Analysis, Adaptive Control and Synchronization of a Novel 3-D Chaotic System with a Quartic Nonlinearity and Two Quadratic Nonlinearities

Sundarapandian Vaidyanathan

Abstract In this work, we announce a seven-term novel 3-D chaotic system with a quartic nonlinearity and two quadratic nonlinearities. The proposed chaotic system is highly chaotic and it has interesting qualitative properties. The phase portraits of the novel chaotic system are illustrated and the dynamic properties of the highly chaotic system are discussed. The novel 3-D chaotic system has three unstable equilibrium points. We show that the equilibrium point at the origin is a saddle point, while the other two equilibrium points are saddle foci. The novel 3-D chaotic system has rotation symmetry about the x_3 axis, which shows that every non-trivial trajectory of the system must have a twin trajectory. The Lyapunov exponents of the novel 3-D chaotic system are obtained as $L_1 = 8.6606$, $L_2 = 0$ and $L_3 = -26.6523$, while the Kaplan-Yorke dimension of the novel chaotic system is obtained as $D_{KY} = 2.3249$. Since the Maximal Lyapunov Exponent (MLE) of the novel chaotic system has a large value, viz. $L_1 = 8.6606$, the novel chaotic system is highly chaotic. Since the sum of the Lyapunov exponents is negative, the novel chaotic system is dissipative. Next, we apply adaptive control method to derive new results for the global chaos control of the novel chaotic system with unknown parameters. We also apply adaptive control method to derive new results for the global chaos synchronization of the identical novel chaotic systems with unknown parameters. The main adaptive control results are established using Lyapunov stability theory. MATLAB simulations are shown to illustrate all the main results derived in this work.

Keywords Chaos · Chaotic systems · Chaos control · Nonlinear control · Adaptive control · Chaos synchronization

S. Vaidyanathan (✉)
Research and Development Centre, Vel Tech University,
Avadi, Chennai 600062, Tamil Nadu, India
e-mail: sundarvtu@gmail.com

A.T. Azar and S. Vaidyanathan (eds.), *Advances in Chaos Theory and Intelligent Control*, Studies in Fuzziness and Soft Computing 337,
DOI 10.1007/978-3-319-30340-6_18

1 Introduction

Chaotic systems are defined as nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and with dense periodic orbits. Sensitivity to initial conditions of chaotic systems is popularly known as the *butterfly effect*.

Chaotic systems are either conservative or dissipative. The conservative chaotic systems are characterized by the property that they are *volume conserving*. The dissipative chaotic systems are characterized by the property that any asymptotic motion of the chaotic system settles onto a set of measure zero, i.e. a strange attractor. In this research work, we shall announce and discuss a novel 3-D dissipative highly chaotic circulant chaotic system with six sinusoidal nonlinearities.

The Lyapunov exponent of a chaotic system is a measure of the divergence of points which are initially very close and this can be used to quantify chaotic systems. Each nonlinear dynamical system has a spectrum of Lyapunov exponents, which are equal in number to the dimension of the state space. The largest Lyapunov exponent of a nonlinear dynamical system is called the *maximal Lyapunov exponent* (MLE).

In the last few decades, Chaos theory has become a very important and active research field, employing many applications in different disciplines like physics, chemistry, biology, ecology, engineering and economics, among others.

Some classical paradigms of 3-D chaotic systems in the literature are Lorenz system [31], Rössler system [45], ACT system [1], Sprott systems [52], Chen system [14], Lü system [32], Cai system [13], Tigan system [64], etc.

Many new chaotic systems have been discovered in the recent years such as Zhou system [128], Zhu system [129], Li system [27], Sundarapandian systems [57, 61], Vaidyanathan systems [72, 74, 76–79, 83, 90, 100, 101, 103, 109, 111, 114, 117, 118, 120], Pehlivan system [37], Sampath system [47], Pham system [39], etc.

Chaos theory and control systems have many important applications in science and engineering [2, 9–12, 130]. Some commonly known applications are oscillators [23, 51], lasers [28, 125], chemical reactions [17, 38, 87, 88, 91, 93, 94, 98], biology [15, 25, 82, 84–86, 89, 92, 96, 97], ecology [18, 54], encryption [26, 127], cryptosystems [44, 65], mechanical systems [4–8], secure communications [16, 34, 126], robotics [33, 35, 122], cardiology [41, 124], intelligent control [3, 29], neural networks [20, 22, 30], finance [19, 53], memristors [40, 123],etc.

The control of a chaotic system aims to stabilize or regulate the system with the help of a feedback control. There are many methods available for controlling a chaotic system such as active control [55, 66, 67], adaptive control [56, 68, 73, 75, 81, 99, 110, 116, 119], sliding mode control [70, 71], backstepping control [36, 113, 121], etc.

Major works on synchronization of chaotic systems deal with the complete synchronization (CS) which has the design goal of using the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically with time.

There are many methods available for chaos synchronization such as active control [21, 48, 49, 104, 106, 112], adaptive control [46, 50, 58–60, 69, 95, 102, 105], sliding mode control [62, 80, 108, 115], backstepping control [42, 43, 63, 107], etc.

In this research work, we announce a seven-term novel 3-D chaotic system with a quartic nonlinearity and two quadratic nonlinearities. Section 2 describes the 3-D dynamic equations and phase portraits of the seven-term novel 3-D chaotic system.

Section 3 details the qualitative analysis and properties of the novel 3-D chaotic system. The Lyapunov exponents of the novel chaotic system are obtained as $L_1 = 8.6606$, $L_2 = 0$ and $L_3 = -26.6523$, while the Kaplan-Yorke dimension of the novel chaotic system is obtained as $D_{KY} = 2.3249$. Since the maximal Lyapunov exponent of the novel chaotic system has a large value, viz. $L_1 = 8.6606$, the novel chaotic system is highly chaotic.

In Sect. 4, we derive new results for the global chaos control of the novel highly chaotic system with unknown parameters. In Sect. 5, we derive new results for the global chaos synchronization of the identical novel highly chaotic systems with unknown parameters. Section 6 contains a summary of the main results derived in this work.

2 A Novel 3-D Chaotic System

In this section, we describe a seven-term novel chaotic system, which is given by the 3-D dynamics

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + qx_3 + x_2x_3 \\ \dot{x}_2 = bx_1 - x_1x_3 \\ \dot{x}_3 = -cx_3 + px_1^4 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b, c, p, q are constant, positive parameters.

The novel 3-D system (1) is a seven-term polynomial system with a quartic nonlinearity and three quadratic nonlinearities.

The system (1) exhibits a *highly chaotic attractor* for the parameter values

$$a = 12, \quad b = 55, \quad c = 6, \quad p = 25, \quad q = 0.2 \tag{2}$$

For numerical simulations, we take the initial conditions as

$$x_1(0) = 2.6, \quad x_2(0) = 1.8, \quad x_3(0) = 2.5 \tag{3}$$

Figure 1 depicts the 3-D phase portrait of the novel 3-D chaotic system (1), while Figs. 2, 3 and 4 depict the 2-D projection of the novel 3-D chaotic system (1) on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes, respectively. Figures 1, 2, 3 and 4 show that the novel 3-D chaotic system (1) exhibits a *highly chaotic* attractor.

