

Global Warming and Economic Externalities

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1 Introduction

Much discussion of the economics of global warming emphasizes the issue of trade-offs in well-being between present and future generations (Nordhaus 2008; Nordhaus and Boyer 2000; Stern 2007). Specifically, is it socially beneficial for present and near future generations to sacrifice their own consumption to mitigate global warming for the benefit of generations yet to come?

In this paper we argue that the intergenerational distribution aspects of climate policy are relevant only when the externality has been corrected, and concern the distribution of welfare gains, not costs. If global warming is a negative externality, standard welfare analysis shows that all generations can benefit from its mitigation. Current generations can direct less of their foregone consumption to physical capital formation and more toward mitigation, thereby maintaining their own levels of welfare while bequeathing a better mix of conventional capital and stock of greenhouse gases (GHG) in the atmosphere to the future.

We illustrate this point by solving a *business-as-usual* economic growth model calibrated to current data for the intertemporal allocation of capital by a representative agent with an uncorrected externality and comparing the results to a solution

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in which the externality is corrected. The results show that the correction can represent a Pareto improvement from an inefficient to an efficient growth path with higher consumption levels and lower environmental damage. There are world political efforts to implement institutions which enforce the social cost of carbon emissions on individual agents, most notably the Kyoto Protocol which created a carbon market and is international law since 2005. Our contribution provides theoretical justification for such and further measures; the existing framework only succeeds in partially internalizing the externality with the emissions restrictions set in generous manners and the majority of emissions not being subject to any caps.

In much of the literature this simple observation is obscured because the optimal path is compared to a reference path along which the externality is partially corrected. This reference path maximizes the present discounted value of the felicity of per-capita consumption subject to the constraint that mitigation expenditure is equal to zero. This *constrained optimum* implicitly includes the marginal social cost of emissions in the representative agent's production and investment decisions, thus partially internalizing the externality. Comparing this solution to the true optimum incorrectly directs attention toward "intergenerational tradeoffs" because typically the constrained optimum shows higher per capita consumption for several decades. While policy economists have persisted in presenting the global warming problem primarily as an issue of intergenerational equity, many scholars emphasize the public-good nature of the problem, which leads to the perspective developed in the present paper (Chichilnisky 1994; Chichilnisky and Heal 1994; Chichilnisky and Sheeran 2009).

When, on the other hand, the optimal and business as usual paths are compared directly, the first-order effect of optimal mitigation is a potential increase in per capita consumption in every time period. Intergenerational equity enters into the problem only as a second-order effect as the optimal program distributes the potential gains from correcting the externality across generations in accordance with the representative agent's preference for consumption smoothing. The comparison of the optimal and constrained optimal paths thus leads to an upward biased estimate of the economic costs of mitigating global warming.

This symposium includes contributions on other important aspects of the global warming problem. The most related to our are those by Karp and Zhang (2016), who discuss the effectiveness of different policy instruments, Ostrom (2016), who discusses means of introducing cost transparency at different governmental levels, and Chipman and Tian (2016), who investigate the assumptions under which markets for pollution rights lead to Pareto improvements. Burniaux and Oliveira Martins (2011) aim at identifying the sensitivity of "carbon leakage" to key parameters in a general equilibrium model of climate policy. Dutta and Radner (2016) and Lecocq and Hourcade (2016) discuss further roadblocks in the way of efficient international climate policy in a multi-region framework. The remaining contributions (Asheim et al. 2016; Chichilnisky 2016; Figuières and Tiball 2016; Lauwers 2016) group around the ethical foundations of criteria for sustainability and their application to economics.

2 The Global Warming Problem

Human (industrial) production entails emissions of GHG. Given scientific evidence like the results presented in the 4th report of the Intergovernmental Panel on Climate Change (IPCC), such emissions impact the world climate negatively. An increase in the concentration of GHG is projected to increase the mean atmospheric temperature implying a higher frequency of disasters and natural catastrophes (such as droughts, floods, and heat waves), higher mortality rates, and a significant loss of biodiversity. These consequences have economic costs, the most apparent being a loss in the productive capacity of the world economy.¹

The world climate is affected by the use of capital which produces a negative externality in the form of emissions. Uninternalized externalities lead to inefficiencies since economic agents do not perceive the true cost of their actions and do not equalize (social) marginal costs and benefits. In our case the representative agent is over-emitting GHG, since she perceives the marginal cost of doing so (to her as an individual) is zero. Under the perfect foresight assumption, she is able to correctly predict the path of GHG (mainly CO₂) concentrations given her (and everybody else's) consumption, production, and investment choices. Although she is aware of the collective consequences of her actions, she thinks her individual contribution to the overall result is negligible. Consequently, she will not reduce her production-related emissions, either through producing less or investing in mitigation, because she knows that nobody else will do so (as they believe their actions to be insignificant, too). All agents end up choosing the same inefficient allocation. This point was made in Foley (2008). Such socially sub-optimal outcomes are well known from simple strategic games, the most prominent being the "Prisoners' Dilemma".

Given the inefficiency of over-accumulation of GHG stock in the atmosphere as a result of capital stock accumulation, the world economy is not operating at the intertemporal production possibility frontier (PPF). Future generations would appreciate lower stocks of CO₂ which implies that current generations should accumulate less conventional capital and consume more (of it) today. There is no intergenerational trade-off despite the fact that such a trade-off is posited in most of the global warming related economic publications. The mutual gains can be illustrated by moving the economy from a point inside the PPF to its boundary. This movement to an efficient equilibrium can be achieved by cost transparency (which amounts to increasing the cost of emitting to its true value).

Creating the correct price signal for GHG emissions (by whatever means, including cap-and-trade permits, Pigouvian taxes, or direct regulation) is sufficient to internalize the negative externality of global warming. As a result our agent will start to invest into mitigation. These mitigation costs, however, are small compared with

¹The worldwide economic implications of climate change are hard to quantify. This task becomes more difficult, the higher the assumed stock of carbon in the atmosphere. The report of the IPCC (2007) presents convincing evidence for the negative relationship between GDP and global warming. Tol (2009) gives an optimistic review.

the gain of obviating GHG emissions. As is shown below, averting climate change can represent a non-trivial Pareto improvement.

3 The Model

The model used here for analyzing the economic aspects of global warming is a standard Ramsey–Cass–Koopmans model of the economy extended to include GHG. In order to maximize the comparability of our results with models in the literature based on neoclassical growth theory, the economy only produces one good using a Cobb–Douglas production function, $F[K, L]$, in conventional capital and effective labor. Effective labor consists of the exogenously given growth paths of population, N , and Harrod-neutral technical change, B , (which can be translated into Hicks-neutral technical progress given the Cobb–Douglas technology), according to $L = BN$. The state equation for conventional capital, K , is in its standard form with capital increasing due to investment, I , and decreasing due to (exponential) depreciation at rate δ .

Following Nordhaus and Boyer (2000), the stock of GHG in the atmosphere, CD (for carbon dioxide, CO_2 , measured in parts per million volume, ppmv), enters the model as an additional state variable. Its dynamics depend on usable output, Y , and are governed by production-related emissions, $G[Y]$, mitigation efforts, $M[m]Y$, and (exponential) depreciation at rate ϵ . m is the share of usable output invested in mitigation. Mitigation efforts, $M[m]Y$, are linear in usable output similar to the abatement cost function, $\Lambda[\cdot]$, used in Nordhaus and Boyer (2000) and Nordhaus (2008).

In the interests of keeping the model as parsimonious as possible, temperature dynamics are omitted and CD in excess of pre-industrial levels lowers productive capacity directly via what we term a damage function, $Z[\text{CD}]$. There are no sinks and no time lags. Emissions fully affect output immediately and directly, $Y = Z[\text{CD}]F[K, L]$. Mitigation can take the form of removing existing CD from the atmosphere or by preventing current emissions. Mitigation does not alter carbon emissions intensity permanently.

