Local Anomaly Cancellation and Equivariant Cohomology of Jet Bundles

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Dedicated to Jaime Muñoz Masqué on the occasion of his 65th birthday, with my best wishes.

Abstract We study the problem, suggested by Singer in [17], and consisting in determining a notion of "local cohomology" adequate to deal with the problem of locality in those approaches to local anomalies based on the Atiyah–Singer index theorem.

Keywords Local cohomology · Equivariant cohomology · Jet bundle · Anomaly cancellation

1 Introduction

An anomaly appears in a theory when a classical symmetry is broken at the quantum level. As we consider only local anomalies, we can assume that the group $\mathcal{G}$ is connected. Let $\mathcal{L}(\psi, s)$ be a $\mathcal{G}$-invariant Lagrangian density depending on bosonic fields $s \in \Gamma(E)$ and fermionic fields ψ. At the quantum level, the corresponding effective action $W(s)$, defined in terms of the fermionic path integral by $\exp(-W(s)) = \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left(-\int_M \mathcal{L}(\psi, s)\right)$ could fail to be $\mathcal{G}$-invariant. We define a form $\mathcal{A} \in \Omega^1(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\Gamma(E)))$ by $\mathcal{A} = \delta W$, i.e. $\mathcal{A}(X)(s) = L_X W(s)$ for $X \in \mathrm{Lie}\,\mathcal{G}$, $s \in \Gamma(E)$. Although W is clearly a non-local functional, $\mathcal{A}$ is local in X and s, i.e. we have $\mathcal{A} \in \Omega^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$. It is clear that $\mathcal{A}$ satisfies the

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condition $\delta\mathcal{A} = 0$ (the Wess–Zumino consistency condition). Moreover, if $\mathcal{A} = \delta\Lambda$ for a *local* functional $\Lambda = \int_M \lambda \in \Omega^0_{\mathrm{loc}}(\Gamma(E))$ then we can define a new lagrangian density $\hat{\mathcal{L}} = \mathcal{L} + \lambda$, such that the new effective action $\hat{W}$ is $\mathcal{G}$-invariant, and in that case the anomaly cancels. If $\mathcal{A} \neq \delta\Lambda$ for every $\Lambda \in \Omega^0_{\mathrm{loc}}(\Gamma(E))$ then we say that there exists an anomaly in the theory. Hence the anomaly is measured by the cohomology class of $\mathcal{A}$ in the BRST cohomology $H^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$ (e.g. see [4, 6–8, 16]).

Local anomalies also admit a nice geometrical interpretation in terms of the Atiyah–Singer index theorem for families of elliptic operators (see [1, 2, 4, 17]). The first Chern class $c_1\,(\det \mathrm{Ind} D) \in H^2(\Gamma(E)/\mathcal{G})$ of the determinant line bundle $\det \mathrm{Ind} D \to \Gamma(E)/\mathcal{G}$ represents an obstruction for anomaly cancellation. However, the condition $c_1\,(\det \mathrm{Ind} D) = 0$ is a necessary but not a sufficient condition for local anomaly cancellation due to the problem of locality. In [17] (see also [1]) Singer proposes the problem of defining a notion of "local cohomology of $\Gamma(E)/\mathcal{G}$", $H^2_{\mathrm{loc}}(\Gamma(E)/\mathcal{G})$, adequate to study local anomaly cancellation. The principal difficulty is the fact that the expression of the curvature of $\det \mathrm{Ind} D$ itself contains non-local terms (Green operators).

Moreover, we recall (see [2, 5, 15]) that the BRST and index theory approaches are related by means of the transgression map t (see Sect. 2), i.e., we have $[\mathcal{A}] = t(c_1\,(\det \mathrm{Ind} D))$. As t is injective, the condition $c_1\,(\det \mathrm{Ind} D) = 0$ on $H^2(\Gamma(E)/\mathcal{G})$ is equivalent to $[\mathcal{A}] = 0$ on $H^1(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\Gamma(E)))$. However, the condition for local anomaly cancellation is $[\mathcal{A}] = 0$ on the BRST cohomology $H^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$. We define $H^\bullet_{\mathrm{loc}}(\Gamma(E)/\mathcal{G})$ in such a way that the preceding condition is equivalent to the vanishing of the class of $c_1\,(\det \mathrm{Ind} D)$ on $H^2_{\mathrm{loc}}(\Gamma(E)/\mathcal{G})$, hence solving Singer's problem.