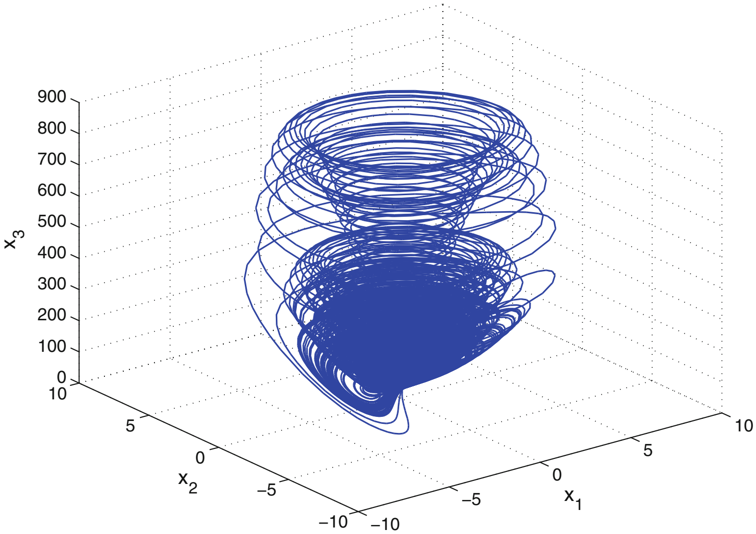

Fig. 1 3-D phase portrait of the novel 3-D chaotic system

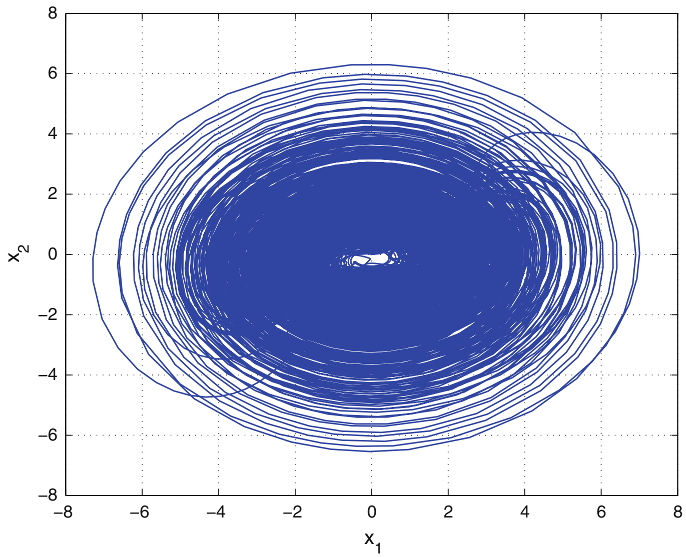

Fig. 2 2-D projection of the novel 3-D chaotic system on the (x_1, x_2) plane

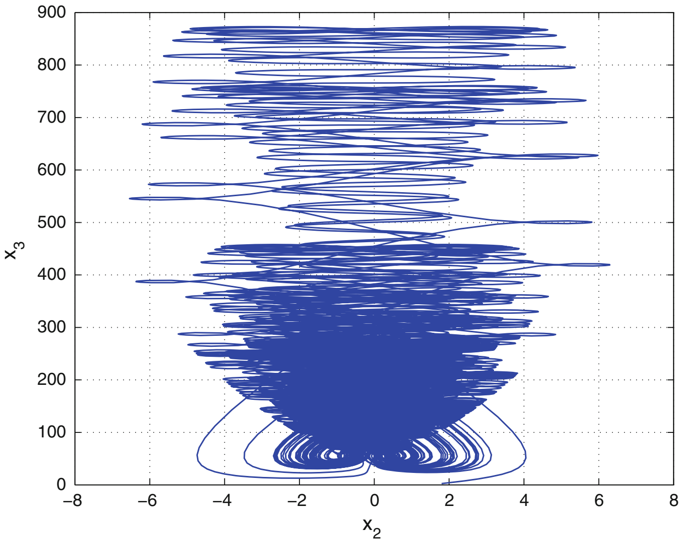

Fig. 3 2-D projection of the novel 3-D chaotic system on the (x_2, x_3) plane

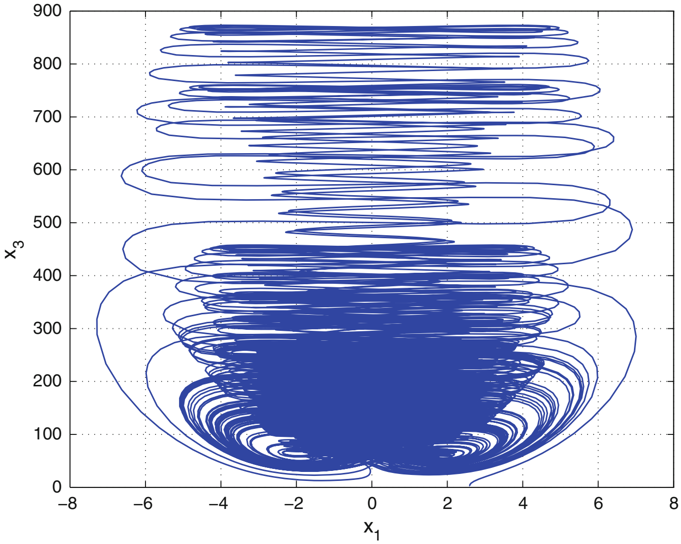

Fig. 4 2-D projection of the novel 3-D chaotic system on the (x_1, x_3) plane

3 Analysis of the Novel 3-D Chaotic System

In this section, we give a dynamic analysis of the 3-D novel highly chaotic system (1). We take the parameter values as in the chaotic case (2), i.e. $a = 12$, $b = 55$, $c = 6$, $p = 25$ and $q = 0.2$.

3.1 Dissipativity

In vector notation, the novel chaotic system (1) can be expressed as

$$\dot{\mathbf{x}} = f(\mathbf{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix}, \tag{4}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) + qx_3 + x_2 x_3 \\ f_2(x_1, x_2, x_3) = bx_1 - x_1 x_3 \\ f_3(x_1, x_2, x_3) = -cx_3 + px_1^4 \end{cases} \tag{5}$$

Let Ω be any region in $\mathbf{R}^3$ with a smooth boundary and also, $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we know that

$$\dot{V}(t) = \int\limits_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3 \tag{6}$$

The divergence of the novel chaotic system (4) is found as

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -(a + c) = -\mu < 0 \tag{7}$$

since $\mu = a + c = 18 > 0$.

Inserting the value of $\nabla \cdot f$ from (7) into (6), we get

$$\dot{V}(t) = \int\limits_{\Omega(t)} (-\mu)\, dx_1\, dx_2\, dx_3 = -\mu V(t) \tag{8}$$

Integrating the first order linear differential equation (8), we get

$$V(t) = \exp(-\mu t) V(0) \tag{9}$$

Since $\mu > 0$, it follows from Eq. (9) that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the novel chaotic system (1) is dissipative.

Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2 Equilibrium Points

We take the parameter values as in the chaotic case (2), i.e. $a = 12$, $b = 55$, $c = 6$, $p = 25$ and $q = 0.2$.

It is easy to see that the system (1) has three equilibrium points, viz.

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1.9061 \\ 0.1772 \\ 55.0000 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -1.9061 \\ -0.5056 \\ 55.0000 \end{bmatrix} \tag{10}$$

The Jacobian of the system (1) at any point $\mathbf{x} \in \mathbf{R}^3$ is calculated as

$$J(\mathbf{x}) = \begin{bmatrix} -a & a + x_3 & q + x_2 \\ b - x_3 & 0 & -x_1 \\ 4px_1^3 & 0 & -c \end{bmatrix} = \begin{bmatrix} -12 & 12 + x_3 & 0.2 + x_2 \\ 55 - x_3 & 0 & -x_1 \\ 100x_1^3 & 0 & -6 \end{bmatrix} = \tag{11}$$

The Jacobian of the system (1) at the equilibrium E_0 is obtained as

$$J_0 = J(E_0) = \begin{bmatrix} -12 & 12 & 0.2 \\ 55 & 0 & 0 \\ 0 & 0 & -6 \end{bmatrix} \tag{12}$$

We find that the matrix $J_0 = J(E_0)$ has the eigenvalues

$$\lambda_1 = -6, \quad \lambda_2 = -32.3818, \quad \lambda_3 = 20.3818 \tag{13}$$

This shows that the equilibrium point E_0 is a saddle-point, which is unstable.

