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Cointegration Portfolios of European Equities for Index Tracking and Market Neutral Strategies

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Introduction

Financial markets are highly interdependent and for many decades portfolio managers have scrutinised the comovements between markets. It is regrettable, however, that traditional quantitative portfolio construction still heavily relies on the analysis of correlations for modelling the complex interdependences between financial assets. Admittedly, the application of the concept of correlation has been improved and, over the last ten years, following the generalised use of the JP Morgan (1994)

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RiskMetrics approach, quantitative portfolio managers have made increasing use of conditional correlations.

Yet, if correlations are indeed time varying, their many changes across time make them a difficult tool to use in practice when managing quantitative portfolios, as the frequent rebalancing they imply may be very costly. Correlation and cointegration are somewhat related concepts, but the key distinction between them is that correlation reflects short-run comovements in returns, while cointegration measures long-run comovements in prices.

Accordingly, the main motivation for this paper is to gauge the benefits of less frequent portfolio rebalancing through the use of the concept of cointegration, which relies on the long-term relationship between time series, and thus assets, to devise quantitative European equities portfolios in the context of two applications: a classic index tracking strategy and a long/short equity market neutral strategy.

When index tracking portfolios are constructed on the basis of returns analysis, ie correlation, it is necessary to rebalance them frequently to keep them in line with the benchmark index to be tracked. Yet, if the allocations in a portfolio are designed such that the portfolio tracks an index, the portfolio should be cointegrated with the index: in the short run the portfolio might deviate from the index, but they should be tied together in the longer run. Optimal cointegration portfolios, as they rely on the long-run trends between asset prices, should therefore not require as much rebalancing.

Market neutral strategies have become popular among investment managers, particularly since the end of the stock market bull run in 2000, as their key characteristic is that, if constructed and implemented properly, the underlying stock market behaviour does not affect the results of the portfolio. In other words, returns generated by an equity market neutral portfolio should be independent of the general stock market returns. A long/short equity market neutral strategy consists in buying a portfolio of attractive stocks, the long portion of the portfolio, and selling a portfolio of unattractive stocks, the short portion of the portfolio. The spread between the performance of the longs and the shorts provides the value added of this investment strategy and, here again, the frequency of rebalancing is a key element in the final performance.

Data are used from the Dow Jones EUROStoxx50 index and its constituent stocks from 4th January, 1999, to 30th June, 2003, to construct cointegration portfolios of European equities, implementing in turn

index tracking and long/short equity market neutral strategies: the results show that the designed portfolios are strongly cointegrated with the benchmark and indeed demonstrate good tracking performance; in the same vein, the long/short market neutral strategy generates steady returns under adverse market circumstances but, contrary to expectations, does not minimise volatility.

The rest of the paper is organised as follows. The second section briefly reviews the literature on common trends in equity markets and cointegration-based trading strategies. The third section describes the techniques and investment strategies retained for this study, while the fourth section documents the data used and the construction of the cointegration portfolios. The estimation results are presented in the fifth section, and the final section closes this paper with a summary of the conclusions.

Literature review

Since the seminal work of Engle and Granger (1987), cointegration has emerged as a powerful technique for investigating common trends in multivariate time series, providing a sound methodology for modelling both long-run and short-run dynamics in a system.

Although models of cointegrated financial time series are now relatively common, their importance for quantitative portfolio optimisation has remained very limited until now, because the traditional starting point for portfolio construction since Markowitz (1952, 1959) is a correlation analysis of returns, whereas cointegration is based on the raw price, rate or yield data: any decision based on long-term common trends in the price data is excluded in standard risk-return modelling.

Recent research on stock market linkages has emphasised finding common stochastic trends for a group of stock markets through testing for cointegrating relationships. Using monthly and quarterly data for the period January 1974 to August 1990 and the Johansen (1988) test for multiple cointegration, Kasa (1992) investigates whether there are any common stochastic trends in the equity markets of the US, Japan, the UK, Germany and Canada. The results indicate the presence of a single common trend driving these countries' stock markets. Corhay *et al.* (1993) study whether the stock markets of different European countries display a common long-run trend. They use static regression models and a VAR-based maximum likelihood framework, which

provides empirical evidence of common stochastic trends among five important European stock markets over the period 1975–1991. Masih and Masih (1997) underline the growing leading role of the US market following the 1987 crash.

Meanwhile, Choudhury (1997) analyses the long-run relationships between six Latin American stock markets and the US market using weekly data for the period January 1989 to December 1993. The cointegration tests indicate the presence of a long-run relationship between the six Latin American indices with and without the US index. Other studies looking at linkages across developing countries include Cheung and Mak (1992), Chowdhury (1994), Garrett and Spyrou (1994), Ng (2002) and Dunis and Shannon (2004).

Yet, these papers focus primarily on stock market linkages. Closer to the preoccupation with optimal portfolio construction, Cerchi and Havenner (1988) and Pindyck and Rothenberg (1992) underline that an equity index is by definition a weighted sum of its constituents, so that there should be a sufficiently large basket of component equities which is cointegrated with the index, provided index weights are reasonably stable across time. Alexander and Dimitriu (2002) build index tracking and market neutral cointegration portfolios for domestic US equities based on the Dow Jones Industrial Average index with daily data from January 1990 to December 2001 whereas, using 12 years of daily data from January 1990 to March 2002, Qiu (2002) devises a cointegration-based portfolio of international bonds from eight different countries to replicate the 13-country JP Morgan global government bond index. Finally, using the same EUROStoxx50 index and constituent series as the present authors do, but with daily data from September 1998 to July 2002, Burgess (2003) develops cointegration-based strategies for hedging a given equity position or implementing statistical arbitrage trading opportunities.

Methodology and investment strategies

Cointegration models

The issues of common trends and the interdependence of financial markets have come under increased scrutiny in recent years, following Engle and Granger (1987), who point out that a linear combination of two or more non-stationary series may be stationary: if such a stationary linear combination exists, the non-stationary time series are said to be cointegrated. The stationary linear combination is called the cointegrating equation and may be interpreted as a long-run

equilibrium relationship between the variables. Thus, cointegration of stock markets means there is a long-run relationship between them: if Y and X are $I(1)$ time series and are cointegrated so that $u = Y - \alpha - \beta X$ is $I(0)$, then, in the long run, Y and X do not drift apart, since u has a constant mean, which is zero. Hence, $Y = \alpha + \beta X$ can be interpreted as an equilibrium or long-run relationship between these markets, and u is referred to as the error-correction term (ECT), since it gives the 'error' value in $Y = \alpha + \beta X$ and so is the deviation from equilibrium which, in the long run, is zero.

Engle and Granger (1987) and Engle and Yoo (1987) propose a two-step estimation method, where the first step consists of estimating a long-run equilibrium relationship, and the second is the estimation of the dynamic error-correction relationship using lagged residuals. Holden and Thompson (1992) claim that this two-step approach has the advantage that the estimation of the two steps is quite separate, so that changes in the dynamic model do not enforce re-estimation of the static model obtained in the first step. As such, it offers a tractable modelling procedure.