Formally, the representative agent allocates shares of output to consumption, c , and investment, s , of which a certain output share, m , is invested into mitigation, in order to maximize utility, measured as the discounted present value of the felicity of per capita consumption over time. Let consumption $C[t] = (1 - s[t])Y[t - 1]$ and world output $Y[t] = Z[\text{CD}[t]]F[K[t], L[t]]$, then total utility is

$$U[C[t], t] = \sum_{t=1}^T \frac{1}{(1 + \rho)^{(t-1)}} U \left[\frac{C[t]}{N[t]} \right]$$

These choices are subject to initial values $K[0]$ and $CD[0]$ and the following state equations:

$$\begin{aligned} K[t+1] &= (1-\delta)K[t] + (s[t+1] - m[t+1])Y[t] \\ CD[t+1] &= (1-\epsilon)CD[t] + G[Y[t]] - M[m[t+1]]Y[t] \end{aligned}$$

With $\lambda[t]$ as the shadow price of capital and $\mu[t]$ the shadow price of CO_2 in the atmosphere, both expressed in terms of undiscounted felicity in the period t , and $c[t] = \frac{C[t]}{N[t]}$ as per capita consumption in period t , the adjoined Lagrangian for the above problem is

$$\begin{aligned} \mathcal{L}[K, CD, \lambda, \mu, t] &= \sum_{t=1}^T \frac{1}{(1+\rho)^{(t-1)}} \\ &\times (U[c[t]] + \lambda[t](K[t] - (1-\delta)K[t-1] - (s[t] - m[t])Y[t-1]) \\ &+ \mu[t](CD[t] - (1-\epsilon)CD[t-1] - G[Y[t-1]] + M[m[t-1]]Y[t])) \quad (1) \end{aligned}$$

Note that $\mu[t] < 0$, as CD affects production negatively. The (negative of the) current dollar price of carbon emissions is given by $\chi[t] = \frac{\mu[t]}{\lambda[t]}$.

3.1 The Optimal Case (OPT)

For optimality the following first-order conditions have to hold, which simultaneously represent a social competitive equilibrium under the assumption that institutions (such as an optimal carbon tax, a universal and optimal cap-and-trade system, or optimal direct regulation) exist to impose the social costs of emission on producers and consumers:

$$\partial_{\lambda} \mathcal{L} = 0 \Leftrightarrow K[t] = (1-\delta)K[t-1] + (s[t] - m[t])Y[t-1] \quad (2)$$

$$\partial_{\mu} \mathcal{L} = 0 \Leftrightarrow CD[t] = (1-\epsilon)CD[t-1] + G[Y[t-1]] - M[m[t]]Y[t-1] \quad (3)$$

$$\begin{aligned} \partial_K \mathcal{L} = 0 \Leftrightarrow \lambda[t] &= \frac{\lambda[t+1]}{1+\rho} (1-\delta + r_K(1-m[t+1]) \\ &+ (G'[Y[t]] - M[m[t+1]])\chi[t+1]) \end{aligned} \quad (4)$$

$$\begin{aligned} \partial_{CD} \mathcal{L} = 0 \Leftrightarrow \mu[t] &= \lambda[t]\chi[t] = \frac{\lambda[t+1]}{1+\rho} (r_{CD}(1-m[t+1]) \\ &+ (1-\epsilon + r_{CD}(G'[Y[t]] - M[m[1+t]]))\chi[1+t]) \end{aligned} \quad (5)$$

$$\partial_s \mathcal{L} = 0 \Leftrightarrow \lambda[t] = \frac{c[t]U'[c[t]]}{C[t]} \quad (6)$$

$$\partial_m \mathcal{L} = 0 \Leftrightarrow \chi[t] = -\frac{1}{M'[m[t]]} \quad (7)$$

The first two equations are simply the state equations for $K[t]$ and $CD[t]$. With $r_K = (1 - Z[CD])F_K[K, L]$ the marginal product of capital, the next equation tells us that in an optimal program the current value of capital must be equal to its marginal benefit, which is the discounted value of its net marginal product factoring in mitigation costs and net emissions resulting from a larger capital stock. Since the time path of $\lambda[t]$ reports the change in the shadow price of capital, its change times the discount factor yields the marginal rates of intertemporal substitution which equal the real interest rate $j[t] = \frac{\lambda[t]}{\lambda[t+1]}(1 + \rho)$.

With $r_{CD} = \frac{-Z'[CD]Y}{Z[CD]}$ the marginal product of CD, the same has to hold in the fourth equation for CD: The price of CO₂ must be equal to the discounted value of its net marginal product, again, factoring in mitigation positively and net emissions resulting from higher output negatively. This equation thus reflects the assumption that an effective system of optimal pricing of emissions is in effect when we calculate the OPT path. The last two equations are the Euler equations and establish optimality with regard to the choice variables, $s[t]$ and $m[t]$. They tell us, first, that marginal utility of consumption per capita in period t has to be equal to cost of capital (which is measured in per capita utils per unit of capital). Through Eq. (4) the marginal utility of consumption per capita is equal to the per capita marginal benefits of accumulating more over the remaining time horizon; second, that marginal cost of mitigating has to be equal to the marginal future benefit of doing so. The capital (usually dollar) price of carbon emissions is fixed by the cost of the marginal emission reduction. This thought will be taken up later. Given the Euler equations and the co-state equations, one can derive Ramsey-Keynes rule equivalents of the system.

3.2 The Business-as-Usual Case (BAU)

We model the business-as-usual case as an equilibrium of the economy in which global warming is the outcome of a negative externality. A state variable is an externality when it has a real impact on the objective function or constraints, but no institutions exist to enforce the social price on individual agent decisions involving it. Each agent assumes that her decisions will not affect the path of the externality, but when all agents make the same decisions the path of the externality changes. On a perfect-foresight equilibrium path with an uncorrected externality, each agent is assumed to correctly forecast the path of the externality, but ignores the effect of her decisions on the path of the externality. Thus on the equilibrium path with CD as an uncorrected externality the typical agent solves the above maximization problem expecting a certain time path of for the external $CD[t]$. The correct forecasting assumption amounts to the side condition that expected $CD^e[t] = CD[t]$, where $CD[t]$ is the path of the externality corresponding to the representative agent's chosen decisions. The difference between the equilibrium path with an uncorrected externality and the optimal path is the fact that the typical agent does not adjust her controls to take account of their effect on the externality. She is not aware of the true social cost of emitting. No social institutions exist to provide the correct price signal to steer the

economy. As a result, the socially competitive equilibrium and the optimum diverge. Also, the social bid price and social ask prices for the externality are not equal.

It is possible to express the second-best equilibrium path with an uncorrected externality through the Lagrangian first-order conditions. The first-order conditions with respect to the shadow prices return the real laws of motion of the system, which must be obeyed. The representative agent, however, ignores the effect of her decisions on the external state variables, which corresponds to setting the shadow price on these variables equal to zero in the first-order conditions with respect to the non-external state and choice variables. The first-order condition with respect to the external state variable then plays no active role in the solution, but does keep track of the real social value of the externality in terms of its shadow-price μ . The BAU path solves these modified first-order conditions:

$$\partial_\lambda \mathcal{L}_{\text{BAU}} = 0 \Leftrightarrow K[t] = (1 - \delta)K[t - 1] + s[t]Y[t - 1] \quad (8)$$

$$\partial_\mu \mathcal{L}_{\text{BAU}} = 0 \Leftrightarrow \text{CD}[t] = (1 - \epsilon)\text{CD}[t - 1] + G[Y[t - 1]] \quad (9)$$

$$\partial_K \mathcal{L}_{\text{BAU}}|_{\mu[t]=0} = 0 \Leftrightarrow \lambda[t] = \frac{\lambda[t + 1]}{1 + \rho} (1 - \delta + r_K) \quad (10)$$

$$\partial_{\text{CD}} \mathcal{L}_{\text{BAU}}|_{\mu[t]=0} = 0 \Leftrightarrow \mu[t] = \lambda[t]\chi[t] = 0 \quad (11)$$

$$\partial_m \mathcal{L}_{\text{BAU}}|_{\mu[t]=0} = 0 \Leftrightarrow m[t] = 0 \quad (12)$$

$$\partial_s \mathcal{L}_{\text{BAU}}|_{\mu[t]=0} = 0 \Leftrightarrow \lambda[t] = \frac{c[t]U'[c[t]]}{C[t]} \quad (13)$$

It is important to notice the subtle differences between these equations and the fully optimal path equations. The state equations (8) and (9) are the same on the optimal and BAU paths. But on the BAU path the controls are optimized without taking account of the externality, since $\mu[t] = 0$ in the first-order condition. As a result $m[t] = 0$. $s[t]$ remains unaltered due to the specific form of the model ($s[t]$ is the total share of income saved). Likewise, Eq. (10) determines the shadow price of the non-external state variables with the shadow price of the externality, $\mu[t] = 0$. In the absence of the externality, the Euler equation for capital and the optimal savings decision reduce to the usual Ramsey-Keynes rule.

