8. Forms of Abduction and an Inferential Taxonomy

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In recent years, the Peircean concept of *abduction* has been differentiated into different forms and made fruitful in a variety of contexts. However, the very notion of abduction still seems to be in need of clarification. The present contribution takes very seriously Peirce's claim (1) that there are only three kinds of reasoning, that is, abduction, deduction, and induction, and (2) that these are mutually distinct. Therefore, the fundamental features of the three inferences canvassed, in particular as regards inferential subprocesses and the validity of each kind of reasoning. It is also argued that forms of abduction have to be distinguished along two dimensions: one concerns *levels* of abstraction (from elementary embodied and perceptual levels to high-level scientific theorizing). The other concerns *domains* of reasoning such as explanatory, instrumental, and moral reasoning. Moreover, Peirce's notion of *theorematic deduction* is taken up and reconstructed as *inverse deduction*. Based on this, *inverse abduction* and *inverse induction* are introduced as complements of the ordinary forms. All in all, the contribution suggests a taxonomy of inferential reasoning, in which different forms of abduction (as well as deduction and induction) can be systematically accommodated. The chapter ends with a discussion on forms of abduction found in the current literature.

For Peirce, not the proverbial misfortune comes in threes, but rather does *fortune*, not least with respect to his inferential triad of abduction, deduction, and induction. On the one hand, they are thought to cover the whole process of scientific reasoning from problem statement to the final adoption of a hypothesis [8.1, CP 5.171 (1903)]. On the other hand, he claimed that there are but these three elementary types of inferences so that all kinds of reasoning must belong to either abduction, deduction, or induction [8.2, CP 8.209 (c. 1905)]. Moreover, and this may sound strange, he explains in the same place that even his earlier classification of inferences dating from 1867 can be understood in the same way, that is, in the sense of the mature Peirce's conception of the three inferences.

Peirce [8.2, CP 8.209 (c. 1905)]:

> "I say that these three are the only elementary modes of reasoning there are. I am convinced of it both a priori and a posteriori. The a priori reasoning is contained in my paper in the Proceedings of the American Academy of Arts and Sciences for

April 9, 1867. I will not repeat it. But I will mention that it turns in part upon the fact that induction is, as Aristotle says, the inference of the truth of the major premiss of a syllogism of which the minor premiss is made to be true and the conclusion is found to be true, while abduction is the inference of the truth of the minor premiss of a syllogism of which the major premiss is selected as known already to be true while the conclusion is found to be true. Abduction furnishes all our ideas concerning real things, beyond what are given in perception, but is mere conjecture, without probative force. Deduction is certain but relates only to ideal objects. Induction gives us the only approach to certainty concerning the real that we can have. In forty years diligent study of arguments, I have never found one which did not consist of those elements."

This is puzzling, if one considers Peirce's own discussion of his earlier conception in his later work where he states explicitly that [8.2, CP 8.221 (1910)]:

"in almost everything I printed before the beginning of this century I more or less mixed up Hypothesis and Induction (i. e., abduction and induction according to his later terminology, G.M.)."

Thus, if he is not contradicting himself, both statements must be true, however, each in a specific respect.

This is one riddle I will try to solve in this chapter, but it is not the only one. I take it as one specific stumbling stone on the way to a full understanding of the very notion and logicality of abduction. In order to achieve a comprehensive account of abduction, however, it is also necessary to accommodate a whole host of different concepts of abduction that have been suggested in recent years. *Magnani*, for instance, not only distinguishes between creative and selective abduction [8.3], but also between sentential and model-based abduction, theoretical and manipulative abduction, explanatory and nonexplanatory abduction [8.4]. The latter distinction is drawn from *Gabbay* and *Woods* [8.5], who maintain that abduction be extended to cover not merely explanatory, but also nonexplanatory abduction, although they remain diffident qualifying their differentiation "as a loose and contextually flexible distinction" [8.5, p. 115].

Another classification is proposed by *Schurz* [8.6] who distinguishes between *factual abduction*, *law-abduction*, *theoretical-model-abduction*, and *second order existential-abduction*, with the first and last being further divided into subclasses. Building on this classification and extending it, *Hoffmann* [8.7] produces a 3×5 matrix containing 15 types. Most importantly, he amends Schurz's main categories by a form focusing on *theoric transformations* that generate or select a new system of representation. However, the idea of theoric transformations relates to Peirce's distinction between theorematic and corollarial *deduction*, which raises the question of whether theoric transformations really belong to the realm of *abductive* reasoning (note that Hoffmann discusses Peirce's analysis of Desargues' theorem in [8.8, NEM III/2, 870–871 (1909)]. Here, another Peircean puzzle enters the scene, because he himself has claimed that theorematic deduction "is very plainly allied to retroduction (i. e., abduction, G.M.), from which it only differs as far as I now see in being indisputable" [8.9, MS 754 (1907)].

Thus, while there seem to be many different forms of abduction, it is unclear how many distinctive forms there really are. However, what is much more important is that the scientific community still seems to grapple with the very notion of abduction, that is, what are the central features of abduction as such or of its specific forms. Above, I started citing Peirce with his claim that there be only three basic and distinct kinds of inferences. However, apart from what has already been mentioned above, a persistent problem seems to be to distinguish between abduction and induction, inasmuch as *inference to the best explanation* (henceforth IBE) has to be understood as a form of induction in the Peircean sense. In [8.10], I have tried to disentangle abduction and IBE, and I have not been alone with this view [8.11]. However, *Gabbay* and *Woods* mention *inference-to-the-best-explanation abductions* [8.5, p. 44], and their schema for abduction [8.5, p. 47] seems to capture both abduction and IBE. *Magnani* [8.3, p. 19], [8.4, pp. 18–22] and *Schurz* [8.6, pp. 201–203] equally subsume IBE to abductive reasoning. I reckon that this has to do with similarities between their notion of *selective abduction* on the one hand and IBE on the other.

In my view, Peirce was right to claim that there are but three kinds of reasoning and that there are clear lines of demarcation between them. Accordingly, I think there is reason to *tighten the Peirce-strings* by integrating different forms of abduction (as well as deduction and induction) within a clear and coherent taxonomy. In Sect. 8.1, I will first point out that abduction and IBE are distinct (Sect. 8.1.1), then show how abduction, deduction, and induction hang together to form a productive inferential cycle from a pragmatist point of view (Sect. 8.1.2), and finally explain how this productivity enables us to construct a hierarchy of conceptual levels (Sect. 8.1.3). Within this context, different forms of abductions can be distinguished in terms of the cognitive levels at which they are located, and in terms of whether *new concepts are invented* or *existing ones applied*.

In Sect. 8.2, I explicate the logicality of each of the three inferential types. This is done in two steps. First

(Sect. 8.2.1), the inferences will be analyzed in terms of three characteristic subprocesses that Peirce assumes for inferences in general, that is, (1) *colligation*, (2) *observation*, and (3) *judgment* ([8.12, CP 2.442–244 (c. 1893)], see also *Kapitan* [8.13, p. 479]). Next, the validity of each inference will be discussed in Sect. 8.2.2.

Based on this analysis, Peirce's notion of *theorematic reasoning* is explored in Sect. 8.3. In Sect. 8.3.1, theorematic deduction is explicated as *inverse deduction*, leading from the result of corollarial deduction to the premise of corollarial deduction, that is, the theoretical point of view from which the result can be deduced. An instructive example is given in Sect. 8.3.2, and in Sect. 8.3.3 the idea of inverse inferences is extended to inverse abduction and induction. As a result, we end up with three ordinary and three inverse forms of *pure* inferential types (note that Peirce has also introduced *analogy* as a compound inference conjoining abduction and induction [8.14, CP 1.65]; see also [8.15] on this issue).

In Sect. 8.4, I will discuss three important distinctions among forms of abductive reasoning: *creative* versus *selective* abduction (Sect. 8.4.1), *factual* versus *theoretical* abduction (Sect. 8.4.2), and *explanatory* versus *nonexplanatory* abduction (Sect. 8.4.3). It turns out that abductions (and other inferences) are to be distinguished in terms of knowledge generation (creative) versus knowledge application (selective) and along two cognitive dimensions: one concerns *levels* of abstraction (from elementary embodied and perceptual levels to high-level scientific theorizing). The other concerns *domains* of reasoning such as explanatory, instrumental, and moral reasoning. In the concluding Sect. 8.5, the main results of my analysis are summarized and routes for further research indicated.

Although I consider my argumentation coherent and in line with Peirce, I do not claim to deliver an exegesis of what Peirce himself might have thought, especially since parts of my inferential taxonomy are clearly not contained in his works.

8.1 Abduction in the Overall Inferential Context

8.1.1 Disentangling Abduction and IBE

In a recent overview of Peirce's theory of abduction, Psillos stresses that abduction, deduction, and induction "constitute the three ultimate, basic and independent modes of reasoning. This is a view that runs through the corpus of the Peircean work" [8.16, p. 121]. So, the task of defining and distinguishing these three kinds of inferences might be assumed to be easy. However, reality is different, not least because [8.16, pp. 136–137]

> "[t]he picture of abduction that Peirce has painted is quite complex. On the face of it, there may be a question of its coherence. Abduction is an inference by means of which explanatory hypotheses are admitted, but it is not clear what this admission amounts to."

