Modeling US Stock Market Volatility-Return Dependence Using Conditional Copula and Quantile Regression

Terence D. Agbeyegbe

1 Introduction

There is a growing literature in economics and finance on methods of dealing with catastrophic risks which can be seen as rare events with major consequences (see Chichilnisky (2009) and the references therein). When attention is on financial econometrics, some of these methods focus on estimating parameters of time series models using quantile regression and copula techniques (see Alexander 2008; Allen et al. Allen et al. 2009, 2012; Badshah 2012; Barnes and Hughes 2002; Bouyé and Salmon 2009; Engle and Manganelli 2004; Koenker and Xiao 2006; Kumar 2012; Patton 2004, 2006a, b, 2009; Taylor 1999; Trivedi and Zimmer 2005; Xiao 2009 among many others). In this chapter we describe the application of quantile regression and copula techniques to United States index stock market price return and volatility data. The quantile regression model we use was initially described in Koenker and Bassett (1978), and is an extension of the classical least squares estimation of the conditional mean to a collection of different conditional quantile function models. It is essentially a statistical technique intended to estimate and conduct inference about conditional quantile functions. It has the additional advantage of being robust to heteroskedasticity, skewness and leptokurtosis which are typical features of financial data.

The main purpose of this chapter is to apply quantile regression methods to investigate the relation between stock returns and implied volatilities. Though such an investigation has been done before, the analysis in this chapter differs in terms of data choice, span and the use of a GARCH filter to control for changes in the volatilities of the series. Two of the series we examine have not been investigated in the quantile regression framework: the Dow Jones Industrial Average Index and the S&P 100 Index. The other two have been examined but for a different time period. We also

T.D. Agbeyegbe (✉)
Department of Economics, Hunter College and the Graduate Center,
City University of New York, New York City, NY 10021, USA
e-mail: tagbeyeg@hunter.cuny.edu

G. Chichilnisky and A. Rezai (eds.), *The Economics of the Global Environment*,
Studies in Economic Theory 29, DOI 10.1007/978-3-319-31943-8_26

focus on tails of the distributions, which is particularly important since volatility and extreme movements are not synonymous. As noted by many others, (see Neftci 2000) the prices of two assets could exhibit the same volatility but very different patterns with regards to their extremes. For this reason, we consider methods that examine the tails of the price distributions. Quantile regression methods are of use when dealing with relationships at the tails of distributions. The relation between stock returns and implied volatilities has long been studied given its practical importance for areas such as risk management, option pricing, and event studies (see for example, the early papers by Cox and Ross 1976; Black 1976; Christie 1982). In several recent papers, the relationship was shown to be asymmetric (see for example, Badshah 2012; Dennis et al. 2006; Fleming et al. 1995; Giot 2005; Hibbert et al. 2008; Low 2004; Whaley 2000; Wu 2001; Allen et al. 2012). An asymmetric relationship means that the negative change in the stock market returns has a higher impact on the volatility index than a positive change, or vice-versa. For this reason, volatility indices are often referred to as being investors gauges of fear (see Whaley 2000). The theoretical basis for this asymmetric volatility-return relationship is the focus of two hypotheses; namely, the leverage hypothesis (see Black 1976; Christie 1982) and the volatility feedback hypothesis (see French et al. 1987; Campbell and Hentschel 1992). The leverage hypothesis states that if the stock price of a firm declines, the relative proportion of equity (debt) value to the firm value decreases (increases), which makes the firm's stock riskier and increases its volatility as a consequence. The volatility feedback hypothesis states that the negative change in expected return tends to be intensified whereas the positive change in the expected return tends to be dampened and these effects generate the asymmetric volatility phenomenon.

The plan of the chapter is as follows. Section 2 discusses quantile regression. Section 3 provides a review of some copula functions and dependence measures. Section 4 deals with non-linear quantile regressions using copula theory. Section 5 deals with the data on US equities and the results. Section 6 contains the conclusion.

2 Quantile Regression

In this section, we provide a brief discussion of quantile regression. For convenience and as a prelude to introducing the simple linear quantile regression model, we briefly discuss a simple linear regression model. A simple bivariate linear regression model may be written as:

$$y_t = \alpha + \beta x_t + \varepsilon_t \tag{1}$$

where the parameters α and β are constants and y is the independent variable, x is the dependent variable, ε is the error term and subscript t is for time period t. The standard assumptions include the provision that the errors are independent and identically distributed with mean zero and that the x is exogenous suggesting that the conditional expectation of ε_t is zero. These conditions mean we can write $E(y \mid x) = \alpha + x\beta$. Assuming further that the y and x is bivariate normal will assure that the distribution function $F(y \mid x)$ is normal and this distribution is completely specified from knowledge of the conditional mean and conditional variance equations. The ordinary least squares estimates are then the solution to the optimization problem

$$min_{\alpha\beta} \sum_t (y_t - \alpha - \beta x_t)^2 \tag{2}$$

When the joint distribution of x and y is not bivariate normal we need more than the conditional mean and conditional variance to specify the conditional distribution of the dependent variable. It is for this reason we need quantiles and by implication a quantile regression framework. The definition of Koenker and Bassett's (1978) linear quantile regression is stated in terms of an optimization problem. Let $q \in (0, 1)$ and the *qth* quantile of the error term be defined as F_ε^{-1}, where the error has a distribution function given as F_ε The simple linear quantile regression model is then given as

$$F^{-1}(q \mid x) = \alpha + x\beta + F_\varepsilon^{-1}(q). \tag{3}$$

where $F^{-1}(q \mid x)$ is the q conditional quantile of the dependent variable in the general case.

More generally, let $(y_1, y_2, \dots, y_T)$ be a random sample on the regression process with $u_t = y_t - x_t\beta$ having distribution function F and $(x_1, x_2, \dots, x_T)$ be a sequence of K-vectors of a known design matrix, the q-th quantile regression will be any solution to the following problem:

$$min_{\beta\varepsilon R^k} (\sum_{t\varepsilon\tau_q} q \mid y_t - x_t\beta \mid + \sum_{t\varepsilon\tau_{1-q}} (1-q) \mid y_t - x_t\beta \mid) \tag{4}$$

with $\tau_q = \{t : y_t \geq x_t\beta\}$ and τ_{1-q} is the complement.

Notice that the median (quantile) regression estimator minimizes the symmetrically weighted sum of absolute errors (where the weight is equal to 0.5). The other conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors, where the weights are now functions of the quantile of interest. The properties of the estimator is provided in Theorem 1 of Koenker and Basset (1978). As noted by Buchinsky (1998), quantile regression models have many useful features: (i) with respect to non-gaussian error terms, quantile regression estimators may be more efficient than least-square estimators, (ii) the entire conditional distribution can be characterized, (iii) different relationships between the regressor and the dependent variable may arise at different quantiles.

Whilst the modern treatment of quantile regression can be traced to Koenker and Basset (1978), the use of the classical least squares' methodology as a modern statistical framework can be traced to Galton (1886). As pointed out by Abdi (2007), Galton used it in his work on the heritability of size, which formed the foundations of correlation and (also gave the name to) regression analysis. For a fuller discussion of the history and pre-history of the classical least squares methodology, the reader is referred to Harper (1974–1976). A distinguishing feature of Galton's regression approach is the minimization of the sum of squares of residuals in order to enable one to estimate models for the conditional mean functions. The least squares methodology framework is not useful if interest is not focused on the conditional mean, to avoid this short-coming researchers developed the quantile regression method. Quantile regression methods provide a way for estimating models for the conditional median function, and the full range of other conditional quantile functions. It is capable of providing a more complete statistical analysis of the stochastic relationships among random variables by supplementing the estimation of conditional mean functions with techniques for estimating an entire family of conditional quantile functions. The estimated conditional quantile functions give a much more complete picture of the effect of covariates on the location, scale and shape of the distribution of a response variable. The method has been extended, and it has found successful application in many areas of applied econometrics. For example, in labor economics, we can find examples based on the works of: Buchinsky and Leslie (1997) who investigated wage structure; Eide and Showalter (1999) together with Buchinsky and Hunt (1999) who investigated earnings mobility; and Eide and Showalter (1998) who considered issues related to educational attainment. In financial econometrics we can find examples based on the works of: Taylor (1999) who estimated the distribution of multiperiod returns using quantile regression; Engle and Manganelli (2004) who proposed estimating value at risk (VaR) using quantile regression; Koenker and Xiao (2006) who proposed a quantile autoregression model and applied it to weekly U.S. gasoline prices; Bouyé and Salmon (2009) who developed a theory of non-linear quantile regression modeling using copula and applied the theory to examine conditional quantile dependency in the foreign exchange market; and Xiao (2009) who developed a theory for quantile cointegration and applied the proposed model to US stock index data.

