A Stochastic Optimal Velocity Model for Pedestrian Flow

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Abstract. We propose a microscopic stochastic model to describe 1D pedestrian trajectories obtained in laboratory experiments. The model is based on optimal velocity (OV) functions and an additive noise determined by the inertial Ornstein-Uhlenbeck process. After statistical estimation of the OV function and noise parameters, we explore the model by simulation. The results show that the stochastic approach gives a good description of the characteristic relation between speed and spacing (fundamental diagram) and its variability. Moreover, it can reproduce the observed stop-and-go waves, bimodal speed distributions, and nonzero speed or spacing autocorrelations.

Keywords: Unidirectional pedestrian streams · Stochastic optimal velocity model · Statistical estimation of the parameters · Ornstein-Uhlenbeck process

1 Introduction

The analysis and modeling of pedestrian dynamics has attracted a lot of attention during the last decades [11,29]. Empirically, data have been obtained from experiments in laboratory conditions [7,15] with software to automatically extract the trajectories from video recordings [9]. These investigations allowed to establish many features of pedestrian dynamics [30], e.g. the unimodal shape of the fundamental flow-density diagram or the presence of stop-and-go waves as characteristics of unidirectional pedestrian streams [30,31]. Interestingly, these phenomena do not only hold for pedestrians but are also observed for vehicle or bike motion in 1D showing a certain universality in streams composed of human agents and related self-driven flows [39].

Numerous models have been developed to understand and analyze the characteristics of self-driven flows [6,11,29]. The unimodal shape of the fundamental diagram is already found in simple models like the Asymmetric Simple Exclusion Process (ASEP) [22] where it is related to the exclusion principle. More generally it is well explained microscopically by phenomenological monotone relations between the agent speed and distance spacing with the neighbor (usually called *optimal velocity* (OV), see [3]). The relation reflects the tendency to

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respect safety spacings to avoid collision due to unexpected movements of the neighbors. It is observed with both pedestrians [2] and drivers [4].

Nonlinear traffic waves and instability were the topics of the pioneering papers in the 1950's and early 1960's [10]. Microscopic continuous models defined by systems of differential equations were initially used [27]. The inertial optimal velocity models based on the OV function and defined by systems of ordinary or delayed equations are ones the most investigated traffic models [3,23]. Traffic waves are analyzed through instability of uniform solutions [25] or mapping to macroscopic soliton equations [20]. Generally speaking, it seems that the introduction of delays and deterministic inertial mechanisms generates instability of the uniform solution and the emergence and stable propagation of stop-and-go waves.

Many microscopic models describing nonuniform dynamics are stochastic [17]. A noise is added to differential systems of continuous models for both pedestrian [13,21] and road vehicle [33,34]. Yet, the stochastic aspect does not seem to be preponderant in the dynamics, especially in the formation of stop-and-go waves. In continuous pedestrian models, the noise is used for ambiguous situations (e.g. conflicts) in which two or more behavioral alternatives are equivalent [13] or to model heterogeneous pedestrian behaviors [28]. Few studies have shown that the noise plays a major role (see [12] for bidirectional streams and the formation of lanes). For road traffic models, probabilistic distributions of the parameters are also used to model heterogeneous driving styles [26], and stochastic noises are introduced to model perception errors [34] or to switch from a stationary state to an other [33]. The use of white noises or time-correlated ones does not impact the global dynamics of the second order models [34].

In this paper, we show that the introduction of a specific additive noise in a first order model can impact the dynamics and generate stop-and-go phenomena without requirement of deterministic instabilities. The noise is relaxed at the second order through a Langevin equation. After calibration, we observe by simulation that the model is able to give a good description of pedestrian dynamics and notably the stop-and-go waves. The paper is organized as following. The stochastic OV model is defined in Sect. 2. The description model calibration is presented in Sect. 3. The simulation of the model and comparison to the real data are done in Sect. 4. Conclusions are proposed in Sect. 5.

