Chapter 1
Introduction

Turbulent flows are omnipresent in nature and in technology. In technology, turbulent flows occur for example in nozzles and pipes, followed closely by the flow in devices such as heat exchangers, combustion engines, and turbo machinery. Also, turbulent flow is almost always observed around moving objects, such as airplanes, trains and cars, influencing the flow resistance of those bodies. At the same time, turbulence plays an important part in a variety of transport phenomena, such as heat and mass transfer, but also in flow-induced mixing. An extra complication in this last case is the occurrence of chemical reactions during the mixing, as is the case in combustion. In nature, turbulence plays a part in flows on a geophysical scale, such as the flows in the atmosphere and in the ocean. Transport phenomena in our atmosphere are, for example, mainly controlled by turbulence. An example of this is the distribution of air pollution by turbulent diffusion. Also, people traveling by plane may experience turbulence at first hand by vigorous agitation of the aircraft. On a somewhat larger scale, our weather and even our climate could be called a turbulent phenomenon. Lastly, turbulence is not restricted to our planet, but also plays an important part in flows occurring in the photosphere of stars, the closest being our own sun, and in the formation of planets in accretion discs.

The interest in turbulent flows has increased considerably in the last several decades. There are two main reasons for this. First, turbulence remains an unsolved problem from both a physical and a mathematical point of view. Second, in many practical flow problems, it appears that an inadequate model of turbulence is the most obstructive factor to a solution of the problem at hand.

In a first introduction to fluid mechanics, the concept of *laminar flow* is often introduced immediately, which is then followed by the definition of a turbulent flow as 'non-laminar' flow. Let us compare the characteristics of both types of flow:

laminar	turbulent
layered, regular	disordered
smooth	fluctuating
ordered	chaotic

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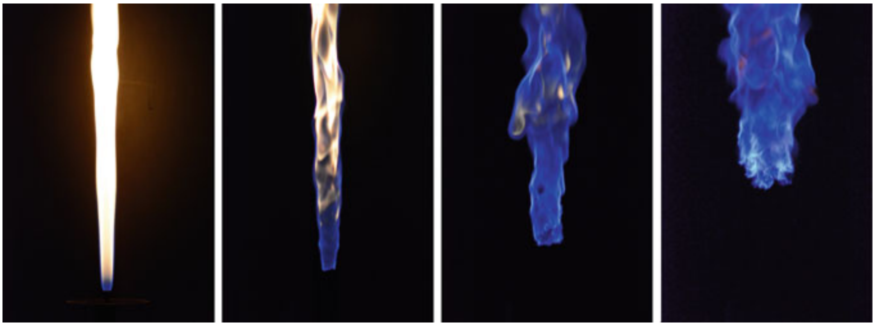

Fig. 1.1 Laminar and turbulent flames. In the laminar flame the combustion is limited by diffusion, whereas in the turbulent flames the combustion is much stronger due to mixing, which results in a higher flame temperature. The laminar flame ($Re \sim 400$) is smooth and has no evident flow structures, whereas the turbulent flames (increasing to $Re \sim 4000$) display a disordered structure and chaotic motion which continuously changes. Images courtesy of: Luis Arteaga and Mark Tummers

On the basis of common experience, almost everyone is somewhat familiar with these qualitative characteristics. An example is shown in Fig. 1.1 for laminar and turbulent flames. The differences between the two types of flow might however lead to the idea that turbulent flow obeys different equations of motion than laminar flow. This idea is not confirmed by experiments, and nowadays there is no doubt that both types of flow obey the same equations of motion. On this basis, we can now ask a simple question: What is the essence of turbulence, and how can we understand turbulent flow as a solution to the equations of motion?

The dynamics of liquids and gases can be described by the laws published by Isaac Newton in 1687 in his *Principia*. At first, Newton devised these equations for the mechanics of solid bodies. However, in subsequent years these laws were extended to frictionless fluids by, among others, Euler and Bernoulli. The formulation of the complete set of equations of motion for a fluid, including flow with friction, did not emerge until the middle of the 19th century. These are the *Navier–Stokes equations* that have the form of a system of nonlinear partial differential equations, describing the relation between the variables of flow, such as velocity and pressure, as a function of position and time.