It should be noted that an important generalization of the basic linear quantile regression to the non-linear case was developed by Powell (1986) using a censored regression modeling framework. The consistency of non-linear quantile regression estimation has been investigated by White (1994), Engle and Manganelli (2004) and Kim and White (2003). For an overview of quantile regression, see the guideline for empirical research by Buchinsky (1998), the surveys by Koenker and Hallock (2001) and Yu et al. (2003) together with the text by Koenker (2005).

3 Review of Copula Functions and Dependence

In this section, we state some well-known properties of copula functions and briefly discuss some measures of dependence. We start with a few definitions and introduce notation and terminology that are consistent throughout this chapter.

The interest in studying the relationship between United States index stock market price return and implied volatility data motivates the need to discuss copula functions. A full treatment of copulas and their properties can be found in Joe (1997) and Nelsen (2006). Nelsen (2006) defines copulas as "functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions." Copula functions are particularly attractive to work with since they allow us to separately model the marginal distribution and the dependence structure. In dealing with dependence, copulas can provide us information on both the degree of dependence and the structure of dependence. In particular, copula functions contain information about the joint behavior of the random variables in the tails of the distribution and can shed light on the symmetric, or asymmetric nature of the dependence. Linear correlation is unable to shed light on tail dependence and/or the symmetry property of dependence. We now provide a definition of a two-dimensional copula and we state the most important result in copula theory, Sklar (1959)'s theorem.

Definition 1 (Nelsen (2006), p. 10) A two-dimensional copula (or 2-copula, or briefly, a copula) is a real function C with the following properties:

1. For every u, v in [0, 1],
$$C(u, 0) = 0 = C(0, v) \tag{5}$$
and
$$C(u, 1) = u, C(1, v) = v; \tag{6}$$
2. For every, u_1, u_2, v_1, v_2in [0, 1] such that $u_1 \leq u_2$ and $v_1 \leq v_2$,
$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1)) \geq 0. \tag{7}$$

Theorem 1 (Sklar (1959)'s Theorem, Nelsen (2006), p. 18) *Let X and Y be two random variables with joint distribution F. Then, there exists a copula C such that for all x,y in* $\bar{\mathbb{R}}$ *satisfying* $F(x, y) = C(F_X(x), F_Y(y))$. *If* F_X, F_Y *are continuous, then C is unique and* F_X, F_Y *represent the marginal distributions of X and Y respectively.*

The above theorem of Sklar is very important, since it provides a way for us to analyse the dependence structure of multivariate distributions without studying marginals distributions. In the case of multivariate continuous distribution functions, the theorem allows us to view the univariate margins and the multivariate dependence structure as separate entities. The underlying dependence structure of the multivariate distribution can be represented by an adequate copula function.

Note from above, any bivariate distribution function whose margins are standard uniform distributions is a copula. Furthermore, copula functions are joint distribution functions of standard uniform random variables: $C(u, v) = Pr(U_1 \leq u, U_2 \leq v)$. They are also subjected to a version of the Fréchet-Hoeffding bounds inequality.

Theorem 2 (Fréchet-Hoeffding bounds inequality, Nelsen (2006), p. 11) *Let M(u, v) = min(u, v) and W(u, v) = max(u + v - 1.0) then for every copula C and every* $(u, v) \in [0, 1]^2$,
$$W(u, v) \leq C(u, v) \leq M(u, v). \tag{8}$$

M is referred to as the Fréchet-Hoeffding upper bound and W as the Fréchet-Hoeffding lower bound.

Definition 2 A parameter θ of a copula is called the dependence parameter if for an m-variate function F, the copula associated with F is a distribution function $C : [0,1]^m \rightarrow [0,1]$ that satisfies

$$F(y_1, y_2, \ldots, y_T) = C(F_1(y_1), \ldots, F_m(y_m); \theta).$$

The copula dependence parameter measures the dependence between the marginals and may be a vector of parameters. In bivariate applications, the dependence parameter is often represented by a scalar parameter and is the focus of estimation.

Copula theory has found successful applications in many fields. For applications and overview of copula to quantitative risk, see Embrechts et al. (2003) and Embrechts et al. (2001), among others. For applications in finance and financial time series, see Cherubini et al. (2004), and Patton (2009).

3.1 Some Dependence Concepts

In this subsection, we discuss the concept of dependence. There is a fairly large literature that deals with this concept and from what has been reported we can view dependence as falling into at least three broad classes. The first discusses dependence in terms of linear dependence relationship between variables in the center of the distribution or rank correlations if interest centers on non-linear monotonic transformation of the variables. The second considers dependence between variables in the tail of the distribution in the presence of extreme events. The third examines dependence along the whole distribution. Examples of the first approach are numerous and they are exemplified in the use of classical least-squares regression to unravel dependence between variables. Measures based on "regular" linear correlation of Pearson's ρ and the rank correlation of Kendall's τ and Spearman's ρ are often reported with this kind of analysis. Pearson's ρ deals with the linear dependence between random variables and when nonlinear transformations are applied to those random variables, linear correlation is not preserved. Instead, a rank correlation coefficient measure, such as Kendall's τ or Spearman's ρ, will be more appropriate. The rank correlations measure the degree to which large or small values of one random variable associates with large or small values of another random variable. Examples of the second approach are found in the works of Longin and Solnik (2001), Ang and Chen (2002) and (Patton 2006a, b) among many others who discuss exceedance correlation and tail dependence. One focus is to discuss dependence in terms of exceedance correlation which is defined as the correlation between two variables X and Y, conditional on both variables being above or below certain thresholds μ_1 and μ_2, respectively. The other focus is in terms of tail dependence a concept which is related to exceedance correlation but it is different. Tail dependence is a key measure for risk management, which mainly focuses on the extreme events of joint distribution. It measures the probability that both variables are simultaneously in their lower or upper tails. The lower (left) and upper (right) tail dependence coefficients, λ_l and λ_r, are defined as below.

Definition 3 $\lambda_l = lim_{u\to 0} Pr[F_Y(y) \leq u \mid F_X(x) \leq u] = lim_{u\to 0} \frac{C(u,u)}{u}$

Definition 4 $\lambda_r = lim_{u\to 1} Pr[F_Y(y) \geq u \mid F_X(x) \geq u] = lim_{u\to 1} \frac{1-2u+C(u,u)}{1-u}$

In both cases λ_l and $\lambda_r \in [0, 1]$. If λ_l or λ_r is positive, X or Y is said to be left (lower) or right (upper) tail dependent. Patton (2009), provide examples of analysis based on tail dependency.

Examples of the third approach can be found in many of the papers on quantile regression and some recent papers in copula quantile regression modeling. In this approach, a copula quantile regression is specified and the dependency between variables of interests are reported for different quantiles. The approach is discussed in Sect. 4.

3.2 *Some Copula Functions*

There are a large number of copulas to work with when modeling data. Each copula imposes a different dependence structure on the data. Joe (1997, Chap. 5), Nelsen (2006: 116–119) and Trivedi and Zimmer (2005) discuss a wide variety of bivariate copulas and their properties. In this sub-section, we discuss some copulas that have appeared frequently in finance applications, and we briefly describe their dependence structures.

The most common copulas can be divided into two broad types: Elliptical and Archimedean Copulas. Examples of the former being-Gaussian Copula and Student's t-Copula and of the latter being Clayton copula, Frank Copula and Gumbel copula.