The Jacobian of the system (1) at the equilibrium E_1 is obtained as

$$J_1 = J(E_1) = \begin{bmatrix} -12 & 67 & 0.3772 \\ 0 & 0 & -1.9061 \\ 692.5275 & 0 & -6 \end{bmatrix} \tag{14}$$

We find that the matrix $J_1 = J(E_1)$ has the eigenvalues

$$\lambda_1 = -53.0246, \quad \lambda_{2,3} = 17.5123 \pm 36.8953i \tag{15}$$

This shows that the equilibrium point E_1 is a saddle-focus, which is unstable.

The Jacobian of the system (1) at the equilibrium E_2 is obtained as

$$J_2 = J(E_2) = \begin{bmatrix} -12 & 67 & -0.3056 \\ 0 & 0 & 1.9061 \\ -692.5275 & 0 & -6 \end{bmatrix} \tag{16}$$

We find that the matrix $J_2 = J(E_2)$ has the eigenvalues

$$\lambda_1 = -52.6091, \quad \lambda_{2,3} = 17.3045 \pm 37.1708i \tag{17}$$

This shows that the equilibrium point E_2 is a saddle-focus, which is unstable.

3.3 Symmetry

It is easy to see that the system (1) is invariant under the change of coordinates

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \tag{18}$$

Thus, it follows that the 3-D novel chaotic system (1) has rotation symmetry about the x_3-axis.

As a consequence, we conclude that any non-trivial trajectory $\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$ of the system (1) must have a twin trajectory $\begin{bmatrix} -x_1(t) \\ -x_2(t) \\ x_3(t) \end{bmatrix}$ of the system (1).

3.4 Lyapunov Exponents and Kaplan-Yorke Dimension

We take the parameter values of the novel system (1) as in the chaotic case (2), i.e.

$$a = 12, \quad b = 55, \quad c = 6, \quad p = 25, \quad q = 0.2 \tag{19}$$

We take the initial state of the novel system (1) as given in (3), i.e.

$$x_1(0) = 2.6, \quad x_2(0) = 1.8, \quad x_3(0) = 2.5 \tag{20}$$

Then the Lyapunov exponents of the system (1) are numerically obtained as

$$L_1 = 8.6606, \quad L_2 = 0, \quad L_3 = -26.6523 \tag{21}$$

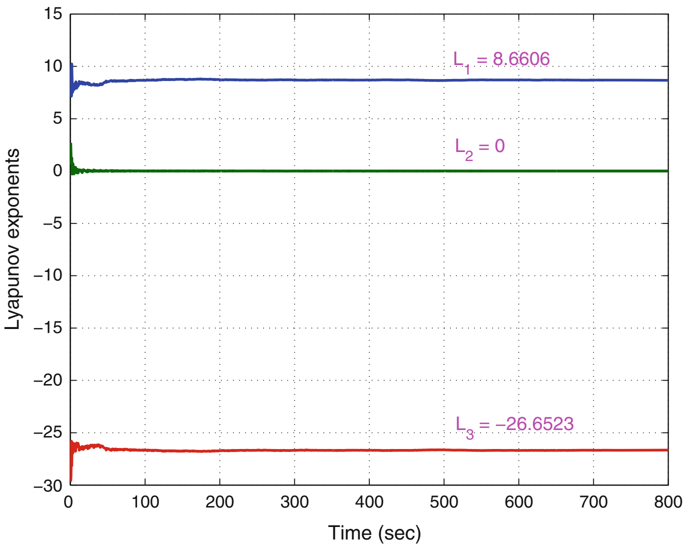

Fig. 5 Dynamics of the Lyapunov exponents of the novel chaotic system

Figure 5 shows the dynamics of the Lyapunov exponents of the novel system (1). From Fig. 5, we note that the Maximal Lyapunov Exponent (MLE) of the novel system (1) is given by $L_1 = 8.6606$, which is a large value. This shows that the novel system (1) is *highly chaotic*.

We also note that the sum of the Lyapunov exponents in (21) is negative, i.e.

$$L_1 + L_2 + L_3 = -17.9917 < 0 \tag{22}$$

This shows that the novel chaotic system (1) is dissipative.

Also, the Kaplan-Yorke dimension of the novel chaotic system (1) is found as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.3249, \tag{23}$$

which is fractional.

Also, the relatively large value of the Kaplan-Yorke dimension of the novel 3-D chaotic system (1), i.e. $D_{KY} = 2.3249$, indicates that the novel 3-D chaotic system exhibits highly complex behaviour. Hence, the novel chaotic system (1) has applications in cryptosystems, secure communication devices, etc.

4 Adaptive Control of the Novel 3-D Chaotic System

In this section, we use adaptive control method to derive an adaptive feedback control law for globally stabilizing the novel 3-D chaotic system with unknown parameters.

Thus, we consider the novel highly chaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + qx_3 + x_2x_3 + u_1 \\ \dot{x}_2 = bx_1 - x_1x_3 + u_2 \\ \dot{x}_3 = -cx_3 + px_1^4 + u_3 \end{cases} \tag{24}$$

In (24), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are the adaptive controls to be found using estimates for the unknown system parameters.

We consider the adaptive feedback control law

$$\begin{cases} u_1 = -\hat{a}(t)(x_2 - x_1) - \hat{q}(t)x_3 - x_2x_3 - k_1x_1 \\ u_2 = -\hat{b}(t)x_1 + x_1x_3 - k_2x_2 \\ u_3 = \hat{c}(t)x_3 - \hat{p}(t)x_1^4 - k_3x_3 \end{cases} \tag{25}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (25) into (24), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = \left[a - \hat{a}(t)\right](x_2 - x_1) + \left[q - \hat{q}(t)\right]x_3 - k_1x_1 \\ \dot{x}_2 = \left[b - \hat{b}(t)\right]x_1 - k_2x_2 \\ \dot{x}_3 = -\left[c - \hat{c}(t)\right]x_3 + \left[p - \hat{p}(t)\right]x_1^4 - k_3x_3 \end{cases} \tag{26}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{27}$$

Using (27), we can simplify the plant dynamics (26) as

$$\begin{cases} \dot{x}_1 = e_a(x_2 - x_1) + e_qx_3 - k_1x_1 \\ \dot{x}_2 = e_bx_1 - k_2x_2 \\ \dot{x}_3 = -e_cx_3 + e_px_1^4 - k_3x_3 \end{cases} \tag{28}$$

Differentiating (27) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{29}$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\mathbf{x}, e_a, e_b, e_c, e_p, e_q) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right) + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2\right) \tag{30}$$

Differentiating V along the trajectories of (28) and (29), we obtain

$$\begin{aligned} \dot{V} &= -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[x_1 x_2 - \dot{\hat{b}}\right] \\ &\quad + e_c\left[-x_3^2 - \dot{\hat{c}}\right] + e_p\left[x_1^4 x_3 - \dot{\hat{p}}\right] + e_q\left[x_1 x_3 - \dot{\hat{q}}\right] \end{aligned} \tag{31}$$

In view of (31), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}}(t) = x_1(x_2 - x_1) \\ \dot{\hat{b}}(t) = x_1 x_2 \\ \dot{\hat{c}}(t) = -x_3^2 \\ \dot{\hat{p}}(t) = x_1^4 x_3 \\ \dot{\hat{q}}(t) = x_1 x_3 \end{cases} \tag{32}$$

Next, we state and prove the main result of this section.

Theorem 1 *The novel 3-D highly chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (25) and the parameter update law (32), where k_1, k_2, k_3 are positive gain constants.*

Proof We prove this result by applying Lyapunov stability theory [24].

