Time-Frequency Analysis of GPR Signal for Cavities Detection Application

Houda Harkat, Saad Bennani Dosse and Abdellatif Slimani

Abstract In a dispersive medium, GPR signal is classified as a nonstationary signal, and which is widely attenuated, as so, some reflected echoes become none visible neither in the time or the frequency representation of the signal. In fact, we are aimed to calculate the travel time inside a cavity, i.e. the time between two transition that are highly attenuated, in order to identify the nature of the dielectric inside this cavity, which become impossible due to the attenuation phenomena. In this outlook, we proposed to analysis this signal using a time-frequency representation. The continuous wavelet transform is the alternative approach to the Fourier transform due to the fact that the spectrogram is limited in the resolution by the width of the window. Besides the Stockwell transform, in addition to the Hilbert Huang transform are widely used for the analyzed of the electrical, biomedical (ECGs), GPR signals, and seismic sections.

Keywords Dispersive medium · GPR signal · Time-frequency representation · Continuous wavelet transform · Fourier transform · Hilbert Huang transform · Stockwell transform · Electrical and biomedical signals

1 Introduction

GPR signals is a nonstationary signal affected by several phenomena, such as noise and clutter signal caused principally by little diffracting objects localized in the subsurface layers. As so, the signal is widely attenuated, and the transitions echo's

H. Harkat (✉) · S. Bennani Dosse · A. Slimani
Faculty of Science and Technology, Laboratory Renewable Energy and Intelligent Systems (LERSI), Road Imouzzer Fez, PO Box 2202, FEZ, Morocco
e-mail: harkat.houda@gmail.com

S. Bennani Dosse
e-mail: bennani.saad.ensaf@gmail.com

A. Slimani
e-mail: slimani.abdellatif.ma@gmail.com

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become undistinguished. In a dispersive medium this phenomena becomes more important, as so, some reflected echoes become none visible neither in the time or the frequency representation of the signal.

In fact, in geophysical data analysis, the concept of a stationary time series is a mathematical idealization that is never realized and is not particularly useful in the detection of signal arrivals. Although the Fourier transform of the entire time series does contain information about the spectral components in a time series, for a large class of practical applications, this information is inadequate. The spectral components of such a time series clearly have a strong dependence on time. It would be desirable to have a joint time-frequency representation (TFR).

Besides, in this study, we must calculate the travel time inside a cavity, i.e. the time between two transition that are highly attenuated, in order to identify the nature of the dielectric inside this cavity.

For time-frequency signal analysis, different techniques can be used. Three techniques that have notably better properties in the analysis of nonstationary signals are Hilbert Huang Transform (HHT), Continuous Wavelet Transform (CWT), and Stockwell Transform.

In fact, Short Time Fourier Transform (STFT) is commonly used. However, the limitation of this technique is that it has a constant resolution in time and in frequency, where the width of the windowing function defines the resolution.

Continuous Wavelet Transform (CWT), is a technique suitable for time localization of frequency content of the signal, has drawn significant attention in the field, especially in seismic trace analyzing, signal denoising and enhancement.

The Stockwell transform is a combination of short-time Fourier (STFT) and wavelet transforms, since it employs a variable window length and the Fourier kernel. The advantage of S-transform is that it preserves the phase information of signal, and also provides a variable resolution similar to the wavelet transform. In addition, the S-transform is a linear transform that can be used as both an analysis and a synthesis tool, which is not the case with some of the bilinear transforms such as Wigner–Ville distribution. However, the S-transform suffers from poor energy concentration at higher frequency and hence poor frequency localization.

However, Hilbert Huang Transform (HHT) is a recent technique for time-frequency analysis, based on the empirical mode decomposition concept, which simultaneously provides excellent resolution in time and frequency [1]. Its essential feature is the use of an adaptive time-frequency decomposition that does not impose a fixed basis set on the data, and therefore it is not limited by the time-frequency uncertainty relation characteristic of Fourier or Wavelet analysis.