3.2.1 Elliptical Copulas

(i) Gaussian Copula.
Let us define $u_i = F_i(x_i)$. The Gaussian (or normal) copula is the copula of the multivariate normal distribution. It takes the form

$$\begin{aligned} C_{Gaussian}(u_1, u_2; \rho) &= \boldsymbol{\Phi}_G(\boldsymbol{\Phi}^{-1}(u_1), \boldsymbol{\Phi}^{-1}(u_2); \rho) \\ &= \int_{-\infty}^{\boldsymbol{\Phi}^{-1}(u_1)} \int_{-\infty}^{\boldsymbol{\Phi}^{-1}(u_2)} \frac{1}{2\pi(1-\rho^2)^{0.5}} e^{\{\frac{-(x_1^2 - 2\rho x_1 x_2 + x_2^2)}{2(1-\rho^2)}\}} dx_1 dx_2 \end{aligned}$$

where $\boldsymbol{\Phi}_G$ is the standard bivariate normal distribution, $\boldsymbol{\Phi}$ is the cumulative distribution function of the standard normal distribution, with Pearson's product moment correlation coefficient $\rho, \rho \in (-1, 1)$. The normal copula is quite flexible and allows for equal degrees of positive and negative dependence and it includes both the lower and upper Fréchet bounds in its permissible range.

(ii) Student's t-copula.
Student's t-copula is based on the multivariate t-distribution in the same way the Gaussian copula is based on the multivariate normal distribution. It adds joint fat tails to the Gaussian copula. The bivariate t-copula takes the form:

$$\begin{aligned} C_t(u_1, u_2; \nu, \rho) &= \phi_\rho(\phi_\nu^{-1}(u_1), \phi_\nu^{-1}(u_2)) \\ &= \int_{-\infty}^{t_\nu^{-1}(u_1)} \int_{-\infty}^{t_\nu^{-1}(u_2)} \frac{1}{2\pi(1-\rho^2)^{0.5}} \\ &\times \{1 + \frac{(x_1^2 - 2\rho x_1 x_2 + x_2^2)}{\nu(1-\rho^2)}\}^{-\frac{(\nu+2)}{2}} dx_1 dx_2 \end{aligned}$$

where t_ν^{-1} denotes the inverse of the cdf of the standard univariate t-distribution with ν degrees of freedom. The dependency parameters are ρ and ν with $\rho \in (-1, 1)$ and $\nu > 2$. The parameter ν controls the heaviness of the tails and when $\nu \leq 3$ the variance does not exist and when $\nu \leq 5$, the fourth moment does not exist. Large values of ν, approximate a Gaussian distribution; $C_t(u_1, u_2; \nu, \rho) \rightarrow \Phi_G(u_1, u_2; \rho)$. The t-copula is attractive because the degree of tail dependency can be set by changing the degrees of freedom. The copula is important in finance and has been recommended by a number of authors. (See, for example, Breymann et. al. 2003).

3.2.2 Archimedean Copulas

Archimedean copulas are an important class of copulas that have a wide range of applications. They are easy to construct from generators. A great variety of families of copulas belongs to this class, and they have many nice properties. (see Nelsen 2006). For a generator ϕ, the Archimedean copula can be defined as:

$$C_{Archimedean}(u_1, u_2; \alpha) = \phi^{-1}(\phi(u_1) + \phi(u_2))$$

and the density is given as:

$$c_{Archimedean}(u_1, u_2; \alpha) = \phi_{(2)}^{-1}(\phi(u_1) + \phi(u_2))\Pi_{i=1}^2 \phi'(u_i).$$

where $\phi_{(2)}^{-1}$ is the 2nd derivative of the inverse generator function, $\phi()$ is a convex decreasing function, with $\phi(1) = 0$. The function $\phi()$ depends on a single parameter α that reflects the degree of dependence. Archimedean copulas allow a wide range of dependence structure. Their mathematical and statistical properties are studied in Genest and Rivest (1993). We will discuss three members of the Archimedean families, namely Gumbel, Clayton and Frank Copula. The copula parameter α of the Archimedean copula is related to Kendall's τ coefficient of correlation which is defined as

$$\tau = \frac{2}{n(n-1)} \sum_{i}^{n} \sum_{j>1} sgn(X_i - X_j)(Y_i - Y_j) \tag{9}$$

where 'sgn' refers to the sign of the term that follows it. Genest and MacKay (1986) show that there is a relationship between τ and α. The relationship is given as $\tau = 4\int_0^1 \frac{\phi(t)}{\phi'(t)}dt + 1$

(i) Clayton copula.
The Clayton (1978) copula is also referred to as the Cook and Johnson (1981) copula and was originally studied by Kimeldorf and Sampson (1975). It takes the form

$$C_{Clayton}(u_1, u_2; \alpha) = \begin{cases} (u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-\frac{1}{\alpha}}, & \alpha \in (0, \infty), \\ u_1 u_2, & \alpha = 0. \end{cases}$$

and α is the dependence parameter. As α approaches zero the marginals become independent and as it approaches infinity the copula attains the Fréchet upper bound. The Clayton copula cannot account for negative dependence, although it does exhibit strong left tail dependence and relatively weak right tail dependence. It has a tail dependence property of $\lambda_r = 0$ and $\lambda_l = 2^{-\frac{1}{\alpha}}$.

(ii) Frank copula.
The Frank copula, which appeared in Frank (1979) takes the form

$$C_{Frank}(u_1, u_2; \alpha) = -\alpha^{-1} log \left\{ 1 + \frac{(e^{-\alpha u_1} - 1)(e^{-\alpha u_2} - 1)}{(e^{-\alpha} - 1)} \right\}, \tag{10}$$

$\alpha \in (-\infty, 0) \bigcup (0, \infty)$. It has a tail dependence property of $\lambda_r = 0$ and $\lambda_l = 0$. The Frank copula is useful in financial modeling for several reasons. First, it allows for negative dependence between marginals. Second, it allows for symmetric tail dependence. Third, it is able to achieve the Fréchet-Hoeffding bounds.

(iii) Gumbel copula.
The Gumbel copula which appeared in Gumbel (1960) takes the form

$$C_{Gumbel}(u_1, u_2; \alpha) = exp(\bar{u_1}^{\alpha} + \bar{u_2}^{\alpha})^{\frac{1}{\alpha}}, \tag{11}$$

$\alpha \in [1, \infty)$ and $\bar{u_j} = -log u_j$. It has a tail dependence property of $\lambda_l = 0$ and $\lambda_r = 2^{\frac{1}{\alpha}}$. Values of 1 and ∞ correspond to independence and the Fréchet-Hoeffding upper bound. The copula does not attain the Fréchet-Hoeffding lower bound for any dependence parameter value. Also it cannot account for negative dependence. The Gumbel copula exhibits strong right tail dependence and relatively weak left tail dependence.

4 Copula Quantile Regression

Both Chen et al. (2006) and Bouyé and Salmon (2009) have built on the quantile regression work of Koenker and Basset (1978) to propose methods for estimating copula based conditional quantile models. The papers assume a correct specification of the parametric copula dependence function without specifying the underlying marginal distribution functions. Chen et al. (2006) use a rescaled empirical cumulative distribution function to obtain the marginals. After this, they employ the method of maximum likelihood to obtain the copula parameter. Their resulting conditional quantile functions are obtained by plugging in the estimated copula parameter and the empirical marginal cumulative distribution function.

The approach we follow is that of Bouyé and Salmon (2009). They estimate several distinct, non-linear quantile regression models implied by their copula specifications and gave closed forms of the quantile curve for several copulas. We begin with some definitions.

Definition 5 (Bouyé and Salmon 2009) Let $p(x, y; \theta)$ be the probability distribution of y conditional on x. Then

$$p(x, y; \theta) = Pr[Y \leq y \mid X = x] \tag{12}$$

$$= C_1[F_X(x), F_Y(y); \theta] \tag{13}$$

with $C_1(u, v; \theta) = \frac{\partial}{\partial u} C(u, v, \theta)$.

Definition 6 (Bouyé and Salmon 2009) For a parametric copula $C(., .; \theta)$, the p-th copula quantile curve of y conditional on x is defined by the following implicit equation

$$p = C_1[F_X(x), F_Y(y); \theta] \tag{14}$$

where $\theta \in \Theta$ the set of parameters.

We give three of these copula quantile regression forms.