We consider the quadratic Lyapunov function defined by (30), which is clearly a positive definite function on $\mathbf{R}^8$.

By substituting the parameter update law (32) into (31), we obtain the time-derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \tag{33}$$

From (33), it is clear that $\dot{V}$ is a negative semi-definite function on $\mathbf{R}^8$.

Thus, we can conclude that the state vector $\mathbf{x}(t)$ and the parameter estimation error are globally bounded i.e.

$$\left[\, x_1(t)\ x_2(t)\ x_3(t)\ e_a(t)\ e_b(t)\ e_c(t)\ e_p(t)\ e_q(t) \,\right]^T \in \mathbf{L}_\infty.$$

We define $k = \min\{k_1, k_2, k_3\}$.

Then it follows from (33) that

$$\dot{V} \le -k\|\mathbf{x}(t)\|^2 \tag{34}$$

Thus, we have

$$k\|\mathbf{x}(t)\|^2 \le -\dot{V} \tag{35}$$

Integrating the inequality (35) from 0 to t, we get

$$k \int_0^t \|\mathbf{x}(\tau)\|^2 \, d\tau \ \le\ V(0) - V(t) \tag{36}$$

From (36), it follows that $\mathbf{x} \in \mathbf{L}_2$.

Using (28), we can conclude that $\dot{\mathbf{x}} \in \mathbf{L}_\infty$.

Using Barbalat's lemma [24], we conclude that $\mathbf{x}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{x}(0) \in \mathbf{R}^3$.

Hence, the novel highly chaotic system (24) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (25) and the parameter update law (32).

This completes the proof. ■

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (24) and (32), when the adaptive control law (25) is applied.

The parameter values of the novel 3-D chaotic system (24) are taken as in the chaotic case (2), i.e.

$$a = 12, \quad b = 55, \quad c = 6, \quad p = 25, \quad q = 0.2 \tag{37}$$

We take the positive gain constants as $k_i = 8$ for $i = 1, 2, 3$.

Furthermore, as initial conditions of the novel highly chaotic system (24), we take

$$x_1(0) = 7.4, \quad x_2(0) = -10.5, \quad x_3(0) = 12.1 \tag{38}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 3.2, \quad \hat{b}(0) = 12.3, \quad \hat{c}(0) = 13.4, \quad \hat{p}(0) = 17.8, \quad \hat{q}(0) = 16.7 \tag{39}$$

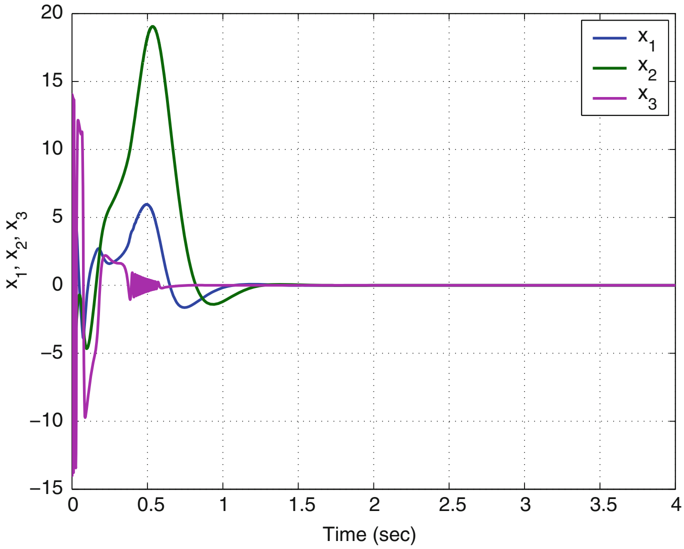

Fig. 6 Time-history of the controlled states x_1, x_2, x_3

In Fig. 6, the exponential convergence of the controlled states of the novel 3-D chaotic system (24) is depicted. From Fig. 6, we see that the controlled states $x_1(t)$, $x_2(t)$, $x_3(t)$ converge to zero in just two seconds.

This shows the efficiency of the adaptive controller designed in this section for the novel 3-D chaotic system (24).

5 Adaptive Synchronization of the Identical Novel 3-D Chaotic Systems

In this section, we apply adaptive control method to derive an adaptive feedback control law for globally synchronizing identical novel 3-D chaotic systems with unknown parameters. The main result is established using Lyapunov stability theory.

As the master system, we consider the novel 3-D chaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + qx_3 + x_2x_3 \\ \dot{x}_2 = bx_1 - x_1x_3 \\ \dot{x}_3 = -cx_3 + px_1^4 \end{cases} \tag{40}$$

In (40), x_1, x_2, x_3 are the states and a, b, c, p, q are unknown system parameters.

As the slave system, we consider the novel 3-D chaotic system given by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + qy_3 + y_2 y_3 + u_1 \\ \dot{y}_2 = by_1 - y_1 y_3 + u_2 \\ \dot{y}_3 = -cy_3 + py_1^4 + u_3 \end{cases} \tag{41}$$

In (41), y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the adaptive controls to be determined using estimates of the unknown system parameters.

The synchronization error between the novel chaotic systems is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{42}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + qe_3 + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 = be_1 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 = -ce_3 + p(y_1^4 - x_1^4) + u_3 \end{cases} \tag{43}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) - \hat{q}(t)e_3 - y_2 y_3 + x_2 x_3 - k_1 e_1 \\ u_2 = -\hat{b}(t)e_1 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 = \hat{c}(t)e_3 - \hat{p}(t)(y_1^4 - x_1^4) - k_3 e_3 \end{cases} \tag{44}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (44) into (43), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 = \left[a - \hat{a}(t)\right](e_2 - e_1) + \left[q - \hat{q}(t)\right]e_3 - k_1 e_1 \\ \dot{e}_2 = \left[b - \hat{b}(t)\right]e_1 - k_2 e_2 \\ \dot{e}_3 = -\left[c - \hat{c}(t)\right]e_3 + \left[p - \hat{p}(t)\right](y_1^4 - x_1^4) - k_3 e_3 \end{cases} \tag{45}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{46}$$

In view of (46), we can simplify the error dynamics (45) as

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) + e_q e_3 - k_1 e_1 \\ \dot{e}_2 = e_b e_1 - k_2 e_2 \\ \dot{e}_3 = -e_c e_3 + e_p(y_1^4 - x_1^4) - k_3 e_3 \end{cases} \tag{47}$$

Differentiating (46) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{48}$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\mathbf{e}, e_a, e_b, e_c, e_p, e_q) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2\right) \tag{49}$$

Differentiating V along the trajectories of (47) and (48), we obtain

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] + e_b\left[e_1 e_2 - \dot{\hat{b}}\right] \\ &+ e_c\left[-e_3^2 - \dot{\hat{c}}\right] + e_p\left[e_3(y_1^4 - x_1^4) - \dot{\hat{p}}\right] + e_q\left[e_1 e_3 - \dot{\hat{q}}\right] \end{aligned} \tag{50}$$

In view of (50), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}}(t) = e_1(e_2 - e_1) \\ \dot{\hat{b}}(t) = e_1 e_2 \\ \dot{\hat{c}}(t) = -e_3^2 \\ \dot{\hat{p}}(t) = e_3(y_1^4 - x_1^4) \\ \dot{\hat{q}}(t) = e_1 e_3 \end{cases} \tag{51}$$

Next, we state and prove the main result of this section.

Theorem 2 *The novel 3-D chaotic systems (40) and (41) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (44) and the parameter update law (51), where k_1, k_2, k_3 are positive gain constants.*

Proof We prove this result by applying Lyapunov stability theory [24].

We consider the quadratic Lyapunov function defined by (49), which is clearly a positive definite function on $\mathbf{R}^8$.