2 Stochastic Optimal Velocity Model

Initially, the optimal velocity model is a second-order model for which the speed is relaxed to an optimal speed depending on the spacing (headway) [3]. The relaxation is determined by an OV function $V : \Delta x \mapsto V(\Delta x)$. Nowadays, any approach based on the OV function is called *OV model* or *extended OV model*. The minimal OV model is [27]

$$\mathrm{d}x_n(t) = V(\Delta x_n(t))\,\mathrm{d}t, \tag{1}$$

with $x_n(t)$ the position of agent n at time t and $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$ the distance spacing, $x_{n+1}(t)$ being the position of the first predecessor $n+1$.

The uniform solutions are stable in this model if the optimal speed function is increasing which is a natural assumption. The minimalist OV model is too simple to reasonably describe wave phenomena. More realistic dynamics are obtained if an inertia is introduced through reaction (or relaxation) time parameters such as in the *ordinary second order OV model* [3]

$$\begin{cases} \mathrm{d}x_n(t) = v_n(t)\,\mathrm{d}t, \\ \mathrm{d}v_n(t) = \frac{1}{b}\big[V(\varDelta x_n(t)) - v_n(t)\big]\mathrm{d}t, \end{cases} \tag{2}$$

with $v_n(t)$ the agent speed and $b > 0$ the relaxation time parameter. The OV function calibrates the fundamental diagram while stop-and-go waves can be obtained if the reaction times are sufficiently high for that the stability condition fails.

In the literature, stochastic OV models are classically related to discrete models of interacting particle systems [18,19]. Here, we propose to use stochastic OV models by adding a stochastic noise to the continuous minimalist model (1). The noise is centered and stationary, with finite variance. It models other random factors affecting the speed besides the spacing. We denote $W(t)$ the Wiener process such that $W(t,s) - W(t)$ is normally distributed with mean zero, variance s, and independent to $W(t)$ for all t and s. In order to introduce a non-vanishing noise autocorrelation, we use the model

$$\begin{cases} \mathrm{d}x_n(t) = V(\varDelta x_n(t))\,\mathrm{d}t + \varepsilon_n(t)\,\mathrm{d}t, \\ \mathrm{d}\varepsilon_n(t) = -\frac{1}{b}\,\varepsilon_n(t)\,\mathrm{d}t + a\,\mathrm{d}W_n(t), \end{cases} \tag{3}$$

with a the amplitude of the noise and $b > 0$ the relaxation time parameter. The noise $\varepsilon_n(t)$ is the solution of a Langevin equation. It is a standard stochastic process called the Ornstein-Uhlenbeck process, for which the autocorrelation tends to zero exponentially. The noise randomly oscillates around zero making positive and negative corrections to the optimal speed at random instants with independent increments. This behavior is consistent with action-point traffic models and observations that drivers react at discrete random times [32,35,36,38]. The model (3) is close to the deterministic second order OV model (2). Yet with the stochastic approach, the inertia only affects the noise. The uniform solutions are linearly stable in the model (3) in the deterministic case where $a = 0$ as soon as $V(\cdot)$ is strictly increasing. However, the trajectories obtained from the model with the additive noise describe nonuniform solutions with stop-and-go waves (see Fig. 1, the simulation details are given in Sect. 4). Yet, oppositely to the unstable deterministic approaches, there are no generic problems of collision and backward motion (see for instance [8,37]).

3 Calibration of the Parameters

The data we use to calibrate and evaluate[1] the models are pedestrian trajectories in a ring over laboratory conditions [1]. The experiments in a ring with length of

[1] There is no split of the data; both calibration and evaluation steps are done with the global data sample.

27 m and width of 0.7 m. Several experiments were carried out with different level of densities (the pedestrians numbers go from 14 to 70 with 11 tested density levels) and uniform initial distribution. The trajectories are measured on two segments with length of 4 m using the software `PeTrack` [5] with a time resolution of 0.04 s (frame-rate 25 fps). The variables used for the model calibration are the distance spacing and speed

$$\Delta x(t) = x_1(t) - x(t) \quad \text{and} \quad v_{\delta t}(t) = \tfrac{1}{\delta t}\big(x(t+\delta t/2) - x(t-\delta t/2)\big). \tag{4}$$

with x_1 the position of the predecessor. The spacings are measured instantaneously while the speeds have to be averaged over time intervals of length $\delta t = 0.8$ s to avoid effects of the pedestrian step frequency that is close to 0.7 s [24].