Normal CQR: The Normal CQR takes the form

$$y = F_Y^{-1}\left[\Phi(\rho\Phi^{-1}(F_X(x)) + \sqrt{1-\rho^2}\Phi^{-1}(q))\right] \tag{15}$$

Student-t CQR: The Student-t CQR takes the form

$$y = F_Y^{-1}\left[t_\nu(\rho t_\nu^{-1}(F_X(x)) + \sqrt{(1-\rho^2)(\nu+1)^{-1}(\nu + t_\nu^{-1}(F_X(x))^2)}) t_{\nu+1}^{-1}(q))\right] \tag{16}$$

Clayton CQR The Clayton CQR takes the form

$$y = F_Y^{-1}\left[(1 + F_X(x)^{-\alpha}(q^{-\frac{\alpha}{1+\alpha}} - 1))^{-\frac{1}{\alpha}}\right]. \tag{17}$$

In the empirical exercise, we aim to estimate a different set of copula parameters $\hat{\theta}_q$ for each quantile regression. Let $(y_1, y_2, \ldots, y_T)$ and $(x_1, x_2, \ldots, x_T)$ be a random sample, the q-th quantile regression curve will be defined as $y_t = \zeta(x_t, q; \hat{\theta}_q)$. The parameters $\hat{\theta}_q$ being any solution to the following optimization problem:

$$min_\theta(\sum_{t=1}^{T}(q - \mathbf{1}_{y_t \leq \zeta(x_t,q;\theta)})(y_t - \zeta(x_t, q; \theta)) \tag{18}$$

See Chap. 7 of Alexander (2008) and Bouyé and Salmon (2009) for details on copula quantile regression modeling.

5 Data and Empirical Estimates

In this section we present the US data and the empirical estimates.

5.1 Preliminary Analysis and Summary Statistics

We examine the return-volatility relationship for indices reported on exchanges in the United States of America. In the empirical analysis, we use daily price data for market and volatility indices of four volatility-return pairs, namely, VXD and DJIA, VIX and S&P 500 (SPX), VXO and S&P 100 (OEX), VXN and NASDAQ (NDX). The daily prices are obtained from the Chicago Board Options Exchange for a period of approximately 11 years from 2/02/2001 to 31/12/2012. For the analysis we use percentage returns computed as 100 times the logarithmic changes. The volatility indices are the VXD, VIX, VXO and the VXN and are discussed below. The CBOE DJIA Volatility Index (VXD) is based on real-time prices of options on the Dow Jones Industrial Average (DJIA), and is designed to reflect investors' consensus view of future (30-day) expected stock market volatility. The SPX VIX, is an index of implied volatility of 30-day options on the S&P 500 calculated from all available stock index option calls and puts bid and ask prices. The index, which was adopted in September 2003 provides an estimate of expected stock market volatility for the subsequent 30 days. According to Hibbert et. al. (2008), the Chicago Board Options Exchange's (CBOE) calculates the VIX from all available stock index option bid and ask prices in the tradable range of these options providing an estimate of expected stock market volatility for the subsequent 30 calendar days (about 21 trading days).

It is based on options on the S&P 500 index (SPX) and it uses options across the tradable range of all strike prices possessing both a bid and ask price; furthermore, it is independent of any option-pricing model. The new method of calculation provides a more robust measure of expected volatility along with option implied volatility skew. The OEX VXO is the original VIX version that was introduced in 1993 and is now disseminated under the ticker symbol VXO, and is based on the S&P 100 index. It considers only near-the-money options, and is calculated using the implied volatilities obtained from the Black-Scholes option-pricing model. The calculation of the CBOE NASDAQ-100 VXN Volatility Index is based on the CBOEs widely accepted VIX methodology. VXN is calculated throughout the trading day based on the near term volatility determined through pricing of NASDAQ-100 Index (NDX) option prices. Like VIX, VXN is a measure of the market's expectation of 30-day volatility, but is based on the NDX rather than the SPX. The CBOE publishes indices of these implied volatilities.

Figures 1 and 2 show the logarithmic return series of the stock return indices and the volatility indices for the period 2/2/2001–31/12/2012. The time series plot

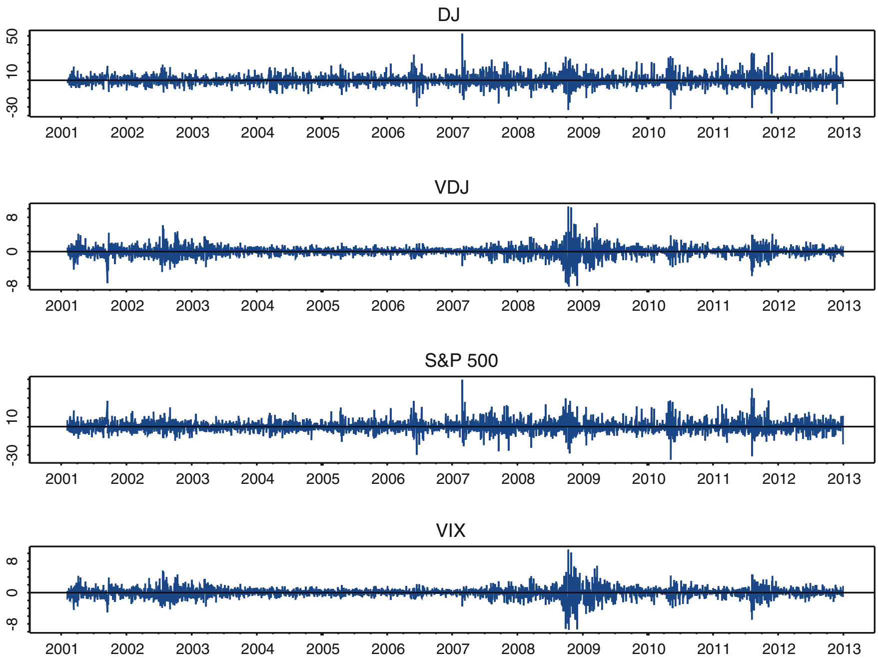


Fig. 1 Time series plot of the stock and volatility indices 2/2/2001–31/12/2012. *Notes* Daily closing percentage returns on the Dow Jones Industrial Average Index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the Dow Jones Volatility Index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the S&P 500 Index (SPX) from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the VIX Index from February 2, 2001 through December 31, 2012

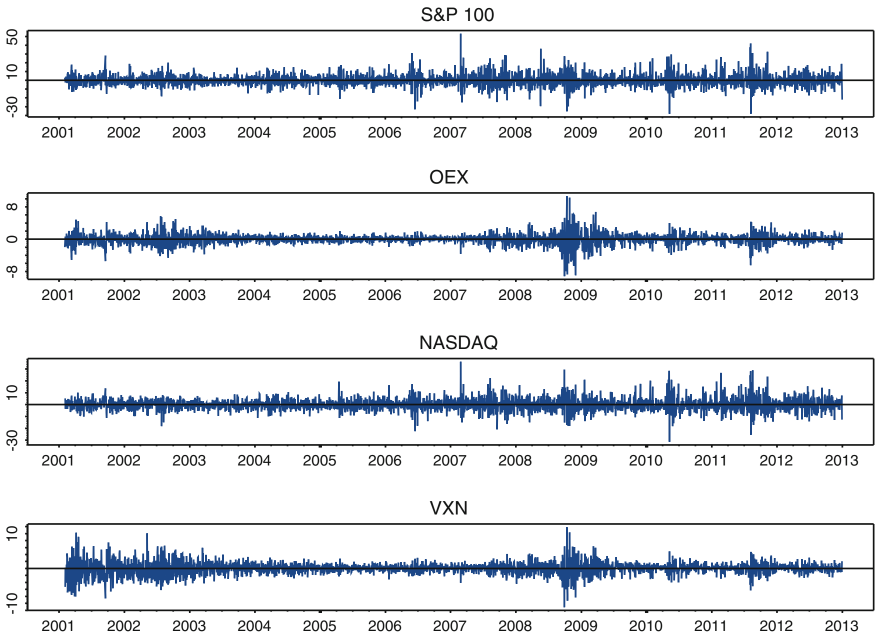


Fig. 2 Time series plot of the stock and volatility indices 2/2/2001–12/31/2012. *Notes* Daily closing percentage returns on the OEX Index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the S&P 100 Volatility Index (VXO) from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the NASDAQ 100 Index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the NASDAQ 100 Volatility Index (VXN) from February 2, 2001 through December 31, 2012

seem to show that the individual volatility index changes according to the respective index return changes. Figure 3 gives the quantile-quantile plots for our series, and none of the data series shows a good fit to the normal distributions. It is well known that when the data distribution is not adequately described by a normal distribution, quantile regression (QR) can provide more efficient estimates for the return-volatility relationships (Badshah 2012). Table 1 gives the descriptive statistics for all the variables. All the variables show excess kurtosis, which indicates fat tails. Looking at the Jarque-Bera test statistics in Table 1, we see that the statistics strongly reject the presence of normal distributions in the series. Thus, we can conclude that all the return time series (both the market and the volatility series) exhibit fat tails and are not normally distributed. The reported ADF test statistics, based on an autoregression of order 8, also reject the presence of unit roots in the time series.