Why is abduction so hard to grasp? To my mind, the main reason is that it is often confounded with (aspects of) induction or IBE and that Peirce himself has given rise to such confusion. At first, he came up with a syllogistic account of this inferential triad [8.12, CP 2.623 (1878)], but told us later that "in almost everything I printed before the beginning of this century I more or less mixed up Hypothesis and Induction" [8.2, CP 8.221 (1910)]. As already quoted above, however, he also maintained that in some sense the early Peirce's concept of *hypothesis* and the mature Peirce's *abduction* were still equivalent. What's more, the passage quoted in the introduction makes explicit that the purpose of abduction is twofold (1) to generate new hypotheses and (2) to select hypotheses for further examination (see also [8.13, p. 477]; [8.17, p. 503]). The same is true for the following passage [8.18, CP 6.525 (1901)]:

> "The first starting of a hypothesis and the entertaining of it, whether as a simple interrogation or with any degree of confidence, is an inferential step which I propose to call abduction. This will include a preference for any one hypothesis over others which would equally explain the facts, so long as this preference is not based upon any previous knowledge bearing upon the truth of the hypotheses, nor on any testing of any of the hypotheses, after having admitted them on probation. I call all such inference by the peculiar name, abduction, because its legitimacy depends upon altogether different principles from those of other kinds of inference."

Thus, the question remains as to whether abduction is associated with IBE at least in some sense, and if so, whether abduction as such bears this feature or whether there are two different basic kinds of abduction – creative and selective – as *Magnani* [8.3, 4] and *Schurz* [8.6] hold. In the succeeding passages, Peirce writes on the testing of hypotheses and explains his concept of induction, in particular as a means to determine which of a number, or even a multitude, of hypotheses is the best explanation and ought to be adopted as true or likely to be true [8.18, CP 6.526–536 (1901)]. Within this elaboration, he is careful to make sure that the selective aspect of abduction is something different [8.18, CP 6.528 (1901)]:

> "These distinctions (among forms of induction, G.M.) are perfectly clear in principle, which is all that is necessary, although it might sometimes be a nice question to say to which class a given inference belongs. It is to be remarked that, in pure abduction, it can never be justifiable to accept the hypothesis otherwise than as an interrogation. But as long as that condition is observed, no positive falsity is to be feared; and therefore the whole question of what one out of a number of possible hypotheses ought to be entertained becomes purely a question of economy."

Elsewhere, Peirce points out that the abductive selection of hypotheses is usually guided by criteria like simplicity and breadth [8.19, CP 7.220–222 (c. 1901)], and his main idea is that we do not consider just any possible hypothesis, but those hypotheses that make most sense to us from the outset (i. e., those with the highest subjective prior probabilities; see also [8.20]). However, this is no IBE, since it is merely a question of economy, and Peirce clearly states that no sound judgment could be based on this kind of selection, for it is "the most deceptive thing in the world" [8.12, CP 2.101 (1902)]. The further course of inquiry, and whether an originally neglected possibility will have to be taken up at a later stage, depends entirely on induction. For when the selected hypothesis is finally evaluated in the light of empirical data it has to be judged [8.12, CP 2.759 (1905)]:

> "whether the hypothesis should be regarded as proved, or as well on the way toward being proved, or as unworthy of further attention, or whether it ought to receive a definite modification in the light of the new experiments and be inductively reexamined ab ovo, or whether finally, that while not true it probably presents some analogy to the truth, and that the results of the induction may help to suggest a better hypothesis."

If the hypothesis is *regarded as proved*, then this is IBE. However, it could not possibly be regarded as proved if there were yet another hypothesis around that could not be excluded based on the evidence gathered so far. Hence, selection in the context of abduction and selection in the context of induction are quite different. In the former case, its role is merely practical, not logical; that is, one hypothesis is tried first, and if it testifies to be beyond any doubt, other alternatives would not have to be considered anymore (however, this implies that by the same token all possible alternatives must, in fact, be refuted). Or if it is to be conceived as *logical*, then the hypothesis to be rejected at this stage has either to be conceived as abductively (here: explanatorily) invalid (see Sect. 8.2.2), or the rejection has to follow from an *inductive* evaluation of the competing hypotheses. From such an inductive evaluation it might follow that the hypothesis currently countenanced is *well on the way of being proved* in the above-quoted sense, in that it is better than a number of other hypotheses, although further testing or further reflection about novel approaches seems appropriate.

Anyhow, it has to be admitted that Peirce is imprecise in this respect. However, in order not to confuse abductive and inductive logic, I would suggest the rigid interpretation just stated. Moreover, I would like to refer to *Aliseda*, who has pointed out very clearly the difficulties of coming to grips with the selection of a *best* or *preferred* inference as an abductive task [8.21, pp. 72–74], even though she herself endorses hypothesis selection as an abductive task [8.21, p. 33].

On this background, let us now consider *Gabbay* and *Woods*' reconstruction of abductive reasoning [8.5, p. 47]. According to them, it starts with a cognitive target T (e.g., to explain a certain phenomenon) that cannot be met based on the reasoner's background knowledge K, and that the reasoner wants to attain (hence $T!$). R denotes the attainment relation on T, and R^{pres} the presumptive attainment relation on T. If $R(K,T)$ is not possible, the reasoner aims at an enhanced successor knowledge base K^* so that $R(K^*,T)$ holds. H denotes a hypothesis, and $K(H)$ a knowledge base revised by H. Furthermore, there is $C(H)$, which means that it is "justified (or reasonable) to conjecture that H" [8.5]. And finally, "H^c denotes the discharge of H. H is discharged when it is forwarded assertively and labelled in ways that reflect its conjectural origins" [8.5]. Based on these definitions they suggest the following schema [8.5, p. 47]:

1.	$T!$	[declaration of T]
2.	$\neg(R(K,T))$	[fact]
3.	$\neg(R(K^*,T))$	[fact]
4.	$R^{\text{pres}}(K(H),T)$	[fact]
5.	H meets further conditions $S_1,\ldots,\ S_n$	[fact]
6.	Therefore, $C(H)$	[conclusion]
7.	Therefore, H^c	[conclusion]

As I try to explain also in Sect. 8.3, the reach of abduction ought to be limited to steps 1 through 4, with (4) establishing a valid abductive inference, that is, that $R^{\text{pres}}(K(H),T)$ holds. This establishes abductive validity in that H is capable of explaining the surprising facts. And this is precisely what $R^{\text{pres}}(K(H),T)$ captures. Steps 5 through 7 ought, to my mind, be attributed to induction, in this case a rather tentative induction that Peirce has labeled "*abductory induction*" [8.18, CP 6.526 (c. 1901)], because it only qualifies a hypothesis H as better than possible alternative hypotheses, but not in the sense of a full-fledged inductive judgment to the truth of H. Further conditions $S_1, \ldots, S_n$ may exist in the form of background knowledge pertaining to Peirce's criteria of simplicity or breadth (see above) or additional empirical evidence in favor of H. However, these further pieces of information clearly go beyond the abductive task; they may be produced by *deductive* reasoning about what certain hypotheses imply (because how do $S_1, \ldots, S_n$ become conscious?), and are finally considered in *inductive* reasoning. Therefore, I propose to repatriate steps 5 through 7 to the realm of *induction*, and to take very seriously the following statement [8.2, CP 8.218 (c. 1901)]:

> "Nothing has so much contributed to present chaotic or erroneous ideas of the logic of science as failure to distinguish the essentially different characters of different elements of scientific reasoning; and one of the worst of these confusions, as well as one of the commonest, consists in regarding abduction and induction taken together (often mixed also with deduction) as a simple argument. Abduction and induction have, to be sure, this common feature, that both lead to the acceptance of a hypothesis because observed facts are such as would necessarily or probably result as consequences of that hypothesis. But for all that, they are the opposite poles of reason [...]."

Recently, *McKaughan* [8.20], *Campos* [8.22], and *Mackonis* [8.23] have argued in favor of a wide notion of IBE, including abduction, although they endorse the sharp disctinction others and myself have made. However, in the light of the subtle, but nonetheless important, distinctions I have tried to highlight in this section, I think there is not much use fitting it all in one global concept of IBE.

8.1.2 The Dynamical Interaction of Abduction, Deduction, and Induction

By the end of the nineteenth century, Peirce rejected his original syllogistic approach and said "I was too much taken up in considering syllogistic forms [...], which I made more fundamental than they really are" [8.12, CP 2.102 (1902)]. However, even more to the point, Peirce realized that induction "never can originate any idea whatever. Nor can deduction. All the ideas of science come to it by the way of Abduction" [8.1, CP 5.145 (1903)]. The crucial point here is that induction can only operate with concepts that are already at hand. On top of this, even simple regularities like $\forall x(Fx \rightarrow Gx)$ do not suggest themselves, but have to be considered by an active mind, before they can be tested and eventually accepted or rejected (see Sect. 8.1.3, relating to Carnap's disposition predicates). This is why the mature Peirce suggests that abduction is the process by which new concepts, laws, and theories are first conceived, before they are investigated further by deductive and inductive processes [8.1, CP 5.171 (1903)]:

> "Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis. Deduction proves that something *must* be; Induction shows that something *actually is* operative; Abduction merely suggests that something *may be*. Its only justification is that from its suggestion deduction can draw a prediction which can be tested by induction, and that, if we are ever to learn anything or to understand phenomena at all, it must be by abduction that this is to be brought about."

Abduction is most important in our overall reasoning, because without it we could not possibly acquire any idea of the world, not even elementary perceptions of objects, let alone scientific theories. Hence, "no new truth can come from induction or from deduction" [8.2, CP 8.219 (c. 1901)]. Whereas abduction is very powerful in terms of the generation of fruitful new ideas, however, it is very week in terms of empirical validity, as Peirce often stresses. He even says that his discovering the true nature of abduction so late was "owing to the extreme weakness of this kind of inference" [8.12, CP 2.102 (1902)]. Empirical validity is gained by deducing necessary consequences from the abduced hypotheses, especially predictions that can be tested empirically, and the inductive evaluation of the experimental results (or other suitable evidence). Figure 8.1 illustrates the dynamical interaction of the three inferential types.