Alexander (1999) suggests nevertheless that the problem of uniqueness arises when there are more than two variables included in the model, ie the possibility of more than one cointegrating vector between the selected variables according to the choice of dependent variable. In the circumstances, the well-documented Johansen (1988) method for multiple cointegration allows testing for a number of cointegrating vectors at the same time. It relies on estimating a vector autoregression (VAR) model in differences, such as

$$\begin{aligned} \Delta X_t = & \mu + \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \dots + \Gamma_{p-1} \Delta X_{t-p-1} \\ & + \Pi X_{t-p-1} + BZ_t = u_t \end{aligned} \quad (1)$$

where X is an $(m \times 1)$ matrix of $I(1)$ variables, Z is an $(s \times 1)$ matrix of $I(0)$ variables, the Γ_j and Π are $(m \times m)$ matrices of unknown parameters, and B is an $(m \times s)$ matrix of unknown parameters. M is the number of variables in X , and p is the maximum lag in the equation, which is a VAR model. If Π has zero rank, no stationary linear combination can be identified and the variables in X_t are not cointegrated. The number of lags to be included within the model is determined by minimizing Akaike's error criterion.

In the current applications, however, the choice of the dependent variable is completely obvious, ie the EUROStoxx50 index for the index tracking application and the ad hoc artificial 'long' and 'short'

benchmarks for the long/short equity market neutral strategy. There is therefore no doubt as to what the endogenous variable in the cointegration equation should be and which cointegrating vector one should be looking for, so the original Engle and Granger (1987) approach can also be applied to estimate cointegration equations such as

$$Y_t = \alpha + \beta X_t + u_t \quad (2)$$

where Y_t and X_t are cointegrated time series, and therefore the residual series and tracking error u_t is stationary.

It is worth noting that, with a large number of stocks, there may be no alternative to using Equation (2), for technical reasons: indeed, multicollinearity may occur, in which case least squares estimates are unbiased, but their variances are large and may be far from the true value. This can be solved using ridge regression (Hoerl and Kennard, 1970a, b), where, by adding a degree of bias to the regression estimates, it is hoped that the net effect will be to give more reliable ones.

Index tracking

The first investment strategy selected in this paper is a classic index tracking strategy which aims to replicate the benchmark in terms of returns and volatility, using cointegration rather than correlation. This allows us to make use of the full information contained in stock prices and base the portfolio weights on the long-run behaviour of stocks.

As with traditional correlation-based portfolio construction, the selection of the stocks to be included in the cointegration portfolio is 'exogenous', so to speak. Obviously, the quality of the index tracking will highly depend on the stock selection, and several alternative combinations should be tried out before choosing the final tracking portfolio.

Then, portfolio weights are determined over the chosen in-sample period by the coefficients of the cointegration equation between the log price of the market index and the portfolio stocks log prices as exogenous variables.

$$\log(\text{STOXX}_t) = a_0 + \sum_{k=1}^n a_k \log(P_{k,t}) + \varepsilon_t \quad (3)$$

where STOXX_t is the EUROStoxx50 index and $P_{k,t}$ is the price of the constituent stock P_k at time t , the series STOXX_t and $P_{k,t}$ are cointegrated, and therefore the residual series, ie the tracking error, ε_t is stationary.

Using log prices has the advantage that the tracking error ε_t is in return format and the a_k coefficients are portfolio weights: they need to be normalised, however, to sum up to one to give the percentage weight of each selected stock in the index tracking portfolio. The index tracking portfolio daily returns are computed as the weighted sum of the daily returns of its constituent stocks.

Long/short equity market neutral strategy

As underlined by Lederman (1996) and Jelcic and Munro (1999), market neutral strategies are often considered by fund managers as state-of-the-art investment strategies. They actually include many different complex trading strategies in the bond and equity markets, and it is beyond the scope of this paper to review them all. This paper concentrates exclusively on long/short equity market neutral strategies.

Long/short equity investment can be traced back to the late 1940s and the A. W. Jones investment partnership that bought and shorted stocks. It was later refined by N. Tartaglia at Morgan Stanley in the late 1980s. It was not until recently, however, that long/short equity strategies gained any real institutional appeal. In fact, these strategies have really become popular among investment managers since the stock market downturn in 2000, because their key characteristic is that, if constructed and implemented properly, the underlying stock market behaviour should not affect the results of the portfolio. In other words, returns generated by an equity market-neutral portfolio should be independent of the general stock market returns.

A long/short equity market neutral strategy consists in buying a portfolio of attractive stocks, the long portion of the portfolio, and selling a portfolio of unattractive stocks, the short portion of the portfolio. The spread between the performance of the longs and the shorts provides the value added of this investment strategy which seeks to provide a return in excess of the risk-free rate. The strategy is not a pure enhanced cash strategy because of the significantly higher risk and return expectations of the strategy, but it is an absolute return investment approach, hence its frequent description as a 'double alpha' strategy.

Indeed, there are two primary sources of return to a long/short equity neutral strategy. The first component is the 'long' portfolio, where the investor is a buyer of stocks: in this 'long' portfolio, the investor profits when the stocks in the portfolio rise in price, on average, and loses when the stock prices fall.¹ The second component is the 'short' portfolio, where the long/short equity investor borrows stocks from another investor and then sells the stocks to generate the short portfolio

(note the self-financing aspect of the long/short strategy): in this 'short' portfolio, the investor profits when the prices of the constituent stocks fall, on average, and loses when these stocks rise in price.

In practice, the construction of both 'long' and 'short' portfolios derives from the index tracking strategy: only this time the aim is to devise two cointegrating portfolios to track two benchmarks, a benchmark 'plus' and a benchmark 'minus' constructed by adding to (respectively, subtracting from) the main benchmark daily returns an annual excess return of x per cent (equally distributed on the daily returns). The two cointegration equations tested are

$$\log(\text{STOXX}_t^+) = a_0 + \sum_{k=1}^n a_k \log(P_{k,t}^+) + \varepsilon_t^+ \quad (4)$$

where STOXX_t^+ is the EUROStoxx50 'plus' index devised as a benchmark for the 'long' portfolio, and $P_{k,t}^+$ is the price of the constituent stock P_k^+ at time t , the series STOXX_t^+ and $P_{k,t}^+$ are cointegrated, and therefore the residual series ε_t^+ is stationary.

$$\log(\text{STOXX}_t^-) = a_0 + \sum_{k=1}^n a_k \log(P_{k,t}^-) + \varepsilon_t^- \quad (5)$$

where STOXX_t^- is the EUROStoxx50 'minus' index devised as a benchmark for the 'short' portfolio and $P_{k,t}^-$ is the price of the constituent stock P_k^- at time t , the series STOXX_t^- and $P_{k,t}^-$ are cointegrated and therefore the residual series ε_t^- is stationary.

Clearly, the choice of the annual excess return to construct the two 'long' and 'short' cointegrated portfolios is critical. If, as mentioned before, there is a good reason to expect *a priori* that a sufficiently large basket of component equities will be cointegrated with the reference market index, this may not be true in the case of ad hoc benchmarks, such as those created for the 'long' and 'short' portfolios. The satisfaction of the cointegration tests in (4) and (5) is therefore essential, but it can be reasonably expected that the larger the annual excess return chosen, the more difficult it will be to satisfy these tests.