There are two types of misallocation on the BAU path. First, because there is no market price for carbon emissions, the typical agent allocates too little (zero) resources to mitigation. Second, the typical agent invests too much in conventional capital because she ignores the impact of increasing output on increasing climate damage.

In our calculations we keep track of how $m[t]$ and $\mu[t]$ would evolve according to their first-best first-order conditions given the other, second-best variables.

$$\begin{aligned} \partial_m \mathcal{L} = 0 &\Leftrightarrow \chi[t] = M'[m[t]] \\ \partial_{\text{CD}} \mathcal{L} = 0 &\Leftrightarrow \mu[t] = \lambda[t]\chi[t] = \frac{\lambda[t + 1]}{1 + \rho} (r_{\text{CD}}(1 - m[t + 1]) \\ &\quad + (1 - \epsilon + r_{\text{CD}}(G'[Y[t]] - M[m[1 + t]])) \chi[1 + t]) \end{aligned}$$

3.3 The Constrained Optimal Case (COPT)

Researchers of the economic consequences of global warming (most prominently and recently Nordhaus and Boyer 2000; Nordhaus 2008) analyze an optimal path under the constraint that no mitigation is undertaken. Because this type of path implicitly partially internalizes the externality, it seems to us that it does not represent “business-as-usual”, and in the remainder of this paper we call this type of path *constrained optimal*.

How exactly the constrained optimal path can be derived as an equilibrium within the representative-agent perfect foresight methodology is somewhat mysterious. Fully rational agents with perfect foresight acting with complete markets adopt the first best solution presented above as the optimal path. When markets are incomplete and there is no price signal for the marginal social value of the externality, the equilibrium is the BAU path described in the last section. The constrained optimal (COPT) path, on the other hand, represents an inconsistent mixture of assumptions about the representative agent’s information. On the one hand, the representative agent on this type of path correctly estimates the marginal social cost of emissions in making her consumption, investment, and production decisions. On the other hand, she seems to ignore the availability of mitigation technologies, despite this understanding of the marginal social cost of emissions. This divergence results in a difference between the marginal social value and marginal social cost of mitigation. The agents in this mixed scenario perceive the marginal social cost of emitting as zero, the only price that justifies no mitigation. At the same time, however, the agent is confronted with the true carbon price in her decision on how much output to consume and how much to re-invest for capital formation. While this inconsistency within the perfect foresight framework is corrected for in the business-as-usual case above, we also solve for the constrained optimal case given its importance in the economic literature on global warming.

The first-order conditions of the constrained optimal case are the special case of the fully optimal with $m[t] = 0$.

$$\partial_{\lambda} \mathcal{L}_{\text{COPT}} = 0 \Leftrightarrow K[t] = (1 - \delta)K[t - 1] + s[t]Y[t - 1] \quad (14)$$

$$\partial_{\mu} \mathcal{L}_{\text{COPT}} = 0 \Leftrightarrow \text{CD}[t] = (1 - \epsilon)\text{CD}[t - 1] + G[Y[t - 1]] \quad (15)$$

$$\partial_K \mathcal{L}_{\text{COPT}} = 0 \Leftrightarrow \lambda[t] = \frac{\lambda[t + 1]}{1 + \rho} (1 - \delta + r_K(1 + G'[Y[t]]\chi[t + 1])) \quad (16)$$

$$\begin{aligned} \partial_{\text{CD}} \mathcal{L}_{\text{COPT}} = 0 \Leftrightarrow \mu[t] = \lambda[t]\chi[t] &= \frac{\lambda[t + 1]}{1 + \rho} \\ &\times (r_{\text{CD}} + (1 - \epsilon + r_{\text{CD}}G'[Y[t]])\chi[1 + t]) \end{aligned} \quad (17)$$

$$\partial_s \mathcal{L}_{\text{COPT}} = 0 \Leftrightarrow \lambda[t] = \frac{c[t]U'[c[t]]}{C[t]} \quad (18)$$

Notable changes include the altered state equation for $CD[t]$ in which carbon dioxide concentration can only be changed through emissions and dissipation and the two modified co-state equations. The change in the marginal benefit of capital from the optimal to the constrained optimal path depends on the functional form of $M[m[t]]$. For any meaningful mitigation function, the marginal benefit of capital will be lower in the constrained optimal case. As a result, less capital will be accumulated. This is intuitive as a reduction in K is the only means by which $CD[t]$ increases can be counteracted with mitigation constrained to zero. Note that while agents are deprived of the choice to mitigate, they still fully respond to changes in the price of carbon in their accumulation decisions. The same logic applies to the price of $CD[t]$. Lower marginal benefit will lead to higher CO_2 concentration, as CD is a “bad”.

In our calculations below we retain the (for this program) superfluous optimality condition for $m[t]$ in order to see what mitigation effort would be under an optimal scenario (at that point in time of the program).

$$\partial_m \mathcal{L} = 0 \Leftrightarrow \chi[t] = -\frac{1}{M'[m[t]]}.$$

3.4 Basic Logic

The basic logic and qualitative features of the three cases can be seen even without specifying functional forms, calibrating the model to reflect current economic values, and solving it over long time horizons.

On the OPT equilibrium path agents will invest enough resources in mitigation to compensate emissions to the point where marginal cost of doing so is equal to the benefit in output due to less environmental damages. This level is defined by the damage function which is key to the quantitative outcomes of the model. The carbon price will be defined by mitigation efforts. It will be equal to the cost of marginal mitigation efforts.

On the BAU equilibrium path agents are not only deprived of the mitigation instrument, but also see themselves incapable of affecting the stock of CO_2 . Their decisions are taken solely with respect to maximizing their intertemporal utility and consumption. The carbon dynamics drive the system. Emissions from rapid capital accumulation will drive up carbon dioxide levels to the point where environmental damage chokes off further accumulation due to the falling profit rate.

On the COPT equilibrium path agents are deprived of the mitigation instrument. In order to move the economy to a steady state equilibrium, the savings decisions and the capital stock will have to do all of the adjustment. It is important to note that in the COPT case (as well as in the BAU case), the climate dynamics dominate the outcome. A steady state can only be achieved when emissions equal to the dissipation of existing carbon stock in the atmosphere. This sets the level of admissible capital stock and the appropriate savings rate.

Given the logic of the three model cases, it is trivial, but nonetheless important, to note that the overall utility will be the greatest on the OPT path, followed by the COPT and BAU paths. Growth in capital stock will be highest on the OPT, followed by the BAU. Investment will be lower in the COPT than in the BAU scenario as agents are aware of the deleterious effects of their accumulation decisions and, hence, more cautious. Climate catastrophes—meaning high equilibrium levels of CO₂—are certain in the BAU and very likely in the COPT case. Over the range of all realistic mitigation functions, CO₂ levels will stabilize at low or moderate levels in the OPT case.

These characteristics are given by the model structure and are, thus, independent of the functional forms and parameter values. We hope that in this light the secondary relevance of much of the current debate on discounting factors will be apparent.

Fast convergence to the steady state implies that the steady state results will drive much of the model's behavior and its transition dynamics. As can be seen below, the optimal path reaches its steady state within 10 decades. Nordhaus (2008) and Stern (2007) do not devote much attention to the steady states of their models and the implications of steady state values on the transition dynamics.