By substituting the parameter update law (51) into (50), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{52}$$

From (52), it is clear that $\dot{V}$ is a negative semi-definite function on $\mathbf{R}^8$.

Thus, we can conclude that the error vector $\mathbf{e}(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[\, e_1(t)\ e_2(t)\ e_3(t)\ e_a(t)\ e_b(t)\ e_c(t)\ e_p(t)\ e_q(t) \,\right]^T \in \mathbf{L}_\infty. \tag{53}$$

We define $k = \min\{k_1, k_2, k_3\}$.

Then it follows from (52) that

$$\dot{V} \leq -k\|\mathbf{e}(t)\|^2 \tag{54}$$

Thus, we have

$$k\|\mathbf{e}(t)\|^2 \leq -\dot{V} \tag{55}$$

Integrating the inequality (55) from 0 to t, we get

$$k \int_0^t \|\mathbf{e}(\tau)\|^2 \, d\tau \ \leq \ V(0) - V(t) \tag{56}$$

From (56), it follows that $\mathbf{e} \in \mathbf{L}_2$.

Using (47), we can conclude that $\dot{\mathbf{e}} \in \mathbf{L}_\infty$.

Using Barbalat's lemma [24], we conclude that $\mathbf{e}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{e}(0) \in \mathbf{R}^3$.

This completes the proof. ■

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (40), (41) and (51), when the adaptive control law (44) is applied.

The parameter values of the novel chaotic systems are taken as in the chaotic case (2), i.e. $a = 12$, $b = 55$, $c = 6$, $p = 25$ and $q = 0.2$.

We take the positive gain constants as $k_i = 8$ for $i = 1, 2, 3$.

Furthermore, as initial conditions of the master system (40), we take

$$x_1(0) = 12.5, \quad x_2(0) = 20.7, \quad x_3(0) = -5.3 \tag{57}$$

As initial conditions of the slave system (41), we take

$$y_1(0) = 6.8, \quad y_2(0) = 4.5, \quad y_3(0) = 11.4 \tag{58}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 3.1, \quad \hat{b}(0) = 4.3, \quad \hat{c}(0) = 10.2, \quad \hat{p}(0) = 5.9, \quad \hat{q}(0) = 7.5 \tag{59}$$

Figures 7, 8 and 9 describe the complete synchronization of the novel 3-D chaotic systems (40) and (41), while Fig. 10 describes the time-history of the complete synchronization errors e_1, e_2, e_3.

From Fig. 7, we see that the states x_1 and y_1 are synchronized in just two seconds. From Fig. 8, we see that the states x_2 and y_2 are synchronized in just two seconds. From Fig. 9, we see that the states x_3 and y_3 are synchronized in just two seconds. From Fig. 10, we see that the errors e_1, e_2, e_3 converge to zero in just two seconds. This shows the efficiency of the adaptive controller developed in this section for the synchronization of identical 3-D chaotic systems.

