The Relationship Between Complex Quantum Hamiltonian Dynamics and Krein Space Quantization

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Abstract Negative energy states are appeared in the structure of complex Hamiltonian dynamics. These states also play the main role in Krein space quantization to achieve a naturally renormalized theory. Here, we will have an overlook on the role of negative energy states in complex mechanics and Krein space. In a previous work, we have shown that the method of complex mechanics provides us some extra wave functions within complex spacetime. We have supported our method of including negative energy states, by referring to the theory of Krein space quantization that by taking the full set of Dirac solutions is able to remove the infinities of quantum field theory (QFT), naturally. Our main proposal here is that particles and antiparticles should be treated as physical entities with positive energy instead of considering antiparticles with negative energy and the unphysical particle and antiparticle with negative energy should be introduced as the complement of the sets of solutions for Dirac equation. Therefore, we infer that the Krein space method which is supposed as a pure mathematical approach, has root on the strong foundations of Hamilton-Jacobi equations and therefore on classical dynamics and it can successfully explain the reason why the renormalization procedure in QFT works.

1 Introduction

In recent years, complex spacetime and complex mechanics has been studied by some physicists. In fact, complex spacetime originates from complex time, as first proposed by Naschie [1], according to a special case of E^{∞} theory [2–6] and then applied by Yang in a series of papers [7–16]. The complex aspects of quantum mechanics has been also dealt with by Bender (see for example [17]).The complex spacetime

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proposed by Yang is in the form $x^\mu = x^{\mu_R} + ix^{\mu_I}, x^{\mu_R}, x^{\mu_I} \in R; x^\mu = (ct, x, y, z)$, showing that quantum mechanics is nothing but an extension of classical mechanics to complex domain and relativistic quantum mechanics is an extension of special relativity to the complex domain, so that considering both relativistic and quantum effects, the Klein-Gordon equation could be derived as a special form of the Hamilton-Jacobi (H-J) equation. Also, the complex spacetime which is a natural consequence of including quantum effects in the relativistic mechanics, is a bridge connecting the causality in special relativity and the non-locality in quantum mechanics, and the entangled state causing the faster-than-light links, is a consequence of an entangled energy plus a quantum potential, i.e. $E^2 + 2m_0c^2Q$, resulting in a constant quantity [15]. Furthermore, it has been shown that negative energy states are appeared in the structure of complex Hamiltonian dynamics [15].

In a previous paper, we have shown that discussing the complex spacetime in a relativistic entangled "space-time" state leads to 12 extra wave functions than the four solutions of Dirac equation for a free particle [18], and then we have presented a new physical interpretation, realizing particles and antiparticles as physical entities with positive energy instead of considering antiparticles with negative energy [19], and introducing unphysical particle and antiparticle with negative energy, as the complement of the sets of solutions for Dirac equation, in accordance to the concept of Krein space quantization, which is a naturally renormalized theory and negative energy states play the main role in its concept [20–42]. Here, our main infer will focus on the connection between complex quantum Hamiltonian dynamics, standard quantum field theory and Krein space quantization emphasizing the point that the Krein space method which is supposed as a pure mathematical approach, has root on the strong foundations of Hamilton-Jacobi equations and therefore on classical dynamics and it can successfully explain the reason why the renormalization procedure in QFT works.

2 A Brief Review on Krein Space Quantization

Here, we have a brief review on the problem of divergence in quantum field theory and its elimination using the method of Krein space quantization. In this method, the auxiliary negative frequency states have been utilized, the modes of which do not interact with the physical states and are not affected by the physical boundary conditions. In Krein space the quantum scalar field is defined as follows [22, 25]:

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_p(x) + \phi_n(x)],$$

where

$$\phi_p(x) = \int d^3\mathbf{k}[a(\mathbf{k})u_p(k, x) + a^\dagger(\mathbf{k})u_p^*(k, x)],$$

$$\phi_n(x) = \int d^3\mathbf{k}[b(\mathbf{k})u_n(k,x) + b^\dagger(\mathbf{k})u_n^*(k,x)].$$

$a(\mathbf{k})$ and $b(\mathbf{k})$ are two independent operators and

$$u_p(k,x) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}-iwt}}{\sqrt{(2\pi)^3 2w}} = \frac{e^{-ik\cdot x}}{\sqrt{(2\pi)^3 2w}}, \quad u_n(k,x) = \frac{e^{-i\mathbf{k}\cdot\mathbf{x}+iwt}}{\sqrt{(2\pi)^3 2w}} = \frac{e^{ik\cdot x}}{\sqrt{(2\pi)^3 2w}},$$

where $w(\mathbf{k}) = k^0 = (\mathbf{k}.\mathbf{k} + m^2)^{\frac{1}{2}} \geq 0$. The positive mode ϕ_p is the scalar field operator as was used in the usual QFT and ϕ_n plays the role of the regularization field. The time-ordered product is defined as:

$$iG_T(x,x') = \langle 0 \mid T\phi(x)\phi(x') \mid 0\rangle = \Re G_F(x,x'),$$

where $G_F(x,x')$ is the Feynman Green function.

