Quickest Multidecision Abrupt Change Detection with Some Applications to Network Monitoring

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Abstract. The quickest change detection/isolation (multidecision) problem is of importance for a variety of applications. Efficient statistical decision tools are needed for detecting and isolating abrupt changes in the properties of stochastic signals and dynamical systems, ranging from on-line fault diagnosis in complex technical systems (like networks) to detection/classification in radar, infrared, and sonar signal processing.

Keywords: Sequential change detection/isolation · Multidecision problems · Anomaly detection · Network monitoring

1 Introduction

The quickest change detection/isolation (multidecision) problem is of importance for a variety of applications. Efficient statistical decision tools are needed for detecting and isolating abrupt changes in the properties of stochastic signals and dynamical systems, ranging from on-line fault diagnosis in complex technical systems (like networks) to detection/classification in radar, infrared, and sonar signal processing. The early on-line fault diagnosis (detection/isolation) in industrial processes (SCADA systems) helps in preventing these processes from more catastrophic failures.

The quickest multidecision detection/isolation problem is the generalization of the quickest changepoint detection problem to the case of $K-1$ post-change hypotheses. It is necessary to detect the change in distribution as soon as possible and indicate which hypothesis is true after a change occurs. Both the rate of false alarms and the misidentification (misisolation) rate should be controlled by given levels.

2 Problem Statement

Let $X_1, X_2, \ldots$ denote the series of observations, and let ν be the serial number of the *last pre-change* observation. In the case of multiple hypothesis, there are several possible post-change hypotheses $\mathcal{H}_j$, $j = 1, 2, \ldots, K-1$. Let $\mathbb{P}_k^j$ and $\mathbb{E}_k^j$

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denote the probability measure and the expectation when $\nu = k$ and $\mathcal{H}_j$ is the true post-change hypothesis, and let $\mathbb{P}_\infty$ and $\mathbb{E}_\infty = \mathbb{E}_0^0$ denote the same when $\nu = \infty$, i.e., there is no change. Let (see [1] for details)

$$\mathbb{C}_\gamma = \left\{ \delta = (T, d)\colon \min_{0 \le \ell \le K-1} \min_{1 \le j \ne \ell \le K-1} \mathbb{E}_0^\ell \left(\inf_{r \ge 1} \{T_r \ : \ d_r = j\} \right) \ge \gamma \right\}, \quad (1)$$

where T is the stopping time, d is the final decision (the number of post-change hypotheses) and the event $\{d_r = j\}$ denotes the first false alarm of the j-th type, be the class of detection and isolation procedures for which the average run length (ARL) to false alarm and false isolation is at least $\gamma > 1$. In the case of detection–isolation procedures, the risk associated with the detection delay is defined analogously to Lorden's worst-worst-case and it is given by [1]

$$\mathsf{ESADD}(\delta) = \max_{1 \le j \le K-1} \sup_{0 \le \nu < \infty} \left\{ \operatorname{esssup} \mathbb{E}_\nu^j [(T-\nu)^+ | \mathcal{F}_\nu] \right\}. \quad (2)$$

Hence, the minimax optimization problem seeks to

$$\text{Find } \delta_{\text{opt}} \in \mathbb{C}_\gamma \text{ such that } \mathsf{ESADD}(\delta_{\text{opt}}) = \inf_{\delta \in \mathbb{C}_\gamma} \mathsf{ESADD}(\delta) \text{ for every } \gamma > 1, \quad (3)$$

where $\mathbb{C}_\gamma$ is the class of detection and isolation procedures with the lower bound γ on the ARL to false alarm and false isolation defined in (1).

Another minimax approach to change detection and isolation is as follows [2,3]; unlike the definition of the class $\mathbb{C}_\gamma$ in (1), where we fixed *a priori* the changepoint $\nu = 0$ in the definition of false isolation to simplify theoretical analysis, the false isolation rate is now expressed by the maximal probability of false isolation $\sup_{\nu \ge 0} \mathbb{P}_\nu^\ell (d = j \ne \ell | T > \nu)$. As usual, we measure the level of false alarms by the ARL to false alarm $\mathbb{E}_\infty T$. Hence, define the class

$$\mathbb{C}_{\gamma,\beta} = \left\{ \delta = (T, d)\colon \mathbb{E}_\infty T \ge \gamma, \ \max_{1 \le \ell \le K-1} \max_{1 \le j \ne \ell \le K-1} \sup_{\nu \ge 0} \mathbb{P}_\nu^\ell (d = j | T > \nu) \le \beta \right\}. \quad (4)$$