Overall, the long/short equity market neutral strategy consists of buying the 'long' portfolio and selling the 'short' portfolio. The global portfolio daily returns are computed as the sum of the daily returns of the 'long' and 'short' portfolios (multiplied by -1 for the 'short' portfolio), where the daily returns of each of these portfolios is the weighted sum of the daily returns of their constituent stocks. In other words, the strategy returns depend on the spread between the benchmarks tracked.

Finally, as the 'long' and 'short' portfolios are both highly correlated with the reference stock market benchmark, and assuming that each tracking error is not correlated with the market, one would expect a low correlation of their difference with the market benchmark, a key characteristic of a market neutral strategy.

Data and portfolio construction

Data

The data used in this paper are the Dow Jones EUROStoxx50 index and its constituent stocks as at 30th June, 2003. The databank spans 4th January, 1999, to 30th June, 2003, four and a half years of data with 1,084 readings in total. It was obtained from the Yahoo financial website (www.finance.yahoo.co.uk). The advantage of taking this stock index is that it covers a panel of international stocks from different European countries, all denominated in a common currency, the euro. Yet, as rightly mentioned by Burgess (2003), the slightly non-synchronous closing times of the different European stock markets would induce distortions in a true trading environment, but, for this paper, it is deemed that these closing prices are good enough and serve well the purpose of demonstrating the use of cointegration portfolios.

The 50 stocks listed in the EUROStoxx50 index, their ticker symbols and their weights in the index as at 30th June, 2003, are given in Appendix 1.

A log transformation is applied to both the benchmark and the underlying stocks, as this ensures that the cointegration equation coefficients can be interpreted as portfolio weights and because, if the level variables are cointegrated, so will be their logarithms. Traditional ADF tests are performed for the EUROStoxx50 index and its constituent time series to confirm that they are all non-stationary.²

Portfolio construction

For both applications, an initial in-sample portfolio is constructed initially for the period from January 1999 to December 2001, and it is progressively expanded monthly until June 2003: the initial portfolio (P0) is constructed over the period from January 1999 to December 2001 and simulated out-of-sample in January 2002 as the first tracking portfolio (P1), then the second tracking portfolio is constructed over the period from January 1999 to January 2002 and simulated out-of-sample in February 2002 (P2), the third tracking portfolio is constructed using data from January 1999 to February 2002 and simulated out-of-sample

in March 2002 (P3), and so on. Therefore 18 out-of-sample portfolios (P1–P18) are obtained.

The initial portfolio P0 is based on three years of daily data, and the coefficients of the cointegration regression are subsequently re-estimated monthly using the Johansen (1988) test procedure (see Appendix 2 for an example). The first cointegration tracking portfolio (P1) is simulated from 2nd to 31st January, 2002, using estimation data from 4th January, 1999, to 28th December, 2001, to determine portfolio weights. The last tracking portfolio (P18) is simulated from 2nd to 30th June, 2003, using data from 4th January, 1999, to 30th May, 2003, to estimate portfolio weights.

To build the index tracking portfolio, it is first necessary to apply a stock selection procedure: for the purpose of diversification, one initially applies the simplest stock selection criterion available, ie the weight of the stocks in the index at the moment of the portfolio construction to construct P0 portfolios containing 5, 10, 15 and 20 constituent stocks that are most highly cointegrated with the EUROStoxx50 index as at 28th December, 2001. Only relative weights are subsequently modified.

The cointegration equation then allows portfolio weights to be determined, using the regression coefficients and normalizing their sum to 1. There is no specific constraint: both long and short positions are allowed.

The stationarity of the tracking error in each regression is then tested with a traditional ADF test, the more stationary the tracking error, the greater the cointegration between the benchmark and the constructed portfolio.

The final stage is the computation and analysis of portfolio results. To gauge portfolio performance, for each tracking portfolio, annualised returns (using portfolio returns, estimated as the first difference in portfolio log prices), annualised volatility, excess returns, information ratio,³ Sharpe ratio⁴ and correlation of the tracking portfolio returns with the index returns are calculated.

This paper devises cointegration portfolios as described above for three different applications: (1) a simple index tracking; (2) the same, but with different rebalancing frequencies; and (3) a long/short market neutral strategy.

Simple index tracking methods

Cointegrated portfolios are constructed, tracking the EUROStoxx50 index, which contain respectively 5, 10, 15 and 20 stocks.

Different rebalancing frequencies

To investigate whether the stock selection method is responsible for potential weight instability, alternative stock selection methods are used, also based on price ranking criteria. To reduce turnover, each portfolio is kept constant for three-month, six-month and one-year investment periods. The initial strategy based on monthly rebalancing is subsequently referred to as RM, while the quarterly, semi-annual and annual rebalancing strategies are denoted by RQ, RSA and RA, respectively. Note that an important difference between the initial stock selection method and the alternative ones proposed here will be associated transaction costs.

Long/short equity market neutral

An extension for exploiting the tracking potential of cointegrated portfolios is to replicate 'plus' and 'minus' benchmarks by creating 'long' and 'short' portfolios. Yet many different 'plus' and 'minus' benchmarks can be devised on the back of the EuroStoxx50 index, leading to alternative tracking portfolios.

Concerning the 'constrained' long/short strategy, one needs to construct two new *artificial* benchmarks by adding/subtracting an annualised return of x per cent uniformly from the daily returns of the EuroStoxx50 index. (For instance, to construct the 'EuroStoxx50–5 per cent' artificial benchmark, one needs to subtract 0.01984 per cent, ie 5 per cent/252, assuming a 252-day trading year, from the EuroStoxx50 daily returns and then find a cointegration relationship between this new benchmark and some of the stocks available.) The methodology for an artificial 'EuroStoxx50 plus' benchmark is obviously similar.

Having ensured that the portfolios pass the cointegration test, one then computes the weights exactly as with the simple index tracking strategy. The long/short portfolio manager gets the sum of the return of the 'long' portfolio and the return (multiplied by -1) of the 'short' portfolio (in fact, less the borrowing cost of the 'short' portfolio, as he/she needs to borrow to 'buy' the stocks of the 'short' portfolio before selling them, and one therefore subtracts 4 per cent p.a. from the 'short' portfolio return to reflect borrowing costs).

Nine combinations of artificial benchmarks are used in order to implement different long/short equity market neutral portfolios: (1) 'plus' 2.5 per cent vs 'minus' 2.5 per cent; (2) 'plus' 2.5 per cent vs 'minus' 5 per cent; (3) 'plus' 2.5 per cent vs 'minus' 10 per cent; (4) 'plus' 5 per cent vs 'minus' 2.5 per cent; (5) 'plus' 5 per cent vs 'minus' 5 per cent; (6) 'plus' 5 per cent vs 'minus' 10 per cent; (7) 'plus' 10 per cent vs 'minus'

2.5 per cent; (8) 'plus' 10 per cent vs 'minus' 5 per cent; and (9) 'plus' 10 per cent vs 'minus' 10 per cent.

Results and performance analysis

This section presents only some of the results obtained for the three strategies followed, ie the simple index tracking, the different rebalancing frequency and the long/short equity market neutral strategies. Complete results are available from the authors upon request.

Simple index tracking

The actual stocks contained in the different tracking portfolios are given in Appendix 3. Table 7.1 documents the in-sample results of the tracking portfolios compared with the benchmark, and Table 7.2 documents the out-of-sample results of the tracking portfolios compared with the benchmark.