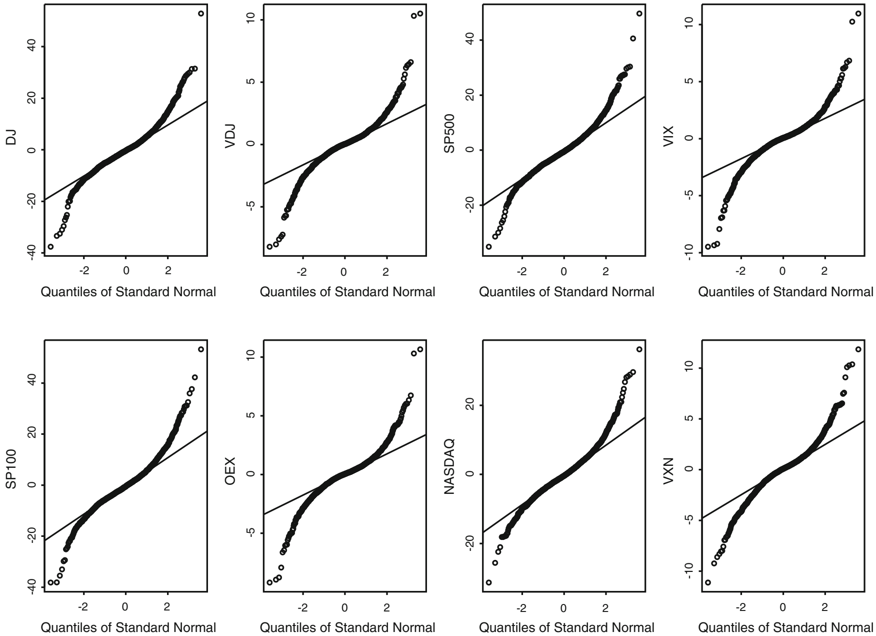


Fig. 3 Quartile-Quartile Plot of the Stock and volatility indices 2/2/2001–12/31/2012. *Notes* Normal qq-plot for Daily closing percentage returns on the Dow Jones Industrial Average Index, the Dow Jones Volatility Index, the S&P 500 Index(SPX), the VIX Index, the SP 100 Index(OEX), the SP 100 Volatility (VXO), the NASDAQ 100 Index and the NASDAQ100 Volatility Index (VXN). The data period is from February 2, 2001 through December 31, 2012

5.2 *Empirical Results Linear Quantile Regression*

Table 2 reports the point estimates of the intercept and regression coefficient for all the volatility-return pairs. The results of the regression coefficients indicate an inverse volatility return relationship. For example, if the DJ index rises by 10 %, then the VDJ will be expected to fall by 34.77 %. Similarly, if the SPX rises by 10 %, then the VIX will be expected to fall by 35.78 %.

Table 3 reports the estimates for the linear quantile regression model, with the intercept α, and the slope coefficient β. The β measures the dependence of volatility on market return. Note that as formulated, the ordinary linear regression model (OLS) is incapable of capturing both the asymmetric and tail dependence between price and implied volatility. In other words, the simple linear regression is incapable of capturing the known empirical facts that (i) volatility increases much more after a large fall in price than it decreases after a large price increase, (ii) volatility reacts more strongly to extreme price moves than normal price moves. One way of address-

Table 1 Descriptive Statistics, Stock and volatility indices 2/2/2001–12/31/2012

	VDJ	DJIA	VIX	SPX	VXO	OEX	VXN	NDX
Observations	2994	2994	2995	2995	2993	2993	2991	2991
Minimum	−37.5750	−8.2005	−35.0589	−9.4695	−38.1487	−9.1862	−31.3049	−11.1149
Quartile 1	−3.6343	−0.5405	−3.7185	−0.5792	−4.0392	−0.5872	−3.0526	−0.8198
Median	−0.3267	0.0409	−0.5013	0.0588	−0.4770	0.0494	-0.4009	0.0825
Arithmetic mean	−0.0049	0.0063	−0.0061	0.0013	−0.0086	−0.0036	−0.0361	0.0008
Quartile 3	3.0311	0.5757	3.1980	0.6130	−4.0392	0.5952	2.7605	0.8504
Maximum	52.8092	10.5084	49.6008	10.9572	53.2274	10.6551	36.2226	11.8493
SE mean	0.1160	0.0230	0.1162	0.0246	0.1287	0.0244	0.0973	0.0331
LCL mean (0.95)	−0.2324	−0.0388	−0.2339	−0.0470	−0.2609	−0.0513	−0.2269	−0.0640
UCL mean (0.95)	0.2226	0.0514	0.2216	0.0495	0.2437	0.0442	0.1547	0.6556
Variance	40.2903	1.5832	40.4082	1.8106	49.5602	1.7740	28.3248	3.2664
Standard deviation	6.3475	1.2583	6.3568	1.3456	7.0399	1.3319	5.3221	1.807
Skewness	0.5564	−0.0232	0.6526	−0.1818	0.5814	−0.1301	0.5425	0.0169
Kurtosis	4.8025	7.8877	4.2860	7.9378	4.3966	7.6032	3.7345	4.3498
ADF	−22.1359	−18.8691	−21.8574	−18.97	−22.3855	−18.9584	−21.5732	−18.3835
Jarque-Bera	3038.111	7776.073	2510.354	7893.879	2584.788	7231.034	1889.112	2363.459

Notes The table reports the various descriptive statistics. LCL is lower control limit. UCL is upper control limit. ADF is augmented Dickey Fuller.
Critical values for ADF: 10 % −3.14; 5 % −3.43; 1 % −4.00
Source for ADF is: MacKinnon (1996)

Table 2 OLS Regression: Stock and volatility indices 2/2/2001–12/31/2012

Model	α	p-value	β	p-value
VDJ-DJ	0.0169	0.841	−3.4770	<0.0001
VIX-SPX	−0.00163	0.983	−3.5778	<0.00015
VXO-OEX	−0.02295	0.787	−3.978	<0.00015
VXN-NDX	−0.034775	0.644	−1.8746	<0.00015

Notes The table reports the OLS regression results for the return volatility pairs. All the estimated β values are significant at the 1 % level

ing this limitation is to employ a linear quantile regression framework. The reported linear quantile regression results are different from those from the OLS. For example, if the DJ index rises by 10 %, then the VDJ will be expected to fall by varying amounts along the quantiles and not by 34.77 % as reported for the OLS. For example, at the 50 % quantile level, we should expect a fall of 35.32 %, and this differs from the 90 % quantile level amount of 37.3 %. Also, the results show that the estimated dependence coefficient (β) values are significant across the quantiles, and are different. Though not reported, we did perform a test to see if the slopes were the same at all the reported quantiles. For the test, we employ the anova command which produces a quantile regression analysis of variance table and is based on tests proposed by Koenker and Bassett (1982). These results indicate that the volatility-return relationship changes across the quantiles and that they are also statistically significant.

