case of a celestial body, creation of a satellite starts

in a fixed point of its parental body where the initial conditions are transferred by the third Kepler's law, which is the consequence of a body creation.

5.1 The Conditions for a Body Separation and Orbiting

It is known that all Solar System bodies (the planets, their satellites, comets and meteoric bodies) are identical in their substantial and chemical content, and in this respect they are of common origin. But the search for a unified mechanism of body creation has encountered an irresistible difficulty in their dynamics. The point is that the planets, having only $\sim 0.015\%$ of the system mass, possess 98% of the orbital angular momentum. At the same time, $\sim 99.85\%$ of the Sun's mass produce no more than 2% of the angular momentum, which is accepted to be a conservative parameter. Also, the specific (for unit of the mass) angular momentum of the planets is increased together with the distance from the Sun. As it was discussed in previous chapters, the above results follow from a calculation model based on the hydrostatic equilibrium state of the system, where the body motion results from the outer forces. It was shown that the hydrostatic equilibrium for celestial body dynamics appeared not to be a correct physical conception.

We analyze the evolutionary problem of the Solar System based on fundamentals of Jacobi dynamics, where the body motion initiates by the inner forces' action. Here, the energy loss in the form of radiation is accepted as the physical basis of the body evolution. And the centrifugal effect of elementary particles collision and scattering appears to be the mechanism of the energy generation and its redistribution. It is clear from observation that all celestial bodies are self-gravitating systems.

It is shown next that creation of a new body occurs within the parental cloud because of its separation in density on shells by the Archimedes law, when the outer shell reaches the state of weightlessness. Here, the orbital moment of momentum of a created secondary body represents the total kinetic moment of the parental body's cloud owing to the energy conservation law, as it was shown in (2.20)–(2.21):

$$Q = \sum_i p_i r_i = \sum_i m_i v_i r_i = \sum_i m_i \dot{r}_i r_i = \frac{d}{dt} \left(\sum_i \frac{m_i r_i^2}{2} \right) = \frac{1}{2} \dot{I}_p, \quad (5.1)$$

where Q is the moment of momentum of the parental cloud; p is the moment of a particle; r is the radius; I_p is the polar moment of inertia of the cloud.

It means that the orbital moment of momentum of each planet represents the kinetic momentum of the protosun at the time of planet separation and orbiting. Kinetic moment of a body is equal to the sum of the rotational and oscillating moments, and the kinetic energy is equal to the sum of the rotational and oscillating energy, which follows from the energy conservation law. At the same time, the planet's orbital moment of momentum is formed by the total potential energy of the

protosun. But the planet's angular moment of the axial rotation is formed by the tangential component of the own planet's potential energy (see below Eqs. (5.8) and (5.9)). Here the energy of axial rotation compiles a small portion of the oscillating energy. As it was noted in Sect. 2.4, kinetic energy of the planets Earth, Mars, Jupiter, Saturn, Uranus and Neptune compiles 10^{-3} – 10^{-2} , and of the Mercury, Venus, the Moon and the Sun is about 10^{-4} from the total kinetic energy of each body. For bodies with uniform mass density distribution the kinetic energy of rotation is equal to zero.

The interaction (collision and scattering) of mass particles is accompanied by continuous redistribution of the body's mass density. According to Roche's tidal dynamics, redistribution of mass density leads to shell separation. It will be shown later that, when the density of the upper shell reaches less than two thirds, with respect to the underlying shells, then the upper shell becomes weightless (i.e. it loses weight). From a physical point of view, it means that its own force field of the upper shell is in dynamical equilibrium with the parental force field. In this case, if the density of the upper shell has non-uniform density distribution, then by the difference in the potentials of the force field, the shell is converted into a secondary body. If the upper shell has uniform mass density distribution, then the shell forms a ring around the equatorial plane of the parental body. In the general case, the upper weightless shell decays into fragments with different amounts of mass. The comets were formed from the solar shell, the satellites and meteorites were created from the planet's shells. During evolution of a non-uniform gaseous body, it undergoes axial and equatorial oblateness by an outer force field of the central parental body. This can be observed by inclination of the planet's and satellite's orbital plane slope relative to the parental equatorial plane. The polar outer force field pressure appears to be higher than the equatorial. As a result, the outer polar force field values appeared to be higher than the equatorial. Because of this, the polar matter of the upper shell is continuously removed to the equatorial plane. This is why the created bodies are formed mainly in the equatorial plane and form into an equatorial disk.

So the orbital motion of a separated secondary body is defined by the outer force field at the surface of the parental body. The value of this field at the body's surface is a fundamental parameter, which is determined by the body's law of energy conservation. That is why the orbital velocity of a newly created body is equal to its parental first cosmic velocity. The direction of the orbital motion is determined by Lenz's law (see Fig. 2.3). In this connection, it is worth noting that from the point of view of the Solar System creation problem, attempts to find an explanation of the observed distribution of the moment of momentum between the axial rotation of the Sun and the planets' orbital motion are not fruitful. This is because the planets' orbital velocity demonstrates parental relationship between the planets and the Sun by proving its identity with the first cosmic velocity and the law of energy conservation.

Thus, it follows from the above scenario that, induced by matter, the interaction of the outer force field of the Sun is responsible for orbital motion of the planets. Analogously, the planets' force field is orbiting their satellites. Doing so, each body with high accuracy records the value of the parent's potential energy at the moment of orbiting. As to the shell's axial rotation, then its potential energy is determined

by the value of its tangential component. The normal and tangential components of body's potential energy comprise the total potential energy, which is a conservative parameter.

Justifications of the above dynamical effects, which take part in creation of the Solar System bodies in the framework of Jacobi's dynamics, are presented below. The main dynamical effect, related to the nature of the Solar System body creation, is proved by observational data seen in Tables 2.1 and 2.2.

