Chapter 9
Position Control via Force Feedback in the Port-Hamiltonian Framework

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Abstract In this chapter, position control strategies via force feedback are presented for standard mechanical systems in the port-Hamiltonian framework. The presented control strategies require a set of coordinate transformations, since force feedback in the port-Hamiltonian framework is not straightforward. With the coordinate transformations force feedback can be realized while preserving the port-Hamiltonian structure. The port-Hamiltonian formalism offers a modeling framework with a clear physical structure and other properties that can often be exploited for control design purposes, which is why we believe it is important to preserve the structure. The proposed control strategies offer an alternative solution to position control with more tuning freedom and exploit knowledge of the system dynamics.

9.1 Introduction

We are honored to write this chapter at the occasion of the 60th birthday of Henk Nijmeijer. I (the second author) know Henk as an influential and stimulating teacher during my study Applied Mathematics at the University of Twente more than 25 years ago. When I performed my traineeship and master thesis project under his supervision, he raised my interest in doing a Ph.D. in the field of systems and control. The corresponding research environment in Twente was open, international and inspiring, and Henk contributed significantly to that, being one of the leaders in the field of nonlinear control systems. In recent years, we started to collaborate and

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publish together. I wish for new collaborations and exchange of ideas in the future. Congratulations Henk!

The current technological advances continuously increase the demand for robots and intelligent systems that are fast, accurate, and able to perform tasks under different circumstances. Sensing and using force measurements are examples of how reliability and performance of such robotic systems can be improved for almost all tasks in which a manipulator comes in contact with external objects [3, 10, 24]. Position control with force feedback for robotic systems has been thoroughly discussed in [3, 10, 18, 19, 25] and the references therein for the Euler–Langrange (EL) framework. In the EL framework, control design is based on selecting a suitable storage function that ensures position control. However, the desired storage function under the EL framework does not qualify as an energy function in any physically meaningful sense as stated in [3, 19].

In this chapter, we present position control strategies via force feedback for standard mechanical systems in the PH framework. The port-Hamiltonian (PH) modeling framework of [13, 26] has received a considerable amount of interest in the last decade due to its insightful physical structure. Moreover, it is well known that a larger class of (nonlinear) physical systems can be described in the PH framework. The popularity of PH systems can be largely accredited to its application for analysis and control design of physical systems, as shown in [6–8, 20, 21, 26] and many others. Control laws in the PH framework are derived with a clear physical interpretation via direct shaping of the closed-loop energy, interconnection, and dissipation structure, see [6, 26]. In this chapter, we apply the PH modeling framework, since it allows extensions on the system coordinates, which facilitates the incorporation of force feedback in the input of the systems. Lastly, the presented control strategy preserves the PH structure, thus granting the aforementioned advantages to the closed-loop system.

The results presented in this chapter are based on [15], and extend the results presented in [16] and [17]. In [16] a class of standard mechanical systems in the PH framework with force feedback and zero external forces has been introduced, for mechanical systems with a constant mass-inertia matrix. However, applying the results from [16] to systems with a nonconstant mass-inertia matrix is not trivial. In [17] preliminary results are presented for the more general class of mechanical systems with a nonconstant mass-inertia matrix. In this chapter, we combine these previous results into a PH framework for position control with force feedback for standard mechanical systems.

The main contribution of this chapter is the introduction of an alternative position control strategy for mechanical systems that includes force feedback, in the PH framework. We present a control approach based on the *modeled* internal forces of a standard mechanical system; for this approach the system is extended with the internal forces into a PH system, which is then asymptotically stabilized. Furthermore, we analyze the disturbance attenuation properties to external forces, i.e., when the external forces are constant we show that the system has a constant steady-state error, and we apply an integral type control to compensate for position errors caused by these constant forces. We reformulate the stability analysis and analyze the robustness against external forces of the control strategy. The resulting controller has nicely tunable properties and interpretations, outperforming most of the existing force feedback control strategies. In addition, we develop a strategy assuming that we have force sensors that give measurements of the (real) total forces in the system, i.e., the internal plus external forces. Those measurements can be used to realize rejection of the external forces in the system.

The chapter is organized as follows. In Sect. 9.2.1, we provide a general background in the PH framework [6]. In Sect. 9.2.2, we apply the results of [27] to equivalently describe the original PH system in a PH form which has a constant mass-inertia matrix in the Hamiltonian via a change of coordinates. This coordinate transformation simplifies the extension of the results in [16] to systems with a nonconstant mass-inertia matrix. A PH model of a robot manipulator of two-DOF is introduced in order to show a mass-inertia decomposition case. Furthermore, in Sect. 9.2.3 we briefly recall the Hamilton–Jacobi inequality related to $\mathscr{L}_2$ analysis. In Sect. 9.2.4, we recap the constructive procedure of [14] to modify the Hamiltonian function of a forced PH system in order to generate Lyapunov functions for nonzero equilibria, i.e., a system in the presence of nonzero constant external forces. In Sect. 9.3, we realize a dynamic extension in order to include the modeled internal forces, while preserving the PH structure. In Sect. 9.4, we present the position control which uses feedback of the modeled forces. We also look at the disturbance attenuation properties when there are external forces, and we apply a type of integral control when the external forces are constant. For constant forces the system converges to a constant position different than the desired one, justifying the application of integral control. In Sect. 9.5, we assume that we have measurements of the total forces in the system, and use these measurements for control. Consequently, we show that we can realize rejection of the total forces in the system while preserving the PH structure. Finally, simulations are given in Sect. 9.6 to motivate our results for position control, and concluding remarks are provided in Sect. 9.7.

9.2 Preliminaries

This section provides the background for the main contributions presented in this chapter. We deal here with the analysis of physical systems described in the PH framework, canonical transformations, and stability analysis in the presence of a disturbance, and a constant force in the input of system.

9.2.1 Port-Hamiltonian Systems

We briefly recap the definition, properties and advantages of modeling and control with the PH formalism.

The PH framework is based on the description of systems in terms of energy variables, their interconnection structure, and power ports. PH systems include a large family of physical nonlinear systems. The transfer of energy between the physical system and the environment is given through energy elements, dissipation elements, and power preserving ports [6, 13], based on the study of Dirac structures.

A class of PH system, introduced in [13], is described by

$$\Sigma = \begin{cases} \dot{x} = [J(x) - R(x)] \dfrac{\partial H(x)}{\partial x} + g(x) w \\ \\ y = g(x)^\top \dfrac{\partial H(x)}{\partial x} \end{cases} \tag{9.1}$$

with $x \in \mathbb{R}^{\mathscr{N}}$ the states of the system, the skew-symmetric interconnection matrix $J(x) \in \mathbb{R}^{\mathscr{N}\times\mathscr{N}}$, the positive-semidefinite damping matrix $R(x) \in \mathbb{R}^{\mathscr{N}\times\mathscr{N}}$, and the Hamiltonian $H(x) \in \mathbb{R}$. The matrix $g(x) \in \mathbb{R}^{\mathscr{N}\times\mathscr{M}}$ weights the action of the control inputs $w \in \mathbb{R}^{\mathscr{M}}$ on the system, and $w, y \in \mathbb{R}^{\mathscr{M}}$ with $\mathscr{M} \leq \mathscr{N}$, form a power port pair. We now restrict the description to a class of standard mechanical systems.

Consider a class of standard mechanical systems of n-DOF as in (9.1), e.g., an n-DOF rigid robot manipulator. Consider furthermore the addition of an external force vector. The resulting system is then given by

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -I_{n\times n} & -D(q,p) \end{bmatrix} \begin{bmatrix} \dfrac{\partial H(q,p)}{\partial q} \\ \dfrac{\partial H(q,p)}{\partial p} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ G(q) \end{bmatrix} u + \begin{bmatrix} 0_{n\times n} \\ B(q) \end{bmatrix} f_e \tag{9.2}$$

$$y = G(q)^\top \frac{\partial H(q,p)}{\partial p}, \tag{9.3}$$

with the vector of generalized configuration coordinates $q \in \mathbb{R}^n$, the vector of generalized momenta $p \in \mathbb{R}^n$, the identity matrix $I_{n\times n}$, the damping matrix $D(q,p) \in \mathbb{R}^{n\times n}$, $D(q,p) = D(q,p)^\top \geq 0$, $y \in \mathbb{R}^n$ the output vector, $u \in \mathbb{R}^n$ the input vector, $f_e \in \mathbb{R}^n$ the vector of external forces, $\mathscr{N} = 2n$, matrix $B(q) \in \mathbb{R}^{n\times n}$, and the input matrix $G(q) \in \mathbb{R}^{n\times n}$ everywhere invertible, i.e., the PH system is *fully actuated*. The Hamiltonian of the system is equal to the sum of kinetic and potential energy,

$$H(q,p) = \frac{1}{2} p^\top M^{-1}(q)\, p + V(q), \tag{9.4}$$

where $M(q) = M^\top(q) > 0$ is the $n \times n$ inertia (generalized mass) matrix and $V(q)$ is the potential energy.

We consider the PH system (9.2) as a class of standard mechanical systems with external forces.

Remark 9.1 The robot dynamics is given in joint space in (9.2), and here the *external forces* $f_e \in \mathbb{R}^n$ are introduced. The geometric Jacobian maps the external forces in the *work space*, F_e, to the (generalized) external forces in the joint space, f_e, [25]. In this chapter the following holds,

$$f_e = \mathscr{J}(q)^\top F_e, \qquad F_e \in \mathbb{R}^N, \tag{9.5}$$

and the geometric Jacobian is given by

$$\mathscr{J}(q) = \begin{bmatrix} \mathscr{J}_v(q) \\ \mathscr{J}_\omega(q) \end{bmatrix} \in \mathbb{R}^{6\times n}, \tag{9.6}$$

where $\mathscr{J}_v(q) \in \mathbb{R}^{3\times n}$, and $\mathscr{J}_\omega(q) \in \mathbb{R}^{3\times n}$ are the linear, and angular geometric Jacobians, respectively, and $N = \{3, 6\}$. If the Jacobian is full rank, we can always find $f_e \in \mathbb{R}^n$ that corresponds to F_e. Then, it is not a limitation to suppose $B(q) = I_n$. This separation between joint and work spaces is important here, because we control the robot by acting on the generalized coordinates q, i.e., in the joint space, but we grasp objects with the end-effector in the work space.