So far, the role of abduction and deduction seems self-evident. However, an explanation should be given for the role of induction in this triad, in particular why it points back to where abduction starts in Fig. 8.1. After all, induction is typically understood as the inference to the truth (or falsity) of the theory in question, and as a consequence it should point back to the theory itself, like for example in *Magnani*'s ST-Model [8.4, p. 16]; also [8.3, p. 23].

However, induction in the Peircean sense is tied back to his pragmatism, which, again, rests on the logic

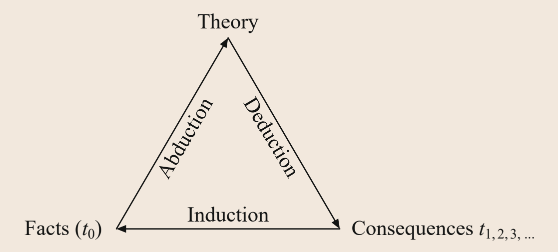

Fig. 8.1 The dynamical interaction of abduction, deduction, and induction

of abduction (cf. also [8.15, pp. 207–212]). *Peirce* [8.1, CP 5.196 (1903)]:

> "That is, pragmatism proposes a certain maxim which, if sound, must render needless any further rule as to the admissibility of hypotheses to rank as hypotheses, that is to say, as explanations of phenomena held as hopeful suggestions."

In other words, all we need in the first place is the confidence that abduction may enable us, by the construction of concepts and hypotheses, to acquire objective knowledge at all. Otherwise, all deductive and inductive examination were futile. However, since abduction is based on experience, induction cannot go beyond that level to infer some kind of "absolute truth" [8.24–27]. Therefore, all that induction can do is to establish habits of expectation and action based on theories.

Peirce [8.1, CP 5.197 (1903)]:

> "What, then, is the end of an explanatory hypothesis? Its end is, through subjection to the test of experiment, to lead to the avoidance of all surprise and to the establishment of a habit of positive expectation that shall not be disappointed."

To be sure, theories in this sense are understood realistically, not as psychological contrivances [8.24, pp. 201–203]. However, finally adopting a theory means to project its content onto all its cases, observed and unobserved, in the past, present, and future. And only in this sense can a theory still be revised or eventually rejected in the future. It is important, therefore, that a circle closes in this very sense of creating habits of expectation and action so that these habits can, in principle, always be broken up again in the future (see also [8.26, pp. 51–57]).

Peirce [8.2, CP 8.270 (1902)] (see also [8.1, CP 5.524 (c. 1905)]):

> "The question is what passes in consciousness, especially what emotional and irritational states of feeling, in the course of forming a new belief. The man has some belief at the outset. This belief is, as to its principal constituent, a habit of expectation. Some experience which this habit leads him to expect turns out differently; and the emotion of *surprise* suddenly appears."

Thus, when an accepted theory is subsequently applied to relevant cases, it is *not only* being applied, but also reassessed over and over. In this very sense, knowledge acquisition and knowledge application are fundamentally tied together and follow the same inferential principles. That is, every application of previously acquired knowledge has to be understood as:

1. Abducing from a certain situational configuration to a suitable interpretation, then
2. Deducing a certain course of action or expectation, and
3. Inducing whether one's actions or expectations were confirmed by experience.

Furthermore, if every application of knowledge constitutes a chance to strengthen or weaken the underlying belief, then by the same token, all failures and situation-specificities in the application of knowledge [8.28] and action-guiding principles, in particular moral principles [8.29, 30], can also be addressed and analyzed within this very frame of reference.

8.1.3 Abduction and Abstraction

As we have seen in the previous sections, the key feature of abduction is to construct new explanatory concepts to accommodate the surprising facts that give rise to them. "By its very definition abduction leads to a hypothesis which is entirely foreign to the data" [8.31, MS 692 (1901)]. Abductive hypotheses are typically of a higher degree of complexity in the sense that phenomena are not just explained by other phenomena that might have brought them about, but by a *theory* of the phenomena. Such theories constitute higher cognitive levels, and this entails that theoretical entities are not observable in the same way as the phenomena they are meant to explain. Of course, *Hanson*'s principle of theory-ladenness of experience [8.32] states that no experience (at least no description of experiences) is theory-free. Nonetheless, there are levels of cognitive architectures building on each other. This has been clear ever since *Carnap* discovered that not even simple disposition predicates could be reduced to observation sentences [8.33, 34].

When *Schurz* [8.6] differentiates between variants like *fact-abduction*, *law-abduction*, or *theoretical abduction*, he also distinguishes such levels, however, without making this aspect of successive theory-building fully explicit. I think it would be a fruitful endeavor to reconstruct how conceptual (or theoretical) levels are built onto one another by successive abductions. For instance, when a simple disposition is discovered (that sugar dissolves in water), this constitutes an empirical law, which is itself a concept of a regularity in nature. It can also be used to explain why someone else does not see the sugar in her drink. We could say that it is very well in there, but cannot be seen, because it has dissolved.

What this simple example shows is that even in simple fact-abduction, we do not just infer to the fact, but to the law from which it then follows that sugar might be in a liquid, even if no sugar can be seen. Thus, dispositional laws and simple action schemes (*When the switch is pushed, the light will go on*) constitute an elementary theory level, that is, regularities in terms of observation language. However, these regularities are themselves phenomena that one may wish to explain, especially when one starts wondering, *how* the switch is causally connected with the light. At first, it was established *that* a natural regularity exists. Now, as the regularity is established as a matter of fact, it becomes the object of theoretical reflection and represents the fact to be explained.

This is what Hintikka highlights when he discusses the difference between abduction and IBE. He says that [8.32, p. 509]:

> "when a dependence law telling us how the observed variable depends on the controlled one the law does not *explain* the result of the experiment. It is the *result* of the experiment, nature's answer to the experimental investigator's question."

Earnan McMullin has made the same point concerning the role of laws in explanation: "Laws are the explanada; they are the questions, not the answers" [8.35, p. 90]. And he continues [8.35, p. 91]:

> "To explain a law, one does not simply have recourse to a higher law from which the original law can be deduced. One calls instead upon a *theory*, using this term in a specific and restricted sense. Taking the observed regularity as effect, one seeks by abduction a causal hypothesis which will explain the regularity. To explain why a particular sort of thing acts in a particular way, one postulates an underlying structure of entities, processes, relationships, which would account for such a regularity. What is ampliative about this, what enables one to speak of this as a strong form of understanding, is that if successful, it opens up a domain that was previously unknown, or less known."

I consider this a strong and important point (see also [8.36, 37] on explanatory hierarchies and explanatory coherence). Laws, in this view, are not the solutions (the explanations) but the problems (the facts to be explained). However, I would not go so far as to deny laws, even simple empirical laws, any explanatory function. It just depends on the point of view and the theoretical level, which is needed and appropriate to solve a particular explanatory problem. If one is looking for causal relationships between events, one is in fact searching for law-like explanations. And this not only applies to children in their early cognitive development. Most adults are content with knowing what keys to press in order to use certain functions of a software; in such cases the question is how a certain result is brought about, and the explanation consists in functional relations between the keys or menu options and the results visible on the screen. The same applies to cookbooks and manuals for technical appliances in which it is explained, how things work or why something I tried did not work. I assume that, for example, *Schurz*'s account of explanation as unification applies not only to scientific theories, but also to such simple forms of explanation [8.10, 38].

Thus, there seems to be an order of theory-levels or levels of abstraction, where the higher ones explain the lower ones, and where abduction is the process that takes the reasoner from a lower level to a higher one. Such a hierarchy of levels may also be the clue to understanding how (intuitive) cognition works below explicit sentential reasoning and how the latter comes about in ontogenetic development.

Peirce famously argued that perceptual judgments are no abductions. However, he seems to have been too strict or narrow-minded in this context (see also [8.3, pp. 42–43], [8.4, pp. 268–276]). While he clearly admits that perceptual judgment "is plainly nothing but the extremest case of Abductive Judgements" [8.1, CP 5.185 (1903)] and that "abductive inference shades into perceptual judgment without any sharp line of demarcation between them" [8.1, CP 5.181 (1903)], he maintains that they are nonetheless distinct, because unlike abductive inferences, perceptual judgments were "absolutely beyond criticism" [8.1, CP 5.181 (1903)]. Peirce points out repeatedly that abduction as an inference requires control and that this misses in perceptual judgment [8.1, CP 5.157, 181, 183, 194 (1903)]. He therefore holds that perceptual judgment is the "starting point or first premiss of all critical and controlled thinking" [8.1, CP 5.181 (1903)], hence something on which abduction is based, but which does not belong to abduction itself.

However, Peirce fails to consider two aspects of perceptual judgments: first, they might be conceivable as established facts in the sense of a complete triad of abduction, deduction, and induction. This would explain why we are (normally) certain of our perceptions. Second, and more importantly, Peirce fails to consider that abductions are well-controlled, not by conscious thought, but by action. Perceptions can be understood as habits of action, that is, of categorization and behavior in accordance with what we perceive. And finally, the abductive or conjectural part of this process is that with every new perception the individual literally *makes sense* of what enters into the sensory system. Sometimes these *creations* are fallacious or even foolish, but this puts them fully in line with abduction in general.