The overall conclusion is that, over an 18-month period where the benchmark lost 24.62 per cent, all tracking portfolios produced better

Table 7.1 In-sample results for EuroStoxx50 and tracking portfolios (January 1999–December 2001)

Portfolio	Annualised return (%)	Annualised volatility (%)	Correlation with benchmark	Information ratio	Sharpe ratio
Benchmark	5.33	23.71	–	0.23	0.06
5 stocks	86.58	91.34	0.21	0.95	0.90
10 stocks	13.05	49.02	0.13	0.27	0.18
15 stocks	19.18	34.30	0.48	0.56	0.44
20 stocks	29.71	45.33	0.44	0.66	0.57

Table 7.2 Out-of-sample results for EuroStoxx50 and tracking portfolios (January 2002–June 2003)

Portfolio	Annualised return (%)	Annualised volatility (%)	Correlation with benchmark	Information ratio	Sharpe ratio
Benchmark	–24.62	34.01	–	–0.72	–0.84
5 stocks	0.23	38.33	0.65	0.01	–0.10
10 stocks	41.75	77.37	0.06	0.54	0.49
15 stocks	–6.28	31.23	0.79	–0.20	–0.33
20 stocks	–9.45	37.28	0.75	–0.25	–0.36

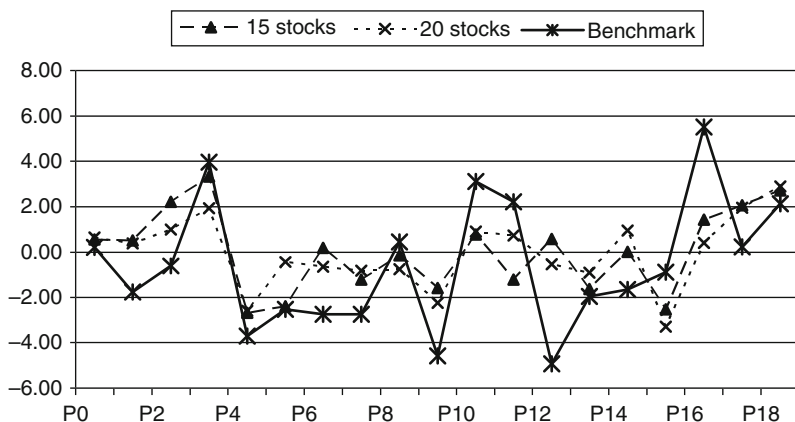


Figure 7.1 Sharpe ratio for EuroStoxx50 and two tracking portfolios (January 2002–June 2003)

out-of-sample returns and risk-adjusted returns. The portfolio comprising ten stocks registers the best performance, but it is also the least correlated with the benchmark.

Figure 7.1 shows that the Sharpe ratios for the 15-stock and 20-stock tracking portfolios are less volatile compared with the benchmark.

Different rebalancing frequencies

The results of the simple index tracking show that the ten-stock tracking portfolio has the best performance out-of-sample. As all tracking errors are stationary throughout the whole period, this portfolio is selected to compare its results when using different rebalancing strategies, monthly (RM), quarterly (RQ), semi-annually (RSA) and annually (RA).

As can be seen from Table 7.3, all portfolios with ten stocks using different rebalancing frequencies have a better performance than the benchmark. In terms of volatility, all tracking portfolios show a higher volatility than the EuroStoxx50 index. Using monthly and quarterly rebalancing produces similar annualised returns of about 42 per cent. The ten-stock tracking portfolio with quarterly rebalancing has the best overall performance, with the highest information ratio and a 0.18 correlation with the benchmark.

It is concluded that quarterly rebalancing is better than monthly rebalancing, especially if transaction costs are included (see Appendix 4 for the weights profiles): true, an important difference between the

Table 7.3 Out-of-sample results for EuroStoxx50 and ten-stock tracking portfolios with various rebalancing frequencies (January 2002–June 2003)

Portfolio	Annualised return (%)	Annualised volatility (%)	Correlation with benchmark	Information ratio	Sharpe ratio
Benchmark	-24.62	34.01	-	-0.72	-0.84
10 (RM)	41.75	77.37	0.06	0.54	0.49
10 (RQ)	42.33	57.21	0.18	0.74	0.67
10 (RSA)	9.90	56.58	0.13	0.18	0.10
10 (RA)	-21.98	48.03	0.25	-0.46	-0.54

rebalancing strategies is transaction costs, which will be lower, the lower the rebalancing frequency retained. Still, with estimated round-trip transaction costs of 12 basis points (b.p.) for the EuroStoxx50 index, and about twice as much for its component stocks in the cash market, these costs are not such a major drawback for the more active strategies.⁵

Long/short equity market neutral

Table 7.4 compares ten-stock tracking portfolios obtained by adding to and subtracting from the benchmark returns an annual excess return of -2.5 per cent, -5 per cent, -10 per cent, +2.5 per cent, +5 per cent and +10 per cent, respectively.

In general, long/short strategies produce better results than the benchmark except for the three strategies replicating the benchmark minus 2.5 per cent which produce very negative performance. Yet, contrary to what one would expect, the long/short strategies do not minimise volatility: annualised volatility is generally higher than the benchmark. The long/short combination 'plus 5 per cent/minus 5 per cent' has the best out-of-sample performance, with a Sharpe ratio of 1.35 compared with -0.84 for the EuroStoxx50 index during this 18-month period.

Still, Table 7.4 shows the out-of-sample results with the benefit of hindsight. In fact, fund managers do not have the benefit of hindsight and would have traded the 'best portfolio' at the end of each calibration period.

Table 7.5 shows that the combination 'plus 5 per cent/minus 5 per cent' has the highest in-sample Sharpe ratio at 0.43 against 0.06 for the EuroStoxx50 index.

Table 7.4 shows that, after 18 months, the combination 'plus 5 per cent/minus 5 per cent' was still the best strategy. In real life, however, as fund managers do not know the future, they would probably have modified their choice of long/short combination every three or six months.

Table 7.4 Average out-of-sample results for ten-stock long/short portfolios (January 2002–June 2003)

	Benchmark	+2.5%/ -2.5%	+2.5%/ -5%	+2.5%/ -10%	+2.5%/ -10%	+5%/ -2.5%	+5%/ -5%	+5%/ -10%	+10%/ -2.5%	+10%/ -5%	+10%/ -10%
Annualised return (%)	-24.62	-486.75	81.15	37.21	37.21	-384.15	183.74	139.80	-368.26	199.64	155.69
Annualised volatility (%)	34.01	179.63	96.83	84.00	84.00	206.63	132.92	119.56	285.72	239.65	225.66
Correlation with benchmark	-	-0.17	0.20	0.18	0.18	-0.13	0.20	0.18	-0.05	0.18	0.17
Information ratio	-0.72	-2.71	0.84	0.44	0.44	-1.86	1.38	1.17	-1.29	0.83	0.69
Sharpe ratio	-0.84	-2.73	0.80	0.40	0.40	-1.88	1.35	1.14	-1.30	0.82	0.67

Table 7.5 In-sample results for ten-stock long/short portfolios (January 1999–December 2001)

	+2.5%/		+2.5%/		+2.5%/		+5%/		+5%/		+10%/		+10%/	
Benchmark	-2.5%	+2.5%	-5%	+2.5%	-10%	+2.5%	-2.5%	+5%	-5%	+5%	-10%	+10%	-5%	+10%
Annualised return (%)	5.33	4.25	5.92	7.35	7.35	5.75	7.42	8.85	78.94	80.61	82.04			
Annualised volatility (%)	23.71	5.67	10.54	21.87	21.87	4.85	7.97	18.36	1761.45	1762.00	1763.34			
Correlation with benchmark	-	0.19	0.23	0.26	0.26	-0.14	0.08	0.21	0.03	0.03	0.03			
Information ratio	0.23	0.75	0.56	0.34	0.34	1.19	0.93	0.48	0.04	0.05	0.05			
Sharpe ratio	0.06	0.04	0.18	0.15	0.15	0.36	0.43	0.26	0.04	0.04	0.04			

It is unlikely that they would leave their portfolio using the same long/short portfolio mix for more than a year.