Example 9.1 Consider the system given by the two-DOF shoulder of the PERA, [23]. A picture of the PERA is shown in Fig. 9.1. A *Denavit–Hartenberg* representation of the PERA, see [25], is given in Fig. 9.2. The shoulder consists of a link actuated by two motors. The model of the shoulder consists of a mass m_s, a link length l_s, and a linear damping $d_s > 0$. The states of the system are $x = (q, p)^\top$, where $(q, p) \in \mathbb{R}^2$ are the generalized coordinates q_1, and q_2, and p_1, p_2 are the generalized momenta of the system. The system is described in the PH form by

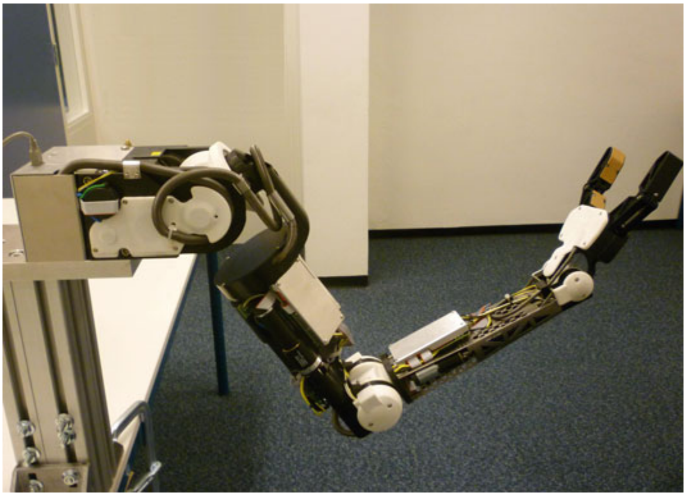

Fig. 9.1 PERA at the University of Groningen

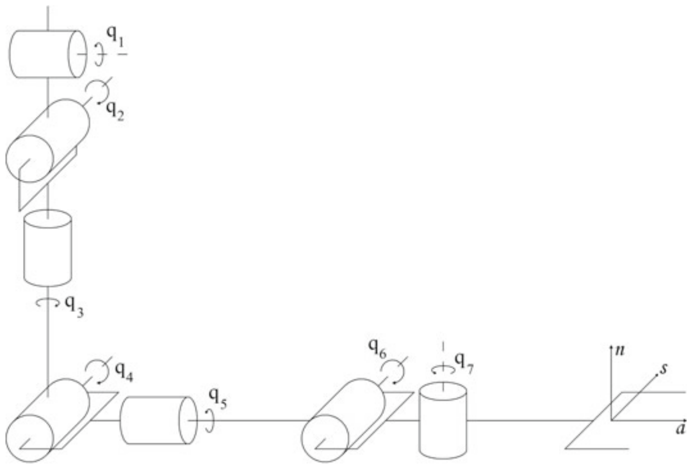

Fig. 9.2 *Denavit–Hartenberg* representation of the PERA [12]

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -I_{2\times2} & D(q,p) \end{bmatrix} \begin{bmatrix} \frac{\partial V(q)}{\partial q} \\ M(q)^{-1}p \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u_s + \begin{bmatrix} 0 \\ B \end{bmatrix} f_e \tag{9.7}$$

$$y_s = G^\top M(q)^{-1} p \tag{9.8}$$

with an input matrix $G = I_{2\times2}$ (fully actuated), a vector of external forces $f_e \in \mathbb{R}^2$, an input–output port pair (u_s, y_s), Hamiltonian of the form

$$H(q,p) = \frac{1}{2} p^\top M(q)^{-1} p + V(q) \tag{9.9}$$

with $V(q)$ the potential energy, and a mass-inertia matrix $M(q) \in \mathbb{R}^{2\times2}$, s.t., $M(q) = diag(a,b)$ where

$$a = m_s l_s^2 \cos(q_2)^2 + \mathscr{I}_1 + \mathscr{I}_2 \tag{9.10}$$

$$b = m_s l_s^2 + \mathscr{I}_2 \tag{9.11}$$

and with $\mathscr{I}_1$, and $\mathscr{I}_2$ the inertias of the joints. Furthermore, the gravity vector is

$$\frac{\partial V(q)}{\partial q} = \begin{bmatrix} g m_s l_s \cos(q_2)\sin(q_1) \\ g m_s l_s \sin(q_2)\cos(q_1) \end{bmatrix} \tag{9.12}$$

with g the gravitational acceleration. The shoulder is experiencing Coulomb friction that we have determined, and validated experimentally, [2, 12]. The dissipation matrix has the form

$$D\left(q,p\right)=D\left(\dot{q}\right)=diag\left(d_{s_1}\left(\dot{q}_1\right),d_{s_2}\left(\dot{q}_2\right)\right), \tag{9.13}$$

where $\dot{q}=M^{-1}\left(q\right)p$, and with

$$d_{s_i}=\left(F_{c_i}+\left(F_{s_i}-F_{c_i}\right)e^{\left|\dot{q}_i\right|\dot{q}_{s_i}^{-1}}\right)\left(\alpha_{f_i}+\dot{q}_i^2\right)^{-0.5}+F_{v_i}\dot{q}_i, \tag{9.14}$$

where F_{c_i}, F_{s_i}, and F_{v_i} are the Coulomb, static, and viscous friction coefficients, respectively, and the Coulomb friction force is approximated as in [9] with positive (small) constants α_i, $\dot{q}_{s_i}$ is the constant due to the Stribeck velocity [1], and $i=1,2$. □

9.2.2 *Canonical Transformations of Port-Hamiltonian Systems*

We recap here the results of [7, 8] in terms of generalized coordinate transformations for PH systems, and we apply the results of [27] to equivalently describe the original PH system in a PH form which has a constant mass-inertia matrix in the Hamiltonian.

A generalized canonical transformation of [7] is applied in (9.1) via a set of transformations

$$\bar{x}=\Phi\left(x\right) \tag{9.15}$$

$$\bar{H}\left(\bar{x}\right)=H\left(x\right)+U\left(x\right) \tag{9.16}$$

$$\bar{y}=y+\alpha\left(x\right) \tag{9.17}$$

$$\bar{u}=u+\beta\left(x\right) \tag{9.18}$$

that changes the coordinates x into $\bar{x}$, the Hamiltonian H into $\bar{H}$, the output y into $\bar{y}$, and the input u into $\bar{u}$. It is said to be a generalized canonical transformation for PH systems if it transforms a PH system (9.1) into another one.

The class of generalized canonical transformations are characterized by the following theorems.

Theorem 9.1 (*[8]*) *Consider the PH system (9.1). For any smooth scalar function $U\left(x\right)\in\mathbb{R}$, and any smooth vector function $\beta\left(x\right)\in\mathbb{R}^{\mathcal{M}}$, there exists a pair of smooth functions $\Phi\left(x\right)\in\mathbb{R}^{\mathcal{N}}$ and $\alpha\left(x\right)\in\mathbb{R}^{\mathcal{M}}$ such that the set of equations (9.15)–(9.18) yields a generalized canonical transformation. The function $\Phi\left(x\right)$ yields a generalized canonical transformation with $U\left(x\right)$ and $\beta\left(x\right)$ if and only if the partial differential equation (PDE)*

$$\frac{\partial \Phi}{\partial (x,t)} \left((J-R) \frac{\partial U}{\partial x}^{\top} + (K-S) \frac{\partial (H+U)}{\partial x}^{\top} + g\beta \right) = 0 \tag{9.19}$$
$$-1$$

holds with a skew-symmetric matrix $K(x)$, *and a symmetric matrix* $S(x)$ *satisfying* $R(x) + S(x) \geq 0$. *We have left out the arguments of* $\Phi(x)$, $H(x)$, $J(x)$, $R(x)$, $S(x)$, $K(x)$, $U(x)$, $g(x)$, *and* $\beta(x)$, *for notational simplicity. Furthermore, the change of output* $\alpha(x)$, *and the matrices* $\bar{J}(\bar{x})$, $\bar{R}(\bar{x})$, *and* $\bar{g}(\bar{x})$, *are given by*

$$\alpha(x) = g(x)^{\top} \frac{\partial U(x)}{\partial x} \tag{9.20}$$

$$\bar{J}(\bar{x}) = \frac{\partial \Phi(x)}{\partial x} (J(x) + K(x)) \frac{\partial \Phi(x)}{\partial x}^{\top} \tag{9.21}$$

$$\bar{g}(\bar{x}) = \frac{\partial \Phi(x)}{\partial x} g(x) \tag{9.22}$$

$$\bar{R}(\bar{x}) = \frac{\partial \Phi}{\partial x} (R(x) + S(x)) \frac{\partial \Phi(x)}{\partial x}^{\top}. \tag{9.23}$$

Theorem 9.2 (*[8]*) *Consider the PH system described by (9.1) and transform it by the generalized canonical transformation with* $U(x)$ *and* $\beta(x)$ *such that* $H(x) + U(x) \geq 0$. *Then, the new input-output mapping* $\bar{u} \to \bar{y}$ *is passive with storage function* $\bar{H}(\bar{x})$ *if and only if*

$$\frac{\partial (H+U)}{\partial (x)}^{\top} \left((J-R) \frac{\partial U}{\partial x}^{\top} - S \frac{\partial (H+U)}{\partial x}^{\top} + g\beta \right) \geq 0. \tag{9.24}$$
$$-1$$

Suppose that (9.19) *holds, that* $H(x) + U(x)$ *is positive-definite and that the system is zero-state detectable. Then, the feedback* $u = -\beta(x) - \mathscr{C}(x)(y + \alpha(x))$ *with* $\mathscr{C}(x) \geq \epsilon I > 0$ *renders the system asymptotically stable. Suppose moreover that* $H + U$ *is decrescent and that the transformed system is periodic, then, the feedback renders the system uniformly asymptotically stable.*

Consider a class of standard mechanical systems (9.2) in the PH framework with a nonconstant mass-inertia matrix $M(q)$. The aim of this section is to transform the original system (9.2) into a PH formulation with a constant mass-inertia matrix via a generalized canonical transformation [7]. The presented change of variables to deal with a nonconstant mass- inertia matrix has first been proposed in [27].