At least, this is what I suggest at this point, and it would certainly have to be examined in more detail. However, *Magnani*'s [8.4] and *Park*'s [8.39] reflections on animal, visual, and manipulative abduction point in the very same direction and could be accommodated as basic forms, not only of abduction, but also of inferential functioning in general. Moreover, reconstructing *the formation* of perceptual judgments in this way makes the pragmatist epistemology even more stronger, because there is no specific level of consciousness, where epistemic processes start (see [8.40] for a discussion of this link between abduction and pragmatism). As for perceptual judgments, they are at this basic level not controlled by conscious reflection, but nonetheless controlled in an embodied and enacted manner [8.41]. Epistemology, then, would rather be embedded in life and would not have to resort to any a priori forms of cognition whatsoever. It also constitutes, to my mind, a sound basis for nonreliabilist epistemological externalism.

Without being able to explain my main point here in detail, I just assume that there are levels over levels of understanding and interaction with one's environment. And even though Peirce has never developed a theory of successive abstraction in this overarching sense, he had a clear idea of the basic principle, which he calls *hypostatic abstraction*. To explain his ideas, Peirce relates to Molière's *Malade imaginaire*, where a medical student takes his oral examination in the last scene. He is asked why opium puts people to sleep and he confidently replies that opium had a dormitive virtue whose nature was to lull the senses to sleep [8.1, CP 5.534 (c. 1905)]:

> "Quia est in eo
> Virtus dormitiva,
> Cujus est natura
> Sensus assoupire.
>
> Whereupon the chorus bursts out,
>
> Bene, bene, bene, bene, respondere:
> Dignus, dignus est entrare,
> In nostro docto corpore."

Peirce explains [8.1, CP 5.534 (c. 1905)]:

> "Even in this burlesque instance, this operation of hypostatic abstraction is not quite utterly futile. For it does say that there is some peculiarity in the opium to which the sleep must be due; and this is not suggested in merely saying that opium puts people to sleep."

Elsewhere, he discusses the same idea, but speaks of *subjectal abstraction* as opposed to *precisive abstraction* (see also [8.42]).

Peirce [8.8, NEM III/2, p. 917 (1904)]:

> "There are two entirely different things that are often confused from no cause that I can see except that the words *abstract* and *abstraction* are applied to both. One is $\alpha\phi\alpha\iota\rho\varepsilon\sigma\iota\varsigma$ leaving something out of account in order to attend to something else. That is *precisive* abstraction. The other consists in making a subject out of a predicate. Instead of saying, Opium puts people to sleep, you say it has dormitive virtue. This is an important proceeding in mathematics. For example, take all *symbolic* methods, in which operations are operated upon. This may be called *subjectal abstraction*."

By *subjectal abstraction* Peirce means that "a transitive element of thought is made substantive, as in the grammatical change of an adjective into an abstract noun" [8.12, CP 2.364 (1901)]. Even though *dormitive virtue* does not explain why opium puts people to sleep, it states an explanatory problem in the sense that a general law (with opium as cause and putting people to sleep as effect) has to be explained at a higher, more abstract level. In this very sense, *dormitve virtue* goes beyond stating a mere disposition of opium (see also *Schurz*'s discussion on this issue [8.6, pp. 219–221]).

Forms of abductive reasoning could, therefore, be distinguished according to levels of abstraction in the sense in which Jean Piaget discusses constructive development and cognitive architectures [8.43]. Among the forms that *Schurz* [8.6] differentiates, some are at the same level, while others belong to different levels. *Factual abduction* and *law abduction* all concern simple empirical laws, the latter establishing them, the former applying them. Higher level abduction is what he calls *theoretical-model abduction*: "The explanandum of a theoretical-model abduction is typically a well-confirmed and reproducible empirical phenomenon expressed by an *empirical law*" [8.6, p. 213].

However, the distinction between empirical laws, on the one hand, and theoretical models, on the other hand, seems to be still rather crude (again, if this is compared to the fine-grained, but highly systematic, distinctions made by *Piaget* and *Garcia* [8.43]). To date, research on cognitive architectures has primarily focused on the lower end of the cognitive hierarchy, i. e., how relative simple conceptual and action schemata are built and grounded in the brain's modal systems for perception, emotions, and actions [8.41, 44, 45]. However, since abduction is the process that leads to successively more abstract cognitions in the sense of hierarchical complexity, there is a promising route for further research and a systematic differentiation of types of abductions according to the cognitive levels, to which they apply (as for moral cognition see [8.46] as an example).

8.2 The Logicality of Abduction, Deduction, and Induction

8.2.1 Inferential Subprocesses and Abduction as Inferential Reasoning

As already mentioned above, Peirce regarded abduction as an extremely weak kind of inference. This raises the question of whether it is an inference at all. On top of this, he says that abduction is "nothing but guessing" [8.19, CP 7.219 (1901)] and its results merely "the spontaneous conjectures of instinctive reason" [8.18, CP 6.475 (1908)]. However, abduction is also said to "cover all the operations by which theories and conceptions are engendered" [8.1, CP 5.590 (1903)], and since it takes us to novel concepts and theories, he cannot mean *guesses* in the ordinary sense of picking out something at random from a range of existing objects of choice. However, the question remains whether abduction is an inference or merely an instinct. In a way, both seems to be true [8.47], but for the present purpose it suffices to stress that abduction has an inferential aspect [8.32, 47, 48]. So, let us try to track this inferential aspect of abduction.

In this respect it may be instructive to consider Peirce's thoughts on inference in general. On his view, all inferences are mental acts of reasoning and as such describe a process with a definite beginning and a definite end. Any inference begins with a question that requires an answer in the form of the respective conclusion. Abduction aims at possible explanations, deduction at necessary consequences following from certain premises, and induction aims at determining whether to accept or reject a hypothesis. Whatever the inference, however, the process of answering these questions contains three distinctive steps, which Peirce calls *colligation*, *observation*, and *judgment*.

Peirce [8.12, CP 2.442 (c. 1893)]:

> "The first step of inference usually consists in bringing together certain propositions which we believe to be true, but which, supposing the inference to be a new one, we have hitherto not considered together, or not as united in the same way. This step is called *colligation*."

Peirce [8.12, CP 2.443–444 (c. 1893)]:

> "The next step of inference to be considered consists in the contemplation of that complex icon ... so as to produce a new icon. [...] It thus appears that all knowledge comes to us by observation. A part is forced upon us from without and seems to result from Nature's mind; a part comes from the depths of the mind as seen from within [...]."

Peirce [8.12, CP 2.444]:

> "A few mental experiments – or even a single one [...] – satisfy the mind that the one icon would at all times involve the other, that is, suggest it in a special way [...] Hence the mind is not only led from believing the premiss to judge the conclusion true, but it further attaches to this judgment another – that *every* proposition *like* the premiss, that is having an icon like it, *would* involve, and compel acceptance of, a proposition related to it as the conclusion then drawn is related to that premiss."

He concludes that "[t]he three steps of inference are, then, colligation, observation, and the judgment that what we observe in the colligated data follows a rule" [8.12, CP 2.444]. The step of colligation is consistently used and explained and thus seems to be rather clear [8.1, CP 5.163 (1903)], [8.1, CP 5.579 (1898)]. However, Peirce is less precise about the other two. In particular, his differentiation, in this context, between a *plan* and the *steps* of reasoning may cause some confusion [8.1, CP 5.158–166 (1903)]. As for the *plan* he says that [8.1, CP 5.162 (1903)]:

> "we construct an icon of our hypothetical state of things and proceed to observe it. This observation leads us to suspect that something is true, which we may or may not be able to formulate with precision, and we proceed to inquire whether it is true or not."

Thus, we observe what is colligated in the premise in order to produce a result. Even though this observation may be guided by strategies and other background knowledge the result will first come about in a spontaneous act as the reasoner becomes conscious of it.

When discussing observation in the context of abduction, he goes on to a general description of observation that brings out this main feature very plainly [8.1, CP 5.581 (1898)]:

> "And then comes an Observation. Not, however, an External observation of the objects as in Induction, nor yet an observation made upon the parts of a diagram, as in Deduction; but for all that just as truly an observation. For what is observation? What is experience? It is the enforced element in the history of our lives. It is that which we are constrained to be conscious of by an occult force residing in an object which we contemplate. The act of observation is the deliberate yielding of ourselves to that *force majeure* – an early surrender at discretion, due to our foreseeing that we must, whatever we do, be borne down by that power, at last."

Thus, the observed result is forced upon us in a rather uncontrolled manner. We just see it and can't help seeing it. However, in order to come to a conclusion as the last step of inference, we have to evaluate whether the result is valid in terms of the respective inference. This constitutes the judgmental step that finalizes each inference (see [8.49] for this matter).

8.2.2 The Validity of Abduction, Deduction, and Induction

Peirce is explicit concerning the validity of an abductive judgment [8.1, CP 5.197 (1903)]:

> "What is good abduction? What should an explanatory hypothesis be to be worthy to rank as a hypothesis? Of course, it must explain the facts. But what other conditions ought it to fulfill to be good? The question of the goodness of anything is whether that thing fulfills its end. What, then, is the end of an explanatory hypothesis? Its end is, through subjection to the test of experiment, to lead to the avoidance of all surprise and to the establishment of a habit of positive expectation that shall not be disappointed. Any hypothesis, therefore, may be admissible, in the absence of any special reasons to the contrary, provided it be capable of experimental verification, and only insofar as it is capable of such verification. This is approximately the doctrine of pragmatism."

Valid *abduction* thus has to satisfy two criteria:

1. It has to *explain the facts*, meaning that the initial surprise be eliminated, and
2. The explanation has to be *capable of experimental verification* in the pragmatist sense.

Any abductively observed result that does not meet these criteria will have to be rejected. If the criteria are met, however, the hypothesis will have to be accepted as a valid abductive conclusion (see also my reflections in Sect. 8.1.1). From this point of view, we can now understand Peirce's famous statement of the abductive inference [8.1, CP 5.189 (1903)]:

> "The surprising fact, C, is observed;
> But if A were true, C would be a matter of course,
> Hence, there is reason to suspect that A is true."