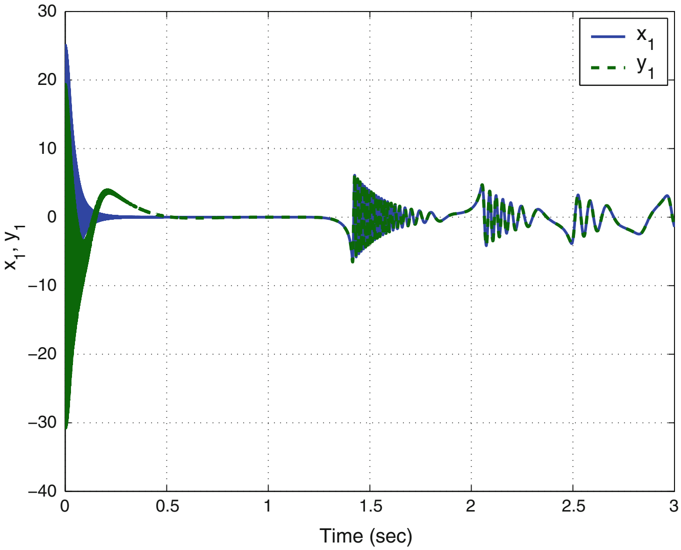

Fig. 7 Synchronization of the states x_1 and y_1

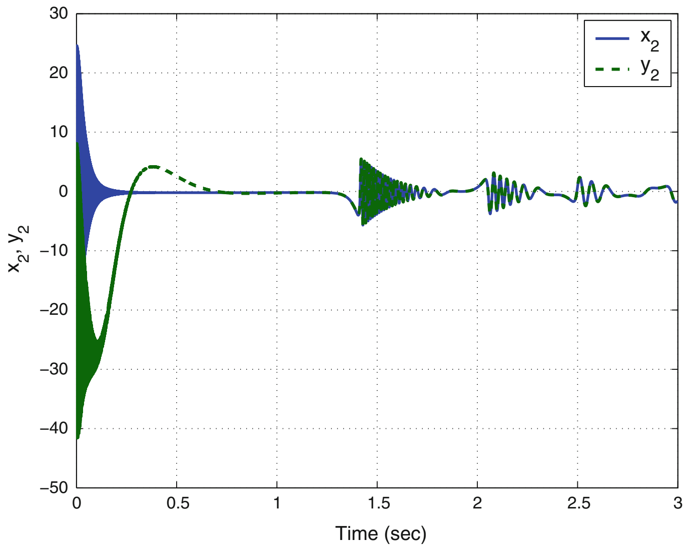

Fig. 8 Synchronization of the states x_2 and y_2

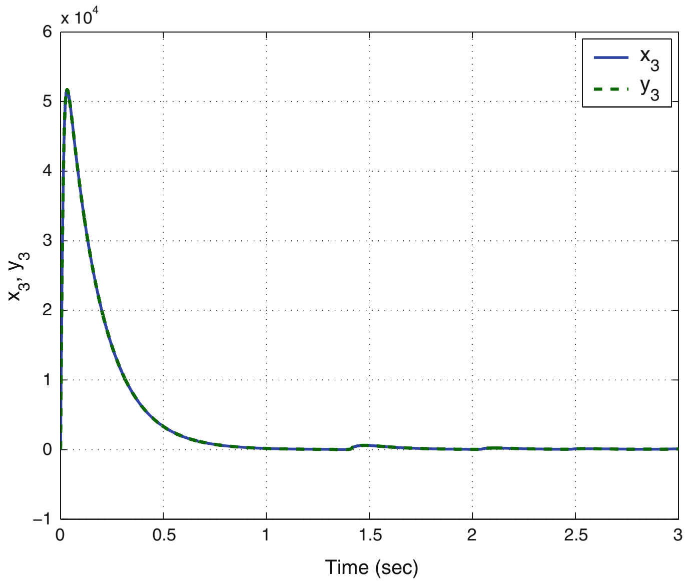

Fig. 9 Synchronization of the states x_3 and y_3

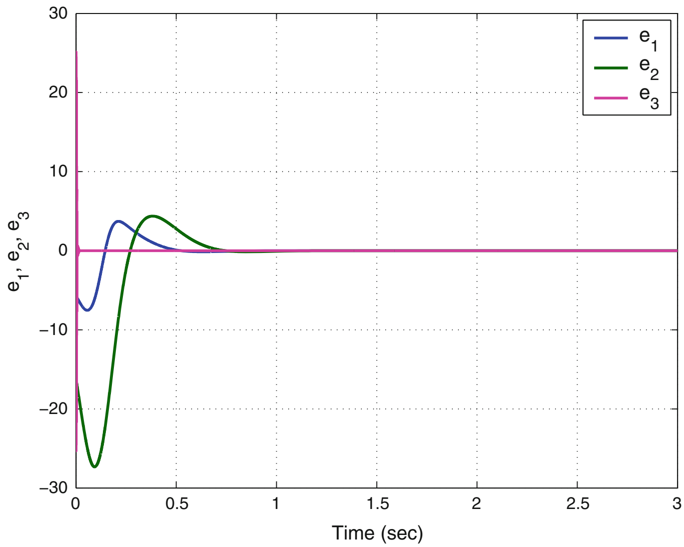

Fig. 10 Time-history of the synchronization errors e_1, e_2, e_3

6 Conclusions

In this work, we announced a seven-term novel 3-D chaotic system with a quartic nonlinearity and two quadratic nonlinearities. The dynamic properties of the novel 3-D chaotic system were discussed and phase portraits of the novel chaotic system were depicted. The novel 3-D chaotic system has three unstable equilibrium points. We showed that the equilibrium point at the origin is a saddle point, while the other two equilibrium points are saddle foci. The novel 3-D chaotic system has rotation symmetry about the x_3 axis, which shows that every non-trivial trajectory of the system must have a twin trajectory. The Lyapunov exponents of the novel 3-D chaotic system have been obtained as $L_1 = 8.6606$, $L_2 = 0$ and $L_3 = -26.6523$, while the Kaplan-Yorke dimension of the novel chaotic system has been obtained as $D_{KY} = 2.3249$. Since the Maximal Lyapunov Exponent (MLE) of the novel chaotic system has a large value, viz. $L_1 = 8.6606$, it follows that the novel chaotic system is highly chaotic. Since the sum of the Lyapunov exponents is negative, the novel chaotic system is dissipative. Next, we derived new results for the global chaos control of the novel chaotic system with unknown parameters via adaptive control method. We also derived new results for the global chaos synchronization of the identical novel chaotic systems with unknown parameters via adaptive control method. The main adaptive control results were proved using Lyapunov stability theory. MATLAB simulations have been shown to illustrate all the main results developed in this work.

References

1. Arneodo A, Coullet P, Tresser C (1981) Possible new strange attractors with spiral structure. Commun Math Phys 79(4):573–576
2. Azar AT (2010) Fuzzy systems. IN-TECH, Vienna, Austria
3. Azar AT (2012) Overview of type-2 fuzzy logic systems. Int J Fuzzy Syst Appl 2(4):1–28
4. Azar AT, Serrano FE (2014) Robust IMC-PID tuning for cascade control systems with gain and phase margin specifications. Neural Comput Appl 25(5):983–995
5. Azar AT, Serrano FE (2015) Adaptive sliding mode control of the Furuta pendulum. In: Azar AT, Zhu Q (eds) Advances and applications in sliding mode control systems. Studies in computational intelligence, vol 576. Springer, Berlin, pp 1–42
6. Azar AT, Serrano FE (2015) Deadbeat control for multivariable systems with time varying delays. In: Azar AT, Vaidyanathan S (eds) Chaos modeling and control systems design. Studies in computational intelligence, vol 576. Springer, Berlin, pp 97–132
7. Azar AT, Serrano FE (2015) Design and modeling of anti wind up PID controllers. In: Zhu Q, Azar AT (eds) Complex system modelling and control through intelligent soft computations. Studies in computational intelligence, vol 319. Springer, Berlin, pp 1–44
8. Azar AT, Serrano FE (2015) Stabilizatoin and control of mechanical systems with backlash. In: Azar AT, Vaidyanathan S (eds) Handbook of research on advanced intelligent control engineering and automation. Advances in computational intelligence and robotics (ACIR). IGI-Global, USA, pp 1–60
9. Azar AT, Vaidyanathan S (2015) Chaos modeling and control systems design. Studies in computational intelligence, vol 581. Springer, Berlin
10. Azar AT, Vaidyanathan S (2015) Computational intelligence applications in modeling and control. Studies in computational intelligence, vol 575. Springer, Berlin
11. Azar AT, Vaidyanathan S (2015) Handbook of research on advanced intelligent control engineering and automation. Advances in computational intelligence and robotics (ACIR). IGI-Global, USA
12. Azar AT, Zhu Q (2015) Advances and applications in sliding mode control systems. Studies in computational intelligence, vol 576. Springer, Berlin
13. Cai G, Tan Z (2007) Chaos synchronization of a new chaotic system via nonlinear control. J Uncertain Syst 1(3):235–240
14. Chen G, Ueta T (1999) Yet another chaotic attractor. Int J Bifurc Chaos 9(7):1465–1466
15. Das S, Goswami D, Chatterjee S, Mukherjee S (2014) Stability and chaos analysis of a novel swarm dynamics with applications to multi-agent systems. Eng Appl Artif Intell 30:189–198
16. Feki M (2003) An adaptive chaos synchronization scheme applied to secure communication. Chaos, Solitons Fractals 18(1):141–148
17. Gaspard P (1999) Microscopic chaos and chemical reactions. Physica A 263(1–4):315–328
18. Gibson WT, Wilson WG (2013) Individual-based chaos: Extensions of the discrete logistic model. J Theor Biol 339:84–92
19. Guégan D (2009) Chaos in economics and finance. Annu Rev Control 33(1):89–93
20. Huang X, Zhao Z, Wang Z, Li Y (2012) Chaos and hyperchaos in fractional-order cellular neural networks. Neurocomputing 94:13–21
21. Karthikeyan R, Sundarapandian V (2014) Hybrid chaos synchronization of four-scroll systems via active control. J Electr Eng 65(2):97–103
22. Kaslik E, Sivasundaram S (2012) Nonlinear dynamics and chaos in fractional-order neural networks. Neural Netw 32:245–256
23. Kengne J, Chedjou JC, Kenne G, Kyamakya K (2012) Dynamical properties and chaos synchronization of improved Colpitts oscillators. Commun Nonlinear Sci Numer Simul 17(7):2914–2923
24. Khalil HK (2001) Nonlinear systems, 3rd edn. Prentice Hall, New Jersey
25. Kyriazis M (1991) Applications of chaos theory to the molecular biology of aging. Exp Gerontol 26(6):569–572

