Study of the Solvability of the Fuzzy Error Matrix Set Equation in Connotative Form of Type II 4

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Abstract The concept of error matrix is presented in this paper, and the types of the fuzzy error matrix equation are presented too. The paper especially researches about the error matrix that consists of general set relations, and the solvability and solutions to it. And theorems about the necessary condition and the necessary and sufficient condition for the solvability of the error matrix equation $XA' = B$ are obtained in the paper. An example of solving this equation would be given in the last part of the paper.

Keywords Fuzzy · Error matrix · Set relation · Set equation

1 Introduction

Errors always happen in our life, work and study, sometimes with huge destructive effect, so it is necessary to study how to avoid and eliminate errors. And in order to avoid and eliminate errors, we have to study causes and laws of errors. And this paper discusses how to utilize the error matrix equation to eliminate errors.

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2 The Concept of Error Matrix

Definition 1 Let

$$\begin{pmatrix} ((U_{11} \quad U_{12} \quad \cdots \quad U_{1k}),\ x_{11}) & \cdots & ((U_{11} \quad U_{12} \quad \cdots \quad U_{1k}),\ x_{1n}) \\ ((U_{21} \quad U_{22} \quad \cdots \quad U_{2k}),\ x_{21}) & \cdots & ((U_{21} \quad U_{22} \quad \cdots \quad U_{2k}),\ x_{2n}) \\ \cdots & \cdots & \cdots \\ ((U_{m1} \quad U_{m2} \quad \cdots \quad U_{mk}),\ x_{m1}) & \cdots & ((U_{m1} \quad U_{m2} \quad \cdots \quad U_{mk}),\ x_{mn}) \end{pmatrix}$$

be an m × n error matrix of K elements.

Definition 2 Let

$$\begin{pmatrix} U_{20} & S_{20} & \vec{P}_{20} & T_{20} & L_{20} & y_{20} & G_{u20} \\ U_{21} & S_{21} & \vec{P}_{21} & T_{21} & L_{21} & y_{21} & G_{u21} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ U_{2t} & S_{2t} & \vec{P}_{2t} & T_{2t} & L_{2t} & y_{2t} & G_{u2t} \end{pmatrix}$$

be a (t + 1) × 7 or m × 7 error matrix. Each element of this kind of error matrix is called a set.

Definition 3 The set relationship containing unknown sets is called set equations.

Definition 4 Let X, A′ and B be m × 7 error matrixes, so XA′ ⊇ (or other relational operators) B is named as set (matrix) equations.

3 Error Matrix Equation

Error matrix equation:

$$\text{Type I}\ AX \supseteq B\ A \cdot X \supseteq B\ A\blacktriangle X \supseteq B\ A \vee X \supseteq B A \wedge X \ \supseteq B$$
$$\text{Type II}\ XA \supseteq B\ X \cdot A \supseteq B\ X\blacktriangle A \supseteq B\ X \vee A \supseteq BX \wedge A \supseteq B$$

4 The Solution of Error Matrix Equation

The solution of Type II 1 XA = B

$$X=(u,v)=(U_1,S_1,\vec{P}_1,T_1,L_1,x)=\begin{pmatrix} U_{10x} & S_{10x} & \vec{P}_{10x} & T_{10x} & L_{10x} & x_{10x} & G_{10x} \\ U_{11x} & S_{11x} & \vec{P}_{11x} & T_{11x} & L_{11x} & x_{11x} & G_{11x} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ U_{1tx} & S_{1tx} & \overrightarrow{P_{1tx}} & T_{1tx} & L_{1tx} & x_{1tx} & G_{1tx} \end{pmatrix}$$

$$A=(U_2,S_2,\vec{P}_2,T_2,L_2,x_2)=\begin{pmatrix} U_{20} & S_{20} & \vec{P}_{20} & T_{20} & L_{20} & y_{20} & G_{u20} \\ U_{21} & S_{21} & \vec{P}_{21} & T_{21} & L_{21} & y_{21} & G_{u21} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ U_{2t} & S_{2t} & \vec{P}_{2t} & T_{2t} & L_{2t} & y_{2t} & G_{u2t} \end{pmatrix}$$