As we know, the origin of divergences in standard quantum field theory lies in the singularity of the Green's function. The divergence appears in the imaginary part of the Feynman propagator, and the real part is convergent [32]:

$$G_F^P(x,x') = -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\,\theta(\sigma_0)\left[\frac{J_1\left(\sqrt{2m^2\sigma_0}\right) - iN_1\left(\sqrt{2m^2\sigma_0}\right)}{\sqrt{2m^2\sigma_0}}\right]$$
$$-\frac{im^2}{4\pi^2}\,\theta(-\sigma_0)\frac{K_1\left(\sqrt{2m^2(-\sigma_0)}\right)}{\sqrt{2m^2(-\sigma_0)}}$$

where, J_1, N_1 and K_1 are Bessel functions:

$$J_1(z) = \frac{z}{2}\sum_{s=0}^{\infty}\frac{(-1)^s}{s!(s+1)!}\left[\frac{z}{2}\right]^{2s}, \quad \lim_{z\to 0}\frac{J_1(z)}{z} = \frac{1}{2}$$

$$N_1(z) = 2J_1(z)\log\frac{z}{2} - \frac{2}{z}, \quad \lim_{z\to 0}\frac{N_1(z)}{z} = -\frac{2}{\pi}\frac{1}{z^2}$$

$$K_1(z) = -\frac{\pi}{2}[J_1(iz) + iN_1(iz)], \quad \lim_{z\to 0}\frac{K_1(z)}{z} = \frac{1}{z^2}$$

Consideration of negative frequency states removes singularity of the Green function with exception of delta function singularity:

$$G_T(x,x') = -\frac{1}{8\pi}\delta(\sigma_0) + \frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1\left(\sqrt{2m^2\sigma_0}\right)}{\sqrt{2m^2\sigma_0}}, \quad \sigma_0 \geq 0$$

However, considering the quantum metric fluctuations removes the latter singularity:

$$\langle G_T(x,x')\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right)+\frac{m^2}{8\pi}\theta(\sigma_0)\frac{J_1\left(\sqrt{2m^2\sigma_0}\right)}{\sqrt{2m^2\sigma_0}}. \tag{1}$$

where $\langle\sigma_1^2\rangle$ is related to the density of gravitons. When $\sigma_0 = 0$, due to the metric quantum fluctuation $\langle\sigma_1^2\rangle \neq 0$, and we have

$$\langle G_T(0)\rangle = -\frac{1}{8\pi}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}+\frac{m^2}{16\pi}.$$

By using the Fourier transformation, we obtain [41]

$$\langle\widetilde{G}_T(p)\rangle = \widetilde{G}_T(p) + PP\frac{m^2}{p^2(p^2-m^2)}$$

However, in the one-loop approximation, the contribution of delta function is negligible and the Green function in Krein space quantization appearing in the transition amplitude is

$$\langle\widetilde{G}_T(p)\rangle\mid_{one-loop} \equiv \widetilde{G}_T(p)\mid_{one-loop} \equiv PP\frac{m^2}{p^2(p^2-m^2)}$$

where $\tilde{G}_1(p)$ is the Fourier transformation of the first part of the Green function (1) and its explicit form is not needed for our discussion here. In a previous paper, it has proved that for the $\lambda\varphi^4$ theory in the one-loop approximation, the Green function in Krein space quantization, which appear in the s-channel contribution of transition amplitude, is the second part of (1) [25]. That means in this approximation, the contribution of the first part (i.e. quantum metric fluctuation) is negligible. It is worth mentioning that in order to improve the UV behavior in relativistic higher-derivative correction theories, the second part of (1) has been used by some authors [43, 44]. This part also appears in the super-symmetry theory [45].