It was shown earlier in Sect. 4.1 that, in the framework of Jacobi dynamics, solution of the Kepler's problem is given by equations:

$$\sqrt{\Phi} = \frac{B}{A} [1 - \varepsilon \cos(\lambda - \psi)], \quad (5.2)$$

$$t = \frac{4B}{(2A)^{3/2}} [\lambda - \varepsilon \sin(\lambda - \psi)] \quad (5.3)$$

$$\omega = \frac{2\pi}{T} = \frac{(2A)^{3/2}}{4B} = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{4}{3}\pi G\rho_0} \quad (5.4)$$

where ε and ψ are the integration constants depending on the initial values of Jacobi's function Φ and its derivative $\dot{\Phi}$ at the time moment t_0 (the time here is an independent variable); T is the period of virial oscillations; ω is the oscillation; λ is the auxiliary an independent variable; $A = A_0 = 1/2E > 0$, $B = B_0 = U\sqrt{\Phi_0}$ for radial oscillations; $A = A_r = 1/2E > 0$, $B = B_r = U\sqrt{\Phi_r}$ for rotation of the body.

The product of the oscillation frequency ω of the outer force field and R of the body gives the value of the first cosmic velocity of an artificial satellite, that is, the velocity with which the satellite undertakes gravity attraction (the pressure induced by the outer force field). In order to undertake the attraction, a satellite uses its own inner energy of the reactive engine. In this way, the satellite reaches the first cosmic velocity and becomes weightless, i.e. its own outer force field reaches equilibrium with the planet's outer force field. After that, the engine is switched off and its motion continues by the energy of an outer force field. In order to be separated from the parental body, its outer shell must reach a state of weightlessness, that is, its own force field reaches dynamical equilibrium with the parental force field. The secondary body, created from the outer shell, being completely in a non-weighty state and in dynamical equilibrium with the parental outer force field, moves farther by that force field with the first cosmic velocity.

The data of Tables 2.1 and 2.2 show that the existing discrepancies in the moment of momentum distribution between the Sun and the planets and also the problems of capture or separation of the planets' cloud are taking off. The secondary body at its creation conserves the parental potential energy through the first cosmic velocity. As to the direction of orbital motion, the Lenz law works, which evidences about common nature of the gravity and electromagnetic fields. The specific value (per mass unit) of the planets' and satellites' orbital moment of

momentum, which increases with the distance from a central body, has found an explanation by the same reasoning.

Now we come to the problem of appearance of the weightlessness for the body's outer shell at the evolution by radiation of energy. First, we discuss the structure of the potential and kinetic energy of a celestial body.

5.2 The Structure of Potential and Kinetic Energies of a Non-uniform Body

In fact, all the celestial bodies of the Solar System, including the Sun, are non-uniform creatures. They have a shell structure and the shells themselves are also non-uniform components of a body. It was shown in Sect. 2.2 that, according to the artificial satellite data, all the measured gravitational moments of the Earth, including tesseral ones, have significant values. In geophysics, this fact is interpreted as a deviation of the Earth from the hydrostatic equilibrium and attendance of the tangential forces which are continuously developed inside the body. From the viewpoint of the planet's dynamical equilibrium, the fact of the measured zonal and tesseral gravitational moments is a direct evidence of permanent development of the normal and tangential volumetric forces which are the components of the inner gravitational force field. In order to identify the above effects the inner force field of the body should be accordingly separated.

The expressions (2.39)–(2.42) in Chap. 2 indicate that the force function and the polar moment of a non-uniform self-gravitating sphere can be expanded with respect to their components related to the uniform mean density mass and its non-uniformities. In accordance with the superposition principle, these components are responsible for the normal and tangential dynamical effects of a non-uniform body. Such a separation of potential energy and polar moment of inertia through their dimensionless form-factors α and β was done by Garcia et al. (1985) with our interpretation (Ferronsky et al. 1996). Taking into account that the observed satellite irregularities are caused by a non-uniform distribution of the mass density, an auxiliary function relative to the radial density distribution was introduced for the separation:

$$\Psi(s) = \int_0^s \frac{(\rho_r - \rho_0)}{\rho_0} x^2 dx \quad (5.5)$$

where $s = r/R$ is the ratio of the running radius to the radius of the sphere R ; ρ_0 is the mean density of the sphere of radius r ; ρ_r is the radial density; x is the running coordinate; the value $(\rho_r - \rho_0)$ satisfies $\int_0^R (\rho_r - \rho_0) r^2 dr = 0$ and the function $\Psi(1) = 0$.

The function $\Psi(s)$ expresses a radial change in the mass density of the non-uniform sphere relative to its mean value at the distance r/R . Now we can write expressions for the force function and the moment of inertia by using the structural form-factors α and β which were introduced in Sect. 2.7:

$$U = \alpha \frac{GM^2}{R} = 4\pi G \int_0^R r \rho(r) m(r) dr, \quad (5.6)$$

$$I = \beta^2 MR^2 = 4\pi \int_0^R r^4 \rho(r) dr. \quad (5.7)$$

By (5.5) we can do a corresponding change of variables. As a result, the expressions for the potential energy U and polar moment of inertia I are found in the form of their components composed of their uniform and non-uniform constituents (Garcia et al. 1985; Ferronsky et al. 1996):

$$\begin{aligned} U &= 4\pi G \int_0^R r \rho(r) m(r) dr = \alpha \frac{GM^2}{R} \\ &= \left[\frac{3}{5} + 3 \int_0^1 \psi x dx + \frac{9}{2} \int_0^1 \left(\frac{\psi}{x} \right)^2 dx \right] \frac{GM^2}{R}, \end{aligned} \quad (5.8)$$

$$I = \beta^2 MR^2 = \left[\frac{3}{5} - 6 \int_0^1 \psi x dx \right] MR^2. \quad (5.9)$$

It is known that the moment of inertia multiplied by the square of the frequency ω of the oscillation-rotational motion of the mass is the kinetic energy of the body. Then Eq. (5.9) can be rewritten as

$$K = I\omega^2 = \beta^2 MR^2 \omega^2 = \left[\frac{3}{5} - 6 \int_0^1 \psi x dx \right] MR^2 \omega^2. \quad (5.10)$$

Let us clarify the physical meaning of the terms in expressions (5.8) and (5.10) of the potential and kinetic energies.