HHT has been applied for target signal enhancement in depth resolution problem for high frequency range. In fact, Hilbert-Huang transform is widely studied and applied in the many fields such as system simulation, spectral data preprocessing, geophysics and the like through being developed for more than ten years. Besides, Feng et al. [2] applied Hilbert transform to convert ground penetrating radar real signal into complex signals. The instantaneous amplitude, instantaneous phase and instantaneous frequency waveform Figs were extracted, and independent profiles of three parameters were formed, thereby improving the accuracy of radar interpretation.

Based on Hilbert transform, the signals must be narrowband when Hilbert transform is used for calculating instantaneous parameters of signals. The GPR data often adopt broadband, and error physical interpretation can be caused through directly adopting Hilbert for calculating instantaneous parameters, such as negative frequency and the like.

This paper gives comparative survey of the three time-frequencies approaches with the intention of giving the guidelines for deciding which of these techniques will be chosen for the analysis of signals obtained from GPR survey, considering the desired outcomes of the analysis, and the specific application for the resolution of the problem of undistinguished reflections in dispersive medium, due to the attenuation phenomena. Besides brief mathematical foundations of the transforms, the paper illustrates their utilization using simulated examples.

2 Time-Frequency Analysis

2.1 Wavelet Transform

A wavelet function, is a function $\psi \in L^2(\mathbb{R})$ with zero average, normalized (i.e. $\int_{\mathbb{R}} \psi = 0$), and centered in the neighborhood of t = 0. Scaling ψ by a positive quantity s, and translating it by $u \in \mathbb{R}$, we define a family of time-frequency atoms, $\psi_{u,s}$, as:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t - u}{s}\right), u \in \mathbb{R}, s > 0 \tag{1}$$

Given a signal $sig \in L^2(R)$, the continuous wavelet transform (CWT) of sig at time u and scale s is defined as:

$$W\,sig(u, s) = \langle sig, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} sig(t)\psi^*_{u,s}(t)dt \tag{2}$$

It provides the frequency component (or details) of sig corresponding to the scale s and time location t.

The revolution of wavelet theory comes precisely from this fact: the two parameters (time u and scale s) of the CWT make possible the study of a signal in both time and frequency domains simultaneously, with a resolution that depends on the scale of interest. According to these considerations, the CWT provides a time-frequency decomposition of sig in the so called time-frequency plane. This method, as it is discussed previously, is more accurate and efficient than the windowed Fourier transform (WFT) basically.

The scalogram of sig is defined by the function :

$$S(s) = \| Wsig(s,u) \| = \sqrt{\int_{-\infty}^{+\infty} |W\ sig\ (s,u)|^2 du} \tag{3}$$

Representing the energy of (W sig) at a scale s. Obviously, S(s) ≥ 0 for all scales s, and if s > 0 we will say that the signal sig has details at scale s. Thus, the scalogram allows the detection of the most representative scales (or frequencies) of a signal, that is, the scales that contribute the most to the total energy of the signal.

Versatile as the wavelet analysis is, the problem with the most commonly used Morlet wavelet is its leakage generated by the limited length of the basic wavelet function, which makes the quantitative definition of the energy-frequency-time distribution difficult. Sometimes, the interpretation of the wavelet can also be counterintuitive. In fact, to define a change occurring locally, one must look for the result in the high-frequency range, for the higher the frequency the more localized the basic wavelet will be. If a local event occurs only in the low-frequency range, one will still be forced to look for its effects in the high-frequency range. Such interpretation will be difficult if it is possible at all. Another difficulty of the wavelet analysis is its non adaptive nature. Once the basic wavelet is selected, one will have to use it to analyze all the data. Since the most commonly used Morlet wavelet is Fourier based, it also suffers the many shortcomings of Fourier spectral analysis, it can only give a physically meaningful interpretation to linear phenomena. In spite of all these problems, wavelet analysis is still the best available non-stationary data analysis method so far [2].

2.2 *Stockwell Transform*

There are several methods of arriving at the S transform. We consider it illuminating to derive the S transform as the phase correction of the CWT [3].