The time-order product of the spinor field is:

$$\langle S_T(x-x')\rangle \equiv (i\not\partial + m)\langle G_T(x,x')\rangle$$

And the time-ordered product propagator in the Feynman gauge for the vector field in Krein space is given by:

$$\langle D^T_{\mu\nu}(x,x')\rangle = -\eta_{\mu\nu}\langle G_T(x,x')\rangle.$$

3 Essential Graphs of QED in Krein Space Quantization

In the standard quantum electrodynamics (QED) the divergent quantities are found in the electron self-energy, the vacuum polarization and the vertex graphs. In the standard QED, we have [46]:

$$\Sigma_{Hi}(p) = \frac{e^2}{8\pi^2}\left\{\ln\left(-\frac{\Lambda^2}{m^2}\right)\left(2m - \frac{\not{p}}{2}\right) + \left(2m - \frac{3}{4}\not{p}\right)\right.$$
$$\left. - \frac{\not{p}}{2}\left[\frac{m^4-(p^2)^2}{(p^2)^2}\ln\left(1-\frac{p^2}{m^2}\right)\right] + 2m\left[\frac{m^2-p^2}{p^2}\ln\left(1-\frac{p^2}{m^2}\right)\right]\right\}.$$

and

$$\Pi_{Hi}(k^2) = \frac{e^2}{12\pi^2}\ln\left(\frac{\Lambda^2}{m^2}\right) - \frac{e^2}{2\pi^2}\int_0^1 dx(1-x)x\ln\left(1-x(1-x)\frac{k^2}{m^2}\right).$$

and

$$F_1^{Hi}(q^2)_{q^2\to 0} = -\frac{e^2}{16\pi^2}\ln\left(\frac{\Lambda^2}{m^2}\right) - \frac{e^2q^2}{12\pi^2m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8}\right).$$

Calculating in Krein space, we get:

$$\Sigma_{kr}(p) = \frac{e^2}{8\pi^2}\left\{\ln\left(-\frac{p^2}{m^2}\right)\left(2m - \frac{\not{p}}{2}\right) - \frac{\not{p}}{2}\left(\frac{m^2}{p^2}\right)\right.$$
$$\left. - \frac{\not{p}}{2}\left[\frac{m^4-(p^2)^2}{(p^2)^2}\ln\left(1-\frac{p^2}{m^2}\right)\right] + 2m\left[\frac{m^2-p^2}{p^2}\ln\left(1-\frac{p^2}{m^2}\right)\right]\right\}.$$

and

$$\Pi^{kr}_{\mu\nu}(k^2) = (k^2g^{\mu\nu} - k^\mu k^\nu)\Pi_{kr}(k^2),$$

where

$$\Pi_{kr}(k^2) = -\frac{e^2}{12\pi^2}\ln\left(-\frac{k^2}{m^2}\right) - \frac{e^2}{6\pi^2}\frac{k^2}{m^2} - \frac{e^2}{2\pi^2}\int_0^1 dx(1-x)x\ln\left(1-x(1-x)\frac{k^2}{m^2}\right).$$

and

$$\Lambda^{\mu}_{kr}(p',p) = \frac{e^2}{8\pi}\int \frac{d^4k}{(2\pi)^4}\gamma^{\nu}(\not{p}' - \not{k} + m)\gamma^{\mu}(\not{p} - \not{k} + m)\gamma_{\nu} PP\frac{1}{k^2-\mu^2}$$

$$PP\left(\frac{m^2}{(p'-k)^2-m^2}\right) PP\left(\frac{m^2}{(p-k)^2-m^2}\right) = F_1^{kr}(q^2)\gamma^{\mu} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2^{kr}(q^2).$$

$F_2^{kr}(q^2)$ in the two different method is the same and $F_1^{kr}(q^2)$ in the Krein regularization is:

$$F_1^{kr}(q^2)_{q^2\to 0} = -\frac{e^2q^2}{16\pi^2m^2} + \frac{3e^2q^2}{64\pi^2m^2} - \frac{e^2q^2}{12\pi^2m^2}\left(\ln\frac{m}{\mu} - \frac{3}{8}\right),$$

where $q^2 = (p-p')^2$. The singular terms of 3 standard graphs of QED are replaced with the two first terms in the resulted graphs in Krein space quantization [47, 48]. By using the value of $F_1(q^2)$ and the photon self energy in Krein space, the value of Lamb Shift is calculated to be 1018.19 MHz, whereas in standard QED it is 1052.1 MHz; and its experimental value has been given as 1057.8 MHz. The small differences may be because of neglecting the linear quantum gravitational effect and working in the one-loop approximation [39]. It should be noted that for QED, the Krein space calculations just eliminate the singularity in the theory without changing the standard physical contents i.e. in calculations of graphs, the unphysical states are eliminated in the external lines and are introduced only in the propagators and eliminate the divergence of the theory automatically.