As it follows from (2.36) and Table 2.5, the first terms in (5.8) and (5.10), numerically equal to $3/5$, represent α_0 and β_0^2 being the structural coefficients of the uniform sphere with radius r , the density of which is equal to its mean value. The ratio of the potential and kinetic energies of such a sphere corresponds to the

condition of the body's dynamical equilibrium when its kinetic energy is realized in the form of oscillations.

The second terms of the expressions can be rewritten in the form

$$3 \int_0^1 \psi x dx \equiv 3 \int_0^1 \left(\frac{\psi}{x} \right) x^2 dx, \quad (5.11)$$

$$-6 \int_0^1 \psi x dx \equiv -6 \int_0^1 \left(\frac{\psi}{x} \right) x^2 dx. \quad (5.12)$$

One can see here that the additive parts of the potential and kinetic energies of the interacting masses of the non-uniformities of each sphere shell, with the uniform sphere having a radius r of the sphere shell that are written there. Note that the structural coefficient β of the kinetic energy is twice as high as the potential energy and has the minus sign. It is known from physics that interaction of mass particles, uniform and non-uniform with respect to density is accompanied by their elastic and inelastic scattering of energy and appearance of a tangential component in their trajectories of motion. In this particular case, the second terms in Eqs. (5.8) and (5.10) express the tangential (torque) component of the potential and kinetic energy of the body. Moreover, the rotational component of the kinetic energy is twice as much as the potential one.

The third term of Eq. (5.8) can be rewritten as

$$\frac{9}{2} \int_0^1 \left(\frac{\psi}{x} \right)^2 dx \equiv \frac{9}{2} \int_0^1 \left(\frac{\psi}{x^2} \right)^2 x^2 dx. \quad (5.13)$$

Here, there is another additive part of the potential energy of the interacting non-uniformities. It is the non-equilibrated part of the potential energy which does not have an appropriate part of the reactive kinetic energy and represents a dissipative component. Dissipative energy represents the electromagnetic energy that is emitted by the body and it determines the body's evolutionary effects. This energy forms the electromagnetic field of the body (see Chap. 7).

Non-uniformity of the density in this case and later is determined as the difference between the density of the given spherical layer and mean value of density of the sphere with radius of the spherical layer.

Thus, by expansion of the expression of the potential energy and the polar moment of inertia, we obtained the components of both forms of energy which are responsible for oscillation and rotation of the non-uniform body. Applying the above results, we can write separate conditions of the dynamical equilibrium for each form of the motion and separate virial equations of the dynamical equilibrium of their motion.

5.3 Equations of Oscillation and Rotation of a Body and Their Solution

Equations (5.8) and (5.10) can be written in the form

$$U = (\alpha_0 + \alpha_t + \alpha_\gamma) \frac{GM^2}{R}, \quad (5.14)$$

$$K = (\beta_0^2 - 2\beta_t^2)MR^2\omega^2, \quad (5.15)$$

where $\alpha_0 = \beta_0^2$ and $\alpha_t = -2\beta_t^2$, the subscripts 0, t , γ define the radial, tangential and dissipative components of the considered values.

Because the potential and kinetic energies of the uniform body are equal ($\alpha_0 = \beta_0^2 = 3/5$) then from (5.8) and (5.10) one has

$$U_0 = K_0, \quad (5.16)$$

$$E_0 = U_0 + K_0 = 2U_0. \quad (5.17)$$

In order to express dynamical equilibrium between the potential and kinetic energies of the non-uniform interacting masses we can write, from (5.8) and (5.10),

$$U_t = 2K_t, \quad (5.18)$$

$$E_t = U_t + K_t = 3U_t, \quad (5.19)$$

where E_t, U_0, K_0, U_t, K_t are the total, potential and kinetic energies of oscillation and rotation accordingly. Note, that the energy is always a positive value.

Equations (5.15)–(5.19) present expressions for uniform and non-uniform components of an oscillating system which serves as the conditions of their dynamical equilibrium. Evidently, the potential energy U_γ of interaction between the non-uniformities, being irradiated from the body's outer shell, is irretrievably lost and provides a mechanism of body's evolution.

In accordance with classical mechanics, for the above-considered non-uniform gravitating body, being a dissipative system, the torque N is not equal to zero, the angular momentum L of the sphere is not a conservative parameter, and its energy is continuously spent during the motion, that is,

$$N = \frac{dL}{dt} \neq 0, \quad L \neq \text{const.}, \quad E \neq \text{const.} > 0.$$

A system physically cannot be conservative if friction or other dissipation forces are present, because Fds due to friction is always positive and an integral cannot vanish (Goldstein 1980),

$$\oint F \cdot ds > 0.$$

After we have found that the resultant of the body's gravitational field is not equal to zero and the system's dynamical equilibrium is maintained by the virial relationship between the potential and kinetic energies, the equations of a self-gravitating body motion can be written.

Earlier we (Ferronsky et al. 1987) used the obtained virial equation for describing and studying the motion of both uniform and non-uniform self-gravitating spheres. Jacobi (1884) derived it from Newton's equations of motion of n mass points and reduced the n -body problem to the particular case of the one-body task with two independent variables, namely, the force function U and the polar moment of inertia Φ , in the form

$$\ddot{\Phi} = 2E - U, \quad (5.20)$$

Equation (5.20) represents the energy conservation law and describes the system in scalar U and Φ volumetric characteristics. In Chap. 3, it was shown that Eq. (5.20) is also derived from Euler's equations for a continuous medium, and from the equations of Hamilton, Einstein, and quantum mechanics. Its time-averaged form gives the Clausius virial theorem for a system with outer source of forces. It was earlier mentioned that Clausius was deducing the theorem for application in thermodynamics and, in particular, as applied to assessment and designing of Carnot's machines. As the machines operate in the Earth's outer force field, Clausius introduced the coefficient $1/2$ to the term of "living force" or kinetic energy, that is,

$$K = \frac{1}{2} \sum_i m_i v_i^2.$$

As Jacobi has noted, the meaning of the introduced coefficient was to taken into account only the kinetic energy generated by the machine, but not by the Earth's gravitational force. That was demonstrated, for instance, by the work of a steam hammer for driving piles. The machine raises the hammer, but it falls down under the action of the force of the Earth's gravity. That is why the coefficient $1/2$ of the kinetic energy of a uniform self-gravitating body in Eqs. (5.8)–(5.10) has disappeared. In its own force field, the body moves due to release of its own energy.