2 The Transgression Maps

First we recall the definition of equivariant cohomology in the Cartan model (*e.g.* see [3]). We consider a left action of a connected Lie group $\mathcal{G}$ on a manifold $\mathcal{N}$. We have an induced Lie algebra homomorphism $\mathrm{Lie}\,\mathcal{G} \to \mathfrak{X}(\mathcal{N})$, $X \mapsto X_{\mathcal{N}} = \frac{d}{dt}\big|_{t=0} \rho(\exp(-tX))$. We denote by $\mathcal{P}^k(\mathrm{Lie}\,\mathcal{G}, \Omega^r(\mathcal{N}))^{\mathcal{G}}$ the space of degree k $\mathcal{G}$-invariant polynomials on $\mathrm{Lie}\,\mathcal{G}$ with values in $\Omega^r(\mathcal{N})$. We recall that $\alpha \in \mathcal{P}^k(\mathrm{Lie}\,\mathcal{G}, \Omega^r(\mathcal{N}))^{\mathcal{G}}$ if and only if $\alpha(\mathrm{Ad}_g X) = \rho(g^{-1})^*(\alpha(X))\ \forall X \in \mathrm{Lie}\,\mathcal{G}, \forall g \in \mathcal{G}$.

The space of $\mathcal{G}$-equivariant differential q-forms is defined by

$$\Omega^q_{\mathcal{G}}(\mathcal{N}) = \bigoplus_{2k+r=q} (\mathcal{P}^k(\mathrm{Lie}\,\mathcal{G}, \Omega^r(\mathcal{N})))^{\mathcal{G}}. \tag{1}$$

The Cartan differential $d_c \colon \Omega^q_{\mathcal{G}}(\mathcal{N}) \to \Omega^{q+1}_{\mathcal{G}}(\mathcal{N})$, $(d_c\alpha)(X) = d(\alpha(X)) - \iota_{X_{\mathcal{N}}}\alpha(X)$ for $X \in \mathrm{Lie}\,\mathcal{G}$, satisfies $(d_c)^2 = 0$, and the $\mathcal{G}$-equivariant cohomology of $\mathcal{N}$, $H^q_{\mathcal{G}}(\mathcal{N})$, is the cohomology of this complex.

We recall (e.g. see [3]) that if $\mathcal{N} \to \mathcal{N}/\mathcal{G}$ is a principal $\mathcal{G}$-bundle we have the (generalized) Chern–Weil homomorphism $\mathrm{ChW}\colon H^\bullet_{\mathcal{G}}(\mathcal{N}) \to H^\bullet(\mathcal{N}/\mathcal{G})$. If A is an arbitrary connection on $\mathcal{N} \to \mathcal{N}/\mathcal{G}$ with curvature F_A, and $\alpha \in \Omega^q_{\mathcal{G}}(\mathcal{N})$, then we have $\mathrm{ChW}([\alpha]) = [\mathrm{hor}_A(\alpha(F_A))]$, where hor_A is the horizontalization with respect to the connection A. We also use the notation $\underline{\alpha} = \mathrm{ChW}(\alpha)$.

If $\omega \in \Omega^2_{\mathcal{G}}(\mathcal{N})$ is a closed $\mathcal{G}$-equivariant 2-form, then we have $\omega = \omega_0 + \mu$ where $\omega_0 \in \Omega^2(\mathcal{N})$ is closed, and $\mu\colon \mathrm{Lie}\,\mathcal{G} \to C^\infty(\mathcal{N})$, is a $\mathcal{G}$-equivariant moment map for ω_0, i.e., $\iota_{X_{\mathcal{N}}}\omega_0 = d(\mu(X))$ for $X \in \mathrm{Lie}\,\mathcal{G}$. A direct computation shows that we have the following

Proposition 1 *Assume that $\mathcal{N} \to \mathcal{N}/\mathcal{G}$ is a principal $\mathcal{G}$-bundle, and let $A \in \Omega^1(\mathcal{N}, \mathrm{Lie}\,\mathcal{G})$ be a connection form. If $\omega = \omega_0 + \mu \in \Omega^2_{\mathcal{G}}(\mathcal{N})$ is a closed $\mathcal{G}$-equivariant 2-form and we define $\alpha \in \Omega^1(\mathcal{N})^{\mathcal{G}}$ by $\alpha(X) = \mu(A(X))(x)$ for $X \in T_x\mathcal{N}$, then we have* $\mathrm{ChW}_A(\omega) = \omega + d_c\alpha$.

Corollary 1 *The map* $\mathrm{ChW}\colon H^2_{\mathcal{G}}(\mathcal{N}) \to H^2(\mathcal{N}/\mathcal{G})$ *is an isomorphism.*

Let us assume now that $H^1(\mathcal{N}) = H^2(\mathcal{N}) = 0$. The cohomology of the Lie algebra $\mathrm{Lie}\,\mathcal{G}$ with values in $\Omega^0(\mathcal{N})$ is denoted by $H^\bullet(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\mathcal{N}))$.