5.3 Empirical Results Quantile Copula

Tables 4 and 5 give estimates for the quantiles for the Normal and Student-t copulas. For the empirical analysis, we assumed the marginals for the bivariate copula quantile regression follow Normal and Student-t distributions. The univariate Student-t distributions are allowed to have different degree of freedom parameters (see Embrechts et al. 2001 or Fang and Fang 2002). Two versions of the regressions are reported. In one, we work with raw volatility and stock return series and in the second, we fit a GARCH (1, 1) with Student-t errors to the data and then work with the standardized residuals. The estimation follows the general procedure outlined by Bouyé and Salmon (2009). See also Appendix A of Koenker (2005). The rugarch package (Version 1.2-3) of Ghalanos (2013) for R is used to extract the degrees of freedom parameters and the standardized residuals of the series. The quantreg pack-

Table 3 Linear quantile regression estimates: Stock and volatility indices 2/2/2001–12/31/2012

	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
VDJ-DJ α	−6.642	−4.815	−3.055	−1.893	−0.981	−0.109	0.750	1.764	2.935	4.890	6.930
(p-value)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VDJ-DJ β	−3.133	−3.166	−3.374	−3.379	−3.510	−3.532	−3,552	−3.544	−3.672	−3.743	−3.795
(p-value)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VIX-SPX α	−6.033	−4.385	−3.026	−1.905	−0.999	−0.198	0.658	1.582	2.821	4.681	6.830
(p-value)	0.0	0.0	0.0	0.0	0.0	0.008	0.0	0.0	0.0	0.0	0.0
VIX-SPX β	−3.25	−3.446	−3.547	−3.583	−3.613	−3.645	−3.672	−3.643	−3.770	−3.736	−3.733
(p-value)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VXO-OEX α	−6.548	−4.738	−3.243	−2.069	−1.199	−0.283	0.703	1.719	2.955	5.225	7.569
(p-value)	0.0	0.0	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
VXO-OEX β	−3.706	−3.845	−3.846	−3.985	−4.020	−3.981	−3.931	−4.02	−4.006	−4.168	−4.275
(p-value)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VXN-NDX α	−6.326	−4.583	−2.912	−1.885	−1.092	−0.246	0.581	1.617	2.76	4.79	6.872
(p-value)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
VXN-NDX β	−1.777	−1.72	−1.756	−1.76	−1.743	−1.733	−1.809	−1.864	−1.90	−1.90	−2.048
(p-value)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Notes The table reports the linear quantile regression results for the return volatility pairs. A p-value of $\leq$ 0:05 shows significance at the 5 % level
Computation is done using the R quantreg package of R. Koenker
R and the package quantreg are open-source software projects and can be freely downloaded from CRAN: http://cran.r-project.org

Table 4 Gaussian copula quantile regression coefficient estimates of volatility-return models 2/2/2001–12/31/2012

ρ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
VDJ-DJ	−0.853	−0.856	−0.855	−0.823	−0.801	−0.801	−0.830	−0.854	−0.862	−0.863	−0.866
VIX-SPX	−0.879	−0.887	−0.878	−0.870	−0.868	−0.878	−0.901	−0.901	−0.888	−0.879	−0.864
VXO-OEX	−0.892	−0.890	−0.881	−0.873	−0.852	−0.873	−0.881	−0.892	−0.889	−0.8880	−0.875
VXN-NDX	−0.811	−0.811	−0.786	−0.746	−0.719	−0.712	−0.771	−0.790	−0.804	−0.806	−0.789
ρ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
VDJ-DJ	−0.853	−0.851	−0.826	−0.781	−0.751	−0.746	−0.759	−0.787	−0.805	−0.817	−0.824
VIX-SPX	−0.887	−0.884	−0.870	−0.841	−0.809	−0.809	−0.830	−0.845	−0.852	−0.851	−0.839
VXO-OEX	−0.887	−0.891	−0.886	−0.857	−0.834	−0.819	−0.832	−0.854	−0.874	−0.875	−0.859
VXN-NDX	−0.795	−0.793	−0.769	−0.726	−0.705	−0.707	−0.729	−0.754	−0.759	−0.763	−0.758

Notes The table reports the Gaussian copula quantile regression coefficient estimates of volatility-return models for different quantiles. Computation is done using the R quantreg package of R. Koenker

R and the package quantreg are open-source software projects and can be freely downloaded from CRAN: http://cran.r-project.org

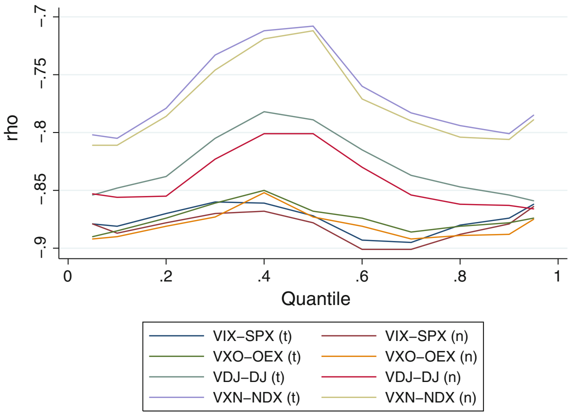
Table 5 Student-t copula quantile coefficient estimates of volatility-return models 2/2/2001–12/31/2012

ρ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
VDJ-DJ	−0.854	−0.848	−0.838	−0.805	−0.782	−0.789	−0.815	−0.837	−0.847	−0.854	−0.859
VIX-SPX	−0.879	−0.881	−0.870	−0.86	−0.861	−0.872	−0.893	−0.895	−0.88	−0.874	−0.862
VXO-OEX	−0.89	−0.885	−0.874	−0.861	−0.85	−0.868	0.874	−0.886	−0.881	−0.878	−0.874
VXN-NDX	−0.802	−0.805	−0.779	−0.733	−0.712	−0.708	−0.76	−0.783	−0.794	−0.801	−0.785
ρ	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95
VDJ-DJ	−0.85	−0.838	−0.802	−0.752	−0.705	−0.701	−0.712	−0.731	−0.775	−0.798	−0.822
VIX-SPX	−0.888	−0.881	−0.853	−0.826	−0.786	−0.778	−0.792	−0.816	−0.826	−0.83	−0.841
VXO-OEX	−0.89	−0.882	−0.868	−0.835	−0.799	−0.788	0.798	−0.82	−0.851	−0.857	−0.861
VXN-NDX	−0.810	−0.788	−0.751	−0.705	−0.676	−0.682	−0.69	−0.705	−0.726	−0.745	−0.764

Notes The table reports the Student-t copula quantile regression coefficient estimates of volatility-return models for different quantiles. Computation is done using the R quantreg package of R. Koenker

R and the package quantreg are open-source software projects and can be freely downloaded from CRAN: http://cran.r-project.org

Fig. 4 Calibration of copula quantile regression of US stock volatility on return: 2/2/2001–12/31/2012. *Notes* Normal copula is (n) and t copula is (t). The data period is from February 2, 2001 through December 31, 2012 unfiltered



age (Version 5.05) of Koenker (2012) for R is used to estimate the parameters of the non-linear quantile regression. The nlrq optimization results of quantreg are dependent on the starting values of the parameters and the algorithm option chosen for optimization.The reported results here are based on using the L-BFGS-B option for the Normal copula and the Brent option for the Student-t copula. In each table, the left panel gives results for the raw data, and right panel gives results for the GARCH (1, 1) filtered data. The estimates for the Clayton CQR are not reported. The GARCH (1, 1)filter allows for control for the changes in volatility. As seen from the tables, negative dependence is greater for low and high quantiles. Furthermore, the lower tail negative dependence is higher than the upper tail negative dependence. The results reported here are similar to those of Allen et al. (2012), who used data from US and European exchanges and a different sample period and reported that for most of the pairs they investigated, the negative dependence is greater for low and high quantiles. It should be noted that they did not consider the Dow-Jones volatilitity-return pair nor the S&P 100 volatility-return pair. They also found that the lower tail negative dependence is also higher than the upper tail negative dependence. Figures 4 and 5 show the calibrated values of rho based on copula quantile regression of US stock volatility on return under both the normal and Student t copulas without and with the GARCH (1, 1) filter. The shape based on the GARCH (1,1) filtered data are much more of an inverted U-shaped as compared to the non-filtered series. Figures 6, 7, 8 and 9 show the corresponding quantile curves with the GARCH (1, 1) filter. We do not present those for the unfiltered series. It should be noted that neither Alexander (2008) nor Allen et al. (2012) used some sort of filter to control for changes in volatility. Neglecting to control for volatility changes can lead to incorrect inference in a VaR analysis. For example, suppose one is interested in a VaR analysis and estimates the 5 % quantile regression to achieve this, if one does not control for changes in the level of volatility, the 5 % quantile regression line cannot be interpreted as a true VaR measure since the probability of witnessing any particular price deviation depends crucially on the variance of the distribution.