Earlier, by means of relation $U\sqrt{\Phi} \approx \text{const}$, an approximate solution of Eq. (5.20) for a non-uniform body was obtained (Ferronsky et al. 1987, 2011). Now, after expansion of the force function and polar moment of inertia, at $U_\gamma = 0$ and taking into account the conditions of the dynamical equilibrium (5.16) and (5.18), Eq. (5.20) can be written separately for the radial and tangential components in the form

$$\ddot{\Phi}_0 = \frac{1}{2}E_0 - U_0, \quad (5.21)$$

$$\ddot{\Phi}_t = \frac{1}{3}E_t - U_t. \quad (5.22)$$

Taking into account the functional relationship between the potential energy and the polar moment of inertia

$$|U|\sqrt{\Phi} = B = \text{const}$$

and also taking into account that the structural coefficients $\alpha_0 = \beta_0^2$ and $2\alpha_0 = \beta_t^2$, both Eqs. (5.21) and (5.22) are reduced to an equation with one variable and have a rigorous solution

$$\Phi_n = -A + \frac{B_n}{\sqrt{\Phi_n}}, \quad (5.23)$$

where A_n and B_n are the constant values and subscript n defines the non-uniform body.

The general solution of Eq. (5.23) is (5.13) and (5.14):

$$\sqrt{\Phi_n} = \frac{B_n}{A_n} [1 - \varepsilon \cos(\xi - \varphi)], \quad (5.24)$$

$$\omega = \frac{2\pi}{T_v} = \frac{(2A_n)^{3/2}}{4B_n}, \quad (5.25)$$

where ε and φ are, as previously, the integration constants depending on the initial values of Jacobi's function Φ_n and its first derivative $\dot{\Phi}_n$ at the time moment t_0 (the time here is an independent variable); T_v is the period of virial oscillations; ω is the oscillation frequency; ξ is the auxiliary independent variable; $A_n = A_0 - 1/2E_0 > 0$; $B_n = B_0 = U_0\sqrt{\Phi_0}$ for radial oscillations; $A_n = A_t = -1/3E_t, > 0$; $B_n = B_t = U_t\sqrt{\Phi_t}$ for rotation of the body.

The expressions for the Jacobi function and its first derivative in an explicit form can be obtained after transforming them into the Lagrange series:

$$\begin{aligned} \sqrt{\Phi_n} &= \frac{B}{A} \left[1 + \frac{\varepsilon^2}{2} + \left(-\varepsilon + \frac{3}{8}\varepsilon^3 \right) \cos M_c - \frac{\varepsilon^2}{2} \cos 2M_c - \frac{3}{8}\varepsilon^3 \cos 3M_c + \dots \right], \\ \Phi_n &= \frac{B^2}{A^2} \left[1 + \frac{3}{2}\varepsilon^2 + \left(-2\varepsilon + \frac{\varepsilon^3}{4} \right) \cos M_c - \frac{\varepsilon^2}{2} \cos 2M_c - \frac{\varepsilon^3}{4} \cos 3M_c + \dots \right], \\ \dot{\Phi}_n &= \sqrt{\frac{2}{A}} \varepsilon B \left[\sin M_c + \frac{1}{2}\varepsilon \sin 2M_c + \frac{\varepsilon^2}{2} \sin M_c (2 \cos^2 M_c - \sin^2 M_c) + \dots \right]. \end{aligned} \quad (5.26)$$

Radial frequency of oscillation ω_{or} and angular velocity of rotation ω_{tr} of the shells of radius r can be rewritten from (5.25) as

$$\omega_{or} = \frac{(2A_0)^{3/2}}{4B_0} = \sqrt{\frac{U_{0r}}{J_{0r}}} = \sqrt{\frac{\alpha_{0r}^2 G m_r}{\beta_{0r}^2 r^3}} = \sqrt{\frac{4}{3} \pi G \rho_{0r}}, \quad (5.27)$$

$$\omega_{tr} = \frac{(2A_t)^{3/2}}{4B_t} = \sqrt{\frac{2U_{tr}}{J_{tr}}} = \sqrt{\frac{2\alpha_{tr}^2 G m_r}{\beta_{tr}^2 r^3}} = \sqrt{\frac{4}{3} \pi G \rho_{0r} k_{er}}, \quad (5.28)$$

where U_{0r} and U_{tr} are the radial and tangential components of the force function (potential energy); J_{0r} and $J_{tr} = 2/3 J_{0r}$ are the polar and axial moment of inertia; $\rho_{0r} = \frac{1}{V_r} \int \rho(r) dV_r$; $\rho(r)$ is the law of radial density distribution; ρ_{0r} is the mean density value of the sphere with a radius r ; V_r is the sphere volume with a radius r ; $2\alpha_{tr} = \beta_{tr}^2$; k_{er} is the dimensionless coefficient of the energy dissipation or tidal friction of the shells equal to the shell's oblateness.

The relations (5.24)–(5.25) represent Kepler's laws of body rotation in dynamical equilibrium. In the case of uniform mass density distribution, the frequency of oscillation of the sphere's shells with radius r is $\omega_{or} = \omega_0 = \text{const}$. It means that here all the shells are oscillating with the same frequency. Thus, it appears that only non-uniform bodies are rotating systems.