The OV function models a phenomenological relation between the speed and the spacing. Two main states are classically distinguished: (1) the free state, when the spacing is large and the speed is equal to the maximal desired speed and (2) the congested (or interactive) state, when the spacing is small and the speed depends on the spacing. Both road traffic and pedestrian observations show clear correlations between speed and spacing in congested regimes. This suggests that the spacing is proportional to the speed to keep a constant safety time gap to react to unexpected behaviors of the predecessor [2,4]. Therefore, we assume that the OV function is piecewise linear

$$V_p(\Delta x) = \min\big\{v_0, \max\{0, (\Delta x - \ell)/T\}\big\}, \qquad p = (v_0, T, \ell), \tag{5}$$

with v_0 the desired (or maximal) speed, T the time gap and ℓ the longitudinal length of the pedestrian. We propose to estimate these parameters microscopically by using $K = 5251$ pseudo-independent measures from the sample of trajectories (by waiting 5 s between each observation). We denote the observations by $(\Delta x_k, v_k)$, $k = 1, \ldots, K$, where the speed v_k is averaged over $\delta t = 0.8$ s. For a given pedestrian k, the residuals $R_k(p)$ of the model are

$$R_k(p) = V_p(\Delta x_k) - v_k. \tag{6}$$

As in [16], the parameters are estimated by minimizing the empirical variance of the residuals

$$\tilde{p} = \arg\min_p \textstyle\sum_k R_k^2(p). \tag{7}$$

This estimation by least squares maximizes the likelihood under the assumption that the residuals are independent and normal, and has in general good properties if the noise repartition is compact. The observations, the estimations of the parameters and the histogram of the residuals are given in Fig. 2. The $\mathrm{R}^2 = 0.78$ of the estimation (the proportion of the variance explained by the model) reveals a good fit of the model. Moreover the distribution of the residuals is relatively compact. Note that the fit can be slightly improved by using sigmoid OV functions with 4 parameters ($\mathrm{R}^2 = 0.80$).

The empirical estimation of the variance of the residuals maximizing the likelihood is $\tilde{\sigma}_R^2 = \frac{1}{K}\sum_k R_k^2(\tilde{p})$. The stationary variance and δt-autocorrelation