Sometimes Lorden's worst-worst-case ADD is too conservative, especially for recursive change detection and isolation procedures, and another measure of the detection speed, namely the maximal conditional average delay to detection $\mathsf{SADD}(T) = \sup_\nu \mathbb{E}_\nu (T - \nu | T > \nu)$, is better suited for practical purposes. In the case of change detection and isolation, the SADD is given by

$$\mathsf{SADD}(\delta) = \max_{1 \le j \le K-1} \sup_{0 \le \nu < \infty} \mathbb{E}_\nu^j (T - \nu | T > \nu). \quad (5)$$

We require that the $\mathsf{SADD}(\delta)$ should be *as small as possible* subject to the constraints on the ARL to false alarm and the maximum probability of false isolation. Therefore, this version of the minimax optimization problem seeks to

$$\text{Find } \delta_{\text{opt}} \in \mathbb{C}_{\gamma,\beta} \text{ such that } \mathsf{SADD}(\delta_{\text{opt}}) = \inf_{\delta \in \mathbb{C}_{\gamma,\beta}} \mathsf{SADD}(\delta)$$
$$\text{for every } \gamma > 1 \text{ and } \beta \in (0, 1). \quad (6)$$

A detailed description of the developed theory and some practical examples can be found in the recently published book [4].

3 Efficient Procedures of Quickest Change Detection/isolation

Asymptotic Theory. In this paragraph we recall a lower bound for the worst mean detection/isolation delay over the class $\mathbb{C}_\gamma$ of sequential change detection/isolation tests proposed in [1]. First, we start with a technical result on sequential multiple hypotheses tests and then we give an asymptotic lower bound for $\mathsf{ESADD}(\delta)$.

Lemma 1. *Let $(X_k)_{k\geq 1}$ be a sequence of i.i.d. random variables. Let $\mathcal{H}_0,\ldots,\mathcal{H}_{K-1}$ be $K\geq 2$ hypotheses, where $\mathcal{H}_i$ is the hypothesis that X has density f_i with respect to some probability measure μ, for $i=0,\ldots,K-1$ and assume the inequality*

$$0<\rho_{ij}\overset{def.}{=}\int f_i \log\frac{f_i}{f_j}d\mu<\infty,\quad 0\leq i\neq j\leq K-1,$$

to be true.

Let $\mathbb{E}_i(N)$ be the average sample number (ASN) in a sequential test (N,δ) which chooses one of the K hypotheses subject to a $K\times K$ error matrix $A=[a_{ij}]$, where $a_{ij}=\mathbb{P}_i(accepting\ \mathcal{H}_j),\ i,j=0,\ldots,K-1$.

Let us reparameterize the matrix A in the following manner :

$$\begin{pmatrix}
1-\sum_{\ell=1}^{K-1}\alpha_\ell & \alpha_1 & \ldots & \alpha_{K-1}\\
\gamma_1 & 1-\sum_{\ell=2}^{K-1}\beta_{1,\ell}-\gamma_1 & \ldots & \beta_{1,K-1}\\
\gamma_2 & \beta_{2,1} & \ldots & \beta_{2,K-1}\\
\ldots & \ldots & \ldots & \ldots\\
\gamma_i & \beta_{i,1} & \ldots & \beta_{i,K-1}\\
\ldots & \ldots & \ldots & \ldots\\
\gamma_{K-1} & \beta_{K-1,1} & \ldots & 1-\sum_{\ell=1}^{K-2}\beta_{K-1,l}-\gamma_{K-1}
\end{pmatrix}.$$

Then a lower bound for the ASN $\mathbb{E}_i(N)$ is given by the following formula :

$$\mathbb{E}_i(N)\geq\max\left\{\frac{(1-\tilde{\gamma}_i)\ln\left(\sum_{\ell=1}^{K-1}\alpha_l\right)^{-1}-\log 2}{\rho_{i0}},\right.$$

$$\left.\max_{1\leq j\neq i\leq K-1}\left(\frac{(1-\tilde{\gamma}_i)\ln\beta_{ji}^{-1}-\log 2}{\rho_{ij}}\right)\right\}$$

for $i = 1, \dots, K-1$, where

$$\tilde{\gamma}_i = \gamma_i + \sum_{\ell=1,\ell\neq i}^{K-1} \beta_{i,\ell}.$$

Theorem 1. *Let $(Y_k)_{k\geq 1}$ be an independent random sequence observed* sequentially *:*

$$\mathcal{L}(Y_k) = \begin{cases} P_0 \text{ if } k \leq \nu \\ P_\ell \text{ if } k \geq \nu+1 \end{cases}, \quad \nu = 0,1,2,\dots, \quad \text{for } 1 \leq \ell \leq K-1$$