Rotation of each body's shell depends on the effect of the potential energy scattering at the interaction of masses of different density. As a result, a tangential component of energy appears which is defined by the coefficient k_{er} . In geodynamics, the coefficient is known as the geodynamical parameter. Its value is equal to the ratio of the radial oscillation frequency and the angular velocity of a shell and can be obtained from Eqs. (5.27)–(5.28), that is,

$$k_e = \frac{\omega_t^2}{\omega_0^2} = \frac{\omega_r^2}{\frac{4}{3} \pi G \rho_0}. \quad (5.29)$$

It was found in the general case of a three-axial (a , b , c) ellipsoid with the ellipsoidal law of density distribution, the dimensionless coefficient $k_e \in [0, 1]$ is equal (Ferronsky et al. 1987, 2011)

$$k_e = \frac{F(\varphi, f)}{\sin \varphi} \bigg/ \frac{a^2 + b^2 + c^2}{3a^2},$$

where $\varphi = \arcsin \sqrt{\frac{a^2 - c^2}{a^2}}$, $f = \sqrt{\frac{a^2 - b^2}{a^2 - c^2}}$, and $F(\varphi, f)$ is an incomplete elliptic integral of the first degree in the normal Legendre form.

Thus, in addition to the earlier obtained solution of radial oscillations (Ferronsky et al. 1987, 2011), now we have a solution of its rotation. It is seen from expression

(5.27) that the shell oscillations do not depend on the phase state of the body's mass and are determined by its density.

It follows from Eqs. (5.24) and (5.28) that in order to obtain the frequency of oscillation and angular velocity of rotation of a non-uniform body, the law of radial density distribution should be revealed. This problem will be considered later on. But before that the problem of the nature of a body shells separation with respect to their density needs to be solved.

5.4 The Nature and Mechanism of Body's Shell Differentiation by Action of the Centrifugal Roche's Dynamics

It is well known that celestial bodies have a quasi-spherical shell structure. This phenomenon has been confirmed by recording and interpretation of seismic longitudinal and transversal wave propagation during earthquakes. The phenomenon is explained by the centrifugal effect of the energy interaction of a body's elementary particles. In order to demonstrate physics and mechanism of a body mass differentiation with respect to its density, we apply Roche's tidal (centrifugal) dynamics.

Newton's theorem of gravitational interaction between a material point and a spherical layer states that the layer does not affect a point located inside the layer. On the contrary, the outside-located material point is affected by the spherical layer. Roche's tidal dynamics is based on the above theorem. His approach is as follows (Ferronsky et al. 1996).

There are two bodies of masses M and m interacting in accordance with Newton's law (Fig. 5.1a)

Let $M \gg m$ and $R \gg r$, where r is the radius of the body m , and R is the distance between the bodies M and m . Assuming that the mass of the body M is uniformly distributed within a sphere of radius R , we can write the accelerations of the points A and B of the body m as

$$q_A = \frac{GM}{(R - r)^2} - \frac{Gm}{r^2}, \quad q_B = \frac{GM}{(R + r)^2} + \frac{Gm}{r^2}.$$

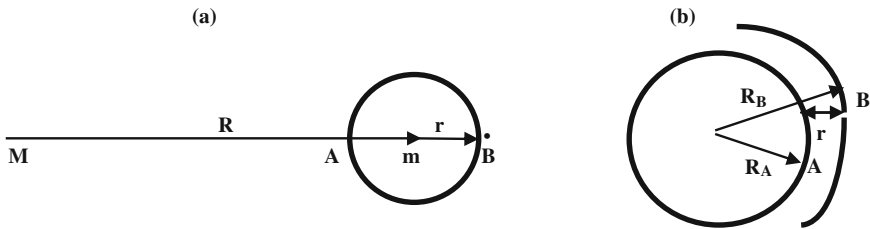


Fig. 5.1 The tidal gravitational stability of a sphere (a) and the sphere layer (b)

The relative tidal acceleration of the points A and B is

$$\begin{aligned}
 q_{AB} &= G \left[\frac{M}{(R-r)^2} - \frac{M}{(R+r)^2} - \frac{2m}{r^2} \right] \\
 &= \frac{4\pi}{3} G \left[\rho_M R^3 \frac{Rr}{(R^2-r^2)^2} - 2\rho_m r \right] \approx \frac{8\pi}{3} Gr(2\rho_M - \rho_m)
 \end{aligned} \tag{5.30}$$

Here $\rho_M = M/\frac{4}{3}\pi R^3$ and $\rho_m = m/\frac{4}{3}\pi r^3$ are the mean density distributions for the spheres of radius R and r . Roche's criterion states that the body with mass m is stable against the tidal force disruption of the body M , if the mean density of the body m is at least twice as high as that of the body M in the sphere with radius R . Roche considered the problem of the interaction between two spherical bodies without any interest in their creation history and in how the forces appeared. From the point of view of the origin of celestial bodies and of the interpretation of dynamical effects, we are interested in the tidal stability of separate envelopes of the same body. For this purpose, we can apply Roche's tidal dynamics to study the stability of a non-uniform spherical envelope.

Let us assess the tidal stability of a spherical layer of radius R and thickness $r = R_B - R_A$ (Fig. 5.1b). The layer of mass m and mean density $\rho_m = m/4\pi R_A^2 r$ is affected at point A by the tidal force of the sphere of radius R_A . The mass of the sphere is M and mean density $\rho_M = M/\frac{4}{3}\pi R_A^3$. The tidal force in point B is generated by the sphere of radius $R+r$ and mass $M+m$. Then the accelerations of the points A and B are

$$q_A = \frac{GM}{R_A^2} \text{ and } q_B = \frac{G(M+m)}{(R_A+r)^2}.$$

Then the accelerations of the points A and B are

$$q_A = \frac{GM}{R_A^2}, \quad q_B = \frac{G(M+m)}{(R_A+r)^2}.$$

The relative tidal acceleration of the points A and B` is

$$\begin{aligned}
 q_{AB} &= GM \left[\frac{1}{R_A^2} - \frac{1}{(R_A+r)^2} \right] - \frac{Gm}{(R_A+r)^2} \\
 &= \left(\frac{8}{3} \pi G \rho_M - 4\pi G \rho_m \right) r = 4\pi \pi G \left(\frac{2}{3} \rho_M - \rho_m \right), (R \gg r).
 \end{aligned} \tag{5.31}$$

Equations (5.30)–(5.31) give the possibility to understand the centrifugal nature of a body shell separation including some other dynamical effects.