This only relates the *final* subprocess of abduction, the judgmental part. However, it is not to be confused with abduction as an inferential cognitive process as a whole [8.50]. *Kapitan* has famously criticized Peirce's concept of abduction, claiming that it was essentially a deductive argument [8.51]. However, he fails to see that the above statement does not describe the entire process of abductive reasoning. And, as far as this *deductive* aspect of the abductive judgment is concerned, Peirce himself expresses this clearly [8.1, CP 5.146 (1903)], [8.40, p. 168]. However, this does not turn the abductive judgment into a deductive inference, because it is not the primary task to derive C, since C is already known and constitutes the premise, whereas A constitutes the conclusion.

Kapitan's later account of abduction [8.51] is largely adequate, but still there is one widespread problem. Like many others, he conflates the abductive judgment with the selective aspect that I have argued above (Sect. 8.1.1) should be excluded. *Hintikka* [8.52, pp. 44–52] grapples with the notion of abductive inference for just the same reason. In my view, this makes it all the more important to drive a deep wedge between accommodating the facts as *the* abductive task, and evaluating abductively valid hypotheses as an inductive task.

The validity of *deduction* seems to be unproblematic. So, let us move straight to *induction*. It has already been pointed out above that induction is the inference that yields factual knowledge, constituting factual truth. However, what is the precise relation between knowledge and truth? The classical notion of knowledge as *justified true belief* requires that a proposition be true in order to be known. However, a main theorem from the point of view of pragmatism is that knowledge is logically prior, that is, knowledge establishes truth rather than requiring it as a condition. After all, this is the basic idea of disquotationalism [8.26, pp. 57–64].

As regards the validity of induction, I adopt an analysis of knowledge and its formation proposed by *Suppe* [8.53] within the framework of a possible-worlds semantics. He suggests a nonreliabilistic externalist approach to knowledge. On this view, we know *p* if it is not causally possible that we perceive the evidence as it is unless the suggested hypothesis is true. This is indicated by a causal possibility operator $\Diamond_{\mathrm{c}}$, where *causal possibility* refers to all logically possible worlds that are consistent with the natural laws of our world (i. e., our current background knowledge regarding natural laws). According to Suppe's approach, the truth of a proposition results from knowing it, and knowing results from the condition stated in (iv), below, being satisfied. Furthermore, "satisfying (iv) entails the satisfaction of condition (iii)" [8.53, p. 402], since R and/or K function as decisive indicators for Φ.

S propositionally knows that θ if and only if

(i) S undergoes a cognitive process R, or S has prior knowledge that K
(ii) S, knowing how to use Φ and knowing how to use θ with the same propositional intent, as a result of undergoing R or having prior knowledge that K entertains the proposition Φ with that propositional intent as being factually true or false
(iii) Φ is factually true
(iv) there exists a conjunction C of partial world state descriptions and probability spaces such that C & $\sim\Diamond_{\mathrm{c}}^{\Phi}\,(C\,\&\,R\,\&\,K\,\&\sim\Phi)\,\&\,\Diamond_{\mathrm{c}}^{\Phi}\,(C\,\&\sim\Phi)\,\&\,\Diamond_{\mathrm{c}}\,R\,\&\,\Diamond\,(R\,\&\sim\Phi)$
(v) as a result of undergoing R or K, S believes that Φ [8.53, p. 405].

As a result, induction can be conceived in terms of an elaborate eliminative inductivism in the sense of *Earman* [8.54]. A theory is to be adopted, if all that has been observed so far supports it and that no alternative hypothesis is conceivable (at the current state of knowledge).

The results of my analysis are condensed in Fig. 8.2 (which is reduced to the essential features). Note that the diagram shows steps in the inferential processes. They are not to be misread as syllogisms.

	Abduction	Deduction	Induction
Colligation	C	$H \wedge P$	$\Box((H \wedge P) \rightarrow E) \wedge E$
Observation	$H \rightarrow \Diamond_{\mathrm{c}} C$	$(H \wedge P) \rightarrow E$	$E \wedge \neg \Diamond_{\mathrm{c}}(E \wedge \neg H)$
Judgement	$\Diamond_{\mathrm{c}} H$	$\Box((H \wedge P) \rightarrow E)$	$\Box_{\mathrm{c}} H$

Fig. 8.2 A formal model of inferential (sub)processes

In abduction we colligate the relevant aspects of a given explanatory problem, that is, what happens to be the case, hence C. C is observed with the intention to find a theoretical hypothesis H that would, if true, explain C, that is, render the previously surprising phenomenon causally possible. As one hits on an idea, one has to make sure that H would really explain C. This is the abductive judgment $\Diamond_{\mathrm{c}} H$.

This result gained from abduction is then used as input for the following deduction, together with suitable premises P available from background knowledge. These are observed so as to generate necessary consequences, in particular empirical hypotheses E. The judgment $\Box((H \wedge P) \rightarrow E)$ states that E follows with necessity from $(H \wedge P)$.

Again, the deductive conclusion is input into induction, where it is colligated with the actual experiment, which are then observed. This observation is more than just recording what happens; in fact, such recording would have to be understood as the main part of the inductive colligation. Observation in the context of induction means to look at these results (maybe at the time when they are actually produced) under the aspect of whether they confirm the tested hypothesis and disconfirm its rivals. If the final outcome is positive, H is accepted as causally necessary, hence $\Box_{\mathrm{c}} H$.

8.3 Inverse Inferences

8.3.1 Theorematic Deduction as Inverse Deduction

Writing about mathematical reasoning, Peirce says: "My first real discovery about mathematical procedure was that there are two kinds of necessary reasoning, which I call the Corollarial and the Theorematic" [8.55, NEM IV, p. 49 (1902)]. However, scholars disagree on what is the content of this discovery. For *Hintikka*, "a valid deductive step is theorematic, if it increases the number of layers of quantifiers in the proposition" [8.56, p. 307]. *Ketner* claims that Hintikka fails to see "the true importance of Peirce's corollarial/theorematic reasoning distinction" [8.57, p. 409], which, according to Ketner, is that "it makes significant contributions toward showing that mathematics and logic are observational, experimental, hypothesis-confirming sciences" [8.57, p. 409]. *Ketner* also maintains that the "production of experiments within theorematic reasoning, on Peirce's view, is done through abduction" [8.57, p. 411]. Referring to this argument, *Hoffmann* says he had "spent some effort to find in Peirce's writings hints at such a connection between abduction and theorematic reasoning, but without much success" [8.48, p. 293]. However, *Hoffmann* acknowledges and discusses obvious similarities between theorematic deduction and abduction and comes to the following result: "It is one thing *to prove* a theorem and another to *formulate* it" and continues that "it would make sense to describe the first task as theorematic deduction and the second task as abduction" [8.48, p. 294].

In view of this puzzlement concerning the proper understanding of theorematic deduction and its relation to abduction, my suggestion is not to subdivide theorematic deduction into abductive and deductive aspects, but to reconstruct theorematic deduction as a form of reasoning of its own, albeit similar to abduction in an important respect. As for the similarity between abduction and theorematic deduction, Peirce himself remarks that "[i]t is very plainly allied to retroduction, from which it only differs as far as I now see in being indisputable" [MS 754 (1907), quoted from [8.48, p. 293]]. And the commonality apparently lies in the creative act of introducing a new idea not present in the premises from which one starts [8.58, p. 97].

Peirce [8.55, NEM IV, 42 (1902)]:

> "What I call the theorematic reasoning of mathematics consists in so introducing a foreign idea, using it, and finally deducing a conclusion from which it is eliminated. Every such proof rests, however, upon judgments in which the foreign idea is first introduced, and which are simply self-evident. As such, they are exempt from criticism."

Peirce [8.55, NEM IV, 49 (1902)]:

> "The peculiarity of theorematic reasoning is that it considers something not implied at all in the conceptions so far gained, which neither the definition of the object of research nor anything yet known about could of themselves suggest, although they give room for it."

Again, the *foreign idea* is what alludes to abduction, and once it is gained, *self-evident* judgments can be taken in order to prove a theorem. Elsewhere in the same text, Peirce has made clear that these self-evident judgments are, in fact, corollarial deductions [8.55, NEM IV, 38 (1902)]:

> "*Theorematic deduction* is deduction in which it is necessary to experiment in the imagination upon the image of the premiss in order from the result of such experiment to make corollarial deductions to the truth of the conclusion."

All these explanations by Peirce can be accommodated, if theorematic abduction is conceived of as an inverse deduction that infers from the result of corollarial deduction to the premises from which the result can be deductively derived. The similarity with abduction results from the fact that theorematic deduction takes the reasoner to a theoretical point of view, which is the point in the above diagram on inferential reasoning (Fig. 8.1) where abduction would take her. Thus, abduction and theorematic deduction both aim at the same point (Fig. 8.3).

Within this frame of reference, it also becomes clear why Peirce thinks that theorematic deduction is *ampliative*. He just did not call it *ampliative deduction*, because he feared that this labeling would have been considered as unacceptable [8.55, NEM IV, 1 (1901)]:

> "It now appears that there are two kinds of deductive reasoning, which might, perhaps, be called *explicatory* and *ampliative*. However, the latter term might be misunderstood; for no mathematical reasoning is what would be commonly understood by *ampliative*, although much of it is not what is commonly understood as *explicative*. It is better to resort to new words to express new ideas. All readers of mathematics must have felt the great difference between *corollaries* and *major theorems*."