4 Functional Forms

There are several choices that need to be made about the forms the production, damage, mitigation, and emissions functions take. The functional forms we use in the simulations reported here are as follows: Utility has its traditional iso-elastic manifestation $U[c[t]] = \frac{c[t]^{1-\eta}}{1-\eta}$, or $U[c[t]] = \text{Log}[c[t]]$ when $\eta = 1$. Potential output is a Cobb-Douglas production function, $F[K[t], L[t]] = AK[t]^\alpha L[t]^{1-\alpha}$. Carbon-related damages are measured on a scale between 0 and 1 with zero damage at the pre-industrial level of 280 ppmv and complete output loss at a CDMax = 780 ppmv, with the damage function $Z[\text{CD}[t]] = \left(1 - \left(\frac{\text{CD}[t]-280}{\text{CDMax}-280}\right)^{\frac{1}{\gamma}}\right)^\gamma$. This functional form implies that even at current CO₂ levels of 380 ppmv a certain fraction of potential output is lost due to environmental degradation. Emissions, $G[Y[t]] = \beta Y[t]$, are linear in output at a constant carbon intensity of production. The mitigation function, $M[m[t]] = \zeta \frac{1-e^{-\nu m[t]}}{\nu}$, where ζ is a scaling parameter and $0 < \nu$ is a semi-elasticity reflecting diminishing returns to m , which converts the unitless proportion of output devoted to mitigation, m , into CO₂ reduction per \$ spent on mitigation, that is ppmv/\$. In our specification of the mitigation function, we diverge from the other studies in assuming positive mitigation costs even at very low mitigation efforts ($\partial_m M[m[t]]|_{m[t]=0} = \zeta \neq 0$). From the first-order condition for $m[t]$, $M'[m[t]]$ equals the carbon price $\chi[t]$ in the first best solution.

Population growth follows Nordhaus (2008) and UN projections in assuming that world population will rise from currently 6500 to 8600 million over the next 10–20 decades, and then stabilize at this level. Labor productivity is assumed to start at

a yearly growth rate of 2 % and to flatten out at 3 times its current value after 30 decades.

Given these functional forms, the adjoined Lagrangian (with $L[t] = B[t]N[t]$) is:

$$\begin{aligned} & \mathcal{L}[K[t], CD[t], \lambda[t], \mu[t], t] \\ &= \sum_{t=1}^T \frac{1}{(1 + \rho)^{t-1}} \left(\frac{\left(\frac{(1-s[t])Z[CD[t-1]]F[K[t-1], L[t-1]]}{N[t]} \right)^{(1-\eta)}}{1 - \eta} \right. \\ & \quad + \lambda[t](K[t] - (1 - \delta)K[t - 1]) \\ & \quad - (s[t] - m[t])Z[CD[t - 1]]F[K[t - 1], L[t - 1]] \\ & \quad + \mu[t] \left(CD[t] - (1 - \epsilon)CD[t - 1] - \left(\beta - \zeta \frac{1 - e^{-vm[t]}}{v} \right) \right. \\ & \quad \left. \left. \times Z[CD[t - 1]]F[K[t - 1], L[t - 1]] \right) \right) \end{aligned}$$

5 Calibration

Given the assumptions of functional forms, the actual functions need to be calibrated to match economic and physical realities. All parameters are geared towards a decadal time interval. In the benchmark case Lagrangian, the discounting factor, $\rho = 0.1$. With output measured in units current (2000–2010) \$ trillion, initial capital stock is assumed to be $K_0 = 200$. CO_2 is measured as parts per million volume (ppmv). Initial CO_2 concentration $CD_0 = 380$. Capital decays radioactively with $\delta = 0.7$.

Currently, 7 Gt carbon are burnt per year. This corresponds to an increase in CD of 3.37 ppmv. With a yearly world output of \$60 trillion this implies a carbon dioxide emission intensity $\beta = \frac{3.37}{60} = 0.0561 \frac{\text{ppmv}}{\text{\$trillion}}$. As the actual increase in atmospheric carbon dioxide concentration is only about 2 ppmv, dissipation is 1.37 ppmv. This yields a depreciation factor $\epsilon = \frac{1.37}{380} = 0.0036$.

The marginal product of capital in the Cobb–Douglas production function α is set at 0.35 in line with standard economic research. Total factor productivity is calibrated to match current world output of \$60 trillion. The elasticity parameter in the utility function η is set at 2, in our baseline simulations.

The damage function $Z[CD[t]]$ also takes an iso-elastic form. This allows us to combine the apparent global warming optimism of economists towards the low damages of global warming at low carbon dioxide concentration with the serious warnings of climate scientists about severe output loss at high carbon dioxide concentration (which is set to 780 ppmv in our model). We set $\gamma = 0.5$, which is at the higher end of potential damages (Barker 2008). The calibration points usually cited are the IPCC (2007) predictions of an increase to a doubling of pre-industrial con-

centrations (about 280 ppmv) leading to a temperature rise of 3 °C and an increase of 4 °C leading to a potential output loss of 1–5 % of current output. Nordhaus (2008) assumes that current damages to the world economy are 0.15 % of output. This corresponds to $\gamma = 0.3$. The results for $\gamma = 0.3$ are reported in the sensitivity analysis section below. We deviate from Nordhaus as his damage function is lacking any reasonable upper limit on temperature and ultimately CO₂ concentration. Rezai (2010) shows that our parametric form of a damage function lies consistently above Nordhaus' for $\gamma = 0.5$ and below it with $\gamma = 0.3$ up to 500 ppmv (which is the relevant range for the optimal program). It is worth mentioning that the asymptotic behavior of the damage function for high concentrations has little impact on the OPT path as long as its shape for low concentrations is close to the mentioned calibration points. This holds for our range of damages.

Figure 1 plots the damage function for $\gamma = 0.5$ and a band of $0.7 \geq \gamma \geq 0.3$, around which a sensitivity analysis is carried out below. It also includes the assumptions on environmental damages from Nordhaus (2008) as a dark gray area up to CD = 580. Note that while our assumptions on γ regarding damages can be regarded as high, so are the assumptions on mitigation costs with the carbon price at \$160 (per t of C).

The parameter ζ represents the marginal reduction in CO₂ concentration (ppmv) per \$T from spending a small amount on mitigation when mitigation is zero. If it costs \$160 to remove one tonne of C at present (carbon markets suggest level between \$75 and \$125), when effectively $m = 0$, then to reduce CO₂ concentration by 1 ppmv through removing 2.07 Gt C from emissions would cost (2.07)(0.160\$ trillion) = 0.331\$ T, so we set $\zeta = 3 \frac{\text{ppmv}}{\$T}$. Using this specification the carbon price is directly linked to and anchored by marginal mitigation efforts. Note that the assumption of a lower current carbon price increases ζ and makes the mitigation function more effective. Figure 2 plots the mitigation function for these parameter values.

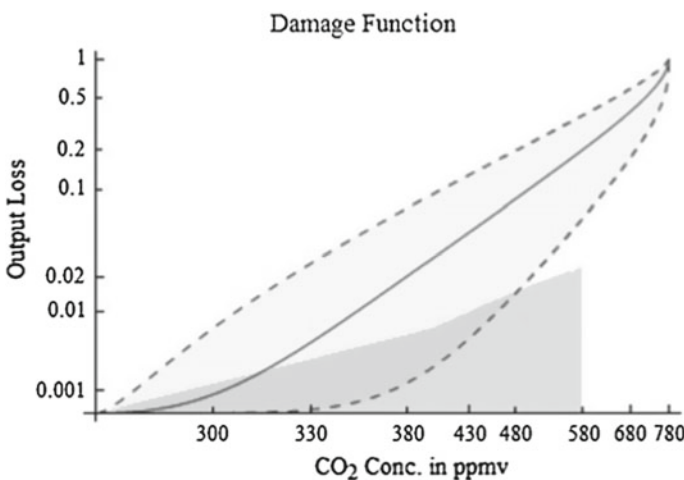


Fig. 1 Damage function with $\gamma = 0.5$

Fig. 2 Mitigation function for $\zeta = 3$ and $\nu = 6$

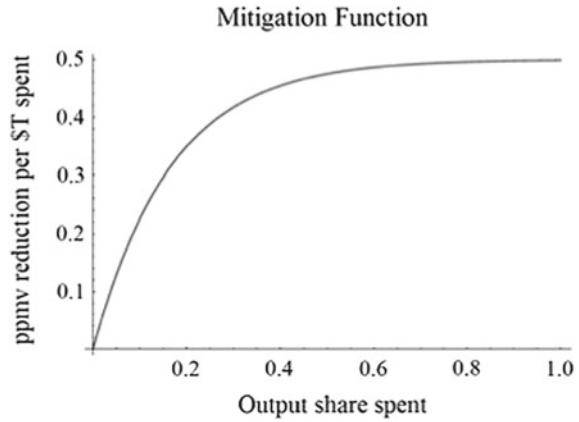


Table 1 Overview of the parameter values used in the numerical simulation

Function	Parameter	Value
$\mathcal{L}[\cdot]$	ρ	0.1
	K_0	200
	CD_0	380
	δ	0.7
	ϵ	0.036
$U[\cdot]$	η	2
$F[\cdot]$	α	0.35
	A	28.4
$Z[\cdot]$	γ	0.5
	CDMax	780
$G[\cdot]$	β	0.056
$M[\cdot]$	ζ	3
	ν	6

Table 1 provides an overview over the parameter assumption.