The distribution P_ℓ has density f_ℓ, $\ell = 0, \dots, K-1$. An asymptotic lower bound for $\mathsf{ESADD}(\delta)$*, which extends the result of Lorden [5] to multiple hypotheses case, is:*

$$\mathsf{ESADD}(T;\gamma) \gtrsim \frac{\log \gamma}{\rho^*} \text{ as } \gamma \to \infty,$$

where

$$\rho^* \stackrel{def.}{=} \min_{1\leq \ell \leq K-1} \min_{0\leq j\neq \ell \leq K-1} \rho_{\ell,j} \text{ and } 0 < \rho_{\ell,j} \stackrel{def.}{=} \mathbb{E}_1^l \left(\log \frac{f_l(Y_i)}{f_j(Y_i)}\right) < \infty$$

is the K-L information.

Generalized CUSUM Test. The generalized CUSUM (non recursive) test asymptotically attains the above mentioned lower bound [1]. Let us introduce the following stopping time and final decision

$$\tilde{N} = \min\{\tilde{N}^1, \dots, \tilde{N}^{K-1}\}; \;\; \tilde{d} = \operatorname{argmin}\{\tilde{N}^1, \dots, \tilde{N}^{K-1}\}$$

of the detection/isolation algorithm. The stopping time $\tilde{N}^\ell$ is responsible for the detection of hypothesis $\mathcal{H}_\ell$:

$$\tilde{N}^\ell = \inf_{k\geq 1} \tilde{N}^\ell(k), \; \tilde{N}^\ell(k) = \inf\left\{ n \geq k : \min_{0\leq j\neq \ell\leq K-1} S_k^n(\ell,j) \geq h \right\}$$

$$\tilde{N}^\ell = \inf\left\{ n \geq 1 \; : \max_{1\leq k\leq n} \min_{0\leq j\neq \ell\leq K-1} S_k^n(\ell,j) \geq h \right\}, \quad S_k^n(\ell,j) = \sum_{i=k}^{n} \log \frac{f_\ell(Y_i)}{f_j(Y_i)}.$$

The generalized matrix recursive CUSUM test, which also attains the asymptotic lower bound, has been considered in [6,7]. Let us introduce the following stopping time and final decision

$$\widehat{N}_r = \min\{\widehat{N}^1, \dots, \widehat{N}^{K-1}\}; \;\; \tilde{d}_r = \operatorname{argmin}\{\widehat{N}^1, \dots, \widehat{N}^{K-1}\}$$

of the detection/isolation algorithm. The stopping time $\widehat{N}^\ell$ is responsible for the detection of hypothesis $\mathcal{H}_l$:

$$\widehat{N}^\ell = \inf\left\{ n \geq 1 : \min_{0\leq k\neq j\leq K-1} Q_n(\ell,j) \geq h \right\},$$

$$Q_n(\ell,j) = \left(Q_{n-1}(\ell,j) + Z_n(\ell,j)\right)^+, \quad Z_n(\ell,j) = \log \frac{f_\ell(Y_n)}{f_j(Y_n)}$$

For some safety critical applications, a more tractable criterion consists in minimizing the maximum detection/isolation delay:

$$\mathsf{SADD}(\delta) = \max_{1\leq j\leq K-1} \sup_{0\leq\nu<\infty} \mathbb{E}_\nu^j(T-\nu|T>\nu). \tag{7}$$

subject to :

$$\mathbb{C}_{\gamma,\beta} = \left\{\delta\ :\mathbb{E}_\infty T \geq \gamma,\ \max_{1\leq\ell\leq K-1} \max_{1\leq j\neq\ell\leq K-1} \sup_{\nu\geq 0} \mathbb{P}_\nu^\ell(d=j|T>\nu) \leq \beta\right\}.$$

for $1\leq \ell, j\neq \ell \leq K-1$. An asymptotic lower bound in this case is given by the following theorem [3].