Proposition 2 *Let $\omega = \omega_0 + \mu \in \Omega^2_{\mathcal{G}}(\mathcal{N})$ be a closed $\mathcal{G}$-equivariant form. If $\rho \in \Omega^1(\mathcal{N})$ satisfies $\omega_0 = d\rho$, then the map $\tau_\rho \in \Omega^1(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\mathcal{N}))$ given by $\tau_\rho(X) = \rho(X_{\mathcal{N}}) + \mu(X)$ determines a linear map $\tau\colon H^2_{\mathcal{G}}(\mathcal{N}) \to H^1(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\mathcal{N}))$ which is independent of the form ρ chosen, and that we call the transgression map τ. If $\mathcal{G}$ is connected, then τ is injective.*

Proof The first part of the Proposition easily follows using that $L_{Y_{\mathcal{N}}}\mu(X) = \mu([Y, X])$ by the invariance of μ. We restrict ourselves to prove that τ is injective. By definition $[\tau_\rho] = 0$ on $H^1(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\mathcal{N}))$ if and only if there exists $\beta \in \Omega^0(\mathcal{N})$ such that for every $X \in \mathrm{Lie}\,\mathcal{G}$ we have $\tau_\rho(X) = L_{X_{\mathcal{N}}}\beta = \iota_{X_{\mathcal{N}}}d\beta$. If we set $\rho' = \rho - d\beta$ then for every $X \in \mathrm{Lie}\,\mathcal{G}$ we have $d\rho' = \omega_0$, $\iota_{X_{\mathcal{N}}}\rho' = -\mu(X)$, $L_{X_{\mathcal{N}}}\rho' = 0$, *i.e.*, $\rho' \in \Omega^1(\mathcal{N})^{\mathcal{G}}$ and $d_c\rho' = \omega$.

Now we assume that $\pi\colon \mathcal{N} \to \mathcal{N}/\mathcal{G}$ is a principal $\mathcal{G}$-bundle. Then we can consider the more familiar transgression map defined as follows

Proposition 3 *Let $\underline{\omega} \in \Omega^2(\mathcal{N}/\mathcal{G})$ be a closed 2-form. If $\eta \in \Omega^1(\mathcal{N})$ is a form such that $\pi^*\underline{\omega} = d\eta$, then the map $t_\eta\colon \mathrm{Lie}\,\mathcal{G} \to \Omega^0(\mathcal{N})$, $t_\eta(X) = \eta(X_{\mathcal{N}})$ determines a linear map $t\colon H^2(\mathcal{N}/\mathcal{G}) \to H^1(\mathrm{Lie}\,\mathcal{G}, \Omega^0(\mathcal{N}))$, which is independent of the form η chosen, and that we call the transgression map t. If $\mathcal{G}$ is connected, then t is injective.*

Proof Again we only prove that t is injective. If $t_\eta = \delta\upsilon$ for certain $\upsilon \in \Omega^0(\mathcal{N})$, then $\eta(X_{\mathcal{N}}) = L_{X_{\mathcal{N}}}\upsilon$. We define $\eta' = \eta - d\upsilon$, and we have $d\eta' = \pi^*\underline{\omega}$, $\iota_{X_{\mathcal{N}}}\eta' = 0$, $L_{X_{\mathcal{N}}}\eta' = 0$. Hence η' is projectable onto a form $\underline{\eta'} \in \Omega^1(\mathcal{N}/\mathcal{G})$ and $d\underline{\eta'} = \underline{\omega}$.

Proposition 4 *If $\omega \in H^2_{\mathcal{G}}(\mathcal{N})$ and $\underline{\omega} = \mathrm{ChW}(\omega)$ then we have $\tau(\omega) = t(\underline{\omega})$.*

Proof If $\omega = \omega_0 + \mu$, by Proposition 1 we have $\omega = \pi^*\underline{\omega} + d_c\alpha$ for some $\alpha \in \Omega^1_{\mathcal{G}}(\mathcal{N}) = \Omega^1(\mathcal{N})^{\mathcal{G}}$, i.e. $\omega_0 = \pi^*\underline{\omega} + d\alpha$ and $\mu(X) = -\alpha(X_{\mathcal{N}})$.

Let $\eta \in \Omega^1(\mathcal{N})$ be a form such that $\pi^*\underline{\omega} = d\eta$. If we set $\rho = \eta + \alpha$ then $\omega_0 = d\rho$ and for every $X \in \mathrm{Lie}\,\mathcal{G}$ we have $\tau_\rho(X) = t_\eta(X)$.

