Choquet Integral with Interval Type 2 Sugeno Measures as an Integration Method for Modular Neural Networks

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Abstract In this paper, a new method for response integration, based on the Choquet integral with interval type 2 Sugeno measures, is presented. Type 1 and interval type 2 fuzzy systems for edge detection based on the Sobel and morphological gradient are used, which is a preprocessing system applied to the training data for better performance in the modular neural network. Fuzzy Sugeno measures are represented by an interval type 2 fuzzy system. The Choquet integral is used as a method to integrate the outputs of the modules of the modular neural networks (MNN). A database of faces was used to perform the preprocessing, the training, and the combination of information sources of the MNN.

1 Introduction

An integration method is a mechanism which takes input as a number n of data and combines them to form a value representative of the information, methods exist which combine information from different sources which can be aggregation operators as arithmetic mean, geometric mean, ordered weighted averaging (OWA) [1], and inter alia.

Artificial neural networks were introduced by W.S. McCullogh and W. Pitts in 1943 [2] and can be used in a variety of applications; however, there are problems that cannot be processed in a single network either because of their complexity or

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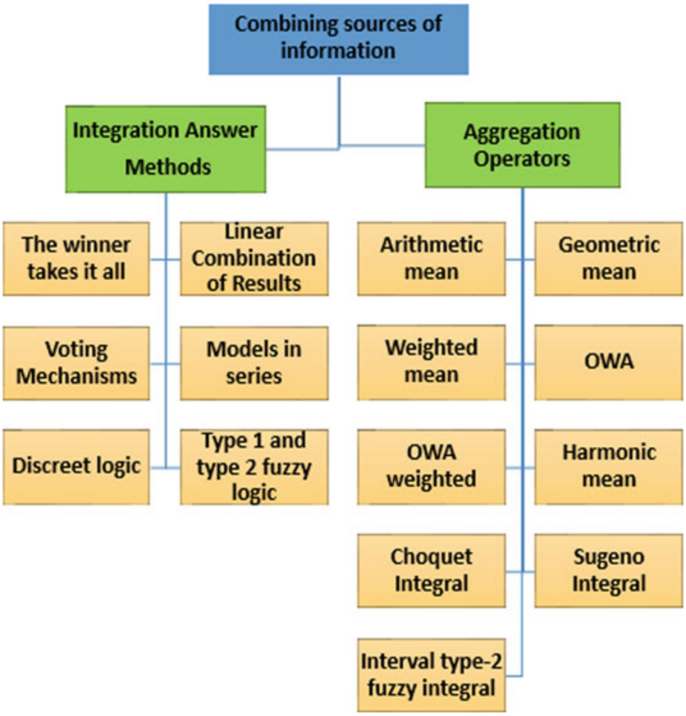

Fig. 1 Methods for combining information from different sources

the amount of information they have; in these cases, a modular neural network (MNN) is used so that each of the modules is responsible for a sub-task or a part of the problem [3–7]. Therefore, it is necessary to have a mechanism to integrate information from different modules to provide the general solution to the problem.

In a MNN, it is common to use some methods such as fuzzy logic type 1 and type 2 [8–10], the fuzzy Sugeno integral [11], interval type 2 fuzzy logic Sugeno integral [12], a probabilistic sum integrator [13], a Bayesian learning method [14], among others, as shown in Fig. 1.

The Choquet integral is an aggregation operator, which has been successfully used in various applications [15–17]. In this paper, the fuzzy Sugeno measures are represented by an interval type 2 fuzzy system in combination with the Choquet integral.

This paper is organized as follows: Sect. 2 shows the concepts of fuzzy measures, interval type 2 fuzzy measures, and Choquet integral which is the technique that was applied for the combination of the several information sources. Section 3 shows edge detection based on Sobel and morphological gradient with interval type 2 fuzzy system. Section 4 shows the modular neural network proposal, and in Sect. 5, the simulation results are shown. Finally, Sect. 6 shows the Conclusions.

2 Fuzzy Measures and Choquet Integral

Initially, Michio Sugeno defined the concept of "fuzzy measures and fuzzy integral" in 1974 [18]. A fuzzy measure is a non-negative monotone function of defined values in "classical sets." Currently, when referring to this topic, the term "fuzzy measures" has been replaced by the term "monotonic measures," "non-additive measures," or "generalized measures" [19–21]. When fuzzy measures are defined on fuzzy sets, we speak about monotonous fuzzified measures [21].