Consider the system (9.1) with nonconstant $M(q)$, and a coordinate transformation as

$$\bar{x} = \Phi(x) = \Phi(q,p) \triangleq \begin{pmatrix} \bar{q} \\ \bar{p} \end{pmatrix} = \begin{pmatrix} q - q_d \\ T(q)^{-1} p \end{pmatrix} = \begin{pmatrix} q - q_d \\ T(q)^{\top} \dot{q} \end{pmatrix} \tag{9.25}$$

with a constant desired position $q_d \in \mathbb{R}^n$, and where $T(q)$ is a lower triangular matrix such that

$$T(q) = T\left(\Phi^{-1}(q, p)\right) = \bar{T}(\bar{q}) \tag{9.26}$$

and

$$M(q) = T(q)\, T(q)^{\top} = \bar{T}(\bar{q})\, \bar{T}(\bar{q})^{\top}. \tag{9.27}$$

Consider now the Hamiltonian $H(q, p)$ as in (9.4), and using (9.25), we realize $\bar{H}(\bar{x}) = H\left(\Phi^{-1}(\bar{x})\right)$ and $\bar{V}(\bar{q}) = V\left(\Phi^{-1}(\bar{q})\right)$ as

$$\bar{H}(\bar{x}) = \frac{1}{2}\bar{p}^{\top}\bar{p} + \bar{V}(\bar{q}). \tag{9.28}$$

The new form of the interconnection and damping matrices of the PH system are realized via the coordinate transformation (9.25), the mass-inertia matrix decomposition (9.27), and the new Hamiltonian (9.28), [26]. The resulting PH system is then given by

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & \bar{T}(\bar{q})^{-\top} \\ -\bar{T}(\bar{q})^{-1} & \bar{J}_2(\bar{q}, \bar{p}) - \bar{D}(\bar{q}, \bar{p}) \end{bmatrix} \begin{bmatrix} \dfrac{\partial \bar{H}(\bar{q}, \bar{p})}{\partial \bar{q}} \\ \dfrac{\partial \bar{H}(\bar{q}, \bar{p})}{\partial \bar{p}} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} \\ \bar{G}(\bar{q}) \end{bmatrix} v + \begin{bmatrix} 0_{n\times n} \\ \bar{B}(\bar{q}) \end{bmatrix} f_e \tag{9.29}$$

$$\bar{y} = \bar{G}(\bar{q})^{\top} \frac{\partial \bar{H}(\bar{q}, \bar{p})}{\partial \bar{p}} \tag{9.30}$$

with a new input $u = v \in \mathbb{R}^n$, and where the skew-symmetric matrix $\bar{J}_2(\bar{q}, \bar{p})$ takes the form

$$\bar{J}_2(\bar{q}, \bar{p}) = \frac{\partial\left(\bar{T}(\bar{q})^{-1}\bar{p}\right)}{\partial \bar{q}}\bar{T}(\bar{q})^{-\top} - \bar{T}(\bar{q})^{-1}\frac{\partial\left(\bar{T}(\bar{q})^{-1}\bar{p}\right)}{\partial \bar{q}}^{\top} \tag{9.31}$$

with

$$(q, p) = \Phi^{-1}(\bar{q}, \bar{p}) \tag{9.32}$$

together with the matrix $\bar{D}(\bar{q}, \bar{p})$, and the input matrices $\bar{G}(\bar{q})$, and $\bar{B}(\bar{q})$, are described by

$$\bar{D}(\bar{q}, \bar{p}) = \bar{T}(\bar{q})^{-1} D\left(\Phi^{-1}(\bar{q}, \bar{p})\right) \bar{T}(\bar{q})^{-\top}, \tag{9.33}$$

$$\bar{G}(\bar{q}) = \bar{T}(\bar{q})^{-1} G(\bar{q}), \tag{9.34}$$

$$\bar{B}(\bar{q}) = \bar{T}(\bar{q})^{-1} B(\bar{q}), \tag{9.35}$$

respectively. Via the transformation (9.25), we then obtain a class of mechanical systems with a constant (identity) mass-inertia matrix in the Hamiltonian function as in (9.28), which equivalently describes the original system (9.2) with nonconstant mass-inertia matrix.

Example 9.2 Consider the robot manipulator of Example 9.1. Given the mass-inertia matrix $M(q) = diag(a, b)$ with a, and b as in (9.10) and (9.11), respectively, we compute a $T(q)$ as in (9.27), s.t.,

$$T(q) = \begin{bmatrix} \sqrt{a} & 0 \\ 0 & \sqrt{b} \end{bmatrix} = \begin{bmatrix} \sqrt{m_s l_s^2 \cos(q_2)^2 + \mathscr{I}_1 + \mathscr{I}_2} & 0 \\ 0 & \sqrt{m_s l_s^2 + \mathscr{I}_2} \end{bmatrix} \tag{9.36}$$

with $\mathscr{I}_1$, and $\mathscr{I}_2$ the inertias of the joints, and m_s, l_s as the mass and the length of the shoulder of the robot, respectively. Based on $T(q)$, we can compute the matrices $\bar{J}_2(\bar{q}, \bar{p})$, $\bar{D}(\bar{q}, \bar{p})$, $\bar{G}(\bar{q})$, and $\bar{B}(\bar{q})$, as in (9.31), (9.33)–(9.35).

The coordinate transformation of this section is used in the rest of this chapter in order to deal with nonconstant mass-inertia matrices.

9.2.3 Hamilton–Jacobi Inequality

In order to show the usefulness of some results on position control with force feedback presented later, we apply the Hamilton–Jacobi inequality useful for $\mathscr{L}_2$ gain analysis of nonlinear systems [26]. Toward this end we analyze the $\mathscr{L}_2$-gain of a closed-loop system w.r.t. an $\mathscr{L}_2$ disturbance δ.

Consider the time-invariant nonlinear system

$$\begin{aligned} \dot{\hat{x}} &= \mathscr{F}(\hat{x}) + \tilde{G}(\hat{x})\,\delta \\ \hat{y} &= h(\hat{x}) \end{aligned} \tag{9.37}$$

with states $\hat{x}$, input disturbance δ, output $\hat{y}$ and continuously differentiable vector functions $\mathscr{F}(\hat{x})$, $\tilde{G}(\hat{x})$ and $h(\hat{x})$. Let γ be a positive constant, then the $\mathscr{L}_2$-gain bound is found if for a γ there exists a continuously differentiable, positive-semidefinite function $\mathscr{W}(\hat{x})$ that satisfies the Hamilton–Jacobi inequality (HJI)

$$\left(\frac{\partial \mathscr{W}\left(\hat{x}\right)}{\partial \hat{x}}\right)^{\top} \mathscr{F}\left(\hat{x}\right)+\frac{1}{2}\frac{1}{\gamma^{2}}\left(\frac{\partial \mathscr{W}\left(\hat{x}\right)}{\partial \hat{x}}\right)^{\top} \tilde{G}\left(\hat{x}\right)\tilde{G}\left(\hat{x}\right)^{\top}\frac{\partial \mathscr{W}\left(\hat{x}\right)}{\partial \hat{x}}+\frac{1}{2}h\left(\hat{x}\right)^{\top}h\left(\hat{x}\right)\leq 0 \tag{9.38}$$

for $\hat{x} \in \mathbb{R}^{\mathscr{N}}$. The system (9.37) is then finite-gain $\mathscr{L}_2$ stable and its gain is less than or equal to γ.

9.2.4 Stability Analysis for Constant External Forces

Consider a class of PH systems as described by (9.1). We now briefly recall the procedure of [14], i.e., we analyze the stability of the system (9.1) for a constant, and nonzero, input $w = \bar{u} \in \mathbb{R}^{\mathscr{M}}$, leading to a forced equilibrium $\breve{x} \in \mathbb{R}^{\mathscr{N}}$. The forced equilibria $\breve{x}$ are solutions of

$$\left[J\left(\breve{x}\right)-R\left(\breve{x}\right)\right]\frac{\partial H}{\partial x}\left(\breve{x}\right)+g\left(\breve{x}\right)\bar{u}=0 \tag{9.39}$$

and if $[J(x) - R(x)]$ is invertible for every $x \in \mathbb{R}^{\mathscr{N}}$, the unique solution of (9.39) is $\frac{\partial H}{\partial x}(x) = \mathscr{K}(x)\,\bar{u}$ where

$$\mathscr{K}\left(x\right)=-\left[J\left(x\right)-R\left(x\right)\right]^{-1}g\left(x\right). \tag{9.40}$$

Based on (9.40), we define the matrices

$$J_s\left(x\right)\triangleq \mathscr{K}^{\top}\left(x\right)J\left(x\right)\mathscr{K}\left(x\right) \tag{9.41}$$

and

$$R_s\left(x\right)\triangleq \mathscr{K}^{\top}\left(x\right)R\left(x\right)\mathscr{K}\left(x\right) \tag{9.42}$$

which we use below to find the embedded Hamiltonian system. Clearly, $J_s(x)$ and $R_s(x)$ satisfy $J_s(x) = -J_s^{\top}(x)$, and $R_s(x) = R_s^{\top}(x) \geq 0$, respectively. Let us now consider the following PH system

$$\begin{bmatrix}\dot{x}\\ \dot{\zeta}\end{bmatrix}=\left[J_a\left(x\right)-R_a\left(x\right)\right]\begin{bmatrix}\dfrac{\partial H_a\left(x\right)}{\partial x}\\ \dfrac{\partial H_a\left(x\right)}{\partial \zeta}\end{bmatrix} \tag{9.43}$$

on the augmented state space $(x, \zeta) \in \mathbb{R}^{\mathscr{N}} \times \mathbb{R}^{\mathscr{M}}$, endowed with the structure matrices

$$J_a(x) = \begin{bmatrix} J(x) & J(x)\mathscr{K}(x) \\ -(J(x)\mathscr{K}(x))^\top & J_s(x) \end{bmatrix} \tag{9.44}$$

$$R_a(x) = \begin{bmatrix} R(x) & R(x)\mathscr{K}(x) \\ (R(x)\mathscr{K}(x))^\top & R_s(x) \end{bmatrix} \tag{9.45}$$

with $\mathscr{K}(x)$, $J_s(x)$, and $R_s(x)$ as in (9.40)–(9.42), respectively, and with an augmented Hamiltonian

$$H_a(x,\zeta) \triangleq H(x) + H_s(\zeta), \quad H_s(\zeta) \triangleq -\bar{u}^\top \zeta. \tag{9.46}$$