5.5 Self-similarity Principle and Radial Component of Non-uniform Sphere

It follows from Eq. (5.31) that in the case of the uniform density distribution ($\rho_m = \rho_M$), all spherical layers of the gravitating sphere move to the centre with accelerations and velocities which are proportional to the distance from the centre. It means that such a sphere contracts without loss of its uniformity. This property of self-similarity of a dynamical system without any discrete scale is unique for a uniform body (Ferronsky et al. 1996).

A continuous system with a uniform density distribution is also ideal from the point of view of Roche's criterion of stability with respect to the tidal effect. That is why there is a deep physical meaning in separation of the first term of potential energy in expression (5.8). A uniform sphere is always similar in its structure in spite of the fact that it is a continuously contracting system. Here, we do not consider the Coulomb forces effect. In this case, we have considered the specific proton and electron branches of the evolution of the body.

Note that in Newton's interpretation the potential energy has a non-additive category. It cannot be localized even in the simplest case of the interaction between two mass points. In our case of a gravitating sphere as a continuous body, for the interpretation of the additive component of the potential energy we can apply Hooke's concept. According to Hooke, there is a linear relationship between the force and the caused displacement. Therefore, the displacement is in square dependence on the potential energy. Hooke's energy belongs to the additive parameters. In the considered case of a gravitating sphere, the Newton force acting on each spherical layer is proportional to its distance from the centre. Thus, here from the physical point of view, the interpretations of Newton and Hooke are identical.

At the same time in the two approaches there is a principal difference even in the case of uniform distribution of the body density. According to Hooke, the cause of displacement, relative to the system, is the action of the outer force. And if the total energy is equal to the potential energy, then equilibrium of the body is achieved. The potential energy here plays the role of elastic energy. The same uniform sphere with Newton's forces will be contracted. All the body's elementary shells will move without change of uniformity in the density distribution. But the first terms of Eqs. (5.8)–(5.10) show that the tidal effects of a uniform body restrict motion of the interacting shells towards the centre. In accordance with Newton's third law and the d'Alembert principle the attraction forces, under the action of which the shells move, should have equally and oppositely direct forces of Hooke's elastic counteraction. In the framework of the elastic gravitational interaction of shells, the dynamical equilibrium of a uniform sphere is achieved in the form of its elastic oscillations with equality between the potential and kinetic energy. The uniform sphere is dynamically stable relative to the tidal forces in all of its shells during the time of the system contraction. Because the potential and kinetic energies of a sphere are equal, then its total energy in the framework of the averaged virial

theorem within one period of oscillation is accepted formally as equal to zero. Equality of the potential and kinetic energy of each shell means the equality of the centripetal (gravitational) and centrifugal (elastic constraint) accelerations. This guarantees the system remaining in dynamical equilibrium. On the contrary, all the spherical shells will be contracted towards the gravity centre which, in the case of the sphere, coincides with the inertia centre but does not coincide with the geometric centre of the masses. Because the gravitational forces are acting continuously, the elastic constraint forces of the body's shells are reacting also continuously. The physical meaning of the self-gravitation of a continuous body consists in the permanent work which applies the energy of the interacting shell masses on one side and the energy of the elastic reaction of the same masses in the form of oscillating motion on the other side. At dynamical equilibrium the body's equality of potential and kinetic energy means that the shell motion should be restricted by the elastic oscillation amplitude of the system. Such an oscillation is similar to the standing wave which appears without transfer of energy into outer space. In this case the radial forces of the shell's elastic interactions along the outer boundary sphere should have a dynamical equilibrium with the forces of the outer gravitational field. This is the condition of the system to be held in the outer force field of a mother's body. Because of this, while studying the dynamics of a conservative system, its rejected outer force field should be replaced by the corresponding equilibrated forces as they do, for instance, in Hooke's theory of elasticity.

Thus, from the point of view of dynamical equilibrium, the first terms in Eqs. (5.8) and (5.10) represent the energy which provides the field of the radial forces in a non-uniform sphere. Here, the potential energy of the uniform component plays the role of the active force function, and the kinetic energy is the function of the elastic constraint forces.

5.6 Charges-like Motion of Non-uniformities and Tangential Component of the Force Function

Let us now discuss the tidal motion of non-uniformities due to their interactions with the uniform body. The potential and kinetic energies of these interactions are given by the second terms in Eqs. (5.8) and (5.10). In accordance with (5.31), the non-uniformity motion looks like the motion of electrical charges interacting on the background of a uniform sphere contraction. Spherical layers with densities exceeding those of the uniform body (positive anomalies) come together and move to the centre in elliptic trajectories. The layers with deficit of the density (negative anomalies) come together, but move from the centre on the parabolic path. Similar anomalies come together, but those with the opposite sign are dispersed with forces proportional to the layer radius. In general, the system tends to reach a uniform and equilibrium state by means of redistribution of its density up to the uniform limit.

Both motions happen not relative to the empty space, but relative to the oscillating motion of the uniform sphere with a mean density. Separate consideration of motion of the uniform and non-uniform components of a heterogeneous sphere is justified by the superposition principle of the forces action which we keep here in mind. The considered motion of the non-uniformities looks like the motion of the positive and negative charges interacting on the background of the field of the uniformly dense sphere (Ferronsky et al. 1996). One can see here that in the case of gravitational interaction of mass particles of a continuous body, their motion is the consequence not only of mutual attraction, but also mutual repulsion by the same law $1/r^2$. In fact, in the case of a real natural non-uniform body it appears that the Newton and Coulomb laws are identical in details. Later on, while considering a body's by-density differentiated masses, the same picture of motion of the positive and negative anomalies will be seen.

If the sphere shells, in turn, include density non-uniformities, then by means of Roche's dynamics it is possible to show that the picture of the non-uniformity motion does not differ from that considered above.