6 Computational Implementation

The above systems of equations can be reduced to 4 laws of motion, 2 for the state variables and 2 for the co-state variables by substituting the optimal expressions for the controls into the other equations and forming thus the maximized Lagrangian. These 4 difference equations, of which some form a subsystem in the COPT and BAU case, have to hold for $t = 1, \dots, T$. In addition, initial conditions on the two state variables and terminal transversality conditions on the co-state variables have to hold. This yields $4(T + 1)$ conditions to determine $4(T + 1)$ variables.

In order to solve this set of equations, we make use of the program *Mathematica* and its root finding command. In this process the specification of initial search parameters is crucial. Generally, *Mathematica* proves to be quite agile in finding the equilibrium path even if state and co-state variables are persistently shifting on a steady growth path. Specifying variables in logarithmic forms some times facilitates the search routine in this case.

7 Growth Paths

The above first-order conditions are sufficient as well as necessary for a global maximum as the maximized Hamiltonian is concave in K and CD for any given λ and μ (that is, the objective function is quasi-concave and the constraints convex in K and CD). The programs below are set up as finite-horizon problems with the additional requirement of the transversality conditions $\lambda[T]K[T] = 0$ and $\mu[T]CD[T] = 0$ with a fixed T at 60 decades. These terminal conditions guarantee that the calculated OPT path is a valid optimum of the primal problem over the finite time horizon. We choose the time horizon sufficiently long such that the paths to approximate the “steady-state” in the middle of the time horizon. This characteristic is known as the turnpike property which assures that finite horizon programs mimic their infinite horizon twins for sufficiently long horizons (Samuelson 1965). The paths below effectively reach their steady state within 30 decades. Solving over 60 periods becomes sufficiently close to the infinite horizon problem. Note that it would be possible to change the transversality end-point conditions to require, for example, some minimal capital and maximal CO_2 stocks at the end of the program, but it is not very easy to see how to choose those levels, or, in fact, to use this method except in the finite time horizon case. (It is not really correct to force the path to converge to the steady state at any finite time, for example.) We also emphasize the illustrative character of the simulations below, since long-run projections of any economic growth model are subject to high uncertainties in key parameters and the possible appearance of intervening factors not included in the model.

Figure 3 is a comparison of the optimal, constrained optimal and business-as-usual paths for these parameters, in terms of world per capita consumption, the damage from global warming, the implied price of carbon, and the CO_2 concentration. The finite-horizon program approaches its steady state very quickly, as to be expected from the turnpike theorem.

On the unconstrained optimal path capital accumulation combined with “small” mitigation efforts of around 1–2 % of GDP enables sustainably rising output and consumption levels. Mitigation efforts are front-loaded, meaning that most of the mitigation is done in the first few decades, such that only current CO_2 emissions have to be mitigated in later periods. The carbon price stabilizes around \$180/t which is close to zero mitigation carbon cost of \$160. In fact, atmospheric carbon concentration decreases almost to its pre-industrial level.

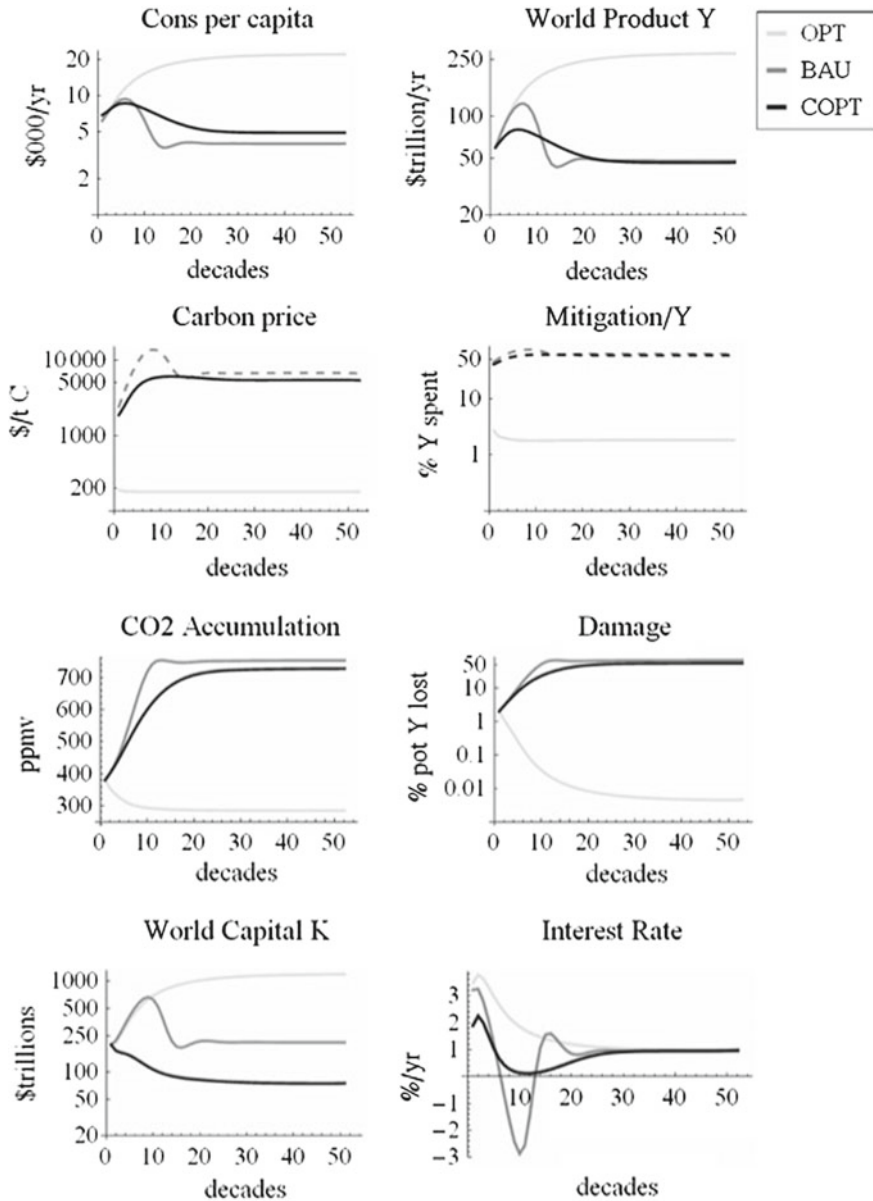


Fig. 3 The optimal equilibrium path, OPT, is plotted in *pale-gray*, the BAU equilibrium in *gray*, and the constrained equilibrium, COPT, in *dark-gray*. The carbon price on the optimal path is about \$200/t, and the damage on the optimal path is less than 1% of potential output. The carbon price is the social marginal value of foregoing the emissions from 1t of carbon. For OPT and COPT the carbon price is effective in economic decisions, but not in BAU (and is plotted as a *dashed line*). For OPT the mitigation percentage is the proportion of world product devoted to mitigation. For COPT and BAU the mitigation percentage is the investment called for by the imputed carbon price (and is plotted as a *dashed line*), while the actual mitigation is zero

On the BAU path agents lack the correct price signals to correct the negative externality. This leads to inefficiencies in several respects. Capital rises at a rate similar to the optimal case during the first 100 years although no mitigation can be carried out; the implication of this over-accumulation is rapidly rising CO₂ concentration and environmental damage. Since the capital accumulation equation for $\lambda[t]_{\text{BAU}}$ is independent of any carbon related costs (or their price signals), accumulation continues until damages are so high that further accumulation cannot occur due to the declining productivity of capital and labor inputs. Output and consumption are bound to decline due to ever higher carbon concentration and damages until the marginal product of capital has fallen sufficiently to approach a stable equilibrium. Equilibrium output and consumption per capita are almost 20 % below current levels and less than 25 % of the OPT levels despite significant technical progress and population growth. The gray dashed lines in the carbon price and mitigation graphs depict the implied carbon price and the mitigation called for by this price. Implicit carbon price and mitigation efforts on the BAU are fifty times higher than their optimal counterparts. The inefficiency of the BAU can also be seen in the equilibrium saving rates. While higher capital stock implies higher saving to compensate the lower marginal product, the equilibrium BAU saving rate with its lower capital stock is higher in our simulations due to the (unnecessary) high carbon concentration.