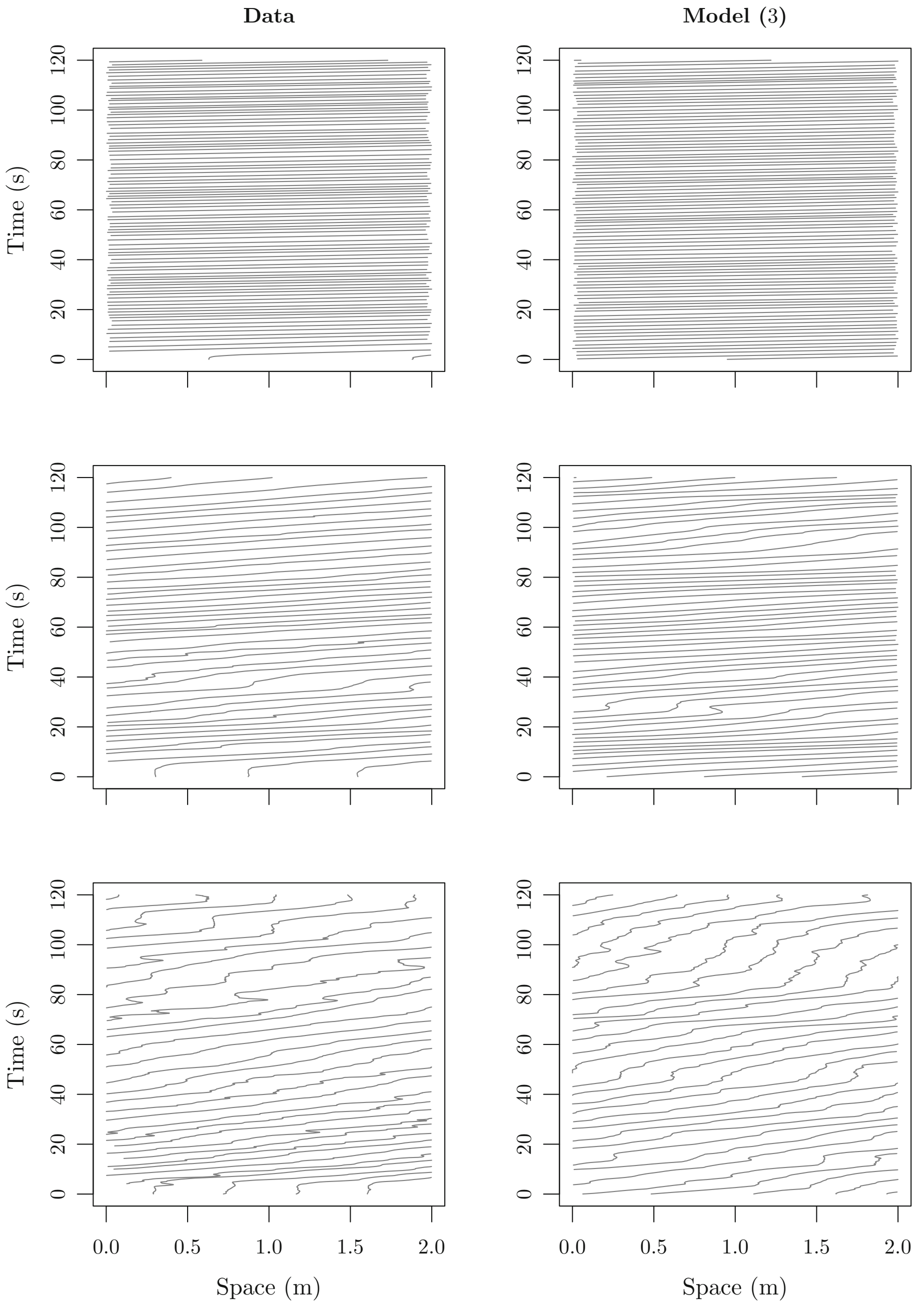

Fig. 1. Trajectories on a segment of length 2 m. From top to bottom: 25 (free state), 45 (slightly congested state) and 62 pedestrians (congested state) on the ring of length 27 m. From left to right: Real data and the calibrated stochastic model (3).

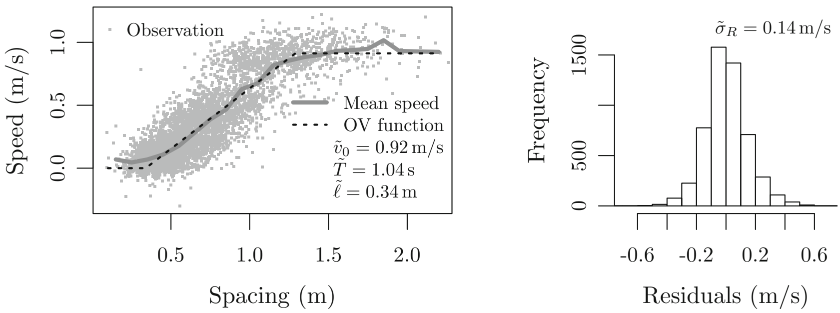

Fig. 2. Statistical estimation of the parameters (left panel) and histogram of the residuals (right panel). $\tilde{\sigma}_R$ is the empirical standard deviation of the residuals. $\delta t = 0.8\,\mathrm{s}$ in the calculus of the speed. Global sample of observations.

of the Ornstein-Uhlenbeck process are $var(\varepsilon) = a^2 b/2$ and $\tilde{c}_{\delta t} = e^{-\delta t/b}$. These relations allow to obtain the estimators for b and a

$$\tilde{b} = -\delta t / \log(\tilde{c}_{\delta t}) \qquad \text{and} \qquad \tilde{a} = \tilde{\sigma}_R \sqrt{2/\tilde{b}}. \tag{8}$$

The estimations for all the data are $\tilde{a} \approx 0.09\,\mathrm{ms}^{-3/2}$ and $\tilde{b} \approx 4.38\,\mathrm{s}$. Note that the value of the relaxation time b is close to 5 s that is approximately 10 times larger than the value $\tau \approx 0.5\,\mathrm{s}$ generally used with force-based pedestrian models based on a relaxation process (see for instance [13]). Estimations by class of spacing show clear relations between the noise parameters and this variable. The results are shown in Fig. 3. We can see for $\tilde{b}$ particular uni-modal shapes in the congested phase where $\Delta x \leq \ell + v_0 T \approx 1.3\,\mathrm{m}$. For the free phase where $\Delta x \geq \ell + v_0$, the values are relatively constant. The shape of the parameter $\tilde{a}$ is more irregular. It will be assumed constant on $\Delta x \leq 0.95$ and $\Delta x \geq 0.95\,\mathrm{m}$ in the simulations.