2.1 Fuzzy Measure

If $x = \{x_1, x_2, \ldots, x_n\}$ is a finite set, a fuzzy measure μ with respect to the data set X is a function $\mu : 2^x \rightarrow [0, 1]$ that must satisfy the following conditions:

(1) $\mu(X) = 1$; $\mu(\varnothing) = 0$
(2) If $A \subseteq B$, then $\mu(\mathrm{A}) \leq \mu(B)$

where in the second condition, A and B are subsets of X.

A fuzzy measure is a Sugeno measure or λ-fuzzy, if it satisfies condition (1) of addition for some $\lambda > -1$.

$$\mu(A \cup B) = \mu(A) + \mu(B) + \lambda\mu(A)\mu(B) \tag{1}$$

where λ can be calculated with (2):

$$f(\lambda) = \left\{ \prod_{i=1}^{n} (1 + M_i(x_i)\lambda) \right\} - (1 + \lambda) \tag{2}$$

The value of the λ parameter is determined by the conditions of the Theorem 1.

Theorem 1 *Let* $\mu(\{x\}) < 1$ *for each* $x \in X$ *and let* $\mu(\{x\}) > 0$ *for at least two elements of* X, *then (2) determines a unique parameter* λ *in the following way:*

If $\sum_{x \in X} \mu(\{x\}) < 1,$ *then is in the interval* $(0, \infty)$.
If $\sum_{x \in X} \mu(\{x\}) = 1,$ *then* $\lambda = 0$; *That is the unique root of the equation.*
If $\sum_{x \in X} \mu(\{x\}) > 1,$ *then* λ *is in the interval* $(-1, 0)$.

The fuzzy measure represents the importance or relevance of the sources when computing the aggregation [22], and the method to calculate Sugeno measures is performed recursively using (3) and (4).

$$\mu(A_1) = \mu(M_1) \tag{3}$$

$$\mu(A_i) = \mu(A_{(i-1)}) + \mu(M_i) + \left(\lambda\mu(M_i) * \mu(A_{(i-1)})\right) \tag{4}$$

where A_i represents the fuzzy measure and M_i represents the fuzzy density determined by an expert, where $1 < M_i \leq \ldots \leq n$ should be permuted with respect to the descending order of their respective $\mu(A_i)$.

2.2 Fuzzy Measures for Interval Type 2 Fuzzy Sets

For the estimation of the fuzzy densities of each information source, we take the maximum value of each X_i, where an interval of uncertainty is added.

You need to add an uncertainty footprint or FOU which will create an interval based on the fuzzy density. Equation (5) can be used to approximate the center of the interval for each fuzzy density, and Eqs. (6) and (7) are used to estimate left and right values of the interval for each fuzzy density. Note that the domain for $\mu_L(x_i)$ and $\mu_U(x_i)$ is given in Theorem 1 [23].

Calculation of the fuzzy densities:

$$\mu_c(x_i) = \max(X_i) \tag{5}$$

$$\mu_L(x_i) = \begin{cases} \mu_c(x_i) - \mathrm{FOU}_\mu/2; & \text{if } \mu_c(x_i) > \mathrm{FOU}_\mu/2 \\ 0.0001 & \text{otherwise} \end{cases} \tag{6}$$

$$\mu_U(x_i) = \begin{cases} \mu_c(x_i) + \mathrm{FOU}_\mu/2; & \text{if } \mu_c(x_i) < (1 - \mathrm{FOU}_\mu/2) \\ 0.9999 & \text{otherwise} \end{cases} \tag{7}$$

Calculating the parameters λ_L and λ_U for each side of the interval with (8) and (9)

$$\lambda_L + 1 = \prod_{i=1}^{n} (1 + \lambda_L \mu_L(\{x_i\})) \tag{8}$$

$$\lambda_U + 1 = \prod_{i=1}^{n} (1 + \lambda_U \mu_U(\{x_i\})) \tag{9}$$

Once the λ_U, λ_L are obtained, parameters can be calculated fuzzy measures left and right by extending the recursive formulas (10, 11) (12, 13):

$$\mu_L(A_1) = \mu_L(x_1) \tag{10}$$

$$\mu_L(A_i) = \mu_L(x_i) + \mu_L(A_{i-1}) + \lambda_L \mu_L(x_i) \mu_L(A_{i-1}) \tag{11}$$

$$\mu_U(A_1) = \mu_U(x_1) \tag{12}$$

$$\mu_U(A_i) = \mu_U(x_i) + \mu_U(A_{i-1}) + \lambda_U \mu_U(x_i)\mu_U(A_{i-1}) \tag{13}$$

There are two types of integral that performed the calculation of Sugeno measures: the integral of Sugeno and Choquet Integral.