Accordingly, it is assumed in the following that investment managers manage their long/short portfolios using three-month and six-month rebalancing frequencies.

Long/short neutral strategies rebalancing every six months

Table 7.6 shows that for the out-of-sample period from January 2002 to June 2002, the combination 'plus 2.5 per cent/minus 10 per cent' produced the best Sharpe ratio at 0.58. Unfortunately, in January 2002, a fund manager would have used the results from Table 7.5 to set up his/her trading strategy using the combination 'plus 5 per cent/minus 5 per cent': six months later, in June 2002, this strategy had produced a Sharpe ratio of -0.58 , still superior to the -1.69 achieved by the EuroStoxx50 index.

If one uses the results from Table 7.6 with the combination 'plus 2.5 per cent/minus 10 per cent' for the following six months, Table 7.7 shows that for the following six-month out-of-sample period from July 2002 to December 2002, the retained strategy produces a Sharpe ratio of -0.23 (still far superior to the -1.13 of the EuroStoxx50 index), whereas the best Sharpe ratio for that period is provided by the combination 'plus 10 per cent/minus 5 per cent' with a Sharpe ratio of 2.36.

Using the results from Tables 7.5–7.7, one can simulate the trading performance of a fund manager rebalancing his/her portfolio every six months. Starting from January 2002 to June 2002, he/she would have traded the combination 'plus 5 per cent/minus 5 per cent' (ie the best in-sample combination), then, from July 2002 to December 2002, the combination 'plus 2.5 per cent/minus 10 per cent' (ie the best strategy between January 2002 and June 2002) and, from January 2003 to June 2003, the combination 'plus 10 per cent/minus 5 per cent' (ie the best strategy between July 2002 and December 2002).

The trading simulation with semi-annual rebalancing yields a Sharpe ratio of 1.03 compared with 1.35 for the best single out-of-sample long/short strategy chosen from the in-sample optimisation (see Table 7.8). This is still far superior to the -0.84 achieved by the EuroStoxx50 index over the same 18-month period.

Long/short neutral strategies rebalancing every three months

A similar approach to that adopted for the six-month rebalancing is used, but this time a trading strategy is assumed whereby the fund manager changes the structure of his/her portfolio every three months (see Appendix 5). Starting from January 2002 to March 2002, a fund

Table 7.6 Out-of-sample results for ten-stock long/short portfolios (January 2002–June 2002)

	Benchmark	+2.5%/ -2.5%	+2.5%/ -5%	+2.5%/ -10%	+5%/ -2.5%	+5%/ -5%	+5%/ -10%	+10%/ -2.5%	+10%/ -5%	+10%/ -10%
Annualised return (%)	-34.52	-22.43	-2.04	19.52	-34.59	-14.21	7.36	-700.02	-679.63	-658.07
Annualised volatility (%)	22.75	22.93	30.22	26.75	25.15	31.50	26.96	136.57	141.33	135.30
Correlation with benchmark	-	-0.08	-0.14	-0.16	-0.07	-0.13	-0.16	-0.21	-0.19	-0.19
Information ratio	-1.52	-0.98	-0.07	0.73	-1.38	-0.45	0.27	-5.13	-4.81	-4.86
Sharpe ratio	-1.69	-1.15	-0.20	0.58	-1.53	-0.58	0.12	-5.16	-4.84	-4.89

Table 7.7 Out-of-sample results for ten-stock long/short portfolios (July 2002–December 2002)

	+2.5%/ -2.5%		+2.5%/ -5%		+2.5%/ -10%		+5%/ -2.5%		+5%/ -5%		+5%/ -10%		+10%/ -2.5%		+10%/ -5%		+10%/ -10%		
Benchmark																			
Annualised return (%)	-47.67	-424.38	14.90	14.90	-28.25	-28.25	-116.57	-116.57	322.70	322.70	279.55	279.55	514.72	514.72	954.00	954.00	910.85	910.85	
Annualised volatility (%)	45.88	198.76	142.41	142.41	143.31	143.31	280.82	280.82	238.67	238.67	238.89	238.89	402.84	402.84	402.90	402.90	402.40	402.40	
Correlation with benchmark	-	-0.05	0.31	0.31	0.28	0.28	0.03	0.03	0.33	0.33	0.31	0.31	0.16	0.16	0.32	0.32	0.31	0.31	
Information ratio	-1.04	-2.14	0.10	0.10	-0.20	-0.20	-0.42	-0.42	1.35	1.35	1.17	1.17	1.28	1.28	2.37	2.37	2.26	2.26	
Sharpe ratio	-1.13	-2.16	0.08	0.08	-0.23	-0.23	-0.43	-0.43	1.34	1.34	1.15	1.15	1.27	1.27	2.36	2.36	2.25	2.25	

Table 7.8 Out-of-sample trading simulation of successive optimal long/short portfolio combinations rebalanced every six months and EuroStoxx50 (January 2002–June 2003)

	Long/short strategies	EuroStoxx50
Annualised return (%)	124.07	–24.62
Annualised volatility (%)	116.55	34.01
Correlation with benchmark	0.19	–
Information ratio	1.06	–0.72
Sharpe ratio	1.03	–0.84

Table 7.9 Out-of-sample trading simulation of successive optimal long/short portfolio combinations rebalanced every three months and EuroStoxx50 (January 2002–June 2003)

	Long/short strategies	EuroStoxx50
Annualised return (%)	90.45	–24.62
Annualised volatility (%)	122.79	34.01
Correlation with benchmark	0.18	–
Information ratio	0.74	–0.72
Sharpe ratio	0.70	–0.84

manager would have traded the combination ‘plus 5 per cent/minus 5 per cent’ (ie the best in-sample combination), then from April 2002 to June 2002, the combination ‘plus 2.5 per cent/minus 5 per cent’ (ie the best strategy between January and March 2002), then from July 2002 to September 2002, the combination ‘plus 2.5 per cent/minus 10 per cent’ (ie the best strategy between April and June 2002), then from October 2002 to December 2002, the combination ‘plus 10 per cent/minus 5 per cent’ (ie the best strategy between July and September 2002), then from January 2003 to March 2003, the combination ‘plus 10 per cent/minus 10 per cent’ (ie the best strategy between September and December 2002) and, finally, from April 2003 to June 2003, the combination ‘plus 10 per cent/minus 5 per cent’ (ie the best strategy between January and March 2003).