The overall process of theorematic deduction can then be analyzed based on the three inferential subprocesses discussed in Sect. 8.2.1, only that the process runs in the inverse direction, starting from a proposition to be proved, say whether p or $\neg p$ is logically true (colligation). This is the premise of theorematic deduction, which is then observed in order to find a conceptual point of view from where to derive either p or $\neg p$ (observation). Once a candidate for this is found, it has to be established by a corollarial deduction to p or $\neg p$, which is equivalent to *judgment* in the context of theorematic deduction. This is how I understand Peirce when he says [8.55, NEM IV, p. 38 (1902)] (see also [8.12, CP 2.267 (c. 1903)]; [8.59, CP 4.233 (1902)]):

> "*Theorematic deduction* is deduction in which it is necessary to experiment in the imagination upon the

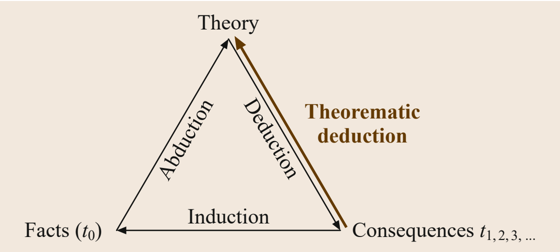

Fig. 8.3 Theorematic deduction in relation to the inferential triad

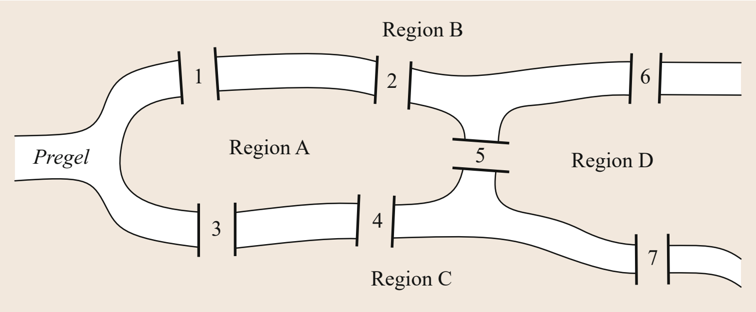

Fig. 8.4 The seven bridges of Königsberg

image of the premiss in order from the result of such experiment to make corollarial deductions to the truth of the conclusion."

Hoffmann, too, stresses that theorematic reasoning (he uses the notion of *theoric transformations*) essentially consists in "looking at facts from a novel point of view" – a phrase taken from [8.60, MS 318] ([8.7, p. 581], [8.48, p. 291], [8.4, p. 181]). And the fact that attaining this novel point of view is first of all the result of *observation* that subsequently has to be subjected to a corollarial deduction as judgment within theorematic deduction may also explain the following passage.

Peirce [8.61, NEM III, p. 869 (1909)]:

"To the Diagram of the truth of the Premisses something else has to be added, which is usually a mere May-be, and then the conclusion appears. I call this *Theorematic* reasoning because all the most important theorems are of this nature."

Ketner [8.57, p. 408] refers to this passage to underpin his view that theorematic deduction is a kind of abduction. However, on my account theorematic deduction is *a May-be*, firstly, in the sense of introducing a theoretical point of view, and secondly, because it is spontaneously generated by *observation* and still has to be submitted to *judgment*. This is my reconstruction of theorematic deduction as inverse deduction. Further refinements might be necessary, in particular analyzing the variants that *Levy* discusses in [8.58, pp. 98–103]. However, this must be left to a separate analysis. Here, I prefer to provide an instructive example and extend the idea of inverse inferences to include inverse abduction and inverse induction.

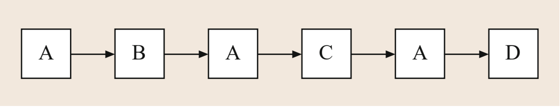

Fig. 8.5 Graph of the state sequence in seven bridges problem

8.3.2 An Example for Theorematic Deduction

In addition to Peirce's examples like Desargues' theorem (discussed in [8.7, pp. 581–584]), I suggest Leonhard Euler's solution of the Königsberg bridge problem as a case in point. In Euler's time, the river Pregel formed the topological shape shown in Fig. 8.4. The question is whether it is possible to pass all seven bridges on a walk while passing each bridge only once.

To solve this problem, Euler used a graph in which the state sequence is shown as transitions from region to region. Figure 8.5 shows how this looks like, if one starts in region A and passes the first five bridges in numerical order. Accordingly, the number of regions in this diagram will always be $N+1$, where N is the number of all bridges. Moreover, with the five bridges connected to region A, this region is mentioned three times. A so-called uneven region, that is, one with an uneven number of bridges, will always appear $(n+1)/2$ times in the graph, independently from whether one starts in this very region or in another region. This is different for even regions. If we only consider regions A and B, there are only two bridges. If one starts in A, A is mentioned twice and B only once. If one starts in B, it is the other way round. In general, the region is mentioned $n/2$ times if one starts outside this region, and $n/2+1$ times if one starts from within.

However, all regions in the seven bridges problem are uneven so that the solution is rather simple. A walk on which one passes each bridge only once encompasses seven transitions between eight states. However, each region must appear $(n+1)/2$ times in the diagram, which means three times for region A and two for regions B through D, that is, nine altogether. Hence, the desired walk is impossible.

This example shows that from an abstract topological point of view it is possible to formulate principles from which the impossibility of the specified walk can be deduced. The diagram in Fig. 8.4, together with the question, represents the *colligated* premise, which is then observed. The result of this *observation* is the approach represented in the graph in Fig. 8.5 and the further reflections based on it. The final *judgment* consists in deriving the solution, that is, the proof as such.

8.3.3 Inverse Abduction and Inverse Induction

Based on the reconstruction of theorematic deduction as an inverse deduction, it follows naturally that there could be two other forms of reasoning: inverse abduction and inverse induction (Fig. 8.6). Moreover, since inverse (theorematic) deduction is similar to abduction in that it aims at the same point in Fig. 8.6, inverse abduction should be similar to induction, and inverse induction should be similar to deduction.

Inverse abduction starts from some theory or abstract concept and searches, for examples, possible instantiations. For instance, the economist *Nicholas Kaldor* [8.62] suggested the cobweb model, which explains how supply (S) and demand (D) develop if time lags are assumed for the reaction of the supply side to a change in demand and vice versa (see Fig. 8.7). If the supply curve is steeper than the demand curve, prices (P) and quantities (Q) will gradually converge to the equilibrium. However, if the slope is the same, supply and demand will fluctuate cyclically.

If it is asked what would be a case in point of such a persistently fluctuating supply and demand, this would require what I call *inverse abduction*. The theoretical model has to be understood on the abstract level, but it is unclear whether there is a concrete case at all to it. An example would be the so-called *pork cycle* that was observed in the 1920s in the United States and in Europe. Kaldor's theoretical model provides a possible explanation for such phenomena, but in this case the argument runs in the opposite direction, from the theory to the case. The similarity to induction consists in the fact that inverse abduction projects a possible explanation onto a case (and all other relevant cases one might think of or encounter). The difference is that it is only a provisional projection (if the theory is true), whereas in induction factual truth in the pragmatist sense is established.

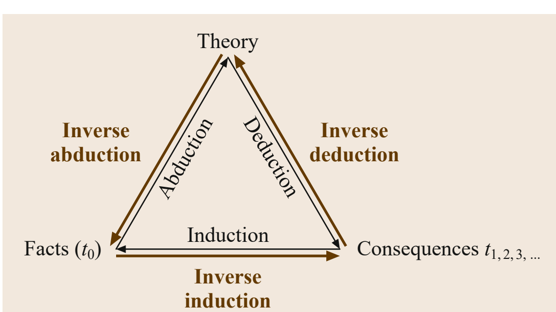

Fig. 8.6 Inverse inferences

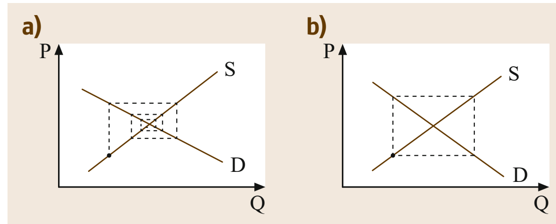

Fig. 8.7a,b The cobweb model: **(a)** successive adjustment of supply and demand; **(b)** cyclical fluctuations of supply and demand

The difference can also be stated in this way: inverse abduction starts from the *colligation* of a theoretical model of some sort, which has a meaning but as yet no reference. This theoretical model is *observed* in order to be able to project it onto some case to which it refers (here is the similarity to induction). Finally, it has to be *judged* (abductively) whether the case that suggested itself can really be subsumed to the theoretical model.

Other examples for inverse abduction are riddles that we give children to solve, once they can use concepts independently from concrete references, for example, *What has a face and two hands, but no arms or legs? A clock*. The task is to find something concrete that satisfies this abstract definition. Again, the definition is first colligated, then observed in order to project it onto some concrete object, and the final part consists in the judgment as to whether the definition really applies to the object and whether this inference is thus abductively valid (in this case, as a *possible* circumscription of a clock).

Turning to *inverse induction*, this inference starts from the purported truth (or falsity) of a theory and tries to infer back to a crucial experiment that determines whether the theory would have to be accepted or rejected. This form of reasoning typically applies when two competing approaches stand against each other, in particular when both are well-confirmed by ordinary induction, but are mutually incompatible. One famous example are Bell's inequalities based on the Einstein–Podolsky–Rosen (EPR) Gedankenexperiment. The EPR assumptions entail the fulfilment of Bell's inequalities, quantum theory entails their violation. Hence, the inequalities did not prove anything in themselves, but were the basis for a decisive empirical test, eventually carried out by Alain Aspect, which established the validity of quantum theory and refuted Einstein in his attempt to save classical physics.