$$\begin{pmatrix} V_{201} & S_{v201} & \vec{P}_{v201} & T_{v201} & L_{v201} & y_{v201} & G_{v201} \\ U_{21j} & S_{v21j} & \vec{P}_{v21j} & T_{v21j} & L_{v21j} & y_{v21j} & G_{v21j} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ U_{2m2m1} & S_{v2m2m1} & \vec{P}_{v2m2m1} & T_{v2m2m1} & L_{v2m2m1} & y_{v2m2m1} & G_{v2m2m1} \end{pmatrix}$$

$$B=\begin{pmatrix} (b_{11},y_{11}) & (b_{12},y_{12}) & \cdots & (b_{1m},y_{1m}) \\ (b_{21},y_{21}) & (b_{21},y_{21}) & \cdots & (b_{2m},y_{2m}) \\ \cdots & \cdots & \cdots & \cdots \\ (b_{m1},y_{m1}) & (b_{m1},y_{m1}) & \cdots & (b_{mm},y_{mm}) \end{pmatrix}$$

Definition 5 Let

$$XA^{'}\supseteq=\begin{pmatrix} (w_{11},z_{11}) & (w_{12},z_{12}) & \cdots & (w_{1m},z_{1m}) \\ (w_{21},z_{21}) & (w_{22},z_{22}) & \cdots & (w_{2m},z_{2m}) \\ \cdots & \cdots & \cdots & \cdots \\ (w_{m1},z_{m1}) & (w_{m2},z_{m2}) & \cdots & (w_{mm},z_{mm}) \end{pmatrix}$$

$$=\begin{pmatrix} V_{201} & S_{v201} & \overrightarrow{P}_{v201} & T_{v201} & L_{v201} & y_{v201} & G_{v201} \\ V_{21j} & S_{v21j} & \overrightarrow{P}_{v21j} & T_{v21j} & L_{v21j} & y_{v21j} & G_{v21j} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ V_{2m2m1} & S_{v2m2m1} & \overrightarrow{P}_{v2m2m1} & T_{v2m2m1} & L_{v2m2m1} & y_{v2m2m1} & G_{v2m2m1} \end{pmatrix}$$

and,

$$(w_{ij},z_{ij})=\begin{pmatrix} U_{1ix}\wedge U_{2j} & S_{1ix}\wedge S_{2j} & \overrightarrow{P}_{1ix}\wedge\overrightarrow{P}_{2j} & T_{1ix}\wedge T_{2j} & L_{1ix}\wedge L_{2j} & x_{1ix}\wedge y_{2j} & G_{U1ix}\wedge G_{U2j} \end{pmatrix}$$

So the following equation is hold.

$$\begin{pmatrix} U_{10x}\wedge U_{20} & S_{10x}\wedge S_{20} & \overrightarrow{P}_{10x}\wedge \overrightarrow{P}_{20} & T_{10x}\wedge T_{20} & L_{10x}\wedge L_{20} & x_{10x}\wedge y_{20} & G_{U10x}\wedge G_{U20} \\ U_{11x}\wedge U_{21} & S_{11x}\wedge S_{21} & \overrightarrow{P}_{11x}\wedge \overrightarrow{P}_{21} & T_{11x}\wedge T_{21} & L_{11x}\wedge L_{21} & x_{11x}\wedge y_{21} & G_{U11x}\wedge G_{U21} \\ \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots \\ U_{1tx}\wedge U_{2t} & S_{1tx}\wedge S_{2t} & \overrightarrow{P}_{1tx}\wedge \overrightarrow{P}_{2t} & T_{1tx}\wedge T_{2t} & L_{1tx}\wedge L_{2t} & x_{1tx}\wedge y_{2t} & G_{U1tx}\wedge G_{U2t} \end{pmatrix}$$
$$= \begin{pmatrix} V_{201} & S_{v201} & \overrightarrow{P}_{v201} & T_{v201} & L_{v201} & y_{v201} & G_{v101} \\ V_{21j} & S_{v21j} & \overrightarrow{P}_{v21j} & T_{v21j} & L_{v21j} & y_{v21j} & G_{v11j} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ V_{2m2m1} & S_{v2m2m1} & \overrightarrow{P}_{v2m2m1} & T_{v2m2m1} & L_{v2m2m1} & y_{v2m2m1} & G_{v1m2m1} \end{pmatrix}$$