This trading simulation with quarterly portfolio rebalancing produces a Sharpe ratio of 0.70 compared with 1.03 for the six-month rebalancing and 1.35 for the best single out-of-sample long/short strategy chosen from the in-sample optimisation. Here again, this trading strategy yields a much better Sharpe ratio than the –0.84 achieved by the EuroStoxx50 index over the same 18-month period (see Table 7.9).

Transaction costs

When analysing the performance of the index tracking strategy, transaction costs will obviously be lower, the lower the rebalancing frequency retained. This is an even more important issue in the case of long/short market neutral strategies, as these entail trading two tracking portfolios, and the self-financing feature offered by the short sale generally implies a leverage of 2:1 and thus double transaction costs. Still, with at most eight round trips in total for quarterly rebalancing and four for semi-annual rebalancing, the transaction costs involved (192 b.p. and 96 b.p., respectively) are minimal compared with the annualised returns, before transaction costs, of the long/short strategies achieved in the trading simulations.

Concluding remarks

The main motivation for this paper was to demonstrate the benefits arising from the use of the concept of cointegration, which relies on the long-term relationship between time series, and thus assets, to devise quantitative European equities portfolios in the context of two applications: a classic index tracking strategy and a long/short equity market neutral strategy. Indeed, its key characteristics, ie a mean-reverting tracking error (ie stationary residuals from the cointegration equation), enhanced portfolio weight stability over time and the full use of the information contained in stock prices, allow for the flexible design of various investment strategies in equity markets, from index tracking to long/short market neutral.

Clearly, the results suffer from some of the simplifying assumptions adopted. First, it was arbitrarily chosen to select at most 20 of the 50 stocks in the EuroStoxx50 index: a larger equity basket would probably have led to better results for the index tracking application. Secondly, the simplest stock selection criterion available are applied, ie the weight of the stocks in the index at the moment of the portfolio construction: the quality of the benchmark tracking highly depends on the stock selection procedure and much improvement could be achieved in this respect. Finally, the slightly non-synchronous closing times of the different European stock markets would induce distortions in a true trading environment, but closing prices serve well the purpose of demonstrating the use of cointegration portfolios.

Yet, the results are quite impressive. Over the 18-month out-of-sample period from January 2002 to June 2003, where the EuroStoxx50 index lost 24.62 per cent, all tracking portfolios produce much better returns

and risk-adjusted returns, with less volatile Sharpe ratio profiles than those of the benchmark.

Strategies based on correlation would require rebalancing portfolios frequently. In contrast, cointegration-based portfolios require less frequent turnover: an analysis of alternative rebalancing frequencies shows that a quarterly portfolio update appears preferable to monthly, semi-annual or annual reallocations.

Furthermore, the tracking capabilities offered by cointegration make it possible to track different benchmarks and thus to implement long/short equity market neutral strategies. Most of the long/short combinations analysed in this paper produce better out-of-sample results and risk-adjusted results than the EuroStoxx50 benchmark, albeit at the cost of higher volatility, which may be linked to the smaller number of stocks included in the 'long' and 'short' portfolios. Two trading simulations with quarterly and semi-annual rebalancing show that, during the adverse market conditions of the January 2002 to June 2003 out-of-sample period, the selected long/short combinations would have attracted Sharpe ratios of 1.03 and 0.70, respectively, against -0.84 for the EuroStoxx50 index. These results are seen to be robust to the introduction of transaction costs.

Overall, the main conclusion from this research is that cointegration portfolios add economic value for investors and fund managers. In the circumstances, the results should go some way towards convincing a growing number of quantitative fund managers to experiment beyond the bounds of correlation analysis for portfolio construction.

Notes

1. Note that, if there are no such constraints imposed on the 'long' and 'short' portfolios, both are likely to include some short equity positions.
2. These results and descriptive statistics are not reproduced here to conserve space. They are available from the authors upon request.
3. The information ratio is simply the average annualised return of an investment strategy divided by its average annualised volatility.
4. The Sharpe ratio was computed as the average annualised return of an investment strategy minus the risk-free rate (assumed at 4 per cent p.a.) divided by the average annualised volatility.
5. Assuming that each time the entire portfolio is reshuffled, which is not the case in this application, monthly rebalancing implies at most 12 round trips per year or 288 b.p., quarterly rebalancing four round trips or 96 b.p., semi-annual rebalancing two round trips or 48 b.p., whereas annual rebalancing entails only one round trip or 24 b.p. For trading costs assumptions, see www.interactive-brokers.com and Bessimbinder (2003).

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Appendix 1: Stock in Dow Jones EUROStoxx50 as at 30th June, 2003 (in descending order according to their weight in EUROStoxx50 index)

Company	ISIN	Market sector	Float factor ^a	Adjusted weight (%)
TOTAL FINA ELF	FR0000120271	Energy	1.00	8.03
ROYAL DUTCH PETROLEUM	NL0000009470	Energy	1.00	7.83
NOKIA	FI0009000681	Technology	1.00	6.10
TELEFONICA	ES0178430E18	Telecommunications	0.94	4.12
ENI	IT0003132476	Energy	0.65	3.19
SIEMENS	DE0007236101	Technology	0.93	3.16
UNILEVER NV	NL0000009348	Food & Beverage	1.00	3.14
BNP	FR0000131104	Banks	0.94	3.11
BCO SANTANDER	ES0113900J37	Banks	1.00	2.81
CENTRAL HIS AVENTIS	FR0000130460	Healthcare	0.87	2.80
BCO BILBAO	ES0113211835	Banks	1.00	2.46
VIZCAYA ARGENT DEUTSCHE	DE0005557508	Telecommunications	0.57	2.45
TELEKOM				
DEUTSCHE BANK R	DE0005140008	Banks	0.95	2.29
E.ON	DE0007614406	Utilities	0.87	2.28
DAIMLERCHRYSLER	DE0007100000	Automobiles	0.81	2.21
ASSICURAZIONI GENERALI	IT0000062072	Insurance	0.86	2.09
GROUPE SOCIETE GENERALE	FR0000130809	Banks	1.00	2.05
CARREFOUR	FR0000120172	Noncyclical Goods & Services	0.80	1.99
SUPERMARCHE				
ABN AMRO	NL0000301109	Banks	0.89	1.91

(Continued)