Inverse induction is similar to deduction because it essentially involves deductive steps to derive the decisive experimental conditions. However, as opposed to Peirce's theorematic deduction, it does not prove anything, and unlike corollarial deduction it does not just derive what follows from a certain theory, but starts from competing theories and the intention to determine which one is true and which one false. From this *colligated* premise, the theories are *observed* in order to find the decisive experimental conditions, and the final *judgment* does not concern the deductive validity but whether the test would really be decisive.

Another, much simpler, example is the so-called *Wason* selection task [8.63], one of the most investigated experimental paradigms. This task consists in determining which two of four cards one has to turn over in order to know whether a certain rule is true or false. There are cards with a yellow or a red back, and on their front-sides they have a number, even or odd. Now, there are four cards showing (1) a *3*, (2) an *8*, (3) a red back, and (4) a yellow back. The rule says that if a card shows an even number on one face, then its opposite face is red. So, which two cards have to be turned over to see whether this rule is violated? The solution is the two cards showing the *8* and the yellow back.

Not even 10% get this right (at least in this rather formal context). The reason may be that they fail to see that they need to use modus ponens (*even number* → *red back*) and modus tollens (¬ (*red back*) → ¬ (*even number*). However, the task is not just to use modus ponens and modus tollens correctly, and therefore it is not just about deduction, as is usually thought. Rather is it the most important part of this reasoning task to *find out* (through *observation*) that these deductive rules allow you to determine which two cards have to be turned over. Moreover, strictly speaking, there are also two competing hypotheses involved: the rule and its negation.

To sum up, all inverse inferences contain elements of its predecessor in the ordinary order, and these elements are important in the *observational* subprocesses. Here, inverse abduction relates to induction, inverse deduction to abduction, and inverse induction to deduction. However, the final judgments are abductive in inverse abduction, deductive in inverse deduction, and inductive in inverse induction.

8.4 Discussion of Two Important Distinctions Between Types of Abduction

8.4.1 Creative Versus Selective Abduction

In this section, some of the important and/or controversial distinctions between abductions shall be discussed starting with the one between creative and selective abduction (see also [8.64]). Most scholars endorse the view that abduction has to fulfil these two purposes, but *Magnani* [8.3, 4] and *Schurz* [8.6], in particular, discuss them even as separate forms of abduction. However, I oppose this view and think that (1) there is no separate form of abduction that is selective, and (2) that the ways in which abduction might be rightly called *selective* are specific forms of one and the same basic form of abduction. As I have pointed out already in Sects. 8.1.1 and 8.2.1 all other candidates for selective abduction would have to be reinterpreted as forms of induction.

Selective abduction is an elusive concept, not only because it is easily confused with IBE, but also because different authors understand it in different ways. In particular, I see a difference in the usage by Magnani and Schurz, on the one hand, and by Kapitan and Hintikka on the other hand. As for the latter, Hintikka [8.17, p. 503] cites *Kapitan*, who claims that "[t]he purpose of *scientific* abduction is both (i) to generate new hypotheses and (ii) to select hypotheses for further examination" [8.13, p. 477]. They state this with particular reference to [8.18, CP 6.525], where *creation* and *selection* do not appear as two different kinds of abduction, but as two aspects of one and the same notion of abduction [8.18, CP 6.525 (c. 1901)]:

> "The first starting of a hypothesis and the entertaining of it, whether as a simple interrogation or with any degree of confidence, is an inferential step which I propose to call *abduction*. This will include a preference for any one hypothesis over others which would equally explain the facts, so long as this preference is not based upon any previous knowledge bearing upon the truth of the hypotheses, nor on any testing of any of the hypotheses, after having admitted them on probation."

The last part of the passage ensures that the selective aspect is not confused with induction. Furthermore, Peirce makes clear that selection does not mean separating stupid ideas from sensible ones, because they all have to *explain the facts*, that is, have to be valid abductions. Thus, selection does not refer to the abductive judgment by which stupid ideas are sorted out. The hypotheses among which to select have already passed this test. However, elsewhere Peirce makes clear that "the whole question of what one out of a number of possible hypotheses ought to be entertained becomes purely a question of economy" [8.18, CP 6.528 (1901)]. Hence, this aspect of selection concerns abduction only from a practical point of view, not from a logical one, as I have argued above in Sect. 8.1.1.

Turning to *Magnani* and *Schurz*, the latter writes [8.6, p. 202]:

> "Following Magnani (2001, p. 20) I call abductions which introduce new concepts or models *creative*, in contrast to *selective* abductions whose task is to choose the best candidate among a given multitude of possible explanations."

This sounds as if IBE were included in this notion of selective abduction. However, *Magnani* is careful to distinguish between these, when he discusses what he calls the "two main epistemological meanings of the word abduction" [8.4, p. 10], that is, creative *and* selective abduction on the one hand, and IBE on the other hand. Then, he goes on to differentiate between creative and selective abduction [8.4, p. 10]; see also [8.3, p. 19]:

> "An illustration from the field of medical knowledge is represented by the discovery of a new disease and the manifestations of causes which can be considered as the result of a creative abductive inference. Therefore, *creative* abduction deals with the whole field of the growth of scientific knowledge. This is irrelevant in medical diagnosis where instead the task is to *select* from an encyclopedia of prestored diagnostic entities."

As it turns out, *selective abduction* in Magnani's sense is nothing else than the application of previously established knowledge. In this sense, some suitable background knowledge is activated or *selected* vis-à-vis a certain problem. As I understand it, medical diagnosis is only one example; such selective abductions seem to be part of everybody's daily routines. I have discussed abduction as knowledge application above in Sect. 8.1.2, so there is nothing more to add here. On this account, selective abduction is to be reconstructed as the abductive step of knowledge application, in particular in the sense that:

1. Specific (explanatory) concepts or theories are activated (selected) from one's background knowledge, triggered by the initial problem at hand.
2. Accepted as the result of abductive judgment (whereas other spontaneously generated ideas may be rejected as abductively invalid).
3. And, if there are more than one abductively valid ideas, ranked in order of a priori plausibility, however, only for economical reasons.

To be sure, the latter aspect is clearly the least central one, since it is merely of practical importance. And it should be noted that *Magnani* does not attribute it to selective abduction when he writes: "Once hypotheses have been selected, they need to be ranked [...] so as to plan the evaluation phase by first testing a certain preferred hypothesis" [8.3, p. 73]. As also Peirce warns in [8.18, CP 6.525, see above], it should by no means be confused with inductive reasoning.

This reconstruction of selective abduction as the abductive step in knowledge application allows us, finally, to solve the riddle highlighted in the introduction. It concerns what Peirce calls *a priori reasoning* in the passage quoted there, and which he associates with his earlier, syllogistic, concept of abduction (i. e., hypothetical reasoning). When Peirce explains that this kind of [8.2, CP 8.209 (c. 1905)]:

> "abduction is the inference of the truth of the minor premiss of a syllogism of which the major premiss is selected as known already to be true while the conclusion is found to be true,"

1. The major premiss to be *selected* is the theory to which one abduces (e.g., $\forall x(Fx \rightarrow Gx)$).
2. Based on the conclusion (of the syllogism), Ga, which is found to be true and which needs to be explained.
3. And Fa results from the assumption that the occurrence of Ga is a case of $\forall x(Fx \rightarrow Gx)$.

With respect to (3), the only question remaining is whether the abduction runs from Ga to $\forall x(Fx \rightarrow Gx)$, as I have suggested, or from Ga to Fa, as *Schurz* [8.6] might perhaps argue based on his notion of *factual abduction*. This is discussed in the following section.

8.4.2 Factual Versus Theoretical Abduction

This is how *Schurz* formalizes the basic form of factual abduction [8.6, p. 206]:

> "*Known Law*: If Cx, then Ex
> *Known Evidence*: Ea has occurred
> ═════════════════
> *Abduced Conjecture*: Ca could be the reason."

Let us take an example that *Aliseda* uses in [8.21]. I wake up in the morning in a hotel, look out of the window, and see that the lawn is wet (w). Wondering about why it is wet, I think that it might have rained (r) or that the sprinklers were on (s) last night. Hence, there are two possible causes, r and s. However, the question is whether I abduce to r and s or to $r \rightarrow w$ and $s \rightarrow w$, respectively. In my view, both is true in a certain way, which becomes clear if we distinguish inferential subprocesses.

Of course, as we look out of the window and wonder about w (*colligation*), either r or s or both spring to our minds (*observation*). However, since we are looking for an explanation of w, we are not interested in r or s as such, but whether w *because of* r ($r \rightarrow w$) or whether w *because of* s ($s \rightarrow w$). In other words, the law must be implicit in observing the fact, because the fact only makes sense as part of the law. What's more, a spontaneous idea is no valid abduction (not yet). In order to abduce that r or that s we have to perform a judgment (explicitly or implicitly) of the type of Schurz's schema. Thus, Schurz's schema fleshes out the abductive judgment in the case of factual abduction. And even though r or s may be our spontaneous ideas they are engendered not as such, but as the antecedents of $r \rightarrow w$ and $s \rightarrow w$, respectively.

This may all appear self-evident. However, since factual abduction is basically abduction to known laws and theories (rather than to facts pure and simple), we can unify Schurz's subforms of factual abduction, namely *observable-fact abduction*, *first-order existential abduction*, and *unobservable-fact abduction* [8.6, pp. 27–210]. Moreover, it reveals that Schurz's distinction between factual abduction, on the one hand, and law abduction, on the other hand, does not refer to entirely different forms of abductive inference. The only difference is that law abduction relates to the *creative* abduction of new laws, whereas factual abduction relates to *selective* abduction as the abductive step of the application of known laws. Schurz sanctions this view when he writes [8.6, p. 207]:

> "In the setting of factual abduction, the problem consists in the *combinatorial explosion* of the search space of possible causes in the presence of a rich background store of laws but in the *absence* of a rich factual knowledge. Thus, factual abductions are primarily *selective* in the sense of Magnani."