2.3 Choquet Integral

The Choquet integral can be calculated using (14) or an equivalent expression (15)

$$\text{Choquet} = \sum_{i=1}^{n} \left\{ \left[A_i - A_{(i-1)} \right] * D_i \right\} \tag{14}$$

with $A_0 = 0$,
or also

$$\text{Choquet} = \sum_{i=1}^{n} A_i * \left\{ \left[D_i - D_{(i+1)} \right] \right\} \tag{15}$$

with $D_{(n+1)} = 0$,
where A_i represents the fuzzy measurement associated with data D_i.

3 Edge Detection

Edge detection can be defined as a method consisting of identifying changes that exist in the light intensity, which can be used to determine certain properties or characteristics of the objects in the image.

We used the ORL Database of Faces [24] to perform the training of the modular neural network, which has images of 40 people with 10 samples of each individual. To each of the images was applied to a preprocessing by making use of Sobel edge detector and morphological gradient with type 1 and type 2 fuzzy logic system [25] in order to highlight features, some of the images can be displayed in Fig. 5b, d.

3.1 The Morphological Gradient

To perform the method of morphological gradient, we need to calculate every one of the four gradients as commonly done in the traditional method using (16–20), see

Fig. 2 Calculation of the gradient in the four directions

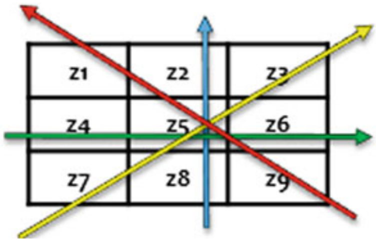

Fig. 2; however, the addition of the gradients is performed by a type 1 and type 2 fuzzy system [25]; in Fig. 3, the membership functions of the fuzzy system are shown, and the resulting image can be viewed in Fig. 5b, d.

$$D1 = \sqrt{(z5 - z2)^2 + (z5 - z8)^2} \tag{16}$$

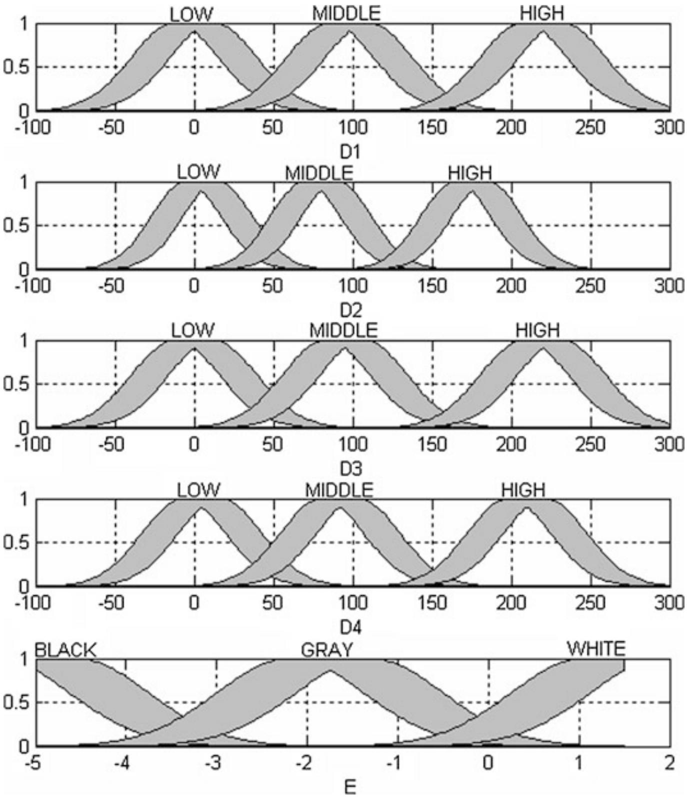

Fig. 3 Variables for the edge detector of morphological gradient the type 2

$$D2 = \sqrt{(z5 - z4)^2 + (z5 - z6)^2} \tag{17}$$

$$D3 = \sqrt{(z5 - z1)^2 + (z5 - z9)^2} \tag{18}$$

$$D4 = \sqrt{(z5 - z7)^2 + (z5 - z3)^2} \tag{19}$$

$$G = D1 + D2 + D3 + D4 \tag{20}$$

3.2 Sobel

The Sobel operator is applied to a digital image in gray scale is a pair of 3 × 3 convolution masks, one estimating the gradient in the x-direction (columns) (21) and the other estimating the gradient in the y-direction (rows) (22) [26].