Company	ISIN	Market sector	Float factor ^a	Adjusted weight (%)
SANOFI SYNTHELABO	FR0000120578	Healthcare	0.56	1.90
ING GROEP	NL0000303600	Insurance	0.88	1.87
PHILIPS ELECTRONICS	NL0000009538	Cyclical Goods & Services	1.00	1.85
FRANCE TELECOM	FR0000133308	Telecommunications	0.43	1.82
BASF	DE0005151005	Chemicals	0.91	1.78
L'OREAL	FR0000120321	Noncyclical Goods & Services	0.47	1.78
AXA UAP	FR0000120628	Insurance	0.82	1.58
GROUPE DANONE	FR0000120644	Food & Beverage	0.95	1.52
UNICREDITO ITALIANO	IT0000064854	Banks	0.69	1.51
TELECOM ITALIA	IT0001127429	Telecommunications	0.45	1.51
TIM	IT0001052049	Telecommunications	0.44	1.39
FORTIS	BE0003801181	Financial Services	0.89	1.37
REPSOL YPF	ES0173516115	Energy	0.82	1.33
VIVENDI UNIVERSAL	FR0000127771	Media	1.00	1.31
AIR LIQUIDE	FR0000120073	Chemicals	1.00	1.23
ENDESA	ES0130670112	Utilities	0.95	1.13
ENEL	IT0003128367	Utilities	0.32	1.01
SUEZ	FR0000120529	Utilities	0.93	1.00
ALLIANZ	DE0008404005	Insurance	0.74	0.90
AEGON	NL0000301760	Insurance	0.88	0.87
SAINT GOBAIN	FR0000125007	Construction	1.00	0.87
BAYER	DE0005752000	Chemicals	0.94	0.86
LVMH MOET HENNESSY	FR0000121014	Cyclical Goods & Services	0.46	0.82
RWE	DE0007037129	Utilities	0.76	0.82
SAN PAOLO IMI	IT0001269361	Banks	0.86	0.78
ALCATEL	FR0000130007	Technology	0.93	0.73
LAFARGE	FR0000120537	Construction	1.00	0.69
VOLKSWAGEN	DE0007664005	Automobiles	0.69	0.65
MUENCHENER RUECKVER R	DE0008430026	Insurance	0.62	0.58
AHOLD	NL0000331817	Noncyclical Goods & Services	1.00	0.29
BAYERISCHE HYPO & VEREINS	DE0008022005	Banks	0.63	0.23

^aThe free float factor is the percentage of shares remaining after the block ownership and restricted shares adjustments are applied to the total number of shares. One has: strategic shareholding (%) = number of shares classified as strategic/total number of shares outstanding free float (%) = 100% - strategic shareholding (%).

Appendix 2: Johansen (1988) cointegration test

Sample(adjusted): 6 716

Included observations: 711 after adjusting endpoints

Trend assumption: Linear deterministic trend

Series: LOG_STOXX LOG_FR_12027 LOG_NL_RD LOG_FI_870737 LOG_IT_ENI

LOG_DE_723610 LOG_FR_13110 LOG_ES_SAN LOG_FR_13046 LOG_ES_BBVA

LOG_DE_555750 Lags interval (in first differences): 1 to 4

Unrestricted cointegration rank test

Hypothesised No. of CE(s)	Eigenvalue	Trace statistic	5% critical value	1% critical value
None**	0.104080	310.8791	277.71	293.44
At most 1	0.086735	232.7369	233.13	247.18
At most 2	0.052139	168.2288	192.89	204.95
At most 3	0.040202	130.1564	156.00	168.36
At most 4	0.036513	100.9820	124.24	133.57
At most 5	0.032318	74.53538	94.15	103.18
At most 6	0.028475	51.17799	68.52	76.07
At most 7	0.017255	30.63820	47.21	54.46

*(**) denotes rejection of the hypothesis at the 5% (1%) level

1 Cointegrating Equation(s): Log likelihood 20651.38

Normalised cointegrating coefficients (std err. in parentheses)

LOG_ STOXX	LOG_ FR_12027	LOG_ NL_RD	LOG_ FI_870737	LOG_ IT_ENI	LOG_ DE_723610	LOG_ FR_13110
1.000000	0.445615 (0.10463)	-0.813099 (0.11559)	-0.072151 (0.03832)	0.942272 (0.13883)	-0.133696 (0.04958)	-0.093449 (0.08731)

Appendix 3: Stocks contained in various tracking portfolios

Company	ISIN	Market Sector
<i>5 stocks tracking portfolio</i>		
TOTAL FINA ELF	FR0000120271	Energy
ROYAL DUTCH PETROLEUM	NL0000009470	Energy
NOKIA	FI0009000681	Technology
ENI	IT0003132476	Energy
SIEMENS	DE0007236101	Technology
<i>10 stocks tracking portfolio</i>		
TOTAL FINA ELF	FR0000120271	Energy
SIEMENS	DE0007236101	Technology

(Continued)

Company	ISIN	Market Sector
BNP	FR0000131104	Banks
AVENTIS	FR0000130460	Healthcare
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks
DEUTSCHE TELEKOM	DE0005557508	Telecommunications
DEUTSCHE BANK R	DE0005140008	Banks
DAIMLERCHRYSLER	DE0007100000	Automobiles
ASSICURAZIONI GENERALI	IT0000062072	Insurance
ABN AMRO	NL0000301109	Banks
<i>15 stocks tracking portfolio</i>		
TOTAL FINA ELF	FR0000120271	Energy
ROYAL DUTCH PETROLEUM	NL0000009470	Energy
NOKIA	FI0009000681	Technology
ENI	IT0003132476	Energy
SIEMENS	DE0007236101	Technology
BNP	FR0000131104	Banks
BCO SANTANDER CENTRAL HIS	ES0113900J37	Banks
AVENTIS	FR0000130460	Healthcare
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks
DEUTSCHE TELEKOM	DE0005557508	Telecommunications
DEUTSCHE BANK R	DE0005140008	Banks
E.ON	DE0007614406	Utilities
DAIMLERCHRYSLER	DE0007100000	Automobiles
ASSICURAZIONI GENERALI	IT0000062072	Insurance
ABN AMRO	NL0000301109	Banks
<i>20 stocks tracking portfolio</i>		
TOTAL FINA ELF	FR0000120271	Energy
ROYAL DUTCH PETROLEUM	NL0000009470	Energy
NOKIA	FI0009000681	Technology
ENI	IT0003132476	Energy
SIEMENS	DE0007236101	Technology
BNP	FR0000131104	Banks
BCO SANTANDER CENTRAL HIS	ES0113900J37	Banks
AVENTIS	FR0000130460	Healthcare
BCO BILBAO VIZCAYA ARGENT	ES0113211835	Banks
DEUTSCHE TELEKOM	DE0005557508	Telecommunications
DEUTSCHE BANK R	DE0005140008	Banks
E.ON	DE0007614406	Utilities
DAIMLERCHRYSLER	DE0007100000	Automobiles
ASSICURAZIONI GENERALI	IT0000062072	Insurance
ABN AMRO	NL0000301109	Banks
ING GROEP	NL0000303600	Insurance
PHILIPS ELECTRONICS	NL0000009538	Cyclical Goods & Services
BASF	DE0005151005	Chemicals
L'OREAL	FR0000120321	Noncyclical Goods & Services
REPSOL YPF	ES0173516115	Energy