The Navier–Stokes equations are not sufficient to completely determine the flow in a defined volume. For this we need to specify additional conditions. These conditions determine what the flow on the volume boundaries should look like, that is the so-called *boundary conditions*. Additionally, we need to know the so-called *initial condition*, which is the complete flow as a function of position at an initial moment.

The essential aspect of Newton's laws, and thus of the Navier–Stokes equations, is that they are *deterministic*. This means that *in principle*, given the equations of motion together with the initial and boundary conditions, the evolution of the flow field can be computed as a function of time; hence, the solution to the equations and conditions that describe the flow is completely determined. In other words, the deterministic character of Newton's mechanics implies full *predictability* of the fluid motion.

This can be regarded as a philosophical world view; the mathematician Pierre-Simon Laplace elaborated on this in detail, which was laid down in his work *Mécanique Céleste* that appeared in three volumes between 1799 and 1825. Allegedly, Laplace stated:

> Give me the velocity and position of every molecule, and I will predict your future.

In the 19th century the mathematician Hadamard formulated the notion of a *well-posed problem*. This means that a problem is well-posed when the solution of a set of differential equations obeys the following conditions:

existence: a solution exists;

uniqueness: there is only a single solution;

stability: small disturbances in the initial or boundary conditions lead only to small variations of the solution.

The first two conditions confirm a deterministic world view: with given initial and boundary conditions, the solution is known. The third condition for a well-posed problem yields an important restriction. Namely, this condition suggests that a deterministic solution is *in practice* only possible when the solution is not susceptible to small disturbances in the initial and boundary conditions. But why do we consider the notion of 'in practice'? Mathematically speaking, there would be no objection against using exactly known initial and boundary conditions. In that case we speak of mathematically *ideal* initial and boundary conditions, and for such a situation the solution would be completely determined due to the first and second conditions. However, this requires the initial and boundary conditions to be known with *infinite accuracy*; this would of course 'in practice' not be feasible, since the initial and boundary conditions are only known with *finite accuracy*. This is what we refer to as *realistic* initial and boundary conditions. Only for a well-posed problem these imperfections in the initial and boundary conditions fail to significantly affect the solution. In that case, the solution is 'in practice' completely predictable. If, however, the third Hadamard condition is not satisfied, we can expect completely different behavior of our solution when we do not exactly know the initial and boundary conditions, which results in what we perceive as *unpredictability*. In that case, the problem is considered to be 'ill-posed'.

These ideas were first elaborated by the French mathematician Henri Poincaré, who published his work *Méthodes Nouvelles de la Mécanique Céleste* in 1892. In this work he tried to solve the famous so-called *three-body problem*. Newton had already solved the *two-body problem*, where he found the elliptic Kepler-trajectories as the solution, which are considered the hallmark of a completely predictable solution. In other words, the two-body problem is well posed. This is in sharp contrast to the three-body problem. Poincaré found that this problem is not integrable; in short, this means that there is no simple solution in terms of a smooth or differentiable function. He found that the solution had irregular and chaotic characteristics. The solution to the three-body problem thus appears to be fundamentally unpredictable; the problem is *ill posed*. This meant the end of Laplace's orderly world view.

The results of Poincaré were further elaborated in the theory of dynamical systems. Dynamical systems can be imagined best as systems of coupled differential equations, describing the behavior of so-called system variables as a function of time. Often the number of system variables, or degrees of freedom, is kept relatively small. An example of a dynamical system is that of two coupled pendulums, but other examples of dynamical systems can be found in economics and certain biological processes. Dynamical systems are deterministic by definition, and therefore full solutions as a function of time can *in principle* be computed.

For these systems, consisting of sets of regular differential equations, it was proven that, given an initial condition, a single and unique solution exists. Hence, the first and second Hadamard conditions are satisfied. The third condition, however, is not always satisfied. The validity of this condition can only be proven for a limited number of systems, often only the linear ones. On the other hand, for many nonlinear dynamical systems it has been found that the solution is extremely sensitive to small variations of the initial conditions. The solution then becomes unpredictable after a certain amount of time, after which it starts to fluctuate. We cannot predict the magnitude, and often not even the sign of these fluctuations. This is called *deterministic chaos*. It is one of the most fundamental new insights that has dominated the developments in mechanics over the last couple of decades. It should be emphasized here that such 'chaotic' behavior is only anticipated for *nonlinear* dynamical systems.