In physics, the process of interaction of particles with different masses without redistribution of their moments is called elastic scattering. The interaction process resulting in redistribution of their moments and change in the inner state or structure is called inelastic scattering. In classical mechanics, while solving the problems of motion of the uniform conservative systems (like motion of the material point in the central field or motion of the rigid body), the effects of the energy scattering do not appear. In the problem of dynamics of the self-gravitating body, where interaction of the shells with different masses and densities are considered, the elastic and inelastic scattering of the energy becomes an evident fact following from consideration of the physical meaning of the expansion of the energy expressions in the form of (5.8) and (5.10). In particular, their second terms represent the potential and kinetic energies of gravitational interaction of masses having a non-uniform density with the uniform mass and express the effect of elastic scattering of density-different shells. Both terms differ only in the numeric coefficient and sign. The difference in the numerical coefficient evidences that the potential energy here is equal to half of the kinetic one ($U_p = 1/2K$). This part of the active and reactive force function characterizes the degree of the non-coincidence of the volumetric centre of inertia and that of the gravity centre of the system. This effect is realized in the form of the angular momentum relative to the inertia centre.

Thus, we find that inelastic interaction of the non-uniformities with the uniform component of the system generates the tangential force field which is responsible for the system rotation. In other words, in the scalar force field of the by-density uniform body the vector component appears. In such a case, we can say that, by analogy with an electromagnetic field, in the gravitational scalar potential field of the non-uniform sphere $U(R, t)$ the vector potential $A(R, t)$ appears for which $U = \text{rot } A$ and the field $U(R, t)$ will be solenoidal. In this field, the conditions for vortex motion of the masses are born, where $\text{div } A = 0$. This vector field, which in electrodynamics is called solenoidal, can be represented by the sum of the potential and vector fields. The fields, in addition to the energy, acquire moments and have a

discrete-wave structure. In our case, the source of the wave effects appears to be the interaction between the elementary shells of the masses by means of which we can construct a continuous body with a high symmetry of forms and properties. The source of the discrete effects can be represented by the interacting structural components of the shells, namely, atoms, molecules and their aggregates. We shall continue discussion about the nature of the gravitational and electromagnetic energy in Chap. 7.

5.7 Centrifugal Nature of the Archimedes and Coriolis Forces

The Archimedes principle states: *The apparent loss in weight of a body totally or partially immersed in a liquid is equal to the weight of the liquid displaced.* We saw in Sect. 5.4 that the principle is described by Eq. (5.31) and the forces that sink down or push out the body or the shell are of a gravitational nature. In fact, in the case of $\rho_m = \rho_M$ the body immersed in a liquid (or in any other medium) is kept in place due to equilibrium between the forces of the body's weight and the forces of the liquid reaction. In the case of $\rho_m > \rho_M$ or $\rho_m < \rho_M$, the body is sinking or floating up depending on the resultant of the above forces. Thus, the Archimedes forces seem to have a gravity nature and are the radial component of the body's inner force field.

It is assumed that the Coriolis forces appeared as an effect of the body motion in the rotational system of co-ordinates relative to the inertial reference system. In this case, rotation of the body is accepted as the inertial motion and the Coriolis forces appear to be the inertial ones. It follows from the solution of Eq. (5.22) that the Coriolis' forces appear to be the tangential component of body's inner force field, and the body rotation is caused by the moment of those forces that are relative to the three-dimensional centre of inertia which also does not coincide with the three-dimensional gravity centre.

In accordance with Eq. (5.31) of the tidal acceleration of an outer non-uniform spherical layer at $\rho_M \neq \rho_m$, the mechanism of the centrifugal density differentiation of masses is revealed. If $\rho_M < \rho_m$, then the shell immerses (is attracted) up to the level where $\rho_M = \rho_m$. At $\rho_M > \rho_m$ the shell floats up to the level where $\rho_M = \rho_m$ and at $\rho_M > 2/3\rho_m$ the shell becomes a self-gravitating one. Thus, in the case when the density increases towards the sphere's centre, which is the Earth's case, then each overlying stratum appears to be in a suspended state due to repulsion by the Archimedes forces which, in fact, are a radial component of the gravitational interaction forces.

The effect of the gravitational differentiation of masses explains the nature of creation of shell-structured celestial bodies and corresponding processes (for instance, the Earth's crust and its oceans, geotectonic, orogenic and seismic processes, including earthquakes). All these phenomena appear to be a consequence of the continuous gravitational differentiation in density of the planet's masses. We

assume that creation of the Earth and the Solar System as a whole resulted from this effect. For instance, the mean value of the Moon's density is less than $2/3$ of the Earth's, i.e., $\rho_M < 2/3\rho_m$. If one assumes that this relation was maintained during the Moon's formation, then, in accordance with Eq. (5.31), this body separated at the earliest stage of the Earth's mass differentiation. Creation of the body from the separated shell should occur by means of the cyclonic eddy mechanism, which was proposed in due time by Descartes and which was unjustly rejected. If we take into account existence of the tangential forces in the non-uniform mass, then the above mechanism seems to be realistic.

Thus, we learned the nature and mechanism of an initially heavy outer shell of a self-gravitating body into a weightlessness state. Such a weightlessness shell, by its own tangential component of the potential energy is transferred into vortex cloud and after reaching dynamical equilibrium (self-gravitating state) becomes a planet, satellite or any other body. In the case of uniform density of the weightless shell, it transfers into a nebula, equatorial ring or diffuse matter. The orbital motion of a newly created planet, or a satellite of another body is determined by the first cosmic velocity of the parental body. And the axial rotation depends on the value of non-uniformity in density.

5.8 Initial Values of Mean Density and Radius of a Secondary Body

Thus, it follows from Eq. 5.31 that the outer shell of a gaseous body, after reaching its density equal to $2/3$ from the mean value of the total body, becomes weightless. If the shell's own density is non-uniform, by its tangential component of the energy the shell is transferred into a secondary body in the form of a vortex creature. As seen from observation, new bodies are formed in different regions of a protoparent body's surface. The large-in-mass bodies like stars, planets and satellites are formed mainly in the equatorial zone due to difference in value for the polar and equatorial outer force field. Because of this a new body inherits the polar and equatorial obliquity, the value of which reflects degree of the non-uniformity of its density. The comets, asteroids and smaller bodies are formed in the other regions of the parental bodies. The high eccentric orbits of such bodies prove this fact. The inclination of the new body's orbital plane relative to the parental equatorial plane can be up to close to 180° .