Appendix 4: Portfolio weights in ten-stocking tracking portfolios

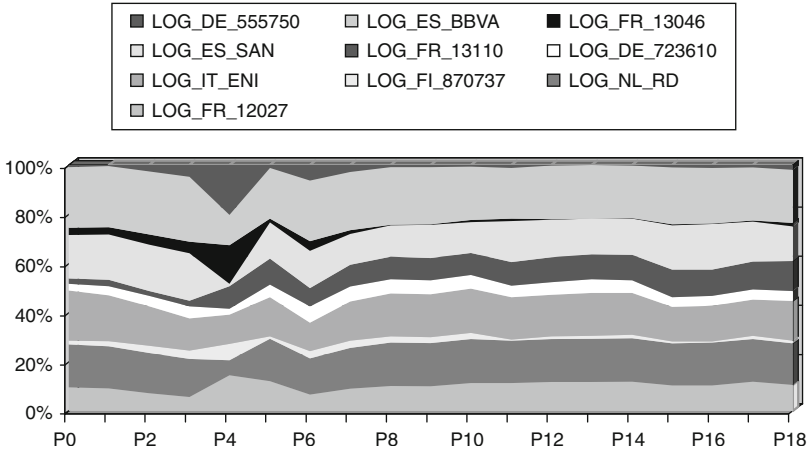


Figure 7A4.1 Portfolio weights in ten-stock tracking portfolio with monthly rebalancing

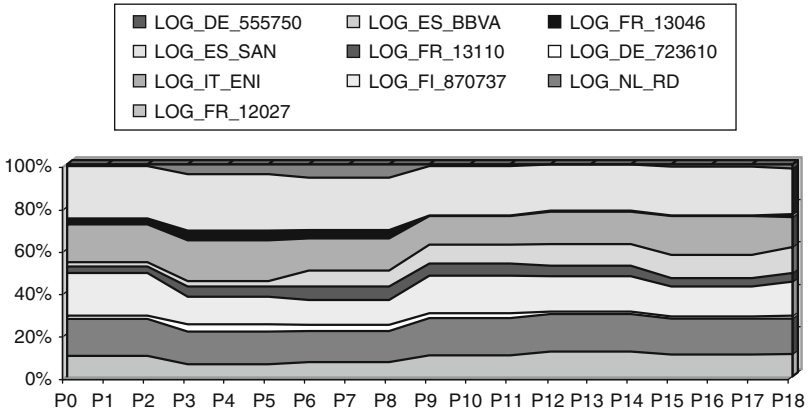


Figure 7A4.2 Portfolio weights in ten-stock tracking portfolio with quarterly rebalancing

Appendix 5: Out-of-sample results for ten-stock long/short portfolios with quarterly rebalancing

Table A5.1 Out-of-sample results for ten-stock long/short portfolios (January–March 2002)

	Benchmark	+2.5%/	+2.5%/	+2.5%/	+2.5%/	+2.5%/	+2.5%/	+5%/	+5%/	+5%/	+5%/	+10%/	+10%/	+10%/	+10%/
		-2.5%	-5%	-10%	-2.5%	-5%	-10%	-5%	-10%	-5%	-10%	-2.5%	-5%	-10%	-10%
Annualised return %	3.94	-31.83	-6.58	-10.46	-37.75	-12.50	-16.37	-1348.00	-1322.75	-1326.62					
Annualised volatility, %	20.52	12.80	17.25	18.37	13.50	17.35	18.21	234.22	236.47	235.93					
Correlation with benchmark	-	0.13	0.14	0.16	0.11	0.12	0.14	-0.23	-0.04	0.04					
Information ratio	0.19	-2.49	-0.38	-0.57	-2.80	-0.72	-0.90	-5.76	-5.59	-5.62					
Sharpe ratio	0.00	-2.80	-0.61	-0.79	-3.09	-0.95	-1.12	-5.77	-5.61	-5.64					

Table A5.2 Out-of-sample results for ten-stock long/short portfolios (April–June 2002)

	Benchmark	+2.5%/ -2.5%	+2.5%/ -5%	+2.5%/ -10%	+2.5%/ -5%	+5%/ -2.5%	+5%/ -5%	+5%/ -10%	+10%/ -2.5%	+10%/ -5%	+10%/ -10%
Annualised return, %	-72.98	-13.02	2.5	49.50	-31.44	-15.91	31.09	-52.04	-36.52	10.48	
Annualised volatility, %	24.97	33.05	43.19	35.14	36.81	45.64	35.72	38.91	46.20	34.66	
Correlation with benchmark	-	-0.29	-0.41	-0.47	-0.24	-0.38	-0.45	-0.19	-0.34	-0.42	
Information ratio	-2.92	-0.39	0.06	1.41	-0.85	-0.35	0.87	-1.34	-0.79	0.30	
Sharpe ratio	-3.08	-0.52	-0.03	1.29	-0.96	-0.44	0.76	-1.44	-0.88	0.19	

Table A5.3 Out-of-sample results for ten-stock long/short portfolios (July–September 2002)

	Benchmark	+2.5%/ -2.5%	+2.5%/ -5%	+2.5%/ -10%	+2.5%/ -10%	+5%/ -2.5%	+5%/ -5%	+5%/ -10%	+10%/ -2.5%	+10%/ -5%	+10%/ -10%
Annualised return, %	-121.08	-834.1	-104.6	-187.6	-233.8	495.7	412.7	710.34	1439.81	1356.80	
Annualised volatility, %	53.48	190.7	87.76	94.86	360.9	258.2	263.9	563.18	461.60	465.98	
Correlation with benchmark	-	0.03	-0.01	-0.07	0.07	0.03	-0.01	0.09	0.05	0.02	
Information ratio	-2.26	-4.37	-1.19	-1.98	-0.65	1.92	1.56	1.26	3.12	2.91	
Sharpe ratio	-2.34	-4.39	-1.24	-2.02	-0.66	1.90	1.55	1.25	3.11	2.90	

Table A5.4 Out-of-sample results for ten-stock long/short portfolios (October–December 2002)

	Benchmark	+2.5%/ -2.5%	+2.5%/ -5%	+2.5%/ -10%	+5%/ -2.5%	+5%/ -5%	+5%/ -10%	+10%/ -2.5%	+10%/ -5%	+10%/ -10%
Annualised return, %	25.73	-14.70	134.39	131.09	0.63	149.7	146.42	319.11	468.19	464.90
Annualised volatility, %	38.28	206.1	197.06	191.76	200.8	219.1	213.92	242.51	344.20	338.82
Correlation with benchmark	-	-0.14	0.64	0.63	0.00	0.63	0.63	0.24	0.60	0.60
Information ratio	0.67	-0.07	0.68	0.68	0.00	0.68	0.68	1.32	1.36	1.37
Sharpe ratio	0.58	-0.09	0.66	0.66	-0.02	0.67	0.67	1.30	1.35	1.36

Table A5.5 Out-of-sample results for ten-stock long/short portfolios (January–March 2003)

	Benchmark	+2.5%/ -2.5%	+2.5%/ -5%	+2.5%/ -10%	+2.5%/ -2.5%	+5%/ -5%	+5%/ -10%	+5%/ -2.5%	+10%/ -2.5%	+10%/ -5%	+10%/ -10%
Annualised return, %	-58.30	-1977	17.74	1.39	-1975	20.06	3.70	-1869.2	125.39	109.03	
Annualised volatility, %	40.94	620.80	129.6	108.39	617.0	146.7	125.66	624.16	227.79	207.20	
Correlation with benchmark	-	-0.26	0.34	0.34	-0.25	0.34	0.33	-0.20	0.34	0.33	
Information ratio	-1.42%	-3.18	0.14	0.01	-3.20	0.14	0.03	-2.99	0.55	0.53	
Sharpe ratio	-1.52%	-3.19	0.11	-0.02	-3.21	0.11	0.00	-3.00	0.53	0.51	