Let us consolidate these findings to the solution of our flow equations. In doing so, we should note that the preceding findings have only been demonstrated for dynamical systems where the number of degrees of freedom is small. Nonetheless, we expect similar findings for systems that have many degrees of freedom, although this has only been proven in a few cases. Here, we interpret the equations of motion for the flow as a system with many degrees of freedom.

Consider a solution of the Navier–Stokes equations for a given flow problem; realistic initial and boundary conditions are given with finite accuracy. Suppose that all conditions for a well-posed problem are be satisfied, so that the solution for the flow is completely *predictable*. We define this as *laminar flow*.

However, the Navier–Stokes equations are nonlinear, and thus we have to expect that only under very special circumstances it is possible to comply with the conditions for a well-posed problem, especially the third one. In all other cases, the equations of motion and initial and boundary conditions for the flow would be ill posed. The solution is then susceptible to small variations in the initial or boundary conditions. We argued above that in this case the solution eventually becomes completely unpredictable, and this now defines a *turbulent flow*. In short, turbulence is associated with the concept of *deterministic chaos*, as mentioned above. So, turbulence is a completely different *kind* of flow than laminar flow, which would be unaffected by small variations in the initial and boundary conditions.

Now what does unpredictability on the basis of susceptibility to initial and boundary conditions mean? Suppose that we consider two solutions of a turbulent flow with the same *realistic* initial and boundary conditions. This would mean that for both solutions the initial and boundary conditions may be different, but within a finite degree of accuracy or tolerance. Such small differences will always be present in practice; for example, consider differences that are the result of molecular fluctuations. Since the third condition for a well-posed problem is not satisfied, the two solutions will, after a certain moment, begin to diverge completely. These two solutions can be considered as two *realizations* of the turbulent flow. Thus, every solution with the same realistic initial and boundary conditions yields a completely new realization. For turbulent flow with given realistic initial and boundary conditions, we are unable to predict the flow temporal and spatial evolution of the flow variables.

It now becomes plausible to consider the *statistics* of the flow variables, rather than the individual realizations. This is the most widely applied approach to describe turbulent flows. However, we cannot reverse this argument; from the turbulence statistics we will never be able to reconstruct the full course of all realizations. This underlies the so-called *closure problem*, which has remained the central fundamental issue in the theoretical description of turbulence.

In the preceding part of this Introduction, an attempt was made to relate flow turbulence to contemporary dynamical systems theory. It should be mentioned here that this relation is mainly qualitative. Except for the *routes to chaos* that are treated in Sect. 3.4, the reader will look in vain for a more quantitative elaboration in the remainder of this book. Nonetheless, the common understanding is that concepts and results in modern *chaos theory* have aided to gain insight in the onset of turbulence in flows. Some even say that this is the right way forward that will eventually lead us to a full solution of the problem of turbulence; see for example the preface in the latest edition of the famous book by Landau and Lifshitz on fluid mechanics. However, many still question whether chaos theory will provide us with a theory of turbulence that could, for example, predict the behavior of a turbulent flow. Note that chaos theory has prepared us to accept that we may never reach a closed-form solution to the problem of turbulence; the Navier–Stokes equations appear to be fundamentally *non-integrable*.

Separate from the modern mathematical insights mentioned above, the field of turbulent flows has passed through a long-term development. This mainly originated from practical questions and problems. Ignoring older and primarily qualitative considerations, research on turbulence commenced in the 18th and 19th centuries. Its origins can be found in the field of hydraulics, because of the interest in studying turbulent flow through pipes. Most of these investigations were empirical. This is why the works of Osborne Reynolds in 1883 and 1895 are considered as the birth of the theory of turbulence. His name will return multiple times in the ensuing chapters.

In this book we first focus on developments in the dynamics of turbulent flow. In doing so, phenomenological considerations are often invoked, because there is in fact no satisfactory theory of turbulence. This is despite the fact that many famous physicists, for example Werner Heisenberg and Richard Feynman, worked on the problem. Quoting from the *Feynman Lectures on Physics*:

> Finally, there is a physical problem that is common to many fields, that is very old, and that has not been solved. It is not the problem of finding new fundamental particles, but something left over from a long time ago – over a hundred years. Nobody in physics has been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of *circulating* or *turbulent fluids*.