Theorem 9.3 (*[14]*) *Consider, a class of PH systems (9.1) with a constant input $w = \bar{u}$, and the matrix $[J(x) - R(x)]$ invertible for every $x \in \mathbb{R}^{\mathscr{N}}$. Define $\mathscr{K}(x)$ by (9.40), and assume the functions $\mathscr{K}_{ij}$ to satisfy*

$$\frac{\partial \mathscr{K}_{ij}}{\partial x_k} = \frac{\partial \mathscr{K}_{kj}}{\partial x_i}, \; i,k \in \bar{n} \triangleq \{1,\ldots,\mathscr{N}\}, \; j \in \bar{m} \triangleq \{1,\ldots,\mathscr{M}\}. \tag{9.47}$$

Also, assume that there exist locally smooth functions $\mathscr{C}_j : \mathbb{R}^{\mathscr{N}} \to \mathbb{R}$, called Casimirs [14], satisfying

$$\mathscr{K}_{ij}(x) = \frac{\partial \mathscr{C}_j}{\partial x_i}(x), \quad j \in \bar{m}, \quad i \in \bar{n} \tag{9.48}$$

and $\zeta_j = \mathscr{C}_j(x) + c_j$, where $c_1, \ldots, c_{\mathscr{M}}$ depend on the initial conditions of $\zeta(t)$ in (9.43). Then, the dynamics of (9.1) with input $u = \bar{u}$ is asymptotically stable at the equilibrium point $\breve{x}$ fulfilling (9.39), and it can be alternatively represented by

$$\dot{x} = [J(x) - R(x)]\frac{\partial H_r}{\partial x}(x) \tag{9.49}$$

where

$$H_r(x) \triangleq H(x) - \sum_{j=1}^{\mathscr{M}} \bar{u}_j \zeta_j \tag{9.50}$$

and H_r qualifies as a Lyapunov function for the forced dynamics (9.49).

Remark 9.2 The $\mathscr{L}_2$-gain analysis of Sect. 9.2.3 gives a bound on the relation between an input δ and a output $\hat{y}$ as in (9.37) of a proposed closed-loop system for a $\mathscr{L}_2$-input disturbance δ. The $\mathscr{L}_2$-gain analysis differs from Theorem 9.3 in the sense that the $\mathscr{L}_2$-gain analysis is related to the output $\hat{y}$ while the analysis in this section is for the case where the system is asymptotically stable, i.e., the system (9.29) has a new equilibrium point caused by a constant f_e.

9.3 Force Feedback via Dynamic Extension

In this section, a force feedback strategy is introduced for a mechanical system in the PH framework. The force feedback is included to bring robustness and better tunable properties in the position control strategy. In comparison with force feedback in the EL framework [3, 24], the force feedback in the PH framework has nicely interpretable control strategies, as well as cleaner tuning opportunities that grant a better performance. The force feedback is achieved via a dynamic extension and a change of variables that introduces a new state for the PH system (9.29). The dynamics of the new state is realized such that it depends on the internal forces of the mechanical system. The internal forces are given by a set of kinetic, potential, and energy dissipating elements. The dynamic extension is realized such that the extended system also has a PH structure. The present work is inspired by the results of [4, 5, 22], which treat position feedback.

Denote the internal forces on the system (9.2) by $f_{in}(q, p)$, i.e.,

$$f_{in}(q, p) = -\frac{\partial H(q, p)}{\partial q} - D(q, p)\frac{\partial H(q, p)}{\partial p} \tag{9.51}$$

with $H(q, p)$ as in (9.4). Define a new state $z \in \mathbb{R}^n$ with dynamics depending on the internal forces $f_{in}(q, p)$, such that,

$$\dot{z} = Y^\top T(q)^{-1} f_{in}(q, p) \tag{9.52}$$

with Y a constant matrix, to be defined later on. Consider now the coordinate transformation

$$\hat{p} = \bar{p} - Az \tag{9.53}$$

with $\bar{p}$ defined in (9.25), and with A a constant matrix that we use later to tune our controller. Furthermore, we can define for system (9.29) the control input

$$v = \bar{G}(\bar{q})^{-1} A\dot{z} + \bar{v} \tag{9.54}$$

where $\bar{v}$ is a new input, which realizes an extended PH system with states $\hat{p}$ and z, i.e.,

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\hat{p}} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{n\times n} & \bar{T}^{-\top} & \bar{T}^{-\top}Y \\ -\bar{T}^{-1} & \bar{J}_2 - \bar{D} & \left(\bar{J}_2 - \bar{D}\right)Y \\ -Y^\top \bar{T}^{-1} & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right) & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right)Y \end{bmatrix}}_{\hat{J}(\bar{q},\hat{p},z) - \hat{R}(\bar{q},\hat{p},z)} \begin{bmatrix} \dfrac{\partial \hat{H}(\bar{q}, \hat{p}, z)}{\partial \bar{q}} \\ \dfrac{\partial \hat{H}(\bar{q}, \hat{p}, z)}{\partial \hat{p}} \\ \dfrac{\partial \hat{H}(\bar{q}, \hat{p}, z)}{\partial z} \end{bmatrix}$$

$$+\begin{bmatrix} 0_{n\times n} \\ \bar{G}(\bar{q}) \\ 0_{n\times n} \end{bmatrix} \bar{v} + \begin{bmatrix} 0_{n\times n} \\ \bar{B}(\bar{q}) \\ 0_{n\times n} \end{bmatrix} f_e \tag{9.55}$$

$$\hat{y} = \bar{G}(\bar{q})^\top \frac{\partial \hat{H}(\bar{q}, \hat{p}, z)}{\partial \hat{p}} \tag{9.56}$$

with Hamiltonian

$$\hat{H}(\bar{q}, \hat{p}, z) = \frac{1}{2}\hat{p}^\top \hat{p} + \frac{1}{2} z^\top K_z^{-1} z + \bar{V}(\bar{q}) \tag{9.57}$$

where $K_z > 0$, and $Y = AK_z$. In (9.55) the arguments of $T(\bar{q})$, $\bar{J}_2(\bar{q}, \hat{p})$, and $\bar{D}(\bar{q}, \hat{p})$, are left out for notational simplicity.

Remark 9.3 Although in (9.55) the $\dot{z}$ dynamics are described in terms of $\bar{J}_2(\bar{q}, \hat{p})$, $\bar{D}(\bar{q}, \hat{p})$, and $\hat{H}(\bar{q}, \hat{p}, z)$, they are still the same as described by (9.52) with (9.51), in the new coordinates (9.25).

It can be verified that system (9.55) is PH, since

$$\hat{J}(\bar{q}, \hat{p}) = \begin{bmatrix} 0_{n\times n} & \bar{T}(\bar{q})^{-\top} & \bar{T}(\bar{q})^{-\top} Y \\ -\bar{T}(\bar{q})^{-1} & \bar{J}_2(\bar{q}, \hat{p}) & \bar{J}_2(\bar{q}, \hat{p}) Y \\ -Y^\top \bar{T}(\bar{q})^{-1} & -Y^\top \bar{J}_2(\bar{q}, \hat{p})^\top & -Y^\top \bar{J}_2(\bar{q}, \hat{p})^\top Y \end{bmatrix} \tag{9.58}$$

is skew-symmetric, while

$$\hat{R}(\bar{q}, \hat{p}) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \bar{D}(\bar{q}, \hat{p}) & \bar{D}(\bar{q}, \hat{p}) Y \\ 0_{n\times n} & Y^\top \bar{D}(\bar{q}, \hat{p}) & Y^\top \bar{D}(\bar{q}, \hat{p}) Y \end{bmatrix} \tag{9.59}$$

can be shown to be positive-semidefinite via the Schur complement. Notice that by extending the dynamics of (9.29) with the internal forces $\dot{z}$ in the input (9.54), we include force feedback and preserve the PH structure.

Remark 9.4 In [16] we present results for the case when the mass-inertia matrix is constant. The case for a constant M does not require the coordinate transformation (9.25), and system (9.55) is then described by $T = I$, $\bar{J}_2 = 0$, $\bar{D} = D$, $\bar{G} = G$, $\bar{B} = B$, $Y = M^{-1}AK_z$ and Hamiltonian

$$\hat{H}_c = \frac{1}{2}\hat{p}^\top M^{-1} \hat{p} + \frac{1}{2} z^\top K_z z + \bar{V}(\bar{q}) \tag{9.60}$$

instead of (9.57).

In this section, we have realized an extended mechanical system that includes force feedback and preserves the PH structure. In the next section, we deal in more detail with position control and stability analysis.

9.4 Position Control with Modeled Internal Forces

In this section, a position control strategy with force feedback is introduced. We feed back the modeled internal forces, and the resulting system preserves the PH structure. The control laws here presented are better tunable and more insightful solutions in comparison with the solutions given in the EL framework [3, 19].

9.4.1 Position Control with Zero External Forces

In this section, energy-shaping [11, 19, 26] and damping injection are combined with force feedback (of modeled forces) to realize position control.

Theorem 9.4 *Consider system (9.55) and assume $f_e = 0$. Then, the control input*

$$v = \bar{G}(\bar{q})^{-1}\left(\frac{\partial \bar{V}(\bar{q})}{\partial \bar{q}} - K_p(\bar{q} - q_d)\right) - C\hat{y} \tag{9.61}$$

with $K_p > 0$, $C > 0$, and q_d being the desired constant position, asymptotically stabilizes the extended system (9.55) *at* $(\bar{q}, \hat{p}, z) = (q_d, 0, 0)$.