However, I see yet another problem with this description. It assumes that there is a multitude of possible hypotheses from which one or a few plausible ones have to be chosen. In the very same sense he explains that [8.6, p. 204]:

> "in abduction problems we are often confronted with thousands of possible explanatory conjectures (or conclusions) – everyone in the village might be the murderer."

To my mind, this misrepresents (factual) abduction. For on the one hand, if we take each of the village's inhabitants as a hypothetical candidate for the murderer, and intend to boil down their number by some kind of inference, this would have to be *induction*. On the other hand, if the problem really is to reduce the search space, then we are not dealing with a multitude of conjectures as abductive solutions to some abductive problem (finding the murderer), but we are dealing with a *problem*. The fact that there is a multitude of possibilities changes the situation. The task is not simply to select one of those *hypotheses*, but to come up with a *theory* that explains the murder and identifies particular individuals as suspects.

The deeper truth is that instead of merely selecting we move to higher level of reasoning, just in the sense that I have described in Sect. 8.1.3. The very first level, in the example of the murderer, is that one understands that the very concept of a murder implies that the victim has been killed by someone. Given that there are certain objective restrictions, not every human being can possibly have committed the crime, but just the set of the villagers. The next step is to move to the level of narratives in the sense of a coherent description of what might have happened. However, there might be still too many possibilities, or also none. Yet another step could consist in applying theoretical knowledge as professional profilers do.

As already expounded in Sect. 8.1.3, my suggestion is to reconstruct different forms of abduction in the dimension of theoretical abstraction. Since factual abduction comes out as applied law or theory abduction, there is no fundamental difference between factual and theoretical abduction. However, what should be distinguished systematically are cognitive levels in reasoning, down from elementary cognitive levels captured by forms like *visual* (or iconic) and *manipulative* abduction [8.3, 4], and up to high-level abductions like *theoretical model abduction*, *common cause abduction* [8.6], or *trans-paradigmatic abduction* [8.65]. Magnani, Schurz, Hoffmann, and others have done pioneering work explicating abductive inferences at both ends concrete versus abstract cognition, a dimension which I prefer to call hierarchical complexity. However, the precise structures of hierarchical complexity have yet to be revealed (cf. Sect. 8.1.3, above).

One also has to be careful to distinguish forms that do not fit entirely in this order. This seems to apply, for example, to Schurz's notions of (extrapolational) *micropart abduction* and *analogical abduction* [8.6, pp. 216–219]. The former consists, for example, in extrapolating from the behavior of observable macroobjects to assume that unobservable microparts like atoms behave in the same (or a similar) way. However, this is equivalent to an analogical inference from macro to micro, and as such both do not indicate a certain level of abstraction or complexity, but, following also Peirce, are to be reconstructed as compound inferences (including an abductive and an inductive step to hit the abductive target), as I have tried to reveal in [8.15]. Moreover, *Schurz*'s concept of *hypothetical (common) cause abduction* [8.6, pp. 219–222], where he draws to the *dormitive virtue* example (see Sect. 8.1.3), is, to my mind, no valid form of abduction, since this kind of reasoning establishes a problem (*Why does opium put people to sleep?* or *What does its dormitive virtue consist of?*), not the solution. It yields the premise of an abductive inference, but not more.

8.4.3 Explanatory Versus Nonexplanatory Abduction

However, *Schurz* [8.6] points to yet another interesting form of abduction, when he discusses "statistical factor analysis" as a kind of "probabilistic common cause abduction" [8.6, pp. 228–231]. He believes that [8.6, p. 228]:

> "factor analysis is a certain generalization of hypothetical common cause abduction, although sometimes it may better be interpreted in a purely instrumentalistic way."

As I have argued a few lines above, hypothetical common cause abduction is no abduction. However, I fully endorse Schurz's interpretation of factor analysis (to be sure, he thinks of *exploratory factor analysis*, not *confirmatory factor analysis*, which also exists). Exploratory factor analysis is a method to reveal correlative structures among numerical representations of empirical items and thus give us a clue as to possible common causes for certain types of effects (the dependent variables). However, factor analyses do not explain anything. *Factors*, once extracted, have to be interpreted, and this is where they are used as hints toward a possible explanation. Hence, their value consists in being instrumental to find interesting patterns in a dataset, but they do not explain anything as such.

Throughout this chapter, I have focused on (forms of) explanatory abduction as the basic purposive context, because this is the received understanding of abduction in general and because most concepts of abductions fall into this category. However, *Gabbay* and *Woods* [8.5] have made the point that there are types of reasoning that do not have an explanatory purpose. In particular, they point to abductions that do not aim at a plausible explanation, because they, in fact, "advance propositionally implausible hypotheses" [8.5, p. 115]. The purpose of such abductions cannot be explanatory, but they can serve to fulfil some other kind of purpose.

As an example, *Gabbay* and *Woods* discuss Newton's action-at-a-distance theorem, which was never conceived of as an explanatory hypothesis by Newton, since he thought that such an action was causally impossible [8.5, p. 116]. From that point of view, it is clear that in the explanatory context an *action at a distance* poses a problem, that is, that of explaining gravitation, not a solution (like Schurz's hypothetical cause abduction discussed above). However, *Gabbay* and *Woods* point out that [8.5, p. 118–119]:

> "[t]he action-at-a-distance equation serves Newton's theory in a wholly instrumental sense. It allows the gravitational theory to predict observations that it would not otherwise be able to predict."

Thus, there are hypotheses that are not set forth in order to explain something, but to serve some practical purpose. Newton used the action-at-a-distance equation as a tool to predict phenomena. Psychologists have used factor analysis as a tool to find basic personality traits (pioneered by [8.66]).

However, technological sciences in general – mechanical, electronic, medical engineering and the like – do not aim at explanations. They aim at practical, though principled, solutions to practical problems. They may be built on explanatory theories, but what they develop does not have to be *true*; it has to be *effective*. Sometimes, technologies have been invented, before the mechanisms that they employed were sufficiently understood (as for instance in the case of x-rays). Moreover, although technological theories are typically based on explanatory theories, the latter ones are *input* in this context and appear in the *colligation* of abductive inferences to technological theories.

For instance, laser technology employs physics in many ways, but the technology itself is *abduced* from these background theories. Before the laser was invented, Charles Townes and Arthur Schawlow developed the *maser* (microwave amplification by stimulated emission of radiation) in 1954 to amplify radio signals. A few years later, the first optical laser was invented. The technological aim was to *produce* focused light, not to explain anything. And even though it was unclear, at the outset, what practical purposes the technology could be used for, it was effective in producing what it was invented for. On top of this, searches for practical applications – of which we know many today – can be easily accommodated as inverse abductions (as suggested in Sect. 8.3.3), from technological theories (here: laser technology) to concrete practical problems which might be solved, either in principle or better than without the technology. Based on technologies, the practical problems and further background knowledge, the functioning of machines and appliances can be deductively derived, and the machines or prototypes so constructed are then evaluated in terms of effectiveness and efficiency.

Hence, there seems to exist (at least) a second kind of cognitive architecture parallel to the explanatory architecture (and, accordingly, *Magnani* [8.4, p. 71] is right to claim that *Gabbay* and *Woods*' [8.5] notion of instrumental abduction is orthogonal to the forms he distinguishes). On the one hand, explanatory concepts and theories aim at true accounts, and truth is the evaluative criterion for induction. On the other hand, there are technological theories, which are inductively evaluated in terms of effectiveness and – as far as economic aspects are concerned – efficiency.

However, technological theories seem to be just one domain of reasoning among others to complement explanatory reasoning. At least moral concepts and ethical theories could be a third domain [8.26, pp. 90–101], [8.30], and they are evaluated neither in terms of truth nor effectiveness, but in terms of *justice*. I can only allude to these domains, here, and a separate paper will be necessary to expound these ideas. However, what seems obvious is that there are distinct realms of abduction and of reasoning in general.

8.5 Conclusion

To sum up, I have argued (as Peirce did) that there are precisely three basic kinds of inferences: abduction, deduction, and induction. I have distinguished three inferential subprocesses and introduced three inverse types of inference, based on the analysis of inferential subprocesses. My claim is that all kinds of real reasoning ought to be reducible to one of these three basic forms, its inverse forms, or a particular subprocesses within one inferential type. However, I also mentioned analogical reasoning as a special compound form of inferential reasoning and referred the reader to my [8.15].

Moreover, I have tried to point out that apart from these fundamental kinds of reasoning, inferences can be distinguished along two dimensions. One is the dimension of hierarchical complexity so that concepts and theories are built upon one another across cognitive levels, from elementary perception and action to high-level scientific theories. The other dimension, discussed in the previous section, is that of *domains*. By a *domain* I do not mean, in this context, issues of content to which one and the same theory is applied, but domains of reasoning. In this respect I distinguished explanatory, technological, and moral/ethical concepts and theories.

This framework opens up a taxonomical system that might be able to accommodate the various forms of reasoning in general, and of abduction in particular, that have been suggested so far. I have discussed a few of them, but by far not all. However, my hope is that this taxonomy allows us to account for all a multitude of varieties of abduction, deduction, and induction, while recognizing them in their particular place and function in an overall system and help us to a distinctive understanding of similarities and differences between these individual forms.

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