4 Consequences of Complex Spacetime in a Relativistic Entangled "Space-Time" State

In a paper by Yang [15], it has been shown that the general form of energy can be written as two sets of positive and negative energies:

$$E(t) = \pm\sqrt{k_0^2 - 2m_0c^2Q(t)} = \pm\sqrt{(m_0c^2)^2 + c^2p^2 - 2m_0c^2Q(t)} \equiv \pm E_{\pm} \tag{2}$$

where $Q(t)$ is quantum potential and is responsible for the quantum mechanical behavior of particles. It is clear that for any time t, there are two momenta ($p > 0, p < 0$) and two energies ($E_+ > 0, E_- < 0$), and in this general form of energy, the quantum potential $Q(t)$ is nonzero.

Considering $Q(t) = 0$, $Q(t) \neq 0$ and discussing the complex space-time in a relativistic entangled "space-time" state, we have realized 16 wave functions i.e. 12 extra ones than the four solutions of Dirac equation for a free particle [18]:

$$\psi_1 = Ce^{\frac{i}{\hbar}(Et - p\cdot r)} \quad (E > 0, p > 0); \quad \psi_2 = Ce^{\frac{i}{\hbar}(Et-(-p)\cdot r)} \quad (E > 0, p < 0)$$
$$\psi_3 = Ce^{\frac{i}{\hbar}((-E)t - p\cdot r)} \quad (E < 0, p > 0); \quad \psi_4 = Ce^{\frac{i}{\hbar}((-E)t-(-p)\cdot r)} \quad (E < 0, p < 0)$$
$$\psi_5 = (C_0^+ e^{i(E_+/\hbar)t} + C_0^- e^{-i(E_+/\hbar)t})C^- e^{-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar} \quad (E_+ > 0, p > 0)$$
$$\psi_6 = (C_0^+ e^{i(E_+/\hbar)t} + C_0^- e^{-i(E_+/\hbar)t})C^- e^{-\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar} \quad (E_+ > 0, p < 0)$$
$$\psi_7 = (C_0^+ e^{i(E_-/\hbar)t} + C_0^- e^{-i(E_-/\hbar)t})C^- e^{-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar} \quad (E_- < 0, p > 0)$$
$$\psi_8 = (C_0^+ e^{i(E_-/\hbar)t} + C_0^- e^{-i(E_-/\hbar)t})C^- e^{-\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar} \quad (E_- < 0, p < 0)$$
$$\psi_9 = C_0^+ e^{i(E_+/\hbar)t}(C^+ e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar})(E_+ > 0, p > 0)$$
$$\psi_{10} = C_0^+ e^{i(E_+/\hbar)t}(C^+ e^{\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar}) \quad (E_+ > 0, p < 0)$$
$$\psi_{11} = C_0^+ e^{i(E_-/\hbar)t}(C^+ e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar}) \quad (E_- < 0, p > 0)$$
$$\psi_{12} = C_0^+ e^{i(E_-/\hbar)t}(C^+ e^{\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar}) \quad (E_- < 0, p < 0)$$
$$\psi_{13} = (C_0^+ e^{i(E_+/\hbar)t} + C_0^- e^{-i(E_+/\hbar)t})(C^+ e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar}) \quad (E_+ > 0, p > 0)$$
$$\psi_{14} = (C_0^+ e^{i(E_+/\hbar)t} + C_0^- e^{-i(E_+/\hbar)t})(C^+ e^{\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar}) \quad (E_+ > 0, p < 0)$$
$$\psi_{15} = (C_0^+ e^{i(E_-/\hbar)t} + C_0^- e^{-i(E_-/\hbar)t})(C^+ e^{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}/\hbar}) \quad (E_- < 0, p > 0)$$
$$\psi_{16} = (C_0^+ e^{i(E_-/\hbar)t} + C_0^- e^{-i(E_-/\hbar)t})(C^+ e^{\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar} + C^- e^{-\frac{i}{\hbar}(-\mathbf{p})\cdot\mathbf{r}/\hbar}) \quad (E_- < 0, p < 0)$$