Proof This is a well-known result, see [26], but we repeat the proof here for notational reasons and for ease of reading. The control input (9.61) applied to system (9.55) with $f_e = 0$ realizes the closed-loop system described by

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\hat{p}} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{n\times n} & \bar{T}^{-\top} & \bar{T}^{-\top}Y \\ -\bar{T}^{-1} & \bar{J}_2 - \bar{D} - \bar{G}C\bar{G}^\top & \left(\bar{J}_2 - \bar{D}\right)Y \\ -Y^\top\bar{T}^{-1} & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right) & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right)Y \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{H}_d}{\partial \bar{q}} \\ \frac{\partial \hat{H}_d}{\partial \hat{p}} \\ \frac{\partial \hat{H}_d}{\partial z} \end{bmatrix}}_{\mathscr{F}(\hat{x})} \tag{9.62}$$

$$\hat{y} = \bar{G}^\top \frac{\partial \hat{H}_d}{\partial \hat{p}} \tag{9.63}$$

with Hamiltonian

$$\hat{H}_d = \frac{1}{2}\hat{p}^\top\hat{p} + \frac{1}{2}(\bar{q} - q_d)^\top K_p(\bar{q} - q_d) + \frac{1}{2}z^\top K_z^{-1}z, \tag{9.64}$$

where the arguments of $\hat{H}_d(\bar{q}, \hat{p}, z)$, $T(\bar{q})$, $\bar{J}_2(\bar{q}, \hat{p})$, $\bar{D}(\bar{q}, \hat{p})$, $\bar{G}(\bar{q})$, and $\bar{B}(\bar{q})$ are left out for simplicity. Take (9.64) as candidate Lyapunov function, which then gives

$$\dot{\hat{H}}_d = -\begin{bmatrix} \frac{\partial \hat{H}_d}{\partial \hat{p}} \\ \frac{\partial \hat{H}_d}{\partial z} \end{bmatrix}^\top \underbrace{\begin{bmatrix} \bar{D} + \bar{G}C\bar{G}^\top & -\bar{D}Y \\ -Y^\top \bar{D} & Y^\top \bar{D}Y \end{bmatrix}}_{K} \begin{bmatrix} \frac{\partial \hat{H}_d}{\partial \hat{p}} \\ \frac{\partial \hat{H}_d}{\partial z} \end{bmatrix}. \tag{9.65}$$

Since $\bar{G}(\bar{q})$ is full rank and $C > 0$, via the Schur complement it can be shown that matrix K in (9.65) is positive definite. Subsequently, via LaSalle's invariance principle we can prove that that the closed-loop system (9.62) is asymptotically stable in $\bar{q} = q_d$. □

Substituting v in (9.54) by (9.61) then gives the total control input u for the original system (9.2), which in terms of the original coordinates (q, p) becomes

$$u = G(q)^{-1}T(q)\left(A\dot{z} + \frac{\partial V(q)}{\partial q} - K_p(q - q_d)\right) - CG(q)^\top\left(M(q)^{-1}p - T(q)^{-\top}Az\right) \tag{9.66}$$

with $\dot{z}$ as in (9.51). The above results correspond to the case when the external forces on the system are zero, i.e., $f_e = 0$. In the next subsection we look more in detail at the case when $f_e \neq 0$.

9.4.2 Disturbance Attenuation Properties

We now show the advantages of the proposed extended system with force feedback for disturbance attenuation to unknown external forces. The closed-loop PH system (9.62) with force feedback is asymptotically stable in the desired position q_d when it has zero forces exerted from the environment, i.e., $f_e = 0$. To look at the effect of f_e being different from zero, we analyze the $\mathscr{L}_2$-gain w.r.t. an $\mathscr{L}_2$ disturbance f_e, [26]. It follows that

Theorem 9.5 *Consider a closed-loop system (9.62), an $\mathscr{L}_2$ disturbance f_e, and a constant matrix C with $\lambda_c \in \mathbb{R}^n$ being its set of eigenvalues. We then obtain a disturbance attenuation of f_e when the following conditions hold:*

$$\Gamma_1(q, p) = -D(q, p) + G(q)^\top\left(-C + \frac{1}{2}I_{n\times n}\right)G(q) + \frac{1}{2}\frac{1}{\gamma^2}B(q)B(q)^\top\frac{1}{2} \leq 0 \tag{9.67}$$

$$\Gamma_2(q) = AT(q)^{-\top}G(q)^\top\left(-C + \frac{1}{2}I_{n\times n}\right)G(q)^\top T(q)^{-1}A + \frac{1}{2}\frac{1}{\gamma^2}AT(q)^{-1}B(q)B(q)^\top T(q)^{-\top}A \leq 0 \tag{9.68}$$

$$\Gamma_3(q) = \frac{1}{2}\frac{1}{\gamma^2}AT(q)^{-1}B(q)B(q)^\top \geq 0 \tag{9.69}$$

$$\lambda_c \geq \frac{1}{2} \tag{9.70}$$

with γ being a positive constant.

Proof Consider the closed-loop system (9.62), but with $f_e \neq 0$, i.e.,

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\hat{p}} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0_{n\times n} & \bar{T}^{-\top} & \bar{T}^{-\top}Y \\ -\bar{T}^{-1} & \bar{J}_2 - \bar{D} - \bar{G}C\bar{G}^\top & \left(\bar{J}_2 - \bar{D}\right)Y \\ -Y^\top\bar{T}^{-1} & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right) & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right)Y \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{H}_d}{\partial \bar{q}} \\ \frac{\partial \hat{H}_d}{\partial \hat{p}} \\ \frac{\partial \hat{H}_d}{\partial z} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{B} \\ 0 \end{bmatrix} f_e \tag{9.71}$$

$$\hat{y} = \bar{G}^\top \frac{\partial \hat{H}_d}{\partial \hat{p}}, \tag{9.72}$$

where the arguments of $\hat{H}_d(\bar{q}, \hat{p}, z)$, $T(\bar{q})$, $\bar{J}_2(\bar{q}, \hat{p})$, $\bar{D}(\bar{q}, \hat{p})$, $\bar{G}(\bar{q})$, and $\bar{B}(\bar{q})$ are left out for notational simplicity. We analyze the HJI (9.38) first for system (9.71) with $\mathscr{W}\left(\hat{x}\right) = \hat{H}_d\left(\hat{x}\right)$ to determine if this could be a solution. Given $\delta = f_e$ we obtain

$$-\left(\frac{\partial \hat{H}_d}{\partial \hat{x}}\right)^\top \tilde{R} \frac{\partial \hat{H}_d}{\partial \hat{x}} + \frac{1}{2}\frac{1}{\gamma^2}\left(\frac{\partial \hat{H}_d}{\partial \hat{p}}\right)^\top \bar{B}\bar{B}^\top \frac{\partial \hat{H}_d}{\partial \hat{p}} + \frac{1}{2}\hat{y}^\top\hat{y} \leq 0 \tag{9.73}$$

with $\hat{x} = (\bar{q}, \hat{p}, z)$, and

$$\tilde{R}\left(\hat{x}\right) = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \bar{D}\left(\bar{q}, \hat{p}\right) + \bar{G}\left(\bar{q}\right)C\bar{G}\left(\bar{q}\right)^\top & \bar{D}\left(\bar{q}, \hat{p}\right)Y \\ 0_{n\times n} & Y^\top\bar{D}\left(\bar{q}, \hat{p}\right) & Y^\top\bar{D}\left(\bar{q}, \hat{p}\right)Y \end{bmatrix}. \tag{9.74}$$

We compute the left-hand side term of the Hamilton–Jacobi inequality (9.38) based on the function $W\left(\hat{x}\right) = \hat{H}_d\left(\hat{x}\right)$ with $\hat{H}_d\left(\hat{x}\right)$ as in (9.64), and on the function $\mathscr{F}\left(\hat{x}\right)$ of the closed-loop (9.62). Consequently, we obtain

$$\frac{\partial \mathscr{W}}{\partial \hat{x}}^\top \mathscr{F} = \begin{bmatrix} \frac{\partial \hat{H}_d}{\partial \bar{q}} \\ \frac{\partial \hat{H}_d}{\partial \hat{p}} \\ \frac{\partial \hat{H}_d}{\partial z} \end{bmatrix}^\top \begin{bmatrix} 0 & \bar{T}^{-\top} & \bar{T}^{-\top}Y \\ -\bar{T}^{-1} & \bar{J}_2 - \bar{D} - \bar{G}C\bar{G}^\top & \left(\bar{J}_2 - \bar{D}\right)Y \\ -Y^\top\bar{T}^{-1} & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right) & -Y^\top\left(\bar{J}_2^\top + \bar{D}\right)Y \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{H}_d}{\partial \bar{q}} \\ \frac{\partial \hat{H}_d}{\partial \hat{p}} \\ \frac{\partial \hat{H}_d}{\partial z} \end{bmatrix}$$

$$= -\left(\frac{\partial \hat{H}_d}{\partial \hat{p}} + Y\frac{\partial \hat{H}_d}{\partial z}\right)^\top \bar{D}\left(\frac{\partial \hat{H}_d}{\partial \hat{p}} + Y\frac{\partial \hat{H}_d}{\partial z}\right) - \frac{\partial \hat{H}_d}{\partial \hat{p}}^\top \bar{G}C\bar{G}^\top \frac{\partial \hat{H}_d}{\partial \hat{p}}, \tag{9.75}$$

where we have left out the arguments of $\mathscr{W}\left(\hat{x}\right)$, $\mathscr{F}\left(\hat{x}\right)$, $\bar{G}\left(\bar{q}\right)$, $\bar{T}\left(\bar{q}\right)$, $\hat{H}_d\left(\bar{q}, \hat{p}, z\right)$, and $\bar{D}\left(\bar{q}, \hat{p}\right)$, for notational simplicity. From $\hat{y}$ as in (9.63), $\hat{p}$ as in (9.53), $\bar{D}\left(\bar{q}, \hat{p}\right)$ as in (9.33), $\hat{x} = \left(\bar{q}, \hat{p}, z\right)$, and $Y = AK_z$, we rewrite (9.75) as

$$\begin{aligned}\frac{\partial \mathscr{W}\left(\hat{x}\right)}{\partial \hat{x}}^\top \mathscr{F}\left(\hat{x}\right) &= -\left(\hat{p} + YK_z^{-1}z\right)^\top \bar{D}\left(\bar{q}, \hat{p}\right)\left(\hat{p} + YK_z^{-1}z\right) - \hat{y}^\top C\hat{y} \\ &= -\frac{\partial H\left(q, p\right)}{\partial p}^\top D\left(q, p\right)\frac{\partial H\left(q, p\right)}{\partial p} - \hat{y}^\top C\hat{y}.\end{aligned} \tag{9.76}$$