$$\mathrm{sobel}_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \tag{21}$$

$$\mathrm{sobel}_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \tag{22}$$

If we have $I_{m,n}$ as a matrix of m rows and r columns, where the original image is stored, then g_x and g_y are matrices having the same dimensions as I, which at each element contains the horizontal and vertical derivative approximations and are calculated by (23) and (24) [25].

$$g_x = \sum_{i=1}^{i=3} \sum_{j=1}^{j=4} \mathrm{Sobel}_{x,ij} * I_{r+i-2,c+j-2} \qquad \begin{matrix} \text{for} = 1, 2, \ldots, m \\ \text{for} = 1, 2, \ldots, n \end{matrix} \tag{23}$$

$$g_y = \sum_{i=1}^{i=3} \sum_{j=1}^{j=4} \mathrm{Sobel}_{y,i,j} * I_{r+i-2,c+j-2} \qquad \begin{matrix} \text{for} = 1, 2, \ldots, m \\ \text{for} = 1, 2, \ldots, n \end{matrix} \tag{24}$$

In the Sobel method, the gradient magnitude g is calculated by (25).

$$g = \sqrt{g_x^2 + g_y^2} \tag{25}$$

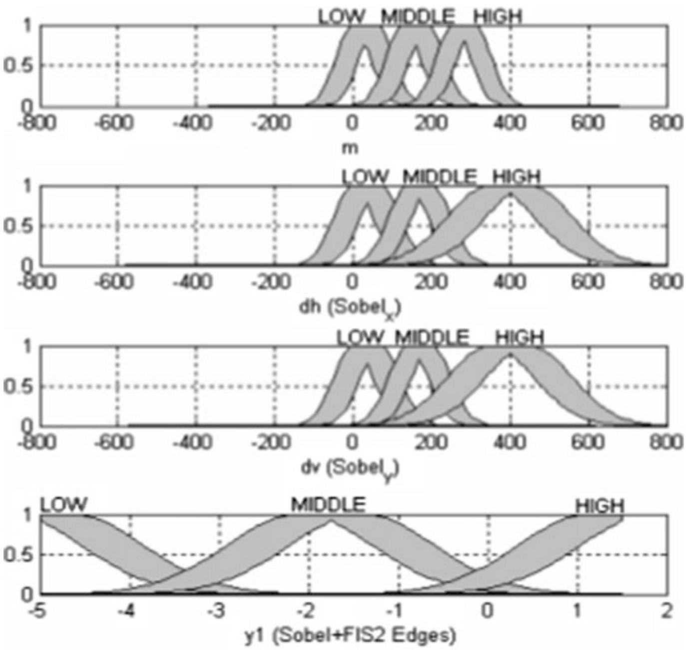

Fig. 4 Variables for the edge detector with the type 2 fuzzy Sobel

For the type 1 and type 2 fuzzy inference systems, 3 inputs can be used, in which 2 of them are the gradients with respect to the x-axis and y-axis, calculated with (23) and (24), which we call DH and DV, respectively. The third variable m is the image after the application of a low-pass filter hMF in (26); this filter allows to detect image pixels belonging to regions of the input where the mean gray level is lower. These regions are proportionally more affected by noise, which it is supposed to be uniformly distributed over the whole image [26]. The membership functions of interval type 2 system are shown in Fig. 4.

$$\mathrm{hMF} = \frac{1}{25} * \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{26}$$

After applying the type 1 and type 2 edge detector with Sobel Method, the resulting image can be viewed in Fig. 5c, e.

Fig. 5 **a** Original image, **b** image with edge detector type 1 morphological gradient, **c** image with edge detector type 1 Sobel, **d** image with interval type 2 morphological gradient, **e** image with edge detector interval type 2 Sobel

4 Modular Neural Networks

We trained a MNN of 3 modules with the data set of ORL. To each image, a methodology of edge detector was applied as described in Sect. 3 and then the image was divided into three horizontal sections, each of which was used as training data in each of the modules, as shown in Fig. 6.