By the definition of equal matrices:if two matrices contain each other,so corresponding elements in both matrices contain each other. So $(w_{ij}, z_{ij}) \supseteq (b_{ij}, y_{ij})$,

$$\begin{pmatrix} U_{1ix}\wedge U_{2j} & S_{1ix}\wedge S_{2j} & \overrightarrow{P}_{1ix}\wedge \overrightarrow{P}_{2j} & T_{1ix}\wedge T_{2j} & L_{1ix}\wedge L_{2j} & x_{1ix}\wedge y_{2j} & G_{U1ix}\wedge G_{U2j} \end{pmatrix} \supseteq (b_{ij}, y_{ij})$$
$$= \begin{pmatrix} V_{2ij} & S_{v2ij} & \overrightarrow{P}_{v2ij} & T_{v2ij} & L_{v2ij} & y_{v2ij} & G_{v2ij} \end{pmatrix}$$

So the following set equations are obtained:

$$\begin{gathered} U_{10x}\wedge U_{20}\supseteq V_{v20} \\ S_{10x}\wedge S_{20}\supseteq S_{v20} \\ \overrightarrow{P}_{10x}\wedge \overrightarrow{P}_{20}\supseteq \overrightarrow{P}_{v20} \\ T_{10x}\wedge T_{20}\supseteq T_{v20} \\ L_{10x}\wedge L_{20}\supseteq L_{v20} \\ x_{10x}\wedge y_{20}\supseteq y_{v20} \\ G_{U10x}\wedge G_{U20}\supseteq G_{v20} \\ \ldots\ldots \\ U_{1ix}\wedge U_{2j}\supseteq V_{v2j} \\ S_{1ix}\wedge S_{2j}\supseteq S_{v2j} \\ \overrightarrow{P}_{1ix}\wedge \overrightarrow{P}_{2j}\supseteq \overrightarrow{P}_{v2j} \\ T_{1ix}\wedge T_{2j}\supseteq T_{v2j} \\ L_{1ix}\wedge L_{2j}\supseteq L_{v2j} \\ x_{1ix}\wedge y_{2j}\supseteq y_{v2j} \\ G_{U1ix}\wedge G_{U2j}\supseteq G_{v2j} \\ \ldots\ldots \\ U_{ttx}\wedge U_{2t}\supseteq V_{v2t} \\ S_{ttx}\wedge S_{2t}\supseteq S_{v2t} \\ \overrightarrow{P}_{ttx}\wedge \overrightarrow{P}_{2t}\supseteq \overrightarrow{P}_{v2t} \\ T_{ttx}\wedge T_{2t}\supseteq T_{v2t} \\ L_{ttx}\wedge L_{2t}\supseteq L_{v2t} \\ x_{ttx}\wedge y_{2t}\supseteq y_{v2t} \\ G_{Uttx}\wedge G_{U2t}\supseteq G_{v2t} \end{gathered}$$

About the operation symbol "∧", if both sides of the equation are sets, then it means "intersection", if both sides of the equation are numbers, then it means "minimum".

As for $(U_{1ix} \wedge U_{2j})\ h_1\ (S_{1ix} \wedge S_{2j})\ h_2\ (\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{2j}) \vee h_3\ (T_{1ix} \wedge T_{2j})\ h_4\ (L_{1ix} \wedge L_{2j})\ h_5\ (x_{1ix} \wedge y_{2j})\ h_6\ (G_{U1ix} \wedge G_{U2j})$, with "$h_i$, i = 1, 2, …6", means that elements have been computed could "compose" a complete matrix element (proposition). The mode of combination depends on different situations. One way is to constitute a new set of error elements or error logic proposition by parameter. And this way is called the multiplication of m × 7 error matrix.

Since what we required in solving practical problems is not $X_iA' = B$, but $X_iA' \supseteq B$, So we find a more general model of error matrix equation, that is to find out the solution of type II 1 equation $XA' \supseteq B$.