Based on a input matrix $\tilde{G}\left(\bar{q}\right)$ defined as

$$\tilde{G}\left(\hat{x}\right) = \begin{bmatrix} 0_{n\times n} \\ \bar{B}\left(\bar{q}\right) \\ 0_{n\times n}\end{bmatrix} \tag{9.77}$$

with $\bar{B}\left(\bar{q}\right)$ as in (9.35), we compute the second term of the left-hand side of the Hamilton–Jacobi inequality (9.38) as

$$\begin{aligned}\tilde{Z}\left(\hat{x}\right) &= \frac{1}{2}\frac{1}{\gamma^2}\left(\frac{\partial \mathscr{W}\left(\hat{x}\right)}{\partial \hat{x}}\right)^\top \tilde{G}\left(\hat{x}\right)\tilde{G}^\top\left(\hat{x}\right)\frac{\partial \mathscr{W}\left(\hat{x}\right)}{\partial \hat{x}} \\ &= \frac{1}{2}\frac{1}{\gamma^2}\hat{p}^\top \bar{B}\left(\bar{q}\right)\bar{B}\left(\bar{q}\right)^\top \hat{p}\end{aligned} \tag{9.78}$$

and we now substitute $\hat{p}$ as in (9.53) in (9.78). Hence, we obtain

$$\begin{aligned}\tilde{Z}\left(\hat{x}\right) &= \frac{1}{2}\frac{1}{\gamma^2}\left(\bar{p} - Az\right)^\top \bar{B}\left(\bar{q}\right)\bar{B}\left(\bar{q}\right)^\top\left(\bar{p} - Az\right) \\ &= \frac{1}{2}\frac{1}{\gamma^2}\left(\Upsilon\left(q, p\right)^\top \Upsilon\left(q, p\right) - \Upsilon\left(q, p\right)^\top Z - Z^\top \Upsilon\left(q, p\right) + Z^\top Z\right)\end{aligned} \tag{9.79}$$

where

$$Z\left(\hat{x}\right) = B\left(q\right)^\top T\left(q\right)^{-\top} Az \tag{9.80}$$

$$\Upsilon\left(q, p\right) = B\left(q\right)^\top \frac{\partial H\left(q, p\right)}{\partial p}. \tag{9.81}$$

Finally, based on the output $\hat{y} = h\left(\hat{x}\right)$, and the results (9.76), and (9.79), the Hamilton–Jacobi inequality (9.38) is rewritten as

$$-\frac{\partial H\left(q,p\right)}{\partial p}^{\top} D\left(q,p\right) \frac{\partial H\left(q,p\right)}{\partial p} - \hat{y}^{\top} C \hat{y} + \tilde{Z} + \frac{1}{2}\hat{y}^{\top}\hat{y} \leq 0 \tag{9.82}$$

with $\tilde{Z}\left(\hat{x}\right)$ as in (9.79). We now rewrite $\hat{y}$ as

$$\hat{y} = \bar{G}\left(\bar{q}\right)^{\top} \frac{\partial \hat{H}\left(\hat{x}\right)}{\partial \hat{p}} = \bar{G}\left(\bar{q}\right)^{\top} \hat{p} = G\left(q\right)^{\top} \frac{\partial H\left(q,p\right)}{\partial p} - G\left(q\right)^{\top} T\left(q\right)^{-1} A\hat{z} \tag{9.83}$$

and we replace (9.83) in (9.82). Lastly, we have that

$$\begin{bmatrix} \frac{\partial H\left(q,p\right)}{\partial p} \\ z \end{bmatrix}^{\top} \underbrace{\begin{bmatrix} \Gamma_1\left(q,p\right) & -\Gamma_3\left(q\right)^{\top} \\ -\Gamma_3\left(q\right) & \Gamma_2\left(q\right) \end{bmatrix}}_{P_{HJi}} \begin{bmatrix} \frac{\partial H\left(q,p\right)}{\partial p} \\ z \end{bmatrix} \leq 0 \tag{9.84}$$

The inequality (9.84) is satisfied when matrix $P_{HJi} \leq 0$, which is the case if matrix C of the control law (9.61) is designed such that the inequalities (9.67)–(9.70) hold, with $\lambda_c \in \mathbb{R}^n$ being the set of eigenvalues of C. □

Remark 9.5 The Hamilton–Jacobi inequality (9.84) based on the closed-loop system (9.71) holds when the set of eigenvalues of the matrix C are chosen such that the conditions for $\Gamma_1\left(q,p\right)$, $\Gamma_2\left(q\right)$, $\Gamma_3\left(q\right)$, and λ_c are satisfied. It follows that increasing the eigenvalues of C allows for a smaller γ, and thus, a smaller $\mathscr{L}_2$-gain bound. Increasing the eigenvalues of C corresponds to increasing the damping injection.

In the next subsection we look at the special case when f_e is unknown, but constant.

9.4.3 Stability Analysis for Constant External Forces

Here, we propose an equivalent description of the system (9.62), with a different Hamiltonian function which can be used as a Lyapunov function for constant nonzero external forces, i.e., $f_e \in \mathbb{R}^n / \{0\}$. We embed the extended system into a larger PH system for which a series of Casimir functions are constructed. The analysis is based on the results of [14].

We proceed to apply the results in Sect. 9.2.4 to the closed-loop system (9.62) with constant nonzero external forces as input, i.e., $\bar{u} = f_e$. We compute matrix $\mathscr{K}\left(\hat{x}\right)$ as in (9.40), and obtain

$$\mathscr{K}\left(\hat{x}\right) = -\begin{bmatrix} \bar{T}\left(-\bar{J}_2 + \bar{D}\right)\bar{T}^{\top} & 0_{n\times n} & \bar{T}Y^{-\top} \\ 0_{n\times n} & \left(\bar{G}C\bar{G}^{\top}\right)^{-1} & -\left(\bar{G}C\bar{G}^{\top}\right)^{-1} Y^{-\top} \\ -Y^{-1}\bar{T}^{\top} & -Y^{-1}\left(\bar{G}C\bar{G}^{\top}\right)^{-1} & Y^{-1}\left(\bar{G}C\bar{G}^{\top}\right)^{-1} Y^{-\top} \end{bmatrix} \begin{bmatrix} 0_{n\times n} \\ \bar{B}\left(\bar{q}\right) \\ 0_{n\times n} \end{bmatrix}. \tag{9.85}$$

Here, we left out the arguments of $\bar{T}(\bar{q})$, $\bar{G}(\bar{q})$, $\bar{J}_2(\bar{q}, \hat{p})$, and $\bar{D}(\bar{q}, \hat{p})$ for notational simplicity. If $\hat{G}(\bar{q}) = \left(\bar{G}(\bar{q})\, C\bar{G}(\bar{q})^\top\right)^{-1}$, then (9.85) leads to

$$\mathscr{K}\left(\hat{x}\right) = \begin{bmatrix} 0_{n\times n} \\ -\hat{G}(\bar{q})\,\bar{B}(\bar{q}) \\ Y^{-1}\hat{G}(\bar{q})\,\bar{B}(\bar{q}) \end{bmatrix}. \tag{9.86}$$

Following, the results of Theorem 9.3, we assume that the local smooth functions $\mathscr{C}_j(x)$, $j \in n$, satisfy the integrability condition (9.47). It follows that the dynamics of (9.71) can be alternatively represented by (9.49) where $H_r\left(\hat{x}\right)$ is

$$\begin{aligned} H_r\left(\hat{x}\right) &= \hat{H}_d\left(\hat{x}\right) - \sum_{j=1}^{n} f_{e_j}\mathscr{C}_j(x) \\ &= \hat{H}_d\left(\hat{x}\right) + f_e^\top \hat{G}(\bar{q})\,\hat{p} - f_e^\top Y^{-1}\hat{G}(\bar{q})\,z + f_e^\top c, \end{aligned} \tag{9.87}$$

where $\hat{x} = \left(\bar{q}, \hat{p}, z\right)$, and $\hat{H}_d\left(\hat{x}\right)$ as in (9.64). If we choose, the constant $c = -K_f f_e \in \mathbb{R}^n$, with $K_f > 0$. Then, we can rewrite (9.87) as

$$H_r\left(\hat{x}\right) = \frac{1}{2}\begin{bmatrix} \bar{q} - q_d \\ \hat{p} \\ z \\ f_e \end{bmatrix}^\top \underbrace{\begin{bmatrix} K_p & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & I_{n\times n} & 0_{n\times n} & \bar{B}^\top\hat{G}^\top \\ 0_{n\times n} & 0_{n\times n} & K_z^{-1} & -\bar{B}^\top\hat{G}^\top Y^{-\top} \\ 0_{n\times n} & \hat{G}\bar{B} & -Y^{-1}\hat{G}\bar{B} & K_f \end{bmatrix}}_{\hat{P}(\bar{q})} \begin{bmatrix} \bar{q} - q_d \\ \hat{p} \\ z \\ f_e \end{bmatrix} > 0 \tag{9.88}$$

where we have left out the arguments of $\bar{G}(\bar{q})$ and $\bar{B}(\bar{q})$ for notational simplicity. Since $\bar{G}(\bar{q})$ and $\bar{B}(\bar{q})$ are full rank, and $C > 0$, via the Schur complement it can be shown that matrix $\hat{P}(\bar{q})$ in (9.88) is positive definite, and then the inequality (9.88) holds. Furthermore, via Theorem 9.3, we have that

$$\dot{H}_r\left(\hat{x}\right) = -\frac{\partial H_r\left(\hat{x}\right)}{\partial \hat{x}}^\top \tilde{R}\left(\hat{x}\right)\frac{\partial H_r\left(\hat{x}\right)}{\partial \hat{x}} \le 0 \tag{9.89}$$

and thus $H_r\left(\hat{x}\right)$ qualifies as a Lyapunov function for the forced dynamics (9.49). Then, $\dot{H}_r\left(\hat{x}\right) \le 0$, and given that $\dfrac{\partial \hat{H}_d\left(\hat{x}\right)}{\partial \hat{p}} = \hat{p}$, and $\dfrac{\partial \hat{H}_d\left(\hat{x}\right)}{\partial z} = K_z^{-1}z$, we know that $\hat{p}, z \to 0$ as $t \to \infty$. Given the dynamics of system (9.49), $\dot{\hat{p}} = \dot{z} = 0$, it can be verified that the largest invariant set for $\dot{H}_r\left(\hat{x}\right) = 0$ equals $\left(\bar{q} - q_d - K_p^{-1}\bar{B}(\bar{q})\right.$ $\left. f_e, \hat{p}, z\right) = (0, 0, 0)$. LaSalle's invariance then implies that the system is asymptotically stable in

$$\bar{q} = q_d + K_p^{-1}\bar{B}(\bar{q})\, f_e. \tag{9.90}$$

Remark 9.6 The $\mathscr{L}_2$-gain analysis of Sect. 9.4.2 gives a bound on the relation between input $\delta = f_e$ and the output $\hat{y}$ of the closed-loop system (9.71) for an $\mathscr{L}_2$-input disturbance δ. The $\mathscr{L}_2$-gain analysis differs from the results of Sect. 9.4.3 in the sense that the $\mathscr{L}_2$-gain analysis evaluates a bound on the output $\hat{y}$ in relation to the size of the input δ, while the analysis in Sect. 9.4.3 is for the case where the system is asymptotically stable, i.e., $\hat{y} \to 0$, with a new equilibrium point caused by a constant f_e, i.e., $\bar{q}$, which is different from the desired equilibrium point q_d. Notice that the $\mathscr{L}_2$-gain bound is related to the amount of damping injected, while the new equilibrium point (steady-state position) is related to the stiffness parameter K_p.