26. Lang J (2015) Color image encryption based on color blend and chaos permutation in the reality-preserving multiple-parameter fractional Fourier transform domain. Opt Commun 338:181–192
27. Li D (2008) A three-scroll chaotic attractor. Phys Lett A 372(4):387–393
28. Li N, Pan W, Yan L, Luo B, Zou X (2014) Enhanced chaos synchronization and communication in cascade-coupled semiconductor ring lasers. Commun Nonlinear Sci Numer Simul 19(6):1874–1883
29. Li Z, Chen G (2006) Integration of fuzzy logic and chaos theory. Studies in fuzziness and soft computing, vol 187. Springer, Berlin
30. Lian S, Chen X (2011) Traceable content protection based on chaos and neural networks. Appl Soft Comput 11(7):4293–4301
31. Lorenz EN (1963) Deterministic periodic flow. J Atmos Sci 20(2):130–141
32. Lü J, Chen G (2002) A new chaotic attractor coined. Int J Bifurc Chaos 12(3):659–661
33. Mondal S, Mahanta C (2014) Adaptive second order terminal sliding mode controller for robotic manipulators. J Franklin Inst 351(4):2356–2377
34. Murali K, Lakshmanan M (1998) Secure communication using a compound signal from generalized chaotic systems. Phys Lett A 241(6):303–310
35. Nehmzow U, Walker K (2005) Quantitative description of robot-environment interaction using chaos theory. Robot Auton Syst 53(3–4):177–193
36. Njah AN, Sunday OD (2009) Generalization on the chaos control of 4-D chaotic systems using recursive backstepping nonlinear controller. Chaos, Solitons Fractals 41(5):2371–2376
37. Pehlivan I, Moroz IM, Vaidyanathan S (2014) Analysis, synchronization and circuit design of a novel butterfly attractor. J Sound Vib 333(20):5077–5096
38. Petrov V, Gaspar V, Masere J, Showalter K (1993) Controlling chaos in Belousov-Zhabotinsky reaction. Nature 361:240–243
39. Pham VT, Vaidyanathan S, Volos CK, Jafari S (2015) Hidden attractors in a chaotic system with an exponential nonlinear term. Eur Phys J: Spec Topics 224(8):1507–1517
40. Pham VT, Volos CK, Vaidyanathan S, Le TP, Vu VY (2015) A memristor-based hyperchaotic system with hidden attractors: dynamics, synchronization and circuital emulating. J Eng Sci Technol Rev 8(2):205–214
41. Qu Z (2011) Chaos in the genesis and maintenance of cardiac arrhythmias. Prog Biophys Mol Biol 105(3):247–257
42. Rasappan S, Vaidyanathan S (2013) Hybrid synchronization of n-scroll Chua circuits using adaptive backstepping control design with recursive feedback. Malays J Math Sci 73(1):73–95
43. Rasappan S, Vaidyanathan S (2014) Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design. Kyungpook Math J 54(1): 293–320
44. Rhouma R, Belghith S (2011) Cryptoanalysis of a chaos based cryptosystem on DSP. Commun Nonlinear Sci Numer Simul 16(2):876–884
45. Rössler OE (1976) An equation for continuous chaos. Phys Lett A 57(5):397–398
46. Vaidyanathan S, VTP, Volos CK, (2015) A 5-D hyperchaotic Rikitake dynamo system with hidden attractors. Eur Phys J: Spec Topics 224(8):1575–1592
47. Sampath S, Vaidyanathan S, Volos CK, Pham VT (2015) An eight-term novel four-scroll chaotic system with cubic nonlinearity and its circuit simulation. J Eng Sci Technol Rev 8(2):1–6
48. Sarasu P, Sundarapandian V (2011) Active controller design for generalized projective synchronization of four-scroll chaotic systems. Int J Syst Signal Control Eng Appl 4(2):26–33
49. Sarasu P, Sundarapandian V (2011) The generalized projective synchronization of hyperchaotic Lorenz and hyperchaotic Qi systems via active control. Int J Soft Comput 6(5): 216–223
50. Sarasu P, Sundarapandian V (2012) Generalized projective synchronization of two-scroll systems via adaptive control. Int J Soft Comput 7(4):146–156
51. Sharma A, Patidar V, Purohit G, Sud KK (2012) Effects on the bifurcation and chaos in forced Duffing oscillator due to nonlinear damping. Commun Nonlinear Sci Numer Simul 17(6):2254–2269

52. Sprott JC (1994) Some simple chaotic flows. Phys Rev E 50(2):647–650
53. Sprott JC (2004) Competition with evolution in ecology and finance. Phys Lett A 325(5–6):329–333
54. Suérez I (1999) Mastering chaos in ecology. Ecol Model 117(2–3):305–314
55. Sundarapandian V (2010) Output regulation of the Lorenz attractor. Far East J Math Sci 42(2):289–299
56. Sundarapandian V (2013) Adaptive control and synchronization design for the Lu-Xiao chaotic system. Lecture notes in electrical engineering, vol 131, pp 319–327
57. Sundarapandian V (2013) Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers. J Eng Sci Technol Rev 6(4):45–52
58. Sundarapandian V, Karthikeyan R (2011) Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control. Int J Syst Signal Control Eng Appl 4(2):18–25
59. Sundarapandian V, Karthikeyan R (2011) Anti-synchronization of Lü and Pan chaotic systems by adaptive nonlinear control. Eur J Sci Res 64(1):94–106
60. Sundarapandian V, Karthikeyan R (2012) Adaptive anti-synchronization of uncertain Tigan and Li systems. J Eng Appl Sci 7(1):45–52
61. Sundarapandian V, Pehlivan I (2012) Analysis, control, synchronization, and circuit design of a novel chaotic system. Math Comput Model 55(7–8):1904–1915
62. Sundarapandian V, Sivaperumal S (2011) Sliding controller design of hybrid synchronization of four-wing chaotic systems. Int J Soft Comput 6(5):224–231
63. Suresh R, Sundarapandian V (2013) Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with recursive feedback. Far East J Math Sci 7(2):219–246
64. Tigan G, Opris D (2008) Analysis of a 3D chaotic system. Chaos, Solitons Fractals 36: 1315–1319
65. Usama M, Khan MK, Alghatbar K, Lee C (2010) Chaos-based secure satellite imagery cryptosystem. Comput Math Appl 60(2):326–337
66. Vaidyanathan S (2011) Output regulation of Arneodo-Coullet chaotic system. Commun Comput Inf Sci 133:98–107
67. Vaidyanathan S (2011) Output regulation of the unified chaotic system. Commun Comput Inf Sci 198:1–9
68. Vaidyanathan S (2012) Adaptive controller and syncrhonizer design for the Qi-Chen chaotic system. Lecture notes of the institute for computer sciences. Social-informatics and telecommunications engineering, vol 84, pp 73–82
69. Vaidyanathan S (2012) Anti-synchronization of Sprott-L and Sprott-M chaotic systems via adaptive control. Int J Control Theory Appl 5(1):41–59
70. Vaidyanathan S (2012) Global chaos control of hyperchaotic Liu system via sliding control method. Int J Control Theory Appl 5(2):117–123
71. Vaidyanathan S (2012) Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system. Int J Control Theory Appl 5(1):15–20
72. Vaidyanathan S (2013) A new six-term 3-D chaotic system with an exponential nonlinearity. Far East J Math Sci 79(1):135–143
73. Vaidyanathan S (2013) A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control. Int J Control Theory Appl 6(2):97–109
74. Vaidyanathan S (2013) Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters. J Eng Sci Technol Rev 6(4):53–65
75. Vaidyanathan S (2013) Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control. Adv Intell Syst Comput 177:1–10
76. Vaidyanathan S (2014) A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities. Far East J Math Sci 84(2):219–226
77. Vaidyanathan S (2014) Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities. Eur Phys J: Spec Topics 223(8): 1519–1529