In absence of a comprehensive theory, we have to resort to heuristics now and then. The use of dimensional analysis, linked with an adequate insight into the physical processes at hand, is then most appropriate. This may leave the reader who expects a comprehensible theory of turbulence somewhat dissatisfied. However, it will be shown that with this approach many important and useful results can be obtained.

To conclude this Introduction, we provide an overview of the material that is covered in this book. After a brief introduction to the governing equations of motion in Chap. 2, we start in Chap. 3 with a short treatise on the emergence of turbulence. On the basis of *linear stability analysis* we discuss the circumstances and conditions under which laminar flow becomes unstable and when we can expect turbulence to appear. Also, we briefly address the transition to fully-developed turbulent flow. This is referred to as the *route to chaos*. As mentioned previously, new insights from chaos theory contributed to this particular topic.

In the next chapter we focus on a particular model of turbulence that has an exact solution, that is *Burgers equation*. Equipped with the knowledge from this model, we discuss the phenomenology of turbulence. Important concepts, such as the *macrostructure* and *microstructure* are introduced, where each is characterized by separate scaling law.

In Chap. 5 we derive the equations of motion for the *mean velocity* in a turbulent flow. Here we are confronted for the first time with the *closure problem*. This closure problem is key to the development of turbulence models; in particular, in this book we pay attention to several closure models.

The first closure model is Ludwig Prandtl's *mixing length hypothesis*, which we apply to turbulent channel flow in Chap. 6. This also serves as an example of turbulence in the vicinity of a solid boundary, or so-called *wall turbulence*. For wall turbulence we can distinguish several regions with different scaling laws. The most important of these regions is the so-called *inertial sublayer* that is characterized by a logarithmic velocity profile. Wall turbulence is opposed to so-called *free turbulence*, which can develop without the restrictive influence of a solid boundary. Examples of free turbulence are discussed in Chap. 6.

The energetic aspects of turbulence are discussed in Chap. 7. These are studied using the equations for the kinetic energy for the mean flow and for the fluctuating velocity in the turbulent flow. These equations lead to two basic results:

- Turbulent kinetic energy is produced in the macrostructure and is dissipated in the microstructure by molecular viscosity;
- Production and dissipation of turbulent kinetic energy are, in a first approximation, in local equilibrium.

We then address the question by what mechanism energy is transferred from the macrostructure to the microstructure. For this, we introduce the vorticity equation in Chap. 8. The process of *vortex stretching* appears to be responsible for the resulting energy transfer from the macrostructure to the microstructure through what is called the *energy cascade process*. Moreover, on the basis of a first-order approximation of the equation for the vorticity fluctuations, we find that the microstructure is indeed in local dynamic equilibrium. This implies that the microstructure is fully decoupled from the macrostructure, and also that the microstructure is *isotropic*. The theory of the turbulence microstructure was first formulated by A.N. Kolmogorov around 1940. A second-order balance of the equation for the fluctuating vorticity leads to an important closure model that is very useful in practice: the so-called k-ϵ model. Subsequently, we discuss also a couple of contemporary turbulence models, such as the *second-order closure model* and the *algebraic stress model*.

Next, in Chap. 9 we discuss the *correlation function* and its Fourier transform: the *spectrum*. Here we mainly limit ourselves to two specific topics. The first topic is the so-called one-dimensional spectrum, which can be interpreted as the spectrum of the fluctuations in a flow property measured at a fixed point. The second topic we discuss is the theory of *isotropic turbulence*. As one of the central results of the theory of turbulence we deduce the so-called $-5/3$-law of the *inertial subrange* of the spectrum. It is demonstrated that this result is directly related to the existence of both the macrostructure and the microstructure, which are dynamically decoupled.

Finally, we conclude our description of turbulence in Chap. 10 with a brief discussion of *turbulent diffusion* and particle-laden turbulent flows.

Almost every section includes selected problems. These are intended to illustrate the covered material, but occasionally expand on more advanced topics.

One final comment. For certain, turbulence is not one of the easiest subjects in fluid mechanics. This book is therefore only an introduction and a gateway to wondrous things beyond.