Theorem 1 *The sufficient and necessary condition for the solvability of error matrix equation* $XA \supseteq B$ *is the solvability of* $X_iA \supseteq B_i$, *i = (1, 2, ……, m2).*

Proof Suppose $XA' \supseteq B$ has solvability, it is can be known by the definitions of $XA' \supseteq B$ and $X_iA' \supseteq B_i$, i = (1, 2, …, m2) that they are the equivalent equations, so it is necessary for $X_iA' \supseteq B_i$, i = (1, 2, …, m2) has solvability; Otherwise, if the solvability of $X_iA' \supseteq B_i$, i = (1, 2, …, m2) exists, similarly it does for $XA' \supseteq B$.

Thereout, we use the method of discussing the solvability of $X_iA' = B_i$, i = (0, 1, 2, …, m2) to discuss the solution of $XA' = B$.

Then in $X_iA' \supseteq B_i$, we can get

$$
\begin{aligned}
&\left(U_{1ix}\quad S_{1ix}\quad \overrightarrow{P}_{1ix}\quad T_{1ix}\quad L_{1ix}\quad v_{1ix}\quad G_{u1ix}\right)A' \supseteq\\
&(U_{1ix} \wedge U_{20}) \vee (S_{1ix} \wedge S_{20}) \vee \left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{20}\right) \vee (T_{1ix} \wedge T_{20}) \vee (L_{1ix} \wedge L_{20})\\
&\vee (x_{1ix} \wedge x_{20}) \vee (G_{U1ix} \wedge G_{U20})\\
&\cdots\cdots\\
&\left(U_{1ix} \wedge U_{2j}\right) \vee \left(S_{1ix} \wedge S_{2j}\right) \vee \left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{2j}\right) \vee \left(T_{1ix} \wedge T_{2j}\right) \vee \left(L_{1ix} \wedge L_{2j}\right)\\
&\vee \left(x_{1ix} \wedge x_{2j}\right) \vee \left(G_{U1ix} \wedge G_{U2j}\right)\\
&\cdots\cdots\\
&(U_{1ix} \wedge U_{2m1}) \vee (S_{1ix} \wedge S_{2m1}) \vee \left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{2m1}\right) \vee (T_{1ix} \wedge T_{2m1})\\
&\vee (L_{1ix} \wedge L_{2m1}) \vee (x_{1ix} \wedge x_{2m1}) \vee (G_{U1ix} \wedge G_{U2m1})\\
&\quad \supseteq \left((b_{i1}, y_{i1})\quad (b_{i2}, y_{i2})\quad \cdots\quad (b_{im1}, y_{im1}) \right)
\end{aligned}
$$

Namely,

$$(U_{1ix} \wedge U_{20}) \vee (S_{1ix} \wedge S_{20}) \vee \left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{20}\right) \vee (T_{1ix} \wedge T_{20})$$
$$\vee (L_{1ix} \wedge L_{20}) \vee (x_{1ix} \wedge x_{20}) \vee (G_{U1ix} \wedge G_{U20}) \supseteq$$
$$\left(V_{20} \quad S_{v20} \quad \overrightarrow{P}_{v20} \quad T_{v20} \quad L_{v20} \quad y_{v20} \quad G_{v20}\right)$$

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$$\left(U_{1ix} \wedge U_{2j}\right) \vee \left(S_{1ix} \wedge S_{2j}\right) \vee \left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{2j}\right) \vee \left(T_{1ix} \wedge T_{2j}\right) \vee \left(L_{1ix} \wedge L_{2j}\right)$$
$$\vee \left(x_{1ix} \wedge x_{2j}\right) \vee \left(G_{U1ix} \wedge G_{U2j}\right) \supseteq$$
$$\left(V_{2j} \quad S_{v2j} \quad \overrightarrow{P}_{v2j} \quad T_{v2j} \quad L_{v2j} \quad y_{v2j} \quad G_{v2j}\right)$$