9.4.4 Integral Position Control

The analysis in the previous section shows that, under the assumption that f_e is constant, we can expect a constant steady-state error in the position of system (9.71). Furthermore, the analysis also justifies the application of integral control, since integral control compensates for constant steady-state errors. The main contribution of this section is to realize a type of integral position control for a class of standard mechanical systems with dissipation in the PH framework. For the extended system (9.55), with f_e constant, we propose a coordinate transformation to include the position error in the new output. By having the position error in the passive output, we can interconnect the closed-loop with an integrator in a passivity-preserving way, i.e., preserving the PH structure. The results of this section are inspired by the works of [4, 5, 22].

Theorem 9.6 *Consider system (9.55) and assume $f_e \neq 0$ and constant. Define the integrator state ξ with dynamics*

$$\dot{\xi} = -\bar{B}(\bar{q})^\top (\hat{p} + K_i(\bar{q} - q_d)) \tag{9.91}$$

q_d the desired constant position and $K_i > 0$ is a constant matrix. Then, the control input

$$v = \bar{G}(\bar{q})^{-1} \left(\frac{\partial \bar{V}(\bar{q})}{\partial \bar{q}} - K_p(\bar{q} - q_d) - K_i \dot{\bar{q}} - \bar{B}(\bar{q})\xi \right) - C\bar{G}(\bar{q})^\top \left(\hat{p} + K_i(\bar{q} - q_d) \right) \tag{9.92}$$

with $K_p > 0$, and $C > 0$, asymptotically stabilizes the extended system (9.55) at $(\bar{q}, \hat{p}, z) = (q_d, 0, 0)$, i.e., zero steady-state error.

Proof We use the results of [5]. Consider first the coordinate transformation

$$\tilde{p} = \hat{p} + K_i(\bar{q} - q_d) \tag{9.93}$$

with a constant matrix $K_i > 0$, which then implies that

$$\dot{\tilde{p}} = \dot{\hat{p}} + K_i \dot{\bar{q}} \tag{9.94}$$

since q_d is constant. The control input (9.92) with integrator dynamics (9.91) then realizes the closed-loop system

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\tilde{p}} \\ \dot{z} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} -K_i K_p^{-1} & \bar{T}^{-\top} & \bar{T}^{-\top} Y & 0 \\ -\bar{T}^{-1} & \bar{J}_2 - \bar{D} - \bar{G} C \bar{G}^\top & \left(\bar{J}_2 - \bar{D}\right) Y & \bar{B} \\ -Y^\top \bar{T}^{-1} & -Y^\top \left(\bar{J}_2^\top + \bar{D}\right) & -Y^\top \left(\bar{J}_2^\top + \bar{D}\right) Y & 0 \\ 0 & -\bar{B}^\top & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{H}_i}{\partial \bar{q}} \\ \frac{\partial \hat{H}_i}{\partial \tilde{p}} \\ \frac{\partial \hat{H}_i}{\partial z} \\ \frac{\partial \hat{H}_i}{\partial \xi} \end{bmatrix} \tag{9.95}$$

$$\tilde{y} = \bar{G}^\top \frac{\partial \hat{H}_i}{\partial \tilde{p}} \tag{9.96}$$

with Hamiltonian

$$\hat{H}_i = \frac{1}{2}\tilde{p}^\top \tilde{p} + \frac{1}{2}(\bar{q} - q_d)^\top K_p (\bar{q} - q_d) + \frac{1}{2} z^\top K_z^{-1} z + \frac{1}{2}(f_e - \xi)^\top (f_e - \xi), \tag{9.97}$$

where the arguments of $\hat{H}_i\ (\bar{q}, \tilde{p}, z, \xi)$, $T(\bar{q})$, $\bar{J}_2(\bar{q}, \tilde{p})$, $\bar{D}(\bar{q}, \tilde{p})$, $\bar{G}(\bar{q})$, and $\bar{B}(\bar{q})$ are left out for notational simplicity. Furthermore, notice that

$$\tilde{y} = \bar{G}(\bar{q})^\top \tilde{p} = \bar{G}(\bar{q})^\top \left(\hat{p} + K_i(\bar{q} - q_d)\right). \tag{9.98}$$

Take (9.97) as candidate Lyapunov function, which then gives

$$\dot{\hat{H}}_i = - \begin{bmatrix} \frac{\partial \hat{H}_i}{\partial \bar{q}} \\ \frac{\partial \hat{H}_i}{\partial \tilde{p}} \\ \frac{\partial \hat{H}_i}{\partial z} \end{bmatrix}^\top \underbrace{\begin{bmatrix} K_i K_p^{-1} & 0 & 0 \\ 0 & \bar{D}\left(\bar{q}, \tilde{p}\right) + \bar{G}\left(\bar{q}\right) C \bar{G}\left(\bar{q}\right)^\top & -\bar{D}\left(\bar{q}, \tilde{p}\right) Y \\ 0 & -Y^\top \bar{D}\left(\bar{q}, \tilde{p}\right)^\top & Y^\top \bar{D}\left(\bar{q}, \tilde{p}\right)^\top Y \end{bmatrix}}_{U(\bar{q}, \tilde{p})} \begin{bmatrix} \frac{\partial \hat{H}_i}{\partial \bar{q}} \\ \frac{\partial \hat{H}_i}{\partial \tilde{p}} \\ \frac{\partial \hat{H}_i}{\partial z} \end{bmatrix}. \tag{9.99}$$

Since $\bar{G}\left(\bar{q}\right)$ is full rank, $\bar{D}\left(\bar{q}, \tilde{p}\right) \geq 0$, $K_i > 0$, $K_p > 0$, $C > 0$, $K_z > 0$, $Y = AK_z$ and A being a constant matrix, via the Schur complement it can be shown that matrix $U\left(\bar{q}, \tilde{p}\right) \geq 0$, and thus $\dot{\hat{H}}_i \leq 0$ holds. Define the set

$$\mathbb{O} = \left\{ (\bar{q}, \tilde{p}, z, \xi) \mid \dot{\hat{H}}_i (\bar{q}, \tilde{p}, z, \xi) = 0 \right\}. \tag{9.100}$$

Given that $\dot{\hat{H}}_i (q_d, 0, 0, \xi) = 0, \forall \xi$, we have that ξ is free. Assume $\xi - f_e = c_1 \neq 0$ constant with $c_1 \in \mathbb{R}^n$, thus $\dot{\xi} = 0$. Then, the dynamics $\dot{\tilde{p}}$ is

$$\dot{\tilde{p}} = \bar{B} (q_d) (c_1 + f_e) \neq 0. \tag{9.101}$$

Since (9.101) is constant, then $\tilde{p}$ will change over time, and hence, we have a contradiction. Thus, the largest invariant set in $\mathbb{O}$ is $\mathbb{M} = \{q_d, 0, 0, f_e\}$. Via LaSalle's invariance principle we conclude that the system (9.95) is asymptotically stable at $(\bar{q}, \tilde{p}, z, \xi) = (q_d, 0, 0, f_e)$, which means that the constant disturbance is compensated by ξ, i.e., $\xi \to f_e$. □

Substituting v in (9.54) by (9.92) then gives the total control input u for the original system (9.2), which in terms of the original coordinates q, p becomes

$$\begin{aligned} u = G(q)^{-1} T(q) \left(A\dot{z} + \frac{\partial V(q)}{\partial q} - K_p(q - q_d) - K_i \dot{q} \right) - G(q)^{-1} B(q) \xi \\ - C G(q)^\top \left(M(q)^{-1} p - T(q)^{-\top} A z - T(q)^{-\top} K_i (q - q_d) \right) \end{aligned} \tag{9.102}$$

with $\dot{z}$ as in (9.52).

Here, we have applied an integral type control law as in (9.92) to compensate for position errors caused by constant forces. We observe in Theorem 9.6 how our integral control strategy follows naturally from the PH structure.

9.5 Position Control with Measured Forces

In the previous section, we have presented a position control strategy that exploits feedback of the modeled internal forces. In other words, the forces used for feedback are based on the dynamical model and the measured positions and velocities. In this section, we assume we have force sensors, which provide the (real) total forces working on the system. Then, we feed back the readings of the force sensors in the input of the system (9.29). Notice that the measured total forces f in the system can be described by

$$f (q, p) = f_{in} (q, p) + B(q) f_e \tag{9.103}$$

with $f_{in} (q, p)$ as in (9.51). In the previous section, we used (9.51) to model and compute the internal forces for feedback control. We can still use (9.51) to describe the internal forces, while adding the external forces to model the total forces in the system. Let $\bar{f} (\bar{q}, \bar{p})$ be the total forces multiplied by the matrix $T(q)$ in (9.25), i.e.,

$$\bar{f} \left(\Phi^{-1} (\bar{q}, \bar{p}) \right) = \bar{f} (q, p) = T(q)^{-1} f(q, p) \tag{9.104}$$

and consider now system (9.29). Notice that in terms of the coordinates $\bar{q}$, $\bar{p}$ that $\bar{f}(\bar{q}, \bar{p})$ is then described by

$$\bar{f}(\bar{q}, \bar{p}) = -\bar{T}(\bar{q})^{-1} \frac{\partial \bar{H}(\bar{q}, \bar{p})}{\partial \bar{q}} + \left(\bar{J}_2(\bar{q}, \bar{p}) - \bar{D}(\bar{q}, \bar{p})\right) \frac{\partial \bar{H}(\bar{q}, \bar{p})}{\partial \bar{p}} + \bar{B}(\bar{q}) f_e. \tag{9.105}$$

Define for system (9.29) the input

$$v = -\bar{G}(\bar{q})^{-1} \bar{f}(\bar{q}, \bar{p}) + \bar{v} \tag{9.106}$$

with $\bar{v}$ being a new input vector, which then changes (9.29) into the PH system

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \end{bmatrix} = \begin{bmatrix} 0 & \bar{T}(\bar{q})^{-\top} \\ -\bar{T}(\bar{q})^{-1} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}_\tau(\bar{q}, \bar{p})}{\partial \bar{q}} \\ \frac{\partial \bar{H}_\tau(\bar{q}, \bar{p})}{\partial \bar{p}} \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{G}(\bar{q}) \end{bmatrix} \bar{v} \tag{9.107}$$

$$\bar{y} = \bar{G}(\bar{q})^\top \frac{\partial \bar{H}_\tau(\bar{q}, \bar{p})}{\partial \bar{p}} \tag{9.108}$$

with Hamiltonian

$$\bar{H}_\tau(\bar{q}, \bar{p}) = \frac{1}{2} \bar{p}^\top \bar{p}. \tag{9.109}$$

We then obtain (9.29), with all forces canceled. We can thus control the system without the problems described in Sect. 9.4.2. Notice that we need to describe (9.2) in the equivalent form (9.29) in order to realize force rejection and preserve the PH structure. In the original coordinates (q, p) the control input (9.106) is given by

$$u = -G(q)^{-1} f(q, p) + \bar{v} \tag{9.110}$$

with $f(q, p)$ as in (9.103). Notice that the advantage here is that we can apply control methods without having to worry about the external forces (disturbances) and internal forces (potential forces and friction). However, (9.110) implies that there is no tuning possible in the application of force feedback. In Sect. 9.4.2 the disturbances are not rejected, however, we have the possibility to tune the force feedback with the matrix A.