The S transform of a function sig(t) is defined as a CWT with a specific mother wavelet multiplied by the phase factor :

$$S(r,\ f) = e^{i2\pi fu} W\ sig(u,s) \tag{4}$$

where W sig (u,s) is the CWT given by (2), and the mother wavelet is defined as :

$$\psi(t,f) = \frac{|sig|}{k\sqrt{2\pi}} e^{-\frac{t^2 f^2}{2k^2}} e^{i2\pi ft}, \quad \forall k > 0 \tag{5}$$

where k is the scaling factor which controls the time-frequency resolution. The wavelet is generally chosen to be positive and normalized Gaussian. In particular, the transform with a Gaussian window can be rewritten in terms of a Morlet wavelet transform [4].

In fact, the S-transform provides a useful extension to the wavelets by having a frequency dependent progressive resolution as opposed to the arbitrary dilations used in wavelets. The kernel of the S-transform does not translate with the localizing window function, in contrast to the wavelet counterpart. For that reason the S-transform retains both the amplitude and absolutely referenced phase information [3]. Absolutely referenced phase information means that the phase information given by the S-transform is always referenced to time. This is the same meaning of the phase given by the Fourier transform, and is true for each S-transform sample of the time-frequency space. The continuous wavelet transform, in comparison can only localize the amplitude or power spectrum, not the absolute phase. There is, in addition, an easy and direct relation between the S-transform and the Fourier transform.

2.3 Hilbert Huang Transform

Hilbert-Huang transform is a new method for analyzing nonlinear and non-stationary signals, which was proposed by Huang in 1998. It mainly includes empirical mode decomposition (EMD) and Hilbert spectral analysis, which firstly decomposes signals into a number of intrinsic mode functions (IMF) through utilizing EMD method, then acts Hilbert transform on every IMF, and obtains corresponding Hilbert instantaneous spectrum, and the multi-scale oscillation change characteristics of original signals are revealed through analyzing each component and its Hilbert spectrum [5].

2.3.1 EMD Decomposition

In fact, we define as IMF any function having the same number of zero-crossings and extrema, and also having symmetric envelopes defined by the local maxima and minima respectively. Since IMFs admit well-behaved Hilbert transforms, the second stage of the algorithm is to use the Hilbert transform to provide instantaneous frequencies as a function of time for each one of the IMF components. Depending on the application, only the first stage of the Hilbert-Huang Transform may be used.

For a signal x(t) the EMD starts by defining the envelopes of its maxima and minima using cubic splines interpolation. Then, the mean of the two envelopes (E_{max} and E_{min}) is calculated:

$$m_1(t) = (E_{max}(t) + E_{min}(t))/2 \tag{6}$$

Accordingly, the mean $m_1(t)$ is then subtracted from the original signal x(t) :

$$h_1(t) = x(t) - m_1(t) \tag{7}$$

And the residual $h_1(t)$ is examined for the IMF criteria of completeness. If it is an IMF then the procedure stops and the new signal under examination is expressed as:

$$x_1(t) = x(t) - h_1(t) \tag{8}$$

However, if $h_1(t)$ if is not an IMF, the procedure, also known as sifting, is continued k times until the first IMF is realized. Thus:

$$h_{11}(t) = h_1(t) - m_{11}(t) \tag{9}$$

where the second subscript index corresponds to sifting number, and finally:

$$IMF_1(t) = h_{1k}(t) = h_{k-1}(t) - m_{1k}(t) \tag{10}$$

In fact, the sifting process is continued until the last residual is either a monotonic function or a constant. It should be mentioned that as the sifting process evolves, the number of the extrema from one residual to the next drops, thus guaranteeing that complete decomposition is achieved in a finite number of steps. The signal given by:

$$x(t) = \sum_{i=1}^{n} IMF_i(t) + r(t) \tag{11}$$

where k is the total number of the IMF components and r(t) is the residual [5].