The constrained optimal path does slightly better than the BAU path. Although the agents in the COPT also are confined to zero mitigation, they conceive the correct price signals and run up GHG in the atmosphere much more cautiously than in the BAU scenario. In fact, overall capital stock is decreasing on this path as current levels are suboptimally high, implying too many carbon emissions. As carbon concentration increases, so must the marginal product of capital which can only be achieved with a lower capital stock. Equilibrium GHG concentration entails a high carbon price which, again, calls for high mitigation efforts. Equilibrium carbon concentration and price are lower than on the BAU path. The carbon price is about twenty times the optimal level.

While the quantitative results of our simulations are dependent on the specific parameter assumptions, it is important to point out that the qualitative results are independent of these assumptions. Especially the finding that moving from the inefficient BAU path to the efficient OPT path through mitigation constitutes a Pareto improvement should be noted. This result implies that there is no cost to mitigation, but there are significant gains from doing so (in our simulations up to 400 % of GDP). Higher or lower discounting rates will not alter this. In light of the magnitude of the avail, we argue that questions of uncertainty and intergenerational equity which are discussed further below can be considered of second-order importance.

The high carbon prices and low, often negative real interest rates implied by these simulation results also underline the important methodological point that the present discounted value of future costs and benefits are conditional on some particular consumption path for the representative agent. On the BAU path simulated here, for example, the implicit real interest rate is negative for a significant part of the transition path and below the discount rate on average over the whole path. The real interest rate is negative because the representative agent would pay a high price to move

consumption from the early part of the BAU path, where capital and labor are highly productive, to the later part of the path, where productivity has been destroyed by the climate catastrophe. Even though the time horizon of the simulations is very long, the present discounted value of carbon emissions on the actual path is, as a result, quite substantial. Much of the existing literature on global warming ignores the dependence of present discounted values on the consumption path and calculates present discounted values of future costs and benefits using steady-state values of real interest rates. Since steady-state real interest rates are positive, discounting at these interest rates results in very low present discounted values even of very significant future damage, if the damage is far enough in the future (see Chichilnisky and Eisenberger 2010). The consistent present discounted values using real interest rates implied by equilibrium paths are much higher.

8 Intergenerational Equity

An important aspect of the current climate change debate in the economic literature centers on the problem of intergenerational equity. This focus on generational equity arises primarily from a failure to appreciate that the business-as-usual path with an uncorrected externality is inefficient. In particular, it is a consequence of the mistaken use of the COPT path rather than the real BAU path as the benchmark with which the OPT path is compared. The most striking difference between the COPT and OPT paths is the generational distribution of consumption. As we have explained above, the COPT path is not a theoretically relevant benchmark, because it represents an inconsistent mixture of partial internalization of the global warming externality and a failure to divert resources from conventional investment to mitigation.² The use of the COPT path as the benchmark comparison leads to the misleading impression that the problem of correcting the global warming externality is primarily an issue of intergenerational equity, how much to sacrifice the consumption of current generations to protect the environment for future generations. In an optimal growth framework the resolution of this trade-off depends on the discount factor, ρ , and the degree of social preference for consumption smoothing expressed in the elasticity of felicity with respect to consumption, η . But with the correct BAU benchmark the correction of the global warming externality can provide an intergenerational Pareto improvement, raising the per capita consumption and felicities of every generation. The parameters ρ and η influence the distribution of this intergenerational gain, which is a

²Shiell and Lyssenko (2008) offers a clear explanation of the logic supporting this point, in the context of a two-state variable model very similar to the present one. This paper also presents an ingenious alternative method to ours of computing approximate BAU paths. Shiell and Lyssenko also find that the asymptotic behavior of COPT and BAU paths is very similar. Because they do not focus on the initial transient levels of consumption on the OPT and BAU paths, however, their discussion does not bring out the critical role of the COPT path in suggesting that an intergenerational distributional tradeoff is at the center of global-warming policy evaluations.

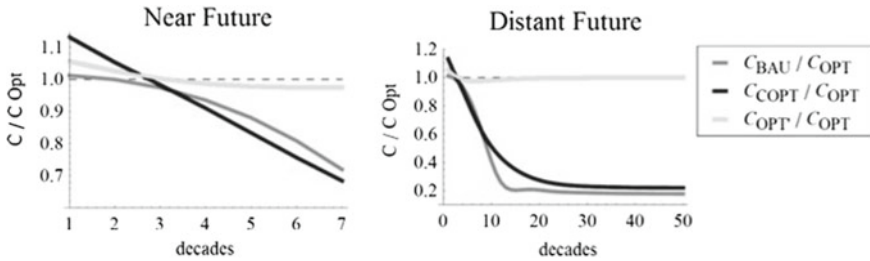


Fig. 4 Comparisons of the consumption paths for the business-as-usual, constrained optimal, and the altered optimal *palegray* OPT' (with $\eta = 3$) cases. All normalized to the optimal consumption stream

second-order consideration.³ In fact, with our benchmark values of $\rho = 0.1/\text{decade}$ and $\eta = 2$, the OPT path exhibits slightly lower per capita consumption than BAU in the first decade. One way to understand this fact is that the representative agent in the BAU equilibrium values current consumption too highly relative to investment in mitigation; when she grasps the full social marginal value of mitigation she prefers to reduce her consumption slightly in the first decade because of the high rate of return of this investment to the consumption of future generations. If the representative agent put more weight on intergenerational smoothing of consumption, for example, if $\eta = 3$, the corresponding OPT path would dominate the BAU path for $\eta = 2$, thus yielding higher utility for any value of η and ρ and demonstrating the inefficiency of the BAU equilibrium.

In Fig. 4, below we plot the initial decades of four paths to underline these points. The COPT, and BAU paths relative to the OPT path all of which are the same as those plotted in Fig. 3 and calculated with the benchmark parameters, in particular with $\eta = 2$. The OPT' path has the same parameters for the technical side of the model, but sets $\eta = 3$, to emphasize the inefficiency of the BAU path. The figure shows that the use of the COPT equilibrium as the benchmark distorts the perception of the economic issues involved in global warming policy by incorrectly suggesting that correction of the global warming externality will depress per capita world consumption significantly for several decades.

Given the inefficiency of over-accumulation of GHG stock in the atmosphere as a result of capital stock accumulation, the world economy is not operating at the intertemporal production possibility frontier (PPF). Future generations would appreciate lower stocks of CO₂ which implies that current generations should accumulate less conventional capital and consume more (of it) today. There is no intergenerational trade-off as is posited in most of the global warming-related economic literature. The mutual gains are illustrated by moving the economy from an inefficient point inside the PPF (the BAU path) to its boundary (the OPT' path with $\eta = 3$).

³On the relation between ρ and η and intergenerational equity, see Arrow (2007), who emphasizes the public good nature of global warming, without, however, noting the inefficiency of the BAU path.

Consumption in the COPT case lies above consumption in the optimal case in the first three decades as agents who are aware of the deleterious effects of global warming wisely choose to accumulate less and consume more. This positive difference in the first few decades forms the basis for the intergenerational equity discussion since the choice of the appropriate program now depends on its parameterization (most importantly the discount factor and the elasticity of felicity with respect to per capita consumption). Nordhaus has placed much emphasis on the dominance of baseline consumption over optimal consumption in the first few decades. This implies that current generations would attain lower utility levels if they started investing into mitigation. The consumption paths of the optimal and business-as-usual scenarios virtually move together in the first few decades with the BAU dominating the OPT path for the first two decades. The OPT' path, however, shows that this effect is due to the representative agent preferring to transfer some of the gain from global-warming mitigation to future generations given the very high rate of return to mitigation investment given the initial conditions.