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$$(U_{1tx} \wedge U_{2m1}) \vee (S_{1tx} \wedge S_{2m1}) \vee \left(\overrightarrow{P}_{1tx} \wedge \overrightarrow{P}_{2m1}\right) \vee (T_{1tx} \wedge T_{2m1}) \vee (L_{1tx} \wedge L_{2m1})$$
$$\vee (x_{1tx} \wedge x_{2m1}) \vee (G_{U1tx} \wedge G_{U2m1}) \supseteq$$
$$\left(V_{2t} \quad S_{v2t} \quad \overrightarrow{P}_{v2t} \quad T_{v2t} \quad L_{v2t} \quad y_{v2t} \quad G_{v2t}\right)$$

A series of set equations be obtained:

$$(U_{1ix} \wedge U_{20}) \supseteq V_{20}$$
$$(S_{1ix} \wedge S_{20}) \supseteq S_{v20}$$
$$\left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{20}\right) \supseteq \overrightarrow{P}_{v20}$$
$$(T_{1ix} \wedge T_{20}) \supseteq T_{v20}$$
$$(L_{1ix} \wedge L_{20}) \supseteq L_{v20}$$
$$(x_{1ix} \wedge x_{20}) \supseteq y_{v20}$$
$$(G_{U1ix} \wedge G_{U20}) \supseteq G_{v20}$$

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$$\left(U_{1ix} \wedge U_{2j}\right) \supseteq V_{2j}$$
$$\left(S_{1ix} \wedge S_{2j}\right) \supseteq S_{v2j}$$
$$\left(\overrightarrow{P}_{1ix} \wedge \overrightarrow{P}_{2j}\right) \supseteq \overrightarrow{P}_{v2j}$$
$$\left(T_{1ix} \wedge T_{2j}\right) \supseteq T_{v2j}$$
$$\left(L_{1ix} \wedge L_{2j}\right) \supseteq L_{v2j}$$
$$\left(x_{1ix} \wedge x_{2j}\right) \supseteq y_{v2j}$$
$$\left(G_{U1ix} \wedge G_{U2j}\right) \supseteq G_{v2j}$$

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$$(U_{1tx} \wedge U_{2m1}) \supseteq V_{2t}$$
$$(S_{1tx} \wedge S_{2m1}) \supseteq S_{v2t}$$
$$\left(\overrightarrow{P}_{1tx} \wedge \overrightarrow{P}_{2m1}\right) \supseteq \overrightarrow{P}_{v2t}$$
$$(T_{1tx} \wedge T_{2m1}) \supseteq T_{v2t}$$
$$(L_{1tx} \wedge L_{2m1}) \supseteq L_{v2t}$$
$$(x_{1tx} \wedge x_{2m1}) \supseteq y_{v2t}$$
$$(G_{U1tx} \wedge G_{U2m1}) \supseteq G_{v2t}$$

Theorem 2 *The sufficient and necessary condition for the solvability of* $X_iA \supseteq B_i$ *is,*

$$\begin{gathered}
U_{20} \supseteq V_{20} \\
S_{20} \supseteq S_{v20} \\
\overrightarrow{P}_{20} \supseteq \overrightarrow{P}_{v20} \\
T_{20} \supseteq T_{v20} \\
L_{20} \supseteq L_{v20} \\
x_{20} \supseteq y_{v20} \\
G_{U20} \supseteq G_{v20} \\
\cdots\cdots \\
U_{2j} \supseteq V_{2j} \\
S_{2j} \supseteq S_{v2j} \\
\overrightarrow{P}_{2j} \supseteq \overrightarrow{P}_{v2j} \\
T_{2j} \supseteq T_{v2j} \\
L_{2j} \supseteq L_{v2j} \\
x_{2j} \supseteq y_{v2j} \\
G_{U2j} \supseteq G_{v2j} \\
\cdots\cdots \\
U_{2m1} \supseteq V_{2t} \\
S_{2m1} \supseteq S_{v2t} \\
\overrightarrow{P}_{2m1} \supseteq \overrightarrow{P}_{v2t} \\
T_{2m1} \supseteq T_{v2t} \\
L_{2m1} \supseteq L_{v2t} \\
x_{2m1} \supseteq y_{v2t} \\
G_{U2m1} \supseteq G_{v2t}
\end{gathered}$$

Proof 1 If one of the conditions above is not satisfied, for example suppose $S_{2j}(t) \supseteq S_{v2j}(t)$ is not satisfied, so in the $(S_{1ix}(t) \wedge S_{2j}(t)) = S_{v2j}(t)$, no mater what value $S_{1ix}(t)$ is, we can not get $(S_{1ix}(t) \wedge S_{2j}(t)) = S_{v2j}(t)$。