78. Vaidyanathan S (2014) Analysis, control and synchronisation of a six-term novel chaotic system with three quadratic nonlinearities. Int J Model Ident Control 22(1):41–53
79. Vaidyanathan S (2014) Generalised projective synchronisation of novel 3-D chaotic systems with an exponential non-linearity via active and adaptive control. Int J Model Ident Control 22(3):207–217
80. Vaidyanathan S (2014) Global chaos synchronisation of identical Li-Wu chaotic systems via sliding mode control. Int J Model Ident Control 22(2):170–177
81. Vaidyanathan S (2014) Qualitative analysis and control of an eleven-term novel 4-D hyperchaotic system with two quadratic nonlinearities. Int J Control Theory Appl 7:35–47
82. Vaidyanathan S (2015) 3-cells cellular neural network (CNN) attractor and its adaptive biological control. Int J PharmTech Res 8(4):632–640
83. Vaidyanathan S (2015) A 3-D novel highly chaotic system with four quadratic nonlinearities, its adaptive control and anti-synchronization with unknown parameters. J Eng Sci Technol Rev 8(2):106–115
84. Vaidyanathan S (2015) Adaptive backstepping control of enzymes-substrates system with ferroelectric behaviour in brain waves. Int J PharmTech Res 8(2):256–261
85. Vaidyanathan S (2015) Adaptive biological control of generalized Lotka-Volterra three-species biological system. Int J PharmTech Res 8(4):622–631
86. Vaidyanathan S (2015) Adaptive chaotic synchronization of enzymes-substrates system with ferroelectric behaviour in brain waves. Int J PharmTech Res 8(5):964–973
87. Vaidyanathan S (2015) Adaptive control of a chemical chaotic reactor. Int J PharmTech Res 8(3):377–382
88. Vaidyanathan S (2015) Adaptive synchronization of chemical chaotic reactors. Int J ChemTech Res 8(2):612–621
89. Vaidyanathan S (2015) Adaptive synchronization of generalized Lotka-Volterra three-species biological systems. Int J PharmTech Res 8(5):928–937
90. Vaidyanathan S (2015) Analysis, properties and control of an eight-term 3-D chaotic system with an exponential nonlinearity. Int J Model Ident Control 23(2):164–172
91. Vaidyanathan S (2015) Anti-synchronization of Brusselator chemical reaction systems via adaptive control. Int J ChemTech Res 8(6):759–768
92. Vaidyanathan S (2015) Chaos in neurons and adaptive control of Birkhoff-Shaw strange chaotic attractor. Int J PharmTech Res 8(5):956–963
93. Vaidyanathan S (2015) Dynamics and control of Brusselator chemical reaction. Int J ChemTech Res 8(6):740–749
94. Vaidyanathan S (2015) Dynamics and control of Tokamak system with symmetric and magnetically confined plasma. Int J ChemTech Res 8(6):795–803
95. Vaidyanathan S (2015) Hyperchaos, qualitative analysis, control and synchronisation of a ten-term 4-D hyperchaotic system with an exponential nonlinearity and three quadratic nonlinearities. Int J Model Ident Control 23(4):380–392
96. Vaidyanathan S (2015) Lotka-Volterra population biology models with negative feedback and their ecological monitoring. Int J PharmTech Res 8(5):974–981
97. Vaidyanathan S (2015) Synchronization of 3-cells cellular neural network (CNN) attractors via adaptive control method. Int J PharmTech Res 8(5):946–955
98. Vaidyanathan S (2015) Synchronization of Tokamak systems with symmetric and magnetically confined plasma via adaptive control. Int J ChemTech Res 8(6):818–827
99. Vaidyanathan S, Azar AT (2015) Analysis and control of a 4-D novel hyperchaotic system. Studies in computational intelligence, vol 581, pp 3–17
100. Vaidyanathan S, Azar AT (2015) Analysis, control and synchronization of a nine-term 3-D novel chaotic system. In: Azar AT, Vaidyanathan S (eds) Chaos modelling and control systems design. Studies in computational intelligence, vol 581. Springer, Berlin, pp 19–38
101. Vaidyanathan S, Madhavan K (2013) Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system. Int J Control Theory Appl 6(2):121–137
102. Vaidyanathan S, Pakiriswamy S (2013) Generalized projective synchronization of six-term Sundarapandian chaotic systems by adaptive control. Int J Control Theory Appl 6(2):153–163

103. Vaidyanathan S, Pakiriswamy S (2015) A 3-D novel conservative chaotic system and its generalized projective synchronization via adaptive control. J Eng Sci Technol Rev 8(2): 52–60
104. Vaidyanathan S, Rajagopal K (2011) Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control. Int J Syst Signal Control Eng Appl 4(3):55–61
105. Vaidyanathan S, Rajagopal K (2012) Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control. Int J Soft Comput 7(1):28–37
106. Vaidyanathan S, Rasappan S (2011) Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control. Commun Comput Inf Sci 198:10–17
107. Vaidyanathan S, Rasappan S (2014) Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback. Arab J Sci Eng 39(4):3351–3364
108. Vaidyanathan S, Sampath S (2012) Anti-synchronization of four-wing chaotic systems via sliding mode control. Int J Autom Comput 9(3):274–279
109. Vaidyanathan S, Volos C (2015) Analysis and adaptive control of a novel 3-D conservative no-equilibrium chaotic system. Arch Control Sci 25(3):333–353
110. Vaidyanathan S, Volos C, Pham VT (2014) Hyperchaos, adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation. Arch Control Sci 24(4):409–446
111. Vaidyanathan S, Volos C, Pham VT, Madhavan K, Idowu BA (2014) Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities. Arch Control Sci 24(3):375–403
112. Vaidyanathan S, Azar AT, Rajagopal K, Alexander P (2015) Design and SPICE implementation of a 12-term novel hyperchaotic system and its synchronisation via active control. Int J Model Ident Control 23(3):267–277
113. Vaidyanathan S, Idowu BA, Azar AT (2015) Backstepping controller design for the global chaos synchronization of Sprott's jerk systems. Studies in computational intelligence, vol 581, pp 39–58
114. Vaidyanathan S, Rajagopal K, Volos CK, Kyprianidis IM, Stouboulos IN (2015) Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system with three quadratic nonlinearities and its digital implementation in LabVIEW. J Eng Sci Technol Rev 8(2):130–141
115. Vaidyanathan S, Sampath S, Azar AT (2015) Global chaos synchronisation of identical chaotic systems via novel sliding mode control method and its application to Zhu system. Int J Model Ident Control 23(1):92–100
116. Vaidyanathan S, Volos C, Pham VT, Madhavan K (2015) Analysis, adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation. Nonlinear Dyn 25(1):135–158
117. Vaidyanathan S, Volos CK, Kyprianidis IM, Stouboulos IN, Pham VT (2015) Analysis, adaptive control and anti-synchronization of a six-term novel jerk chaotic system with two exponential nonlinearities and its circuit simulation. J Eng Sci Technol Rev 8(2):24–36
118. Vaidyanathan S, Volos CK, Pham VT (2015) Analysis, adaptive control and adaptive synchronization of a nine-term novel 3-D chaotic system with four quadratic nonlinearities and its circuit simulation. J Eng Sci Technol Rev 8(2):181–191
119. Vaidyanathan S, Volos CK, Pham VT (2015) Analysis, control, synchronization and SPICE implementation of a novel 4-D hyperchaotic Rikitake dynamo system without equilibrium. J Eng Sci Technol Rev 8(2):232–244
120. Vaidyanathan S, Volos CK, Pham VT (2015) Global chaos control of a novel nine-term chaotic system via sliding mode control. In: Azar AT, Zhu Q (eds) Advances and applications in sliding mode control systems. Studies in computational intelligence, vol 576. Springer, Berlin, pp 571–590
121. Vincent UE, Njah AN, Laoye JA (2007) Controlling chaos and deterministic directed transport in inertia ratchets using backstepping control. Physica D 231(2):130–136

122. Volos CK, Kyprianidis IM, Stouboulos IN (2013) Experimental investigation on coverage performance of a chaotic autonomous mobile robot. Robot Auton Syst 61(12):1314–1322
123. Volos CK, Kyprianidis IM, Stouboulos IN, Tlelo-Cuautle E, Vaidyanathan S (2015) Memristor: A new concept in synchronization of coupled neuromorphic circuits. J Eng Sci Technol Rev 8(2):157–173
124. Witte CL, Witte MH (1991) Chaos and predicting varix hemorrhage. Med Hypotheses 36(4):312–317
125. Yuan G, Zhang X, Wang Z (2014) Generation and synchronization of feedback-induced chaos in semiconductor ring lasers by injection-locking. Optik: Int J Light Electron Opt 125(8):1950–1953
126. Zaher AA, Abu-Rezq A (2011) On the design of chaos-based secure communication systems. Commun Nonlinear Syst Numer Simul 16(9):3721–3727
127. Zhang X, Zhao Z, Wang J (2014) Chaotic image encryption based on circular substitution box and key stream buffer. Sig Process Image Commun 29(8):902–913
128. Zhou W, Xu Y, Lu H, Pan L (2008) On dynamics analysis of a new chaotic attractor. Phys Lett A 372(36):5773–5777
129. Zhu C, Liu Y, Guo Y (2010) Theoretic and numerical study of a new chaotic system. Intell Inf Manag 2:104–109
130. Zhu Q, Azar AT (2015) Complex system modelling and control through intelligent, soft computations. Studies in fuzzines and soft computing, vol 319. Springer, Berlin