The following initial values of density ρ_i and radius R_i of the protosun and protoplanets can be obtained on the basis of their dynamic equilibrium state.

The protosolar gaseous cloud has separated from the protogalaxy body when its outer shell in the equatorial domain has reached the state of weightlessness. In fact, the gaseous cloud should represent a chemically non-homogeneous rotating body. As it follows from Roche's dynamics (Eq. 5.31), the mean density of the gaseous protogalaxy outer shell should be $\rho_s = 2/3\rho_g$. The condition $\rho_s = 2/3\rho_g$ is the starting point of separation and creation of the protosun from the outer protogalaxy shell.

Accepting the above described mechanism of formation of the secondary body, we can find the mean density of the protogalaxy at the moment of the protosun separation as

$$\begin{aligned}\rho_g &= \frac{m_\Gamma}{\frac{4}{3}\pi R^3} = \frac{2.5 \times 10^{41}}{\frac{4}{3} \times 3.14 \times (2.5 \times 10^{20})^3} = 1.67 \times 10^{-21} \text{ kg/M}^3 \\ &= 1.67 \times 10^{-24} \text{ g/cm}^3.\end{aligned}$$

Here the protogalaxy radius is equal to the semi-major orbital axis of the protosun, i.e. $R_u = 2.5 \times 10^{20}$ m.

The mean density of the separated protogalaxy shell is

$$\rho_c = 2/3\rho_g = 2/3 \times 1.67 \times 10^{-24} = 1.11 \times 10^{-24} \text{ g/cm}^3.$$

In accordance with Eq. (5.30), the mean density and radius of the initially created protosun body should be

$$\begin{aligned}\rho_s &= 2\rho_g = 2 \times 1.67 \times 10^{-24} = 3.34 \times 10^{-24} \text{ g/cm}^3; \\ R_c &= \sqrt[3]{\frac{2 \times 10^{33}}{\frac{4}{3} \times 3.34 \times 10^{-24}}} = 7.5 \times 10^{18} \text{ cm} = 7.5 \times 10^{16} \text{ m}\end{aligned}$$

The mean density and the radius of the initially created protojupiter, protoearth and Protomoon are as follows:

the protojupiter: $\rho_j = 2 \times 10^{-9} \text{ g/cm}^3$, $R_j = 6.2 \times 10^{13} \text{ cm} = 6.2 \times 10^{11} \text{ m}$;
the protoearth: $\rho_e = 2.85 \times 10^{-7} \text{ g/cm}^3$, $R_e = 1.9 \times 10^{11} \text{ cm} = 1.9 \times 10^9 \text{ m}$;
the Protomoon: $\rho_m = 5 \times 10^{-4} \text{ g/cm}^3$, $R_m = 1.1 \times 10^9 \text{ cm} = 1.1 \times 10^7 \text{ m}$;

An analogous unified process was repeated for all the planets and their satellites.

From the analysis of the above observational and calculated data, the following conclusions are made:

1. The planets of the Solar System were created from a common non-uniform in density self-gravitating protosolar cloud, which has separated during evolution on shells with different densities. In accordance with Roche's tidal dynamics, after the outer shell has reached a density equal to 2/3 from the cloud's mean value (the condition of the weightlessness relative to the total body), by the inner force field and the tangential component of the potential energy, the protoplanets after became self-gravitating bodies, were formed and separated. Analogous processes have taken place at creation of satellites from planets. In addition, accumulation of "light" matter in the outer shells took place gradually and accompanied by separation of small portions in the form of comets and other bodies and dust matter being weightlessness relative to the surrounding weighted bodies.

2. The orbital velocities of the planets and satellites, which corresponds to the first cosmic velocity of the parental bodies, appears to be an effect of the outer force field, which is realized at the moment when the shell reaches its weightlessness state. The orbital motion of the planets, satellites and other bodies in the outer force field results by the laws of electrodynamics.
3. The small planets of the asteroid belt have created from the protosolad cloud by the common law. Appraising by orbital velocities, there are no features of their creation because of a body destruction.
4. The axial rotation of the Sun, planets and satellites has taken and takes place by tangential component of the inner force field. The axial rotation has never been inertial like a rigid body. The body's angular moment depends on the friction (weight) of the rotating masses and, to the contrary of the orbital moment of momentum, it does not remain a conservative value. The orbital angular momentum is the fundamental and conservative parameter because it expresses the law of the body's energy conservation law. The angular momentum of the Sun itself expresses only the tangential component of its potential energy which is a small part of the total potential energy of the body (see minus sign in Eq. 5.10). The direction of revolution and rotation of all the planets and satellites is governed by the force field of the parental body and determined, as in electrodynamics, by Lenz's law.

The discussed physics and kinematics of creation and separation of Solar System bodies prove the Huygens' law of motion on semi-cubic parabola of his watch pendulum, which synchronously follows the Earth's motion. Relationship between the evolute and Huygens pendulum clockevolent represents the relationship between a function and its derivative or between function and its integral. For the Huygens' oscillating pendulum the suspension filament starts unrolling in a fixed point. In the case of a celestial body, creation of a satellite starts in a fixed point of its parental body where the initial conditions are transferred by the third Kepler's law, which is the consequence of a body creation.

References

- Ferronsky VI, Denisik SA, Ferronsky SV (1987) *Jacobi dynamics*. Reidel, Dordrecht
- Ferronsky VI, Denisik SA, Ferronsky SV (2011) *Jacobi dynamics*, 2nd edn. Springer, Dordrecht/Heidelberg
- Ferronsky VI, Denisik SA, Ferronsky SV (1996) Virial oscillations of celestial bodies: V. The structure of the potential and kinetic energies of a celestial body as a record of its creation History. *Celest Mech Dyn Astron* 64:167–183
- Garcia LD, Mosconi MB, Sersic JL (1985) A global model for violent relaxation. *Astrophys Space Sci* 113:89–98
- Goldstein H (1980) *Classical mechanics*, 2nd edn. Addison-Wesley, Reading, Massachusetts
- Jacobi CGJ (1884) *Vorlesungen über Dynamik*. Klebsch, Berlin