Proof 2 Since we could only take union operation between the corresponding element of A and X_i in the $X_iA' \supseteq B_i$, that is

$$\begin{gathered}
U_{1ix} = U_{20} \cup U_{21} \cup \cdots U_{2j} \cup \cdots \cup U_{2t} \\
S_{1ix} = S_{20} \cup S_{21} \cup \cdots S_{2j} \cup \cdots \cup S_{2t} \\
\overrightarrow{P}_{1ix} = \overrightarrow{P}_{20} \cup \overrightarrow{P}_{21} \cup \cdots \overrightarrow{P}_{2j} \cup \cdots \cup \overrightarrow{P}_{2t} \\
T_{1ix} = T_{20} \cup T_{21} \cup \cdots T_{2j} \cup \cdots \cup T_{2t} \\
L_{1ix} = L_{20} \cup L_{21} \cup \cdots L_{2j} \cup \cdots \cup L_{2t} \\
x_{1ix} = x_{20} \cup x_{21} \cup \cdots x_{2j} \cup \cdots \cup x_{2t} \\
G_{U1ix} = G_{U20} \cup G_{U21} \cup \cdots G_{U2j} \cup \cdots \cup G_{U2t}
\end{gathered}$$

Then we discuss all the solutions of $X_iA' \supseteq B_i$ and $XA' \supseteq B$.

When $X(x_1, x_2, x_n)$ are obtained, we take intersection operation between X 与 Kg, rw, xq, so we can get $X'(x'_1, x'_2, \ldots, x'_n) \in X'$。

5 Examples of Error Matrix Equation

Suppose $A' = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, and

$$\begin{aligned}
a_{11} &= \begin{pmatrix} U_{201} & S_{201} & \overrightarrow{P}_{201} & T_{201} & L_{201} & y_{201} & G_{U201} \end{pmatrix} \\
a_{12} &= \begin{pmatrix} U_{202} & S_{202} & \overrightarrow{P}_{202} & T_{202} & L_{202} & y_{202} & G_{U202} \end{pmatrix} \\
a_{21} &= \begin{pmatrix} U_{211} & S_{211} & \overrightarrow{P}_{211} & T_{211} & L_{211} & y_{211} & G_{U211} \end{pmatrix} \\
a_{22} &= \begin{pmatrix} U_{212} & S_{212} & \overrightarrow{P}_{212} & T_{212} & L_{212} & y_{212} & G_{U212} \end{pmatrix}
\end{aligned}$$

When n = 11, we get the following equations:

$$\begin{aligned}
U_{201} &= \{u_{201}, u_{202}, \ldots, u_{20n}\} \\
S_{201} &= \{s_{201}, s_{202}, \ldots, s_{20n}\} \\
\overrightarrow{P}_{201} &= \{\overrightarrow{p}_{201}, \overrightarrow{p}_{202}, \ldots, \overrightarrow{p}_{20n}\} \\
T_{201} &= \{t_{201}, t_{202}, \ldots, t_{20n}\} \\
L_{201} &= \{l_{201}, l_{202}, \ldots, l_{20n}\} \\
y_{201} &= \{y_{201}, y_{202}, \ldots, y_{20n}\} \\
G_{U201} &= \{g_{201}, g_{202}, \ldots, g_{20n}\}
\end{aligned}$$

When n = 9, we get the following equations:

$$\begin{aligned}
U_{202} &= \{u_{201}, u_{202}, \ldots, u_{20n}\} \\
S_{202} &= \{s_{201}, s_{202}, \ldots, s_{20n}\} \\
\overrightarrow{P}_{202} &= \{\overrightarrow{p}_{201}, \overrightarrow{p}_{202}, \ldots, \overrightarrow{p}_{20n}\} \\
T_{202} &= \{t_{201}, t_{202}, \ldots, t_{20n}\} \\
L_{202} &= \{l_{201}, l_{202}, \ldots, l_{20n}\} \\
y_{202} &= \{y_{201}, y_{202}, \ldots, y_{20n}\} \\
G_{U202} &= \{g_{201}, g_{202}, \ldots, g_{20n}\}
\end{aligned}$$