3 Local Equivariant Cohomology

Let $p\colon E \to M$ be a bundle over a compact, oriented n-manifold M without boundary. We denote by $J^r E$ its r-jet bundle, by $J^\infty E$ the infinite jet bundle and by $\Gamma(E)$ be the manifold of global sections of E (assumed to be not empty).

We denote by $\mathrm{Proj}E$ the space of projectable diffeomorphism of E, and by Proj^+E the subgroup of elements preserving the orientation of M. The space of projectable vector fields on E is denoted by $\mathrm{proj}E$. We consider the natural actions of $\mathrm{Proj}E$ on $J^\infty E$ and $\Gamma(E)$.

Let $\mathrm{j}^\infty\colon M \times \Gamma(E) \to J^\infty E, \mathrm{j}^\infty(x,s) = j^\infty_x s$ be the evaluation map. We define a map $\Im\colon \Omega^{n+k}(J^\infty E) \to \Omega^k(\Gamma(E))$, by $\Im[\alpha] = \int_M (\mathrm{j}^\infty)^* \alpha$, for $\alpha \in \Omega^{n+k}(J^\infty E)$. The map $\Im$ commutes with the exterior differential and is Proj^+E-equivariant (see [9]). We define the space of local k-forms on $\Gamma(E)$, as the image of the map $\Im$, i.e. $\Omega^k_{\mathrm{loc}}(\Gamma(E)) = \Im(\Omega^{n+k}(J^\infty E)) \subset \Omega^k(\Gamma(E))$. The cohomology $H^\bullet_{\mathrm{loc}}(\Gamma(E))$ of the complex $(\Omega^\bullet_{\mathrm{loc}}(\Gamma(E)), d)$ is called the local cohomology of $\Gamma(E)$. We have $H^k_{\mathrm{loc}}(\Gamma(E)) \cong H^{n+k}(E)$ for $k > 0$ (see [10]).

Let $\mathcal{G}$ be a Lie group acting on E by elements Proj^+E. In order to define an adequate notion of local equivariant cohomology we made the following

Assumption 1 We assume that $\mathrm{Lie}\,\mathcal{G}$ is isomorphic to the space of sections of a Lie algebroid $V \to M$, i.e. $\mathrm{Lie}\,\mathcal{G} \cong \Gamma(V)$. We also assume that the map $\mathrm{Lie}\,\mathcal{G} \cong \Gamma(V) \to \mathrm{proj}E$, $X \mapsto X_E$ is a differential operator. Finally, in the definition of $\mathcal{G}$-equivariant cohomology $H^{n+k}_{\mathcal{G}}(J^\infty E)$, we assume that the polynomial maps $\alpha\colon \mathrm{Lie}\,\mathcal{G} \to \Omega^\bullet(J^\infty E)$ are differential operators.

We extend the integration operator to a map $\Im\colon \Omega^{n+k}_{\mathcal{G}}(J^\infty E) \to \Omega^k_{\mathcal{G}}(\Gamma(E))$, by setting $(\Im[\alpha])(X) = \Im[\alpha(X)]$ for every $\alpha \in \Omega^{n+k}_{\mathcal{G}}(J^\infty E)$, $X \in \mathrm{Lie}\,\mathcal{G}$. The map $\Im$ commutes with the Cartan differential and induces a homomorphism in equivariant cohomology $\Im\colon H^{n+k}_{\mathcal{G}}(J^\infty E) \to H^k_{\mathcal{G}}(\Gamma(E))$ (see [9]).

We define the space of local $\mathcal{G}$-equivariant k-forms by

$$\Omega^k_{\mathcal{G},\mathrm{loc}}(\Gamma(E)) = \Im(\Omega^{n+k}_{\mathcal{G}}(J^\infty E)) \subset \Omega^k_{\mathcal{G}}(\Gamma(E)). \tag{2}$$

The local $\mathcal{G}$-equivariant cohomology of $\Gamma(E)$, $H^\bullet_{\mathcal{G},\mathrm{loc}}(\Gamma(E))$, is defined as the cohomology of the complex $(\Omega^\bullet_{\mathcal{G},\mathrm{loc}}(\Gamma(E)), d_c)$.