The integration of the modules of the MNN was performed with the Choquet integral (14) and (15). In Table 1, we can appreciate the data distribution of the database.

4.1 Training Parameters

Training method: gradient descendent with momentum and adaptive learning rate back-propagation (Traingdx).
Each module of the MNN has two hidden layers [200 200].
Error goal: 0.00001.
Epochs: 500.

In Table 2, the distribution of the training data in the MNN is shown; 70 % of data are used for training, 15 % for validation, and the other 15 % for testing.

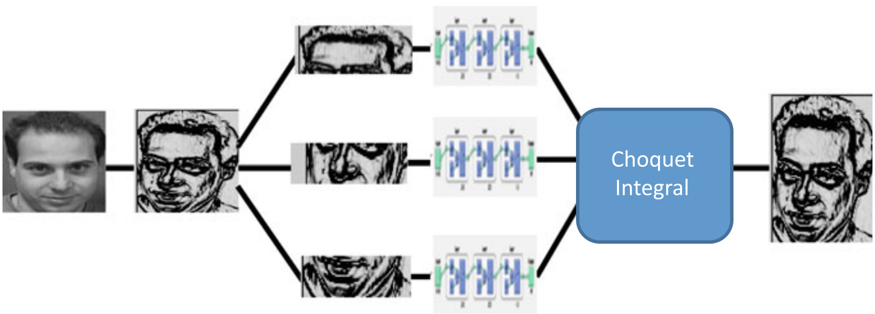

Fig. 6 Proposed architecture of the modular neural network

Table 1 Procedure performed in the experiment

Database	People quantity	Samples per people	Size of training set 80 %	Size of test set 20 %
ORL	40	10	320	80

Table 2 Distribution of the training data

Training 70 %	Validation 15 %	Test 15 %
224	48	48

Table 3 Procedure performed in the experiment

1. Define the database of images
2. Define the edge detector
3. Detect the edges of each image
4. Add the edges to the train set
5. Divide the images in three parts
6. Calculate the recognition rate using the *k*-fold cross-validation method
(a) Calculate the indices for *k*-folds
(b) Train the modular neural network $k - 1$ times for each training fold
(c) Simulate the modular neural network with the *k* test fold
7. Calculate the mean of rate for all the *k*-folds using Choquet integral with interval type 2 fuzzy measures

4.2 The Experiment Consists of a Modular Neural Network Recognition System and Choquet Integral for the Modules Fusion

The experiment consist on apply a preprocessing at the images through an edge detector on database of faces ORL to obtain a data set with which we train a modular neural network, this with the purpose of compare the recognition rate obtained using the k-fold cross validation method [27], see Table 3.

In the experiments, we performed 27 tests in each simulation of the trainings with each edge detector making variations in fuzzy densities and performing the calculation of the parameter $\boldsymbol{\lambda_U}$ **and** $\boldsymbol{\lambda_L}$ with the bisection method.

In Table 4, the parameters shown are used for the integration of information; the first column shows the tests number performed, the second shows the diffuse density associated with each information source—in this case each one of the 3 modules—and the third and fourth columns show the value of lambda for the upper $\boldsymbol{\lambda_U}$ and lower $\boldsymbol{\lambda_L}$ intervals calculated from the fuzzy densities.

5 Simulation Results

In Table 5, are shown an example of the results obtained for face recognition. For this case, the preprocessing of the image is done with the interval type-2 morphological gradient edge detector, and the aggregation method used for the MNN is the interval type-2 Choquet integral.