Figure 5 illustrates these equilibrium allocations in terms of PPFs and indifference curves. The curve closest to the origin represents all attainable allocations with $\mu = 0$ (the BAU PPF). The dark line next to it is the COPT PPF and represents all supportable allocations with $m = 0$. The internalization of the externality in the absence of mitigation allows a welfare maximizing COPT allocation with higher present and future consumption levels compared to BAU. The outmost curve represents what is commonly understood as a PPF: all technologically feasible, unconstrained first-best allocations. It is apparent that there are many allocation along the OPT PPF

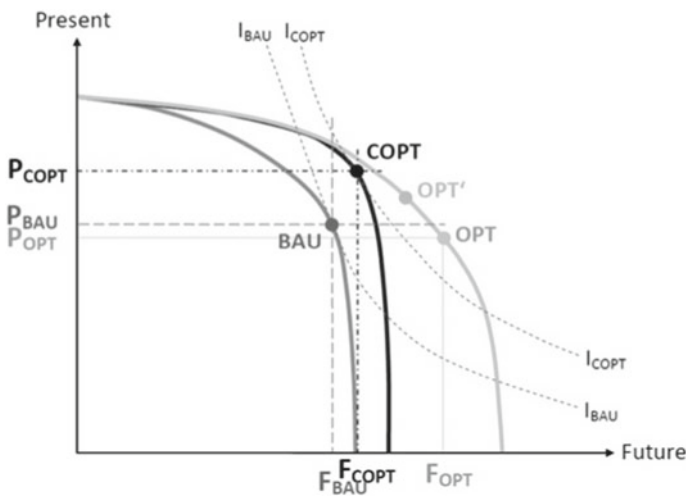


Fig. 5 Comparisons of the equilibrium allocations for the business-as-usual, constrained optimal, the optimal and the altered optimal OPT' scenarios. Given the parameter values for η and ρ), OPT has the lowest present consumption level. OPT' distributes the gains suboptimally, but such that consumption levels rise in both periods

which yield higher utility than COPT and BAU. The preference parameters ρ and η pin down the welfare-maximizing one. In our results they are such that the optimal intertemporal allocation (OPT) features the lowest present consumption level. As Fig. 5 demonstrates, the gains from moving from the BAU to the OPT allocation are much greater than from the COPT to the OPT. In particular, the set of allocations with non-decreasing consumption in either period is greater in the former than in the latter. This is due to the fact that the high saving rate in the BAU allocation provides more room to reshuffle the investment portfolio toward green investment in the form of mitigation. While the OPT' allocation does not lie on the highest indifference curve and is suboptimal, it distributes the gains from moving from the BAU to the OPT PPF such that consumption in neither period falls.

9 Parameter Sensitivity

In order to gauge the effect of parametric changes on the quantitative results of our simulations, this section explores sensitivity analysis for some model parameters. Generally, consumption smoothing dominates most of the adjustment in parametric changes. Due to spatial limitations, only the selected parameters are reported here. A complete set can be obtained from the authors. Table 2 presents an overview over the effects of parametric changes (rows) on selected variables (columns) relative to their values with standard parameters (which are reported in the bottom row). The sensitivity table reports lower and upper bounds for each parameter.

For example, the first Table in Table 2 captures the effects on the OPT path. The first two rows report changes in the discount rate of which the standard value is 0.1. The first row has $\rho = 0.05$ and the second $\rho = 0.15$. The first column reports the changes on consumption per capita. A lower time preference increases consumption by 0.5 % at the median and the 20 % quantile compared to the consumption stream under standard parameter assumptions. Consumption rises by 0.6 % above the corresponding standard consumption level at the 80 % quantile. An increase in the discount rate to $\rho = 0.15$ decreases consumption by 0.6 % at the 20 % quantile and by 0.7 % at the median and 80 % quantile. The next column reports effects of the changes on the carbon price. The change in pure time preference has virtually no effect on the carbon price. The third column reports the effects of parametric changes on mitigation efforts, the fourth on atmospheric carbon concentration, and so on. The third and fourth row the changes in the severity of damage, γ , the fifth and the sixth changes in the upper bound of atmospheric changes in CD, and so on.

The table does not report parameter changes in ν and ζ for the BAU and COPT paths, since these parameters concern the mitigation function and there is no mitigation available in these scenarios. Varying the discount factor has very different effects in the three scenarios. In the optimal case higher time preference yields higher consumption today and higher CO2 concentration and damage in the future. This is the standard result which most of the intergenerational literature rests on. In the BAU the higher consumption induced by higher time preference leads to less capital ac-

Table 2 Effects of parametric changes on time paths of selected variables relative to their standard value

	Cons per cap (\$/year)		Carbon price (\$/t)		Mitigation/Y (%)		CO ₂ (ppmv)		K (\$trillions)		Interest rate (%/year)				
	Q20	Median	Q80	Median	Q80	Median	Q20	Median	Q80	Median	Q20	Median	Q80		
OPT															
ρ	0.05	1.005	1.006	1.0	1.0	1.003	1.003	0.989	0.993	1.099	1.104	1.104	0.539	0.604	0.762
	0.15	0.992	0.993	1.0	1.0	0.997	0.997	1.007	1.008	1.011	0.911	0.914	1.209	1.38	1.439
γ	0.3	0.97	0.971	0.998	0.999	0.981	0.982	0.992	1.229	1.244	0.969	0.97	0.999	1.0	1.002
	0.7	1.086	1.086	1.0	1.001	0.997	1.007	0.949	0.979	1.086	1.086	1.086	0.999	1.0	1.0
CDMax	580	1.061	1.061	1.0	1.001	0.998	1.005	0.966	0.986	1.061	1.061	1.061	1.0	1.0	1.0
	980	0.959	0.984	0.984	1.0	1.0	0.998	1.004	1.017	0.969	0.984	0.984	1.0	1.0	1.011
ν	3	1.001	1.001	0.945	0.946	0.972	0.973	0.997	0.999	1.001	1.001	1.001	1.0	1.0	1.0
	12	0.998	0.998	1.13	1.132	1.061	1.062	1.002	1.003	0.998	0.998	0.998	1.0	1.0	1.001
ζ	1.5	0.964	0.967	2.261	2.262	2.119	2.139	1.024	1.029	1.064	0.961	0.965	1.0	1.0	1.005
	6	1.015	1.015	0.472	0.473	0.482	0.487	0.972	0.989	1.016	1.016	1.017	0.998	1.0	1.0
BAU															
ρ	0.05	0.965	0.972	0.977	1.093	1.129	1.025	1.033	1.002	1.002	1.062	1.068	0.504	0.51	0.879
	0.15	1.015	1.025	1.028	0.848	0.913	0.963	0.976	0.994	0.998	0.923	0.94	1.099	1.467	1.477
γ	0.3	1.019	1.027	1.044	0.978	1.197	1.201	1.048	1.032	1.035	0.989	1.026	0.822	1.005	1.036
	0.7	0.939	0.955	0.971	0.855	0.858	0.91	0.959	0.974	0.951	0.949	0.956	1.0	0.568	0.997
CDMax	580	0.718	0.755	0.76	1.107	1.121	1.027	1.03	0.756	0.833	0.736	0.755	0.898	0.293	1.007
	980	1.172	1.235	1.301	0.7	0.894	0.901	0.92	1.231	1.234	0.988	1.235	0.244	0.998	1.107
COPT															
ρ	0.05	0.98	1.019	1.042	1.122	1.137	1.15	1.032	0.985	0.991	0.725	0.803	-0.209	0.345	0.482
	0.15	0.969	0.991	1.015	0.894	0.905	0.91	0.969	1.005	1.009	1.086	1.092	1.476	1.617	2.173
γ	0.3	1.063	1.083	1.117	1.063	1.166	1.181	1.017	1.041	1.049	0.711	0.716	0.922	1.012	1.069
	0.7	0.896	0.925	0.93	0.901	0.91	0.975	0.97	0.944	0.961	0.947	1.096	1.11	0.848	0.993
CDMax	580	0.683	0.765	0.787	1.11	1.158	1.172	1.029	0.776	0.872	0.467	0.54	0.212	1.024	1.51
	980	1.199	1.228	1.298	0.814	0.892	0.915	0.942	1.201	1.226	1.474	1.496	0.8	0.976	1.541

$\alpha \rightarrow 0.35, \beta \rightarrow 0.056, \gamma \rightarrow 0.5, \delta \rightarrow 0.7, \epsilon \rightarrow 0.036, \zeta \rightarrow 3, \eta \rightarrow 2, \nu \rightarrow 6, \rho \rightarrow 0.1, A \rightarrow 28.4, \text{CDMax} \rightarrow 780$

cumulation and less carbon emissions which implies less carbon damage later on. This lowers the extent of the climate crisis. A lower valuation of the future, thus, leads to (marginally) higher consumption today and tomorrow as the externality is mitigated. In the constrained optimal case the increase in ρ leads to the standard intertemporal reallocation. A higher consumption level is sustained by higher accumulation with higher carbon concentrations early on. The shift also implies that the marginal rates shift. In the future a higher damage level has to be compensated with higher maintenance investments. Consumption is, consequently, lowered.