4 Application to Local Anomaly Cancellation

Let us define the BRST cohomology (see [6, 16]). Recall (see Assumption 1) that we assume $\mathrm{Lie}\,\mathcal{G} \cong \Gamma(V)$ for some vector bundle $V \to M$. A map $\alpha\colon \bigwedge^k \mathrm{Lie}\,\mathcal{G} \to \Omega^0_{\mathrm{loc}}(\Gamma(E))$ is said to be local if there exists a differential operator $A\colon \bigwedge^k \mathrm{Lie}\,\mathcal{G} \to \Omega^n(J^\infty E)$ such that $\alpha(X_1, \ldots, X_k) = \Im[A(X_1, \ldots, X_k)]$ for every $X_1, \ldots, X_k \in \mathrm{Lie}\,\mathcal{G}$. We denote by $\Omega^k_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E))$ the space of local k-forms on $\mathrm{Lie}\,\mathcal{G}$ with values on $\Omega^0_{\mathrm{loc}}(\Gamma(E))$. The differential δ on the complex $\Omega^\bullet(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E))$ induces a differential on $\Omega^\bullet_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E))$. The corresponding cohomology $H^\bullet_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$ is called the BRST cohomology. We assume that $H^2(\Gamma(E)) = H^1(\Gamma(E)) = 0$ and also that $H^2_{\mathrm{loc}}(\Gamma(E)) = H^1_{\mathrm{loc}}(\Gamma(E)) = 0$.

Proposition 5 *The restriction of the transgression map* τ *to* $H^2_{\mathcal{G},\mathrm{loc}}(\Gamma(E))$ *takes values on the BRST cohomology* $H^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$. *The map* $\tau\colon H^2_{\mathcal{G},\mathrm{loc}}(\Gamma(E)) \to H^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$ *is injective for* $\mathcal{G}$ *connected.*

Proof Let $\omega = \omega_0 + \mu \in \Omega^2_{\mathcal{G},\mathrm{loc}}(\Gamma(E))$ be a closed local $\mathcal{G}$-equivariant 2-form. As $H^2_{\mathrm{loc}}(\Gamma(E)) = 0$, we have $\omega_0 = d\rho$, for certain $\rho \in \Omega^1_{\mathrm{loc}}(\Gamma(E))$. By our assumption in the definition of local equivariant cohomology (Assumption 1), the map $\tau_\rho\colon \mathrm{Lie}\,\mathcal{G} \to \Omega^0_{\mathrm{loc}}(\Gamma(E))$, $\tau_\rho(X) = \rho(X_{\Gamma(E)}) + \mu(X) \in \Omega^0_{\mathrm{loc}}(\Gamma(E))$ is a local map. The injectiveness of τ follows from Proposition 2.

Assume that $\Gamma(E) \to \Gamma(E)/\mathcal{G}$ is a principal $\mathcal{G}$-bundle. Then we define the local cohomology by $H^k_{\mathrm{loc}}(\Gamma(E)/\mathcal{G}) = \mathrm{ChW}(H^k_{\mathcal{G},\mathrm{loc}}(\Gamma(E)))$. By Proposition 4 we have the following

Proposition 6 *Let* $\omega \in \Omega^2_{\mathcal{G},\mathrm{loc}}(\Gamma(E))$ *be a closed local* $\mathcal{G}$*-equivariant* 2*-form and let* $\underline{\omega} = \mathrm{ChW}(\omega)$. *Then we have* $\tau(\omega) = t(\underline{\omega})$, *and in particular we conclude that* $t(\underline{\omega}) \in H^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$. *Moreover, the following conditions are equivalent*

(a) $[\omega] = 0$ *on* $H^2_{\mathcal{G},\mathrm{loc}}(\Gamma(E))$.
(b) $[\underline{\omega}] = 0$ *on* $H^2_{\mathrm{loc}}(\Gamma(E)/\mathcal{G})$.
(c) $[\tau(\omega)] = [t(\underline{\omega})] = 0$ *on* $H^1_{\mathrm{loc}}(\mathrm{Lie}\,\mathcal{G}, \Omega^0_{\mathrm{loc}}(\Gamma(E)))$.

Proposition 6 applied to $\underline{\omega} = c_1(\det \mathrm{Ind} D)$ shows that our definition of $H^2_{\mathrm{loc}}(\Gamma(E)/\mathcal{G})$ solves Singer's problem. We also note that if $\omega \in \Omega^2_{\mathcal{G},\mathrm{loc}}(\Gamma(E))$ is closed, the form $\underline{\omega} \in \Omega^2(\Gamma(E)/\mathcal{G})$ determining the class $\mathrm{ChW}([\omega])$ could contain non-local terms, as $\underline{\omega}$ depends on the curvature of a connection Θ on the principal $\mathcal{G}$-bundle $\Gamma(E) \to \Gamma(E)/\mathcal{G}$, and Θ usually contains non-local terms. This fact explains the appearance of non-local terms on the expression of the curvature of $\det \mathrm{Ind} D$ commented on the Introduction.

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