The above mentioned wave functions represent different entanglements of particles and antiparticles. In [18], we have argued that the entanglement of two particles or two antiparticles could be done only with the opposite momenta and the entanglement of particle and antiparticle could be done only with the same momenta, where the latter is in contradiction with experiments. Since empirical experiments have shown the quantum correlation at a distance of a particle-antiparticle system like kaon and antikaon system which are entwined. Therefore, introducing a parallel approach we corrected all the being results, considering the point that something was missed there. According to the theory of Dirac, antiparticles are believed to be particles of negative energy. But, due to the fact that antiparticles are detectable, so the physical antiparticles must be of positive energies. Moreover, according to (2) there are both positive and negative energy states. However, it seems that taking the negative energies as antiparticles, is not covering all the underlying physics [19]. So, we proposed that it is rational to accept that positive energy belongs to physical particles and negative energy belongs to unphysical particles. Then, we deduced that the solutions of Dirac equation describe both physical particles and antiparticles with positive energy and both unphysical particles and antiparticles with negative energy. Consequently, we modified the descriptions of wave functions $\psi_1, \psi_2, \ldots, \psi_{16}$ and as an application and verification of unphysical negative energy states, we referred to two famous paradoxes of physics, EPR and Klein [18]. In 1929, Klein [49] calculated the reflection and transmission coefficients for an incident beam of electrons of energy E, falling on a potential barrier of strength V_0. He found out that the unexpected amount of reflected electrons or transmitted electrons with a steady rate causes paradoxical results. Treating Klein's paradox with full set of Dirac solutions i.e. using the unphysical negative energy states in addition to physical positive energy states, we have removed the Klein's paradox without the need of any further explanations or justifications like backwardly moving electrons and gain equal values for reflected and transmitted electrons and positrons [18, 19, 50]. Also, in [18], we have explained that the correct and unique solution to Einstein-Podolsky-Rosen (EPR) paradox [51], can also be verified due to the new results based on quantum Hamiltonian dynamics approach, i.e. the unique solution to the original version of EPR paradox is a particle and its antiparticle moving in opposite direction.

As an interesting result, we observe that negative energy solutions necessarily appear within the structure of the theory of complex quantum Hamiltonian dynamics and their interpretation as unphysical particles and antiparticles is vital for achieving consistence results.

5 Discussion About the Relationship Between Complex Hamiltonian Dynamics and Krein Space Quantization

In this part, we want to establish the connection between the Sects. 2, 3 and 4, in order to discuss about the relationship between complex Hamiltonian dynamics and Krein space quantization. As told before, negative energy states are appeared in the structure of complex Hamiltonian dynamics. On the other hand, negative energy states play the main role in Krein space quantization approach to achieve a naturally renormalized theory. In this method, the auxiliary negative frequency states have been utilized, the modes of which do not interact with the physical states and are not affected by the physical boundary conditions and since it is similar to Pauli-Villars regularization, so it is called the "Krein regularization", too. Considering the QED in Krein space quantization, it has been shown that the theory is automatically regularized [39]. Calculation of the three primitive divergent integrals, the vacuum polarization, electron self energy and vertex function using Krein space method leads to finite values, since the infrared and ultraviolet divergencies do not appear. Also, this method could be easily generalized to non-Abelian gauge theory and quantum gravity in the background field method, and could be used as an alternative way for solving the non-renormalizability of quantum gravity in the linear approximation. However, since Krein space quantization is a purely mathematical theory and its appearance and extension i.e. applying negative energy states is based on a historical background and not a strong theoretical foundation, so the results have been under debate by most of the physicists, up to now.

But, whereas Krein quantization is a pure mathematical approach, complex quantum Hamiltonian dynamics is based on the strong foundations of Hamilton-Jacobi equations and therefore on classical dynamics. The negative energy solutions necessarily appear within the structure of the theory of complex quantum Hamiltonian dynamics and as we referred in this paper, their interpretation as unphysical particles and antiparticles is vital for achieving consistence results. Due to the theory of Dirac, antiparticles are believed to be particles of negative energy. But, according to the fact that antiparticles are detectable, so the physical antiparticles must be of positive energies i.e. taking the negative energies as antiparticles, is not covering all the underlying Physics [19]. Then, our main proposal in this paper is due to the results of Sect. 4, in which we deduced that particles and antiparticles should be realized as physical entities with positive energy instead of considering antiparticles with negative energy, and also unphysical particle and antiparticle with negative energy should be introduced as the complement of the sets of solutions for Dirac equation.