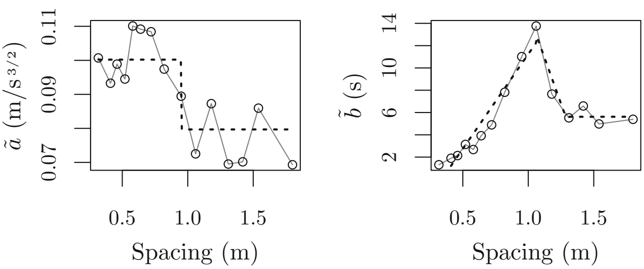

Fig. 3. Statistical estimation of the noise parameters by class of spacing. The dotted lines are the linear approximations used in the simulations.

4 Simulation Results

In the analysis of complex systems, the *top-down* method consists in calibrating the parameters of a microscopic model in order to reproduce observed macroscopic behaviors. It requires knowledge about the relation between the parameter values and the macroscopic properties of the model, or to implement a sensitivity analysis. The top-down approach has been mainly used in particle physics where the microscopic particle behaviors are unknown (only macroscopic quantities such as the temperature are measured). In this study, the microscopic performances (i.e. the trajectories) are observed and directly used to calibrate the parameters. The macroscopic behaviors are observed by simulation and used to validate the calibrated models. This *bottom-up* method allows to control both local and global dynamics.

We evaluate the model (3) by comparing simulation results to the real data. A similar setup as in the real experiments is reproduced for the simulations (from 14 to 70 pedestrians in a ring of length 27 m). The models are simulated by using explicit Euler-Maruyama schemes [14]. The discretisation of the relaxed noise model (3) is

$$\begin{cases} x_n(t+dt) = x_n(t) + dt\, V_{\tilde{p}}(\Delta x_n(t)) + dt\, \varepsilon_n(t), \\ \varepsilon_n(t+dt) = (1 - dt/\tilde{b})\, \varepsilon_n(t) + \sqrt{dt}\, \tilde{a}\, \xi_n(t), \end{cases} \tag{9}$$

with $(\xi_n(t), n, t)$ independent normal random variables. The time step dt is set to 1e-3 s.

The stochastic model is firstly evaluated by looking at the mean, standard deviation and correlation of the speed and spacing for the global sample of observations (see Table 1). The trajectories for 25, 45 and 62 pedestrians are presented in Fig. 1. Some stop-and-go waves propagate when the density increases as in real data (see Fig. 1, middle and bottom panels). Yet we do not observe the collision and backward motion problems frequently related with the unstable deterministic approaches [8,37]. The autocorrelations of the speed and spacing also give good fits to the data (see Fig. 4). The speed distributions by class of spacing are plotted in Fig. 5. We clearly observe bimodal distributions for intermediate

Table 1. Mean, standard deviation (in m and m/s) and correlation for the spacing Δx and speed $v_{\delta t}$ of a pedestrian and his/her predecessor (Δx^1 and $v^1_{\delta t}$) for global sample of observations. $\delta t = 0.8$ s.