When n = 10, we get the following equations:

$$
\begin{aligned}
U_{211} &= \{u_{211}, u_{212}, \ldots, u_{21n}\} \\
S_{211} &= \{s_{211}, s_{212}, \ldots, s_{21n}\} \\
\overrightarrow{P}_{211} &= \{\overrightarrow{p}_{211}, \overrightarrow{p}_{212}, \ldots, \overrightarrow{p}_{21n}\} \\
T_{211} &= \{t_{211}, t_{212}, \ldots, t_{21n}\} \\
L_{211} &= \{l_{211}, l_{212}, \ldots, l_{21n}\} \\
y_{211} &= \{y_{211}, y_{212}, \ldots, y_{21n}\} \\
G_{U211} &= \{g_{211}, g_{212}, \ldots, g_{21n}\}
\end{aligned}
$$

When n = 15, we get the following equations:

$$
\begin{aligned}
U_{212} &= \{u_{211}, u_{212}, \ldots, u_{21n}\} \\
S_{212} &= \{s_{211}, s_{212}, \ldots, s_{21n}\} \\
\overrightarrow{P}_{212} &= \{\overrightarrow{p}_{211}, \overrightarrow{p}_{212}, \ldots, \overrightarrow{p}_{21n}\} \\
T_{212} &= \{t_{211}, t_{212}, \ldots, t_{21n}\} \\
L_{212} &= \{l_{211}, l_{212}, \ldots, l_{21n}\} \\
y_{212} &= \{y_{211}, y_{212}, \ldots, y_{21n}\} \\
G_{U212} &= \{g_{211}, g_{212}, \ldots, g_{21n}\}
\end{aligned}
$$

And suppose $X = \begin{pmatrix} x_1 & x_2 \end{pmatrix}$, where

$$
\begin{aligned}
x_1 &= \begin{pmatrix} U_{10x} & S_{10x} & \overrightarrow{P}_{10x} & T_{10x} & L_{10x} & x_{10x} & G_{U10x} \end{pmatrix} \\
x_2 &= \begin{pmatrix} U_{11x} & S_{11x} & \overrightarrow{P}_{11x} & T_{11x} & L_{11x} & x_{11x} & G_{U11x} \end{pmatrix}
\end{aligned}
$$

And suppose $B' = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, where

$$
\begin{aligned}
b_{11} &= \begin{pmatrix} V_{201} & S_{v201} & \overrightarrow{P}_{v201} & T_{v201} & L_{v201} & y_{v201} & G_{V201} \end{pmatrix} \\
b_{12} &= \begin{pmatrix} V_{202} & S_{v202} & \overrightarrow{P}_{v202} & T_{v202} & L_{v202} & y_{v202} & G_{V202} \end{pmatrix} \\
b_{21} &= \begin{pmatrix} V_{211} & S_{v211} & \overrightarrow{P}_{v211} & T_{v211} & L_{v211} & y_{v211} & G_{V211} \end{pmatrix} \\
b_{22} &= \begin{pmatrix} V_{212} & S_{v212} & \overrightarrow{P}_{v212} & T_{v212} & L_{v212} & y_{v212} & G_{V212} \end{pmatrix}
\end{aligned}
$$

And when k = 7, we get the following equations:

$$
\begin{aligned}
V_{201} &= \{u_{201}, u_{202}, \ldots, u_{20k}\} \\
S_{v201} &= \{s_{201}, s_{202}, \ldots, s_{20k}\} \\
\overrightarrow{P}_{v201} &= \{\overrightarrow{p}_{201}, \overrightarrow{p}_{202}, \ldots, \overrightarrow{p}_{20k}\} \\
T_{v201} &= \{t_{201}, t_{202}, \ldots, t_{20k}\} \\
L_{v201} &= \{l_{201}, l_{202}, \ldots, l_{20k}\} \\
y_{v201} &= \{y_{201}, y_{202}, \ldots, y_{20k}\} \\
G_{v201} &= \{g_{201}, g_{202}, \ldots, g_{20k}\}
\end{aligned}
$$