Table 4 Parameters of the fuzzy densities, λ_U and λ_L

Test	Fuzzy densities			λ_U	λ_L
1	0.1	0.1	0.1	9.76E−18	5.04E−17
2	0.1	0.1	0.5	1.40E−16	0.00E+00
3	0.1	0.1	0.9	1.60E−16	−1.00E+00
4	0.1	0.5	0.1	1.40E−16	0
5	0.1	0.5	0.5	1.236	−0.7307
6	0.1	0.5	0.9	−0.6262	−1.00E+00
7	0.1	0.9	0.1	−8.35E−18	−0.9998
8	0.1	0.9	0.5	−0.6262	−1
9	0.1	0.9	0.9	−9.38E−01	−1
10	0.5	0.1	0.1	1.40E-16	0
11	0.5	0.1	0.5	1.236	−0.7307
12	0.5	0.1	0.9	−0.6262	−1
13	0.5	0.5	0.1	1.236	−0.7307
14	0.5	0.5	0.5	−0.4428	−0.9043
15	0.5	0.5	0.9	−0.8715	−1
16	0.5	0.9	0.1	−0.6262	−1
17	0.5	0.9	0.5	−0.8715	−1
18	0.5	0.9	0.9	−0.9691	−1
19	0.9	0.1	0.1	−8.35E−18	−1.00E+00
20	0.9	0.1	0.5	−0.6262	−1
21	0.9	0.1	0.9	−0.6262	−1
22	0.9	0.5	0.1	−0.6262	−1
23	0.9	0.5	0.5	−8.72E−01	−1
24	0.9	0.5	0.9	−0.9691	−1
25	0.9	0.9	0.1	−0.9376	−1
26	0.9	0.9	0.5	−0.9691	−1
27	0.9	0.9	0.9	−0.9911	−1

In Table 6, the percentage of recognition of the Choquet integral with each interval edge detector is displayed, and a higher percentage of recognized with the usage of the type 2 edge detector with the gradient morphological was obtained with a 0.9549 %.

In Table 7, the percentage of recognition of the Choquet integral using interval fuzzy measures presents a small increase with respect to Choquet integral without interval fuzzy measures.

Table 5 Results obtained in the experiment

Test	Training data			Test data		
	Mean rate	Std rate	Max rate	Mean rate	Std rate	Max rate
1	1	0	1	0.865	0.006	0.875
2	1	0	1	0.863	0.015	0.888
3	1	0	1	0.858	0.014	0.875
4	1	0	1	0.868	0.014	0.888
5	1	0	1	0.863	0.023	0.9
6	1	0	1	0.86	0.016	0.888
7	1	0	1	0.868	0.014	0.888
8	1	0	1	0.868	0.021	0.9
9	1	0	1	0.865	0.014	0.888
10	1	0	1	0.86	0.011	0.875
11	1	0	1	0.855	0.014	0.875
12	1	0	1	0.853	0.011	0.863
13	1	0	1	0.865	0.011	0.875
14	1	0	1	0.858	0.019	0.888
15	1	0	1	0.858	0.011	0.875
16	1	0	1	0.87	0.014	0.888
17	1	0	1	0.87	0.014	0.888
18	1	0	1	0.865	0.011	0.875
19	1	0	1	0.86	0.011	0.875
20	1	0	1	0.863	0.009	0.875
21	1	0	1	0.86	0.014	0.875
22	1	0	1	0.86	0.006	0.863
23	1	0	1	0.86	0.011	0.875
24	1	0	1	0.86	0.014	0.875
25	1	0	1	0.865	0.014	0.888
26	1	0	1	0.868	0.014	0.888
27	1	0	1	0.865	0.011	0.875
	1	**0**	**1**	**0.863**	**0.013**	**0.881**

Table 6 Results with the Choquet integral using type 1 and type 2 edge detector

Method	mean_rate	std_rate	max_rate
T1-Sobel	0.93125	0.0385	0.925
T1-Morphological gradient	0.94	0.0677	0.975
T2-Sobel	0.9431	0.0193	0.9625
T2-Morphological gradient	0.9549	0.0482	0.9625

Table 7 Results with the Choquet integral using type 2 edge detector with interval fuzzy measures

Uncertainty footprint	mean_rate	std_rate	max_rate
FOU 0.1	0.955	0	0.925
FOU 0.2	0.957	0.04825	0.9625

6 Conclusions

The use of Choquet integral as an integration method of responses of a modular neural network applied to face recognition has yielded favorable results when performing the aggregation process of the preprocessed images with the detectors type 1 and type 2 of Sobel edges and morphological gradient; the use of the Sugeno measure by intervals allowed increase of the percentage of data recognition; however, it is still necessary to use a method that optimizes the value of the Sugeno measure assigned to each source of information, and also how to calculate the interval because these were designated arbitrarily. Future work could be considering the optimization of the proposed method, as in [28–30].

7 Future Works

Although good results were obtained by applying the Choquet integral as an aggregation operator of the MNN, more testing is needed on other benchmark databases to verify results obtained also to find another way to generate an interval of uncertainty among the data, fuzzy measures, value of lambda, and fuzzy densities.

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