	Δx		$v_{\delta t}$		Δx^1		$v^1_{\delta t}$	
	Data	Mod. (3)	Data	Mod. (3)	Data	Mod. (3)	Data	Mod. (3)
Mean	0.68	0.67	0.32	0.32	0.68	0.67	0.32	0.31
Std-dev	0.33	0.34	0.30	0.30	0.33	0.35	0.30	0.30
Corr. Δx	1	1	0.87	0.87	0.79	0.76	0.87	0.87
$v_{\delta t}$	0.87	0.87	1	1	0.85	0.84	0.97	0.97

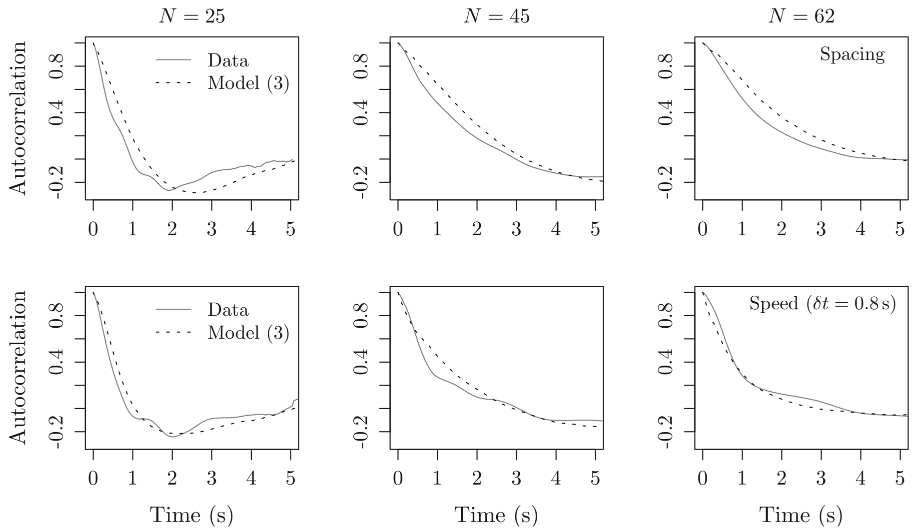

Fig. 4. Autocorrelation function for the spacing (top panels) and the speed (bottom panels) for $N = 25$ (free state, left panels), $N = 45$ (slightly congested state, middle panels) and $N = 62$ (congested state, right panels).

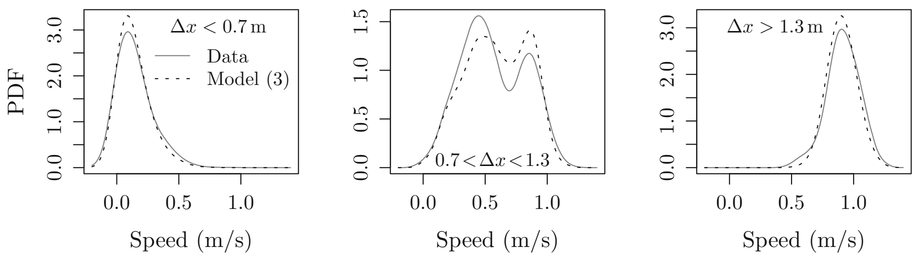

Fig. 5. Probability density function of the speed $v_{\delta t}$ with $\delta t = 0.8\,\mathrm{s}$ by class of spacing Δx. Global sample of observations.

spacings within the data and model (3) (see Fig. 5, middle panel). This result is consistent with stable propagation of the stop-and-go waves.

5 Conclusion

A first order pedestrian model based on Optimal Velocity functions and additive stochastic noise is proposed and calibrated using real pedestrian data on a ring. The model gives realistic descriptions of pedestrian trajectories in one dimension. Mean values and correlations of the speed and spacing are relatively well fitted through piecewise linear OV function with three parameters. Stop-and-go phenomena at congested density levels, bimodal speed distributions, and nonzero speed and spacing autocorrelations are obtained thanks to the relaxed noise at the second order.

As the classical deterministic OV models, inertia mechanisms are used to generate collective waves. Yet the inertia here is stochastic, without deterministic instability of the uniform solution. Also, and oppositely to classical deterministic models, there is no requirement of using nonlinear dynamics to obtain (nonlinear) traffic waves within the stochastic OV approach. Moreover, we do not observe the generic problems of collision and motion backward that are unfortunately frequently obtained with the unstable deterministic approaches. The statistical estimation of the relaxation time is close to 5 s for the noisy model, while it is generally around 0.5 s for the deterministic Ansatz. The relaxation mechanism of the stochastic approach is clearly not that of the classical models. This makes the stochastic OV model a new way to describe accurately stop-and-go phenomena. For pedestrian dynamics in two dimensions, it has to be completed by a direction model.

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