Now, Comparing the two approaches i.e. complex quantum Hamiltonian dynamics and Krein space quantization we can point out to the existence of a connection between quantum Hamiltonian dynamics, standard quantum field theory, and Krein space quantization. As we know, there are some gaps between the theories of classical mechanics, quantum mechanics, special relativity, Relativistic quantum mechanics, standard quantum field theory and quantum field theory in Krein space. However, the theory of complex quantum Hamiltonian dynamics has shown that quantum mechanics is nothing but the extension of classical mechanics into complex domain, so that in the viewpoint of complex H-J theory, quantum mechanics does not seem strange anymore and simplifies into an understandable theory. Also, the complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics, and is a bridge connecting the causality in special relativity and the non-locality in quantum mechanics, i.e. extending special relativity to the complex domain leads to relativistic quantum mechanics. On the other hand, Krein space quantization is a parallel approach and without sufficient and strong base to quantum field theory in order to show and explain the hidden part of the theory, which is purposely omitted by physicists and then has led to infinities in QFT. In other words, in the viewpoint of Krein space quantization, the procedure of ugly mathematics of renormalization can be explained by the hidden part of theory i.e. negative energy solutions. However, it can be seen that the base of Krein space quantization i.e. appearance and applying negative energy states has root in the theory of complex quantum Hamiltonian dynamics. Hence, it seems as if complex quantum Hamiltonian dynamics can construct a connecting bridge between standard quantum field theory and Krein space quantization in order to explain the reason for the practical ugly mathematics of renormalization and provide an answer to the Feynman reply: "A Nobel prize for hiding the rushes (infinities) under the carpet"?. So, the other important result is that Krein space method is nothing but an extension of complex mechanics into the theory of quantum fields and it can successfully explain the reason why the renormalization procedure in QFT works. So that, it should not be considered as a pure mathematical approach and it is necessary to devote more efforts to include more physics in the concept of negative energy states.

It could be inferred here that investigation in complex aspects of quantum mechanics and quantum field theory may open the doors to bridge between the being theories in physics and fill their gaps, and as a result our main discussion here is that negative energies should be considered as important as positive ones, since they contribute in variety of processes and their importance has been pointed out by some famous physicists, e.g. Feynman has discussed the negative probabilities as viable concepts in quantum physics [52, 53] and Dirac has replied that Negative energies and probabilities should not be considered as nonsense, since they are well-defined concepts mathematically like a negative sum of money, and important properties of them can still be used when they are negative [54, 55].

6 Conclusion

Negative energy states are applied in Krein space quantization approach to achieve a naturally renormalized theory. For example, this theory by taking the full set of Dirac solutions, is able to remove the propagator Green function's divergences and automatically without any normal ordering, to vanish the expected value for vacuum state energy.

On the other hand, negative energy states are also appeared in the structure of complex Hamiltonian dynamics. However, whereas Krein quantization is a pure mathematical approach, complex quantum Hamiltonian dynamics is based on strong foundations of Hamilton-Jacobi (H-J) equations and therefore on classical dynamics. Due to complex quantum Hamilton-Jacobi theory, complex spacetime is a natural consequence of including quantum effects in the relativistic mechanics. Characterizing the complex time involved in an entangled energy state and writing the general form of energy considering quantum potential, two sets of positive and negative energies could be realized that are in accordance with Krein space Quantization. Realizing new states for both positive and negative values of energy and momentum and then discussing the complex space-time in a relativistic entangled "space-time" state leading to 12 extra wave functions than the four solutions of Dirac equation for a free particle, we observed that negative energy solutions necessarily appear within the structure of the theory of complex quantum Hamiltonian dynamics and their interpretation as unphysical particles and antiparticles is vital for achieving consistence results. So, along with a previous investigation [19], we realized particles and antiparticles as physical entities with positive energy instead of considering antiparticles with negative energy. Finally, Comparing the two approaches i.e. complex quantum Hamiltonian dynamics and Krein space quantization we concluded that Krein space method is nothing but an extension of complex mechanics into the theory of quantum fields and along with the physicists desire, it can successfully explain the reason why the renormalization procedure in QFT works. Therefore, it should not be considered as a pure mathematical approach.

The main idea is that investigation in complex aspects of quantum mechanics and quantum field theory may open the doors to bridge between the being theories in physics and fill the unavoidable gaps between the theories of classical mechanics, quantum mechanics, special relativity, relativistic quantum mechanics, and quantum field theory.

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