And when k = 8, we get the following equations:

$$
\begin{aligned}
V_{202} &= \{u_{201}, u_{202}, \ldots, u_{20k}\} \\
S_{v202} &= \{s_{201}, s_{202}, \ldots, s_{20k}\} \\
\overrightarrow{P}_{v202} &= \{\overrightarrow{p}_{201}, \overrightarrow{p}_{202}, \ldots, \overrightarrow{p}_{20k}\} \\
T_{v202} &= \{t_{201}, t_{202}, \ldots, t_{20k}\} \\
L_{v202} &= \{l_{201}, l_{202}, \ldots, l_{20k}\} \\
y_{v202} &= \{y_{201}, y_{202}, \ldots, y_{20k}\} \\
G_{v202} &= \{g_{201}, g_{202}, \ldots, g_{20k}\}
\end{aligned}
$$

And when k = 2, we get the following equations:

$$
\begin{aligned}
V_{211} &= \{u_{211}, u_{212}, \ldots, u_{21k}\} \\
S_{v211} &= \{s_{211}, s_{212}, \ldots, s_{21k}\} \\
\overrightarrow{P}_{v211} &= \{\overrightarrow{p}_{211}, \overrightarrow{p}_{212}, \ldots, \overrightarrow{p}_{21k}\} \\
T_{v211} &= \{t_{211}, t_{212}, \ldots, t_{21k}\} \\
L_{v211} &= \{l_{211}, l_{212}, \ldots, l_{21k}\} \\
y_{v211} &= \{y_{211}, y_{212}, \ldots, y_{21k}\} \\
G_{v211} &= \{g_{211}, g_{212}, \ldots, g_{21k}\}
\end{aligned}
$$

And when k = 3, we get the following equations:

$$V_{212} = \{u_{211}, u_{212}, \ldots, u_{21k}\}$$
$$S_{v212} = \{s_{211}, s_{212}, \ldots, s_{21k}\}$$
$$\overrightarrow{P}_{v212} = \{\overrightarrow{p}_{211}, \overrightarrow{p}_{212}, \ldots, \overrightarrow{p}_{21k}\}$$
$$T_{v212} = \{t_{211}, t_{212}, \ldots, t_{21k}\}$$
$$L_{v212} = \{l_{211}, l_{212}, \ldots, l_{21k}\}$$
$$y_{v212} = \{y_{211}, y_{212}, \ldots, y_{21k}\}$$
$$G_{v212} = \{g_{211}, g_{212}, \ldots, g_{21k}\}$$

By the Theorem 2, when n = 11, we get the solution of XA′ ⊇ B, which is

$$U_{10x} = \{u_{201}, u_{202}, \ldots, u_{20n}\}$$
$$S_{10x} = \{s_{201}, s_{202}, \ldots, s_{20n}\}$$
$$\overrightarrow{P}_{10x} = \{\overrightarrow{p}_{201}, \overrightarrow{p}_{202}, \ldots, \overrightarrow{p}_{20n}\}$$
$$L_{10x} = \{l_{201}, l_{202}, \ldots, l_{20n}\}$$
$$x_{10x} = \{y_{201}, y_{202}, \ldots, y_{20n}\}$$
$$G_{U10x} = \{g_{201}, g_{202}, \ldots, g_{20n}\}$$

Similarly when n = 8 we get the following equations:

$$U_{11x} = \{u_{211}, u_{212}, \ldots, u_{20n}\}$$
$$S_{11x} = \{s_{201}, s_{202}, \ldots, s_{20n}\}$$
$$\overrightarrow{P}_{11x} = \{\overrightarrow{p}_{201}, \overrightarrow{p}_{202}, \ldots, \overrightarrow{p}_{20n}\}$$
$$T_{11x} = \{t_{201}, t_{202}, \ldots, t_{20n}\}$$
$$x_{11x} = \{y_{211}, y_{212}, \ldots, y_{21n}\}$$
$$G_{U11x} = \{g_{211}, g_{212}, \ldots, g_{21n}\}$$

6 Conclusion

We get the necessary and sufficient condition for the solvability of the error matrix equation XA′ = B, and they are also can be proved by the case studies.

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