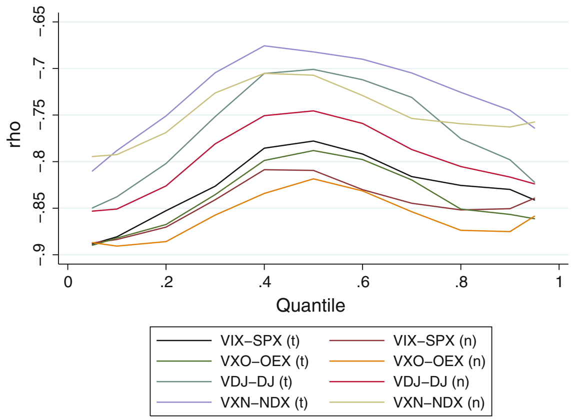


Fig. 5 Calibration of copula quantile regression of US stock volatility on return: 2/2/2001–12/31/2012. *Notes* Normal copula is (n) and t copula is (t). The data period is from February 2, 2001 through December 31, 2012 filtered with a GARCH(1, 1) specification

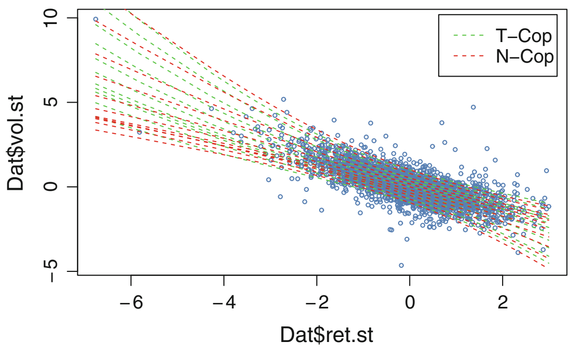


Fig. 6 DJ volatility-return quantile curves of normal and Student t copulas. *Notes* Daily closing percentage returns on the Dow Jones Industrial Average Index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the Dow Jones Volatility Index from February 2, 2001 through December 31, 2012

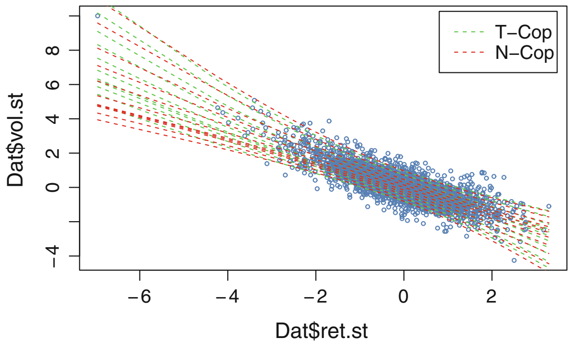


Fig. 7 S&P 500 volatility-return quantile curves of normal and Student t copulas. *Notes* Daily closing percentage returns on the S&P 500 Index (SPX) from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the VIX Index from February 2, 2001 through December 31, 2012

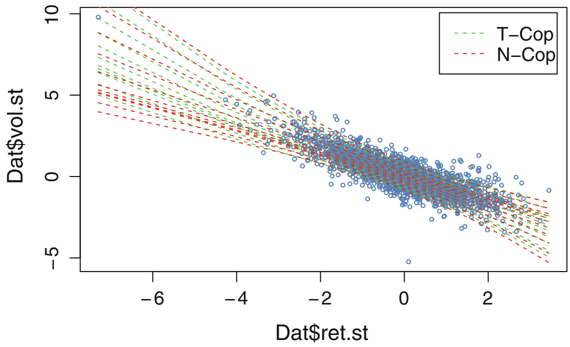


Fig. 8 S&P 100 volatility-return quantile curves of normal and Student t copulas. *Notes* Daily closing percentage returns on the OEX Index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the S&P 100 Volatility Index (VXO) from February 2, 2001 through December 31, 2012

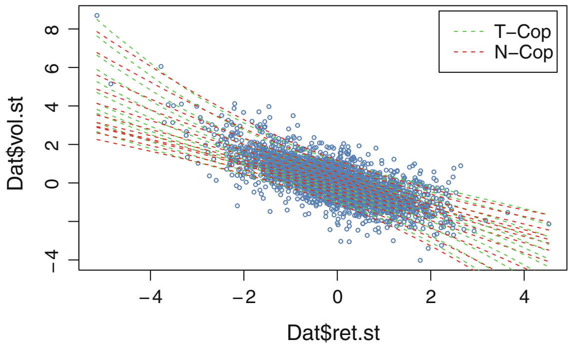


Fig. 9 NASD volatility-return quantile curves of normal and Student t copulas. *Notes* Daily closing percentage returns on the NASADAQ 100 index from February 2, 2001 through December 31, 2012. Daily closing percentage returns on the NASDAQ 100 Volatility Index (VXN) from February 2, 2001 through December 31, 2012

6 Conclusion

In this article, we have applied quantile copula regression techniques to examine the return-volatility relationship for indices reported on exchanges in the United States of America. We adopt the approach of Bouyé and Salmon (2009), which allows one to estimate copula based conditional quantile models. We utilize both linear quantile regression and copula quantile regression to evaluate the asymmetric volatility-return relationship between changes in the volatility index (VXD, VIX, VXO and VXN) and the corresponding stock index return series (DJIA, S&P 500, the S&P 100 and NASDAQ). The data period is from February 2, 2001 through December 31, 2012. We find, firstly, that the relationship between stock return and implied volatility depends on the quartile at which the relationship is being investigated. Secondly, we obtain results similar to those reported for European exchanges that show the existence of an inverted U-shaped relationship between stock return and implied volatility. This result was obtained even after controlling for changes in volatilities of return using a GARCH (1, 1) filter. This conclusion holds for all the US stock and implied volatility indices examined. Models that assumed otherwise are misspecified because ignoring the role of quartiles will result in errors in any attempt to forecast the relationship between returns and implied volatilities.

There are several issues that have not been addressed in the chapter. First, unlike Giot (2005), who examined the relationship between returns and volatility based on high volatility bull market, low volatility bull market, high volatility bear market sub-period classification, we have not concerned ourselves with such sub-period analysis in this chapter. It will be interesting to find out if the relationship is different across sub-periods. Second, the entire focus here is on the stock markets. Understanding the relationship between returns and implied volatilities for other commodities should be interesting.

Acknowledgements I like to thank Professor David Allen, Professor Graciela Chichilnisky, Professor Roger Koenker, Professor Randall Filer, a referee and the editor, for comments and suggestions. I also like to thank Lucas Dowiak for comments, suggestions and very useful computing assistance. I alone am responsible for any remaining errors.

References

Abdi, H. (2007). Method of least squares. In Neil Salkind (Ed.), *Encyclopedia of measurement and statistics*. Thousand Oaks (CA): Sage.

Alexander, C. (2008). *Market risk analysis: practical financial econometrics* (Vol. 2). Chichester: Wiley.

Allen, D. E., Gerrans, P., Singh, A. K., & Powell, R. (2009). Quantile regression and its application in investment analysis. *The Finsia Journal of Applied Finance (JASSA)*, *4*, 7–12.

Allen, D., Singh, A. K., Powell, R. J., McAleer, M., Taylor, J., & Thomas, L. (2012). *The volatility-return relationship: insights from linear and non-linear quantile regression*. Working Paper, School of Accounting Finance & Economics, Edith Cowan University, Retrieved from Complutense. http://eprints.ucm.es/16688/.

Ang, A., & Chen, J. (2002). Asymmetric correlations of equity portfolios. *Journal of Financial Economics*, *63*(3), 443–494.

Badshah, I. U. (2012). Quantile regression analysis of the asymmetric return-volatility relation. *Journal of Futures Markets*, *33*(3), 235–265.

Barnes, M. L., & Hughes, W. A. (2002). *Quantile regression analysis of the cross section of stock market returns* (Working Paper). Retrieved from Social Science Research. http://ssrn.com/abstract=458522.

Black, F. (1976). Studies of stock market volatility changes, *Proceedings of the American Statistical Association, Business and Economic Statistics Section* (pp. 177–181)

Bouyé, E., & Salmon, M. (2009). Dynamic copula quantile regressions and tail area dynamic dependence in Forex markets. *The European Journal of Finance*, *15*(7–8), 721–750.

Breymann, W., Dias, A., & Embrechts, P. (2003). Dependence structures for multivariate high-frequency data in finance. *Quantitative Finance*, *3*(1), 1–16.

Buchinsky, M. (1998). Recent advances in quantile regression models: A practical guideline for empirical research. *The Journal of Human Resources*, *33*(1), 88–126.