Theorem 2. *Let* $(Y_k)_{k\geq 1}$ *be an independent random sequence observed* sequentially*:*

$$\mathcal{L}(Y_k) = \begin{cases} P_0 \ \text{if}\, k\leq\nu \\ P_\ell \ \text{if}\, k\geq\nu+1 \end{cases},\quad \nu=0,1,2,\ldots,\quad \text{for}\ \ 1\leq\ell\leq K-1$$

Then

$$\mathsf{SADD}(N;\gamma,\beta) \gtrsim \max\left\{\frac{\log\gamma}{\rho_{\mathrm{d}}^*},\ \frac{\log\beta^{-1}}{\rho_{\mathrm{i}}^*}\right\}\ \ as\ \ \min\{\gamma,\beta^{-1}\}\to\infty,$$

where $\rho_{\mathrm{d}}^* = \min_{1\leq j\leq K-1}\rho_{j,0}$ *and* $\rho_{\mathrm{i}}^* = \min_{1\leq\ell\leq K-1}\min_{1\leq j\neq\ell\leq K-1}\rho_{\ell,j}$.

Vector Recursive CUSUM Test. If $\gamma\to\infty$, $\beta\to 0$ and $\log\gamma\geq\log\beta^{-1}(1+o(1))$, then the above mentioned lower bound can be realized by using the following recursive change detection/isolation algorithm [2] :

$$N_r = \min_{1\leq\ell\leq K-1}\{N_r(\ell)\},\quad d_r = \arg\min_{1\leq\ell\leq K-1}\{N_r(\ell)\},$$

where $N_r(\ell) = \inf\{n\geq 1 : \min_{0\leq j\neq\ell\leq K-1}[S_n(\ell,j) - h_{\ell,j}]\geq 0\}$,

$$S_n(\ell,j) = g_n(\ell,0) - g_n(j,0),\ g_n(\ell,0) = \left(g_{n-1}(\ell,0) + Z_n(\ell,0)\right)^+,$$

with $Z_n(\ell,0) = \log\frac{f_\ell(Y_n)}{f_0(Y_n)}$, $g_0(\ell,0)=0$ for every $1\leq\ell\leq K-1$ and $g_n(0,0)\equiv 0$,

$$h_{\ell,j} = \begin{cases} h_d \ \text{if}\ \ 1\leq\ell\leq K-1\ \ \text{and}\ j=0 \\ h_i \ \text{if}\ 1\leq j,\ell\leq K-1\ \text{and}\ j\neq\ell \end{cases}.$$

4 Applications to Network Monitoring

In this section the above mentioned theoretical results are illustrated by application of the proposed detection/isolation procedures to the problem of network monitoring.

Let us consider a network composed of r nodes and n mono-directional links, where y_ℓ denotes the volume of traffic on the link ℓ at discrete time k (see details in [8,9]). For the sake of simplicity, the subscript k denoting the time is omitted now. Let $x_{i,j}$ be the Origin-Destination (OD) traffic demand from node i to node j at time k. The traffic matrix $X = \{x_{i,j}\}$ is reordered in the lexicographical order as a column vector $X = \left[(x_{(1)}, \ldots, x_{(m)})\right]^T$, where $m = r^2$ is the number of OD flows.

Let us define an $n \times m$ routing matrix $A = [a_{\ell,k}]$ where $0 \leq a_{\ell,k} \leq 1$ represents the fraction of OD flow k volume that is routed through link ℓ. This leads to the linear model

$$Y = A\,X,$$

where $Y = (y_1, \ldots, y_n)^T$ is the Simple Network Management Protocol (SNMP) measurements. Without loss of generality, the known matrix A is assumed to be of full row rank, i.e., $\operatorname{rank} A = n$.

The problem consists in detecting and isolating a significant volume anomaly in an OD flow $x_{i,j}$ by using only SNMP measurements $y_1, \ldots, y_n$. In fact, the main problem with the SNMP measurements is that $n \ll m$. To overcome this difficulty a parsimonious linear model of non-anomalous traffic has been developed in the following papers [10–17].

The derivation of this model includes two steps: *(i)* description of the ambient traffic by using a spatial stationary model and *(ii)* linear approximation of the model by using piecewise polynomial splines.

The idea of the spline model is that the non-anomalous (ambient) traffic at each time k can be represented by using a known family of basis functions superimposed with unknown coefficients, i.e., it is assumed that

$$X_k \approx B\mu_k, \quad k = 1, 2, \ldots,$$

where the $m \times q$ matrix B is assumed to be known and $\mu_t \in \mathbb{R}^q$ is a vector of unknown coefficients such that $q < n$. Finally, it is assumed that the model residuals together with the natural variability of the OD flows follow a Gaussian distribution, which leads to the following equation:

$$X_k = B\mu_k + \xi_k \tag{8}$$

where $\xi_k \sim \mathcal{N}(0, \varSigma)$ is Gaussian noise, with the $m \times m$ diagonal covariance matrix $\varSigma = \operatorname{diag}(\sigma_1^2, \ldots, \sigma_m^2)$. The advantages of the detection algorithm based on a parametric model of ambient traffic and its comparison to a non-parametric approach are discussed in [11,14], (see also [18] for PCA based approach). Hence, the link load measurement model is given by the following linear equation :

$$Y_k = A\,B\mu_k + A\xi_k = H\mu_k + \zeta_k + [\theta_\ell], \tag{9}$$

where $Y_k = (y_1, \ldots, y_n)_k{}^T$ and $\zeta_k \sim \mathcal{N}(0, A\varSigma A^T)$. Without any loss of generality, the resulting matrix $H = A\,B$ is assumed to be of full column rank. Typically, when an anomaly occurs on OD flow ℓ at time $\nu + 1$ (change-point), the vector θ_ℓ has the form $\theta_\ell = \varepsilon\, a(\ell)$, where $a(\ell)$ is the ℓ-th normalized column of A and ε is the intensity of the anomaly. The goal is to detect/isolate the presence of an anomalous vector θ_ℓ, which cannot be explained by the ambient traffic model $X_k \approx B\mu_k$.

Therefore, after the de-correlation transformation, the change detection/isolation problem is based on the following model with nuisance parameter X_k :

$$Y_k = HX_k + \xi_k + \theta(k,\nu), \;\; \xi_k \sim \mathcal{N}(0,\sigma^2 I_n), \;\; k = 1,2,\ldots, \tag{10}$$

where H is a full rank matrices of size $n \times q$, $n > q$, and $\theta(k,\nu)$ is a change occurring at time $\nu + 1$, namely :

$$\theta(k,\nu) = \begin{cases} 0 & \text{if } k \neq \nu \\ \theta_\ell & \text{if } k \geq \nu + 1 \end{cases}, \;\; 1 \leq \ell \leq K - 1.$$

This problem is invariant under the group $G = \{Y \to g(Y) = Y + HX\}$ (see details in [19]). The invariant test is based on maximal invariant statistics. The solution is the projection of Y on the orthogonal complement $R(H)^\perp$ of the column space $R(H)$ of the matrix H. The parity vector $Z = WY$ is a maximal invariant to the group G.

$$WH = 0, \;\; W^T W = P_H = I_r - H(H^T H)^- H^T, \;\; WW^T = I_{n-q}.$$

Transformation by W removes the interference of the nuisance parameter X

$$Z = WY = W\xi \; (+W\theta).$$

Hence, the sequential change detection/isolation problem can be re-written as

$$Z_k = WY_k = W\xi_k + \; W\theta(k,\nu), \;\; \xi_k \sim \mathcal{N}(0,\sigma^2 I_{n-q}), \;\; k = 1,2,\ldots.$$

Theorem 3. *Let $(Y_k)_{k\geq 1}$ be the output of the model given by (10) observed* sequentially*. Then the generalized CUSUM or matrix recursive CUSUM tests attain the lower bound corresponding to the minimax setup :*

$$\mathsf{ESADD}(N;\gamma) \gtrsim \frac{\log\gamma}{\overline{\rho^*}} \;\; \textit{as} \;\; \gamma \to \infty, \;\; \overline{\rho^*} \overset{def.}{=} \inf_{X^\ell, X^j} \min_{1\leq \ell \leq K-1} \; \min_{0\leq j\neq \ell \leq K-1} \; \rho_{\ell,j}(X^\ell, X^j)$$

where X^ℓ (resp. X^j) corresponds to the hypothesis $\mathcal{H}_\ell$ (resp. $\mathcal{H}_j$). The vector recursive CUSUM test attains the lower bound

$$\mathsf{SADD}(N;\gamma,\beta) \gtrsim \max\left\{\frac{\log\gamma}{\overline{\rho^*}_{\mathrm{d}}}, \; \frac{\log\beta^{-1}}{\overline{\rho^*}_{\mathrm{i}}}\right\} \;\textit{as}\; \gamma \to \infty, \; \beta \to 0, \log\gamma \geq \log\beta^{-1}(1 + o(1)),$$

where

$$\overline{\rho^*}_{\mathrm{d}} = \inf_{X^j, X^0} \min_{1\leq j \leq K-1} \rho_{j,0}(X^j, X^0) \;\; \textit{and} \;\; \overline{\rho^*}_{\mathrm{i}} = \inf_{X^\ell, X^j} \min_{1\leq \ell \leq K-1} \min_{1\leq j\neq \ell \leq K-1} \rho_{\ell,j}(X^\ell, X^j).$$

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