In the next section we illustrate, via simulation of the system (9.7), the results of Sects. 9.4 and 9.5 for obtaining asymptotic stability in a desired position.

9.6 Simulation Results: Two-DOF Shoulder System

Consider the system of Examples 9.1 and 9.2. We have determined the parameters of the two-DOF shoulder system of Fig. 9.1 as $\mathscr{I}_i = \{0.013, 1.692\}$, $F_{c_i} = \{0.005, 0.025\}$, $F_{s_i} = \{1.905, 2.257\}$, $F_{v_i} = \{4.119, 4.973\}$, and $\dot{q}_{s_i} = \{0.167, 0.170\}$.

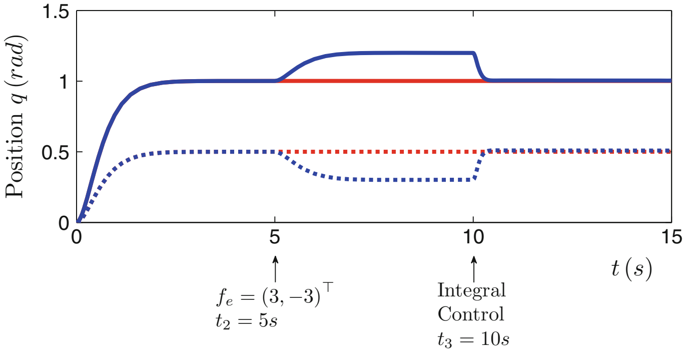

Fig. 9.3 Position control via force feedback (*blue line*) with $f_e = col\,(0, 0)^\top$ at $t \le 5s$. Integral control (*blue line*) with $f_e = col\,(3, -3)^\top$ at $t \ge 10s$. Total force rejection (*red line*) at $t \ge 0$. Initial conditions $(q\,(0)\,, p\,(0))^\top = (0, 0, 0, 0)^\top$. *Solid line* q_1. *Dashed line* q_2

We have a link length of $l_c = 0.249m$, $m = 3.9\,kg$; matrices $A = diag\,(0.5, 0.7)$, $K_z = diag\,(2, 2)$, $K_p = diag\,(15, 15)$, and $C = diag\,(10, 10)$; an initial position $q(0) = (0, 0)$, and desired position $q_d = (1, 0.5)\,rad$. We obtain the desired position $q_d = (1, 0.5)^\top$ from an initial position $q\,(0) = (0, 0)^\top$ at $t = t_1 \ge 3s$ with the control law (9.61). Then, we apply a constant nonzero force, i.e., $f_e = (3, -3)^\top$, at $t_2 =\ge 5s$, to the closed-loop system (9.62). Results are shown in Fig. 9.3. The new position is $q = K_p^{-1} f_e + q_d = (1.2, 0.3)^\top$ (blue line) which corresponds to a different equilibrium point as in (9.90). The results presented here validate the fact that the PH system (9.62) remains stable with a constant nonzero input $\bar{u} = f_e$. Furthermore, we want to recover the desired position by applying the integral control law (9.102) to the PH system (9.62) at $t_3 \ge 10s$, with a matrix $K_i = diag\,(1, 0.5)$. We observe how the system is stabilized again at the desired position q_d at $t \ge 11s$ without a steady-state error.

Finally, we apply a constant nonzero force, i.e., $f_e = (3, -3)^\top$, to the two-DOF inputs of the to the system (9.7), and apply (9.110) at $t = t_2 \ge 5s$, which includes the measured forces of the sensors. Then, the equilibrium is achieved immediately, independent of f_e as seen in Fig. 9.3.

9.7 Concluding Remarks

We have provided a method for position control via force feedback in the PH setting. The method relies on a structure preserving extension of the system. Disturbance attenuation is studied, and robustness is obtained by extending the system once more in a structure preserving way with integral type dynamics. Finally, we present a method when forces are reconstructed and fed back directly from the sensor information.

The robotic arm example shows that performance of the method is very good. Tests with the robotic arm are under way, and show promising results, also in comparison with other control methods.

References

1. Andersson, S., Söderberg, A., Björklund, S.: Friction models for sliding dry, boundary and mixed lubricated contacts. Tribol. Intern. **40**(4), 580–587 (2007)
2. Bol, M.: Force And Position Control of the Philips Experimental Robot Arm in a Energy-Based Setting. University of Groningen, Groningen (2012)
3. Canudas de Wit, C., Siciliano, B., Bastin, G.: Theory of Robot Control. Springer, London(1996)
4. Dirksz, D.A., Scherpen, J.M.A.: Power-based control: canonical coordinate transformations. Integr. Adapt. Control Autom. **48**(6), 1046–1056 (2012)
5. Donaire, A., Junco, S.: On the addition of integral action to port-controlled Hamiltonian systems. Automatica **45**, 1910–1916 (2009)
6. Duindam, V., Macchelli, A., Stramigioli, S., Bruyninckx, H. (eds.): Modeling and Control of Complex Physical Systems: The Port-Hamiltonian Approach. Springer, Berlin (2009)
7. Fujimoto, K., Sugie, T.: Canonical transformation and stabilization of generalized hamiltonian systems. Syst. Control Lett. **42**(3), 217–227 (2001)
8. Fujimoto, K., Sakurama, K., Sugie, T.: Trajectory tracking of port-controlled Hamiltonian systems via generalized canonical transformations. Automatica **39**(12), 2059–2069 (2003)
9. Gömez-Estern, F., van der Schaft, A.J.: Physical damping in IDA-PBC controlled underactuated mechanical systems. Eur. J. Control **10**(5), 451–468 (2004)
10. Gorinevsky, D., Formalsky, A., Scheiner, A.: Force Control of Robotics Systems. CRC Press LLC, Moscow (1997)
11. Khalil, H.: Nonlinear Systems, 2nd edn. Prentice-Hall, New York (2001)
12. Koop, F.: Trajectory Tracking Control of the Philips Experimental Robot Arm in the Port-Hamiltonian Framework. University of Groningen, Groningen (2014)
13. Maschke, B.M., van der Schaft, A.J.: Port-controlled hamiltonian systems: modeling origins and system-theoretic properties. IN: Procedings of the IFAC Symposium on Nonlinear Control Systems, pp. 282–288. Bordeaux, France (1992)
14. Maschke, B.M., Ortega, R., van der Schaft, A.J.: Energy-based lyapunov functions for forced hamiltonian systems with dissipation. IEEE Trans. Autom. Control **45**(8), 1498–1502 (2000)
15. Muñoz-Arias, M., Scherpen, J.M.A., Dirksz, D.A.: Force feedback of a class of standard mechanical system in the Port-Hamiltonian framework. In: Proceedings of the 20th International Symposium on Mathematical Theory of Networks and Systems, Melbourne, Australia (2012)
16. Muñoz-Arias, M., Scherpen, J.M.A., Dirksz, D.A.: A class of standard mechanical systems with force feedback in the Port-Hamiltonian framework. In: Proceedings of the 4th IFAC Workshop on Lagrangian and Hamiltonian Methods for Nonlinear Control, pp. 90–95. Bertinoro, Italy (2012)
17. Muñoz-Arias, M., Scherpen, J.M.A., Dirksz, D.A.: Position control via force feedback for a class of standard mechanical systems in the Port-Hamiltonian framework. In: Proceedings of the 52nd IEEE Conference on Decision and Control, pp. 1622–1627. Florence, Italy (2013)
18. Murray, R., Li, Z., Sastry, S.S.: A Mathematical Introduction to Robotic Manipulation. CRC Press, Boca Raton (1994)
19. Ortega, R., Loría, A., Nicklasson, P.J., Sira-Ramirez, H.: Passivity-Based Control of Euler-Lagrange Systems. Springer, Heidelberg (1998)
20. Ortega, R., van der Schaft, A.J., Maschke, B., Escobar, G.: Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems. Automatica **38**, 585–596 (2002)

21. Ortega, R., Spong, M., Gomez, F., Blankenstein, G.: Stabilization of underactuated mechanica systems via interconnection and damping assignment. IEEE Trans. Autom. Control **47**(8), 1218–1233 (2002)
22. Ortega, R., Romero, J.G.: Robust integral control of port-Hamiltonian systems: the case of non-passive outputs with unmatched disturbances. Syst. Control Lett. **61**, 11–17 (2011)
23. Rijs, R., Beekmans, R., Izmit, S., Bemelmans, D.: Philips Experimental Robot Arm: User Instructor Manual, Version 1.1. Koninklijke Philips Electronics N.V., Eindhoven (2010)
24. Siciliano, B., Kathib, O.: Springer Handbook of Robotics. Springer, Berlin (2008)
25. Spong, M., Hutchinson, S., Vidjasagar, M.: Robot Modeling and Control. Wiley, Hoboken (2006)
26. van der Schaft, A.J.: L_2-Gain and Passivity Techniques in Nonlinear Control: Lecture Notes in Control and Information Sciences 218. Springer, London (1999)
27. Viola, G., Ortega, R., Banavar, R., Acosta, J.A., Astolfi, A.: Total energy shaping control of mechanical systems: simplifying the matching equations via coordinate changes. IEEE Trans. Autom. Control **52**(6), 1093–1099 (2007)