Buchinsky, M., & Leslie, P. (1997). *Educational attainment and the changing U.S. wage structure: Some dynamic implications* (Working Paper No. 97–13). Department of Economics, Brown University.

Buchinsky, M., & Hunt, J. (1999). Wage mobility in the united state. *The Review of Economics and Statistics*, *8*(3), 351–368.

Campbell, J. Y., & Hentschel, L. (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics*, *31*(3), 281–318.

Chen, X., Fan, Y., & Tsyrennikov, V. (2006). Efficient estimation of semiparametric multivariate copula models. *Journal of the American Statistical Association*, *101*, 1228–1240.

Cherubini, U., Luciano, E., & Vecchiato, W. (2004). *Copula methods in finance*. New York: Wiley.

Chichilnisky, G. (2009). The topology of fear. *Journal of Mathematical Economics*, *45*(11–12), 807–816.

Christie, A. (1982). The stochastic behaviour common stock variances: Value, leverage and interest rate. *Journal of Financial Economics*, *10*, 407–432.

Clayton, D. (1978). A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. *Biometrika*, *65*, 141–151.

Cook, R. D., & Johnson, M. E. (1981). A family of distributions for modelling non-elliptically symmetric multivariate data. *Journal of Royal Statistical Society B*, *43*(2), 210–218.

Cox, J., & Ross, S. (1976). The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, *3*, 145–166.

Dennis, P., Mayhew, S., & Stivers, C. (2006). Stock returns, implied volatility innovations, and the asymmetric volatility phenomenon. *Journal of Financial and Quantitative Analysis*, *41*(2), 381–406.

Eide, E., & Showalter, M. H. (1998). The effect of school quality on student performance: A quantile regression approach. *Economics Letters*, *58*(3), 345–350.

Eide, E., & Showalter, M. H. (1999). Factors affecting the transmission of earnings across generations. A quantile regression approach. *Journal of Human Resources*, *34*(2), 253–267.

Embrechts, P., Höing, A., & Juri, A. (2003). Using copulae to bound the value-at- risk for functions of dependent risks. *Finance & Stochastics*, *7*, 145–167.

Embrechts, P., McNeil, A., & Straumann, D. (2001). Correlation and dependence in risk management: Properties and pitfalls. In M. A. H. Dempster (Ed.), *Risk management: Value at risk and beyond* (pp. 176–223). Cambridge: Cambridge University Press.

Engle, R. F., & Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economics Statistics*, *22*(4), 367–381.

Fang, H., Fang, K., & Kotz, S. (2002). The metaelliptical distributions with given marginals. *Journal of Multivariate Analalysis*, *82*, 1–16.

Fleming, J., Ostdiek, B., & Whaley, R. E. (1995). Predicting stock market volatility: A new measure. *Journal of Futures Markets*, *15*(3), 265–302.

Frank, M. J. (1979). On the simultaneous associativity of F(x, y) and x + y − F(x, y). *Aequationes Math*, *19*, 194–226.

French, Kenneth R., William Schwert, G., & Stambaugh, Robert F. (1987). Expected stock returns and volatility. *Journal of Financial Economics*, *19*, 3–29.

Galton, F. (1886). Regression towards mediocrity in hereditary stature. *The Journal of the Anthropological Institute of Great Britain and Ireland*, *15*, 246–263.

Genest, C., & Mackay, J. (1986). The joy of copulas: Bivariate distributions with uniform marginals. *The American Statistician*, *40*, 280–283.

Genest, C., & Rivest, L. P. (1993). Statistical inference procedures for bivariate Archimedean copulas. *Journal of the American Statistical Association*, *88*, 1034–1043.

Ghalanos, A. (2013). rugarch: A garch r package (Version 1.2-3). Retrieved from r-project.org. http://www.r-project.org.

Giot, P. (2005). Relationships between implied volatility indices and stock index returns. *Journal of Portfolio Management*, *31*, 92–100.

Gumbel, E. J. (1960) Distributions des Valeurs Extremes en Plusieurs Dimensions. Publications de 1Institute de Statistique de 1Universite de Paris 9:171–173.

Harper, H. L. (1974–1976). The method of least squares and some alternatives. Part I, II, II, IV, V, VI. International Statistical Review *42*, 147–174; *42*, 235–264; *43*, 1–44; *43*, 125–190; *43*, 269–272; *44*, 113–159.

Hibbert, A., Daigler, R., & Dupoyet, B. (2008). A behavioural explanation for the negative asymmetric return-volatility relation. *Journal of Banking and Finance*, *32*, 2254–2266.

Joe, H. (1997). *Multivariate models and dependence concepts*. New York: Chapman and Hall.

Kim, T. H., & White, H. (2003). Estimation, inference, and specification testing for possibly misspecified quantile regressions. In T. Fomby & R. C. Hill (Eds.), *Maximum Likelihood Estimation of Misspecified Models: Twenty Years Later* (pp. 107–132). New York: Elsevier.

Kimeldorf, G., & Sampson, A. R. (1975). Uniform representations of bivariate distributions. *Communications in Statistics*, *4*, 617–627.

Koenker, R. (2005). *Quantile regression*., Econometric society monograph series New York: Cambridge University Press.

Koenker, R. (2012). Quantreg: A quantile regression R package (Version 5.05). Retrieved from r-project.org. http://www.r-project.org.

Koenker, R. W., & Bassett, G, Jr. (1978). Regression quantiles. *Econometrica*, *46*(1), 33–50.

Koenker, R. W., & Bassett, G, Jr. (1982). Robust tests for heteroscedasticity based on regression quantiles. *Econometrica*, *50*(1), 1577–1584.

Koenker, R. W., & Hallock, K. F. (2001). Quantile regression. *Journal of Economic Perspectives*, *15*(4), 143–156.

Koenker, R. W., & Xiao, Z. (2006). Quantile autoregression. *Journal of the American Statistical Association*, *101*(475), 980–990.

Kumar, S. (2012). A first look at the properties of India's volatility index. *International Journal of Emerging Markets*, *7*(2), 160–176.

Longin, F., & Solnik, B. (2001). Extreme correlation of international equity markets. *Journal of Finance*, *56*, 649–676.

Low, C. (2004). The fear and exuberance from implied volatility of S&P 100 index options. *Journal of Business*, *77*, 527–546.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics*, *11*, 601–618.

Neftci, S. (2000). Value at risk calculations, extreme events, and tail estimation. *Journal of Derivatives*, *7*(3), 23–37.

Nelsen, R. B. (2006). *Introduction to copulas*. New York: Springer Verlag.

Patton, A. J. (2004). On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. *Journal of Financial Econometrics*, *2*(1), 130–168.

Patton, A. J. (2006a). Estimation of multivariate models for time series of possibly different lengths. *Journal of Applied Econometrics*, *21*(2), 147–173.

Patton, A. J. (2006b). Modelling asymmetric exchange rate dependence. *International Economic Review*, *47*(2), 527–556.

Patton, A. J. (2009). Copula-based models for financial time series. In T. G. Andersen, R. A. Davis, J. P. Kreiss, & T. Mikosch (Eds.), *Handbook of financial time series*. New York: Springer Verlag.

Powell, J. L. (1986). Censored regression quantiles. *Journal of Econometrics*, *32*, 143–155.

Sklar, A. (1959). Fonctions de Riépartition án Dimensions et Leurs Marges. *Publications de lInstitut de Statistique de lUniversité de Paris*, *8*, 229–231.

Taylor, J. (1999). A quantile regression returns. *Journal of Derivatives*, *7*(1), 64–78.

Trivedi, P. K., & Zimmer, D. M. (2005). Copula modeling: An introduction for practitioners. *Foundations and Trends in Econometrics*, *1*(1), 1–111.

Whaley, R. (2000). The investor fear gauge. *Journal of Portfolio Management*, *26*, 12–17.

White, H. (1994). *Estimation inference and specification analysis*. New York: Cambridge University Press.

Wu, G. (2001). The determinants of asymmetric volatility. *The Review of Financial Studies*, *14*(3), 837–859.

Xiao, Z. (2009). Quantile cointegrating regression. *Journal of Econometrics*, *150*(2), 248–260.

Yu, K., Lu, Z., & Stander, J. (2003). Quantile regression: Applications and current research areas. Journal of the Royal Statistical Society, Series D. (The Statistician) *52*(3), 331–350.