A reduction in γ to 0.3 leads to lower environmental damages. In the optimal case capital, output, and consumption can rise, while mitigation efforts fall. For marginal rates to equalize, CO₂ concentration and damages rise. In the constrained optimal case lower damages, again, lead to higher output and consumption and higher CO₂ concentrations. Damages and capital stock, however, fall. A milder damage function allows higher GHG concentrations in the atmosphere although these are not run up to the same damage level as before. Less damage requires less capital stock for the same level of consumption and less maintenance investment. The resources freed increase consumption even further. In the BAU case a milder damage function implies a milder climate crisis. More capital and CD can be accumulated. In the BAU capital accumulation is only slowed down by falling profitability due to environmental damages. A softer damage function allows a higher level of consumption to be obtained before and after a climate catastrophe. The climate crisis itself is more pronounced as can be seen in the steep fall of output and the reversal of the interest rate to almost -6%. World output and consumption increase while CO₂ is run up, but fall more drastically thereafter.

In the optimal case agents are able to fully internalize the externality. As a result the effects of increases in the maximal permissible CO₂ level are negligible. In the BAU the increase of CD_{Max} increases output and consumption considerably. Generally the transition to the steady state occurs much smoother which is due to the relative decrease of carbon emissions to carbon dissipation at higher CO₂ levels. This allows a higher capital stock and higher economic activity. The same rationale holds true for the constrained optimal case. Capital stock, output and consumption increase together with CD, while damages decrease.

The mitigation function is only relevant in the optimal case and consists of two parameters. ζ measures the cost of mitigation as it ties marginal mitigation to the carbon price. ν measures the effectiveness of mitigation for different levels of mitigation investment, m . An increase in the cost structure of mitigation leads to a higher carbon price and higher mitigation costs, with slight increases in CD and environmental damages to offset some of the cost increase. A decrease in the effectiveness of mitigation leads, again, to increased mitigation costs and a higher carbon price. This movement is counteracted by a slight increase in CO₂ and environmental damage. Note that front-loading of mitigation efforts is not a robust result and can change depending on parametric assumptions regarding the mitigation and damage functions. Convergence to full mitigation levels remains robust at 6 decades.

10 Conclusions

Human emissions of carbon dioxide into the atmosphere are a negative externality; despite world political efforts such as the Kyoto Protocol and its carbon market, the majority of individual producers do not take into account the impact of their emissions on atmospheric CO₂, but atmospheric concentration of CO₂ affects their (or their descendants') collective well-being through the deleterious effects of global warming. Such a negative externality leads to a market failure and an inefficient allocation of resources. In the case of global warming the inefficiency takes the form of over-accumulation of capital and under-investment in mitigation. The correction of this market failure through the implementation of institutions which enforce cost transparency represents a Pareto improvement: the current generation invests less, spending the retrenchment on consumption and mitigation such that future generations enjoy higher output, higher consumption combined with lower GHG concentration.

Our simulations show that the gains from such a movement to the intertemporal production possibility frontier are large. While this reallocation will imply significant changes of our life-style and the technology used, the mitigation costs to return to pre-industrial carbon dioxide levels are, however, relatively small with their peak at around 2% of world product.

The question of how such cost transparency can be achieved and which mechanisms to use are important, but cannot be answered in our framework.⁴ This is also true for the Pareto improvement question of who needs to compensate whom. Given our representative agent assumption, such a realistic question is clearly unanswerable. These problems will need to be solved by the world political system.⁵

If global warming is a real economic problem, there is no economic cost to correct it. In principle, the costs of reducing emissions in the current generation can be shifted to the future generations who will benefit from a cooler planet by reducing conventional investment.

References

- Arrow, K. (2007, June). Global climate change: a challenge to policy. *Economists' Voice* 1–5.
- Asheim, G., Mitra, T., & Tungodden, B. (2016). Sustainable recursive social welfare function. *Econ Theory*. reprinted in this volume.
- Barker, T. (2008). The economics of dangerous climate change. *Climatic Change*, 173–194.
- Burniaux, J.-M., & Oliveira Martins, J. (2011). Carbon Leakages: a general equilibrium view. *Economic Theory*. this issue.
- Chichilnisky, G. (1994). North-south trade and the global environment. *American Economic Review*, 84, 851–874.

⁴See Ostrom (2016) for a discussion of the former and Karp and Zhang (2016) and Chipman and Tian (2016) of the latter.

⁵See Lecocq and Hourcade (2016) and Dutta and Radner (2016) for a discussion of these issues.

- Chichilnisky, G. (2016). Sustainable markets with short sales. *Economic Theory*. reprinted in this volume.
- Chichilnisky, G., & Eisenberger, P. (2010). Asteroids: assessing catastrophic risks. *Journal of Probability and Statistics* 954750, 15.
- Chichilnisky, G., & Heal, G. (1994). Who Should abate carbon emission? An international perspective. *Economic Letters*, 44, 443–449.
- Chichilnisky, G., & Sheeran, K. (2009). *Saving kyoto*. UK: New Holland.
- Chipman, J. S., & Tian, G. (2016). Detrimental externalities, pollution rights, and the “Coarse Theorem”. *Economic Theory*. reprinted in this volume.
- Dutta, P. K., & Radner, R.: Capital growth in a global warming model will china and india sign a climate treaty? *Economic Theory*. reprinted in this volume.
- Figuières, C., & Tidball, M. (2016). Sustainable exploitation of a natural resource: A satisfying use of chichilnisky’s criterion. *Economic Theory*. reprinted in this volume.
- Foley, D. K. (2008). The economic fundamentals of global warming. In: J. M. Harris, N. R. Goodwin (Eds.) *Twenty-first century macroeconomics: responding to the climate challenge*, ch. 5. Cheltenham and Northampton: Edward Elgar Publishing.
- IPCC. (2007). *Climate Change 2007, the Fourth IPCC Assessment Report*. Retrieved August 10, 2008 from <http://www.ipcc.ch/ipccreports/ar4-syr.htm>.
- Karp, L., Zhang, J. (2016). Taxes versus quantities for a stock pollutant with endogenous abatement costs and asymmetric information. *Economic Theory*. reprinted in this volume.
- Lauwers, L. (2016). Intergenerational equity, efficiency and constructibility. *Economic Theory*. reprinted in this volume.
- Lecocq, F., Hourcade, J. -C. (2016). Unspoken ethical issues in the climate affair insights from a theoretical analysis of negotiation mandates. *Economic Theory*. reprinted in this volume.
- Nordhaus, W. D. (2008). *A question of balance: weighing the options on global warming policies*. New Haven: Yale University Press.
- Nordhaus, W. D., & Boyer, J. (2000). *Warming the world*. Cambridge: MIT Press.
- Ostrom, E. (2016). Nested externalities and polycentric institutions: must we wait for global solutions to climate change before taking actions at other scales? *Economic Theory*. reprinted in this volume.
- Rezai, A. (2010). Recast DICE and its policy implications, *Macroeconomic Dynamics*.
- Samuelson, P. A. (1965). A catenary turnpike theorem involving consumption and the golden rule. *American Economic Review*, 55, 486–496.
- Shiell, M. L., Lyssenko, N. (2008). Computing business-as-usual with a representative agent and a pollution externality. *Journal of Economic Dynamics and Control*, 32, 1543–1568.
- Stern, N. (2007). *The Economics of climate change: The stern review*. Cambridge: Cambridge University Press.
- Tol, R. (2009). The economic effects of climate change. *Journal of Economic Perspectives*, 23, 29–51.