The final product is a wavelet-like decomposition going from higher to lower oscillation frequencies, with the frequency content of each mode decreasing as the order of the IMF increases. The big difference however, with the wavelet analysis is that while modes and residuals can intuitively be given a spectral interpretation in the general case, their high versus low frequency discrimination applies only locally and corresponds in no way to a predetermined sub-band filtering. Selection of modes instead, corresponds to an automatic and adaptive time variant filtering.

2.3.2 Hilbert Spectrum

For given decomposed signal, sig(t), the Hilbert transform, HT (t), is defined as:

$$HT(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{sig(\tau)}{t - \tau} d\tau \tag{12}$$

HT (t) can be combined to form analytical signal A (t), given by:

$$A(t) = sig(t) + iHT(t) = a(t)e^{\theta(t)} \tag{13}$$

From the polar coordinate expression, the instantaneous frequency can be defined

$$\omega = \frac{d\theta}{dt} \tag{14}$$

Applying the Hilbert transform to the n IMF components, the signal sig(t) can be written as :

$$sig(t) = R \sum_{j=1}^{n} a_j(t) e^{i\theta \int \omega_j(t)dt} \tag{15}$$

where R is the real part of the value to be calculated and a_j the analytic signal associated with the jth IMF.

The above Equation is written in terms of amplitude and instantaneous frequency associated with each component as functions of time. The time-dependent amplitude and instantaneous frequency might not only improve the flexibility of the expansion, but also enable the expansion to accommodate nonstationary data [5].

The frequency-time distribution of the amplitude is designated as the Hilbert amplitude spectrum, H(ω, t), or Hilbert spectrum simply, defined as:

$$H(\omega, t) = \sum_{j=1}^{n} a_{jd}(t)dt \tag{16}$$

The marginal spectrum, h(ω), is defined as :

$$h(\omega) = \sum_{j=1}^{n} \int_{0}^{T} a_{jd}(t)dt \tag{17}$$

It is provides a measure of total amplitude contribution from each frequency value, in which T denotes the time duration of data.

3 Simulation

3.1 Example of Simulation

An example of simulation for a prospection type surface-surface, is realized for a cavity embedded in a dispersive clay (Fig. 1) using Matlab. The received signal is analyzed with the three time-frequency techniques discussed bellow.

3.2 Results and Discussion

We present the result of the GPR signal analyzed by wavelet transform (Fig. 2), Stockwell transform (Fig. 3), and Hilbert Huang Transform (Fig. 4).

CWT (Fig. 2) shows excellent time localization for different frequencies, but it is limited by the choice of the mother wavelet. Nevertheless, although CWT is able to provide sharper time localization of appearance of different frequencies than HHT (Fig. 4), it is not capable to determine the instantaneous frequency of the signal. But, otherwise, the Hilbert transform, with EMD algorithm, is unstable with noisy data (Fig. 5), which is the case for real data.

The S-transform (Fig. 3) improves the short-time Fourier transform and the continuous wavelet transform by merging the multi-resolution and frequency-dependent analysis properties of wavelet transform with the absolute phase retaining of Fourier transform.

The common S-transform applies a Gaussian window to provide appropriate time and frequency resolution and minimizes the product of these resolutions. However, the Gaussian S-transform is unable to obtain uniform time and frequency

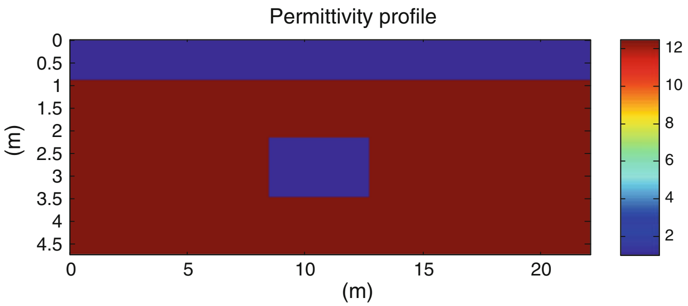

Fig. 1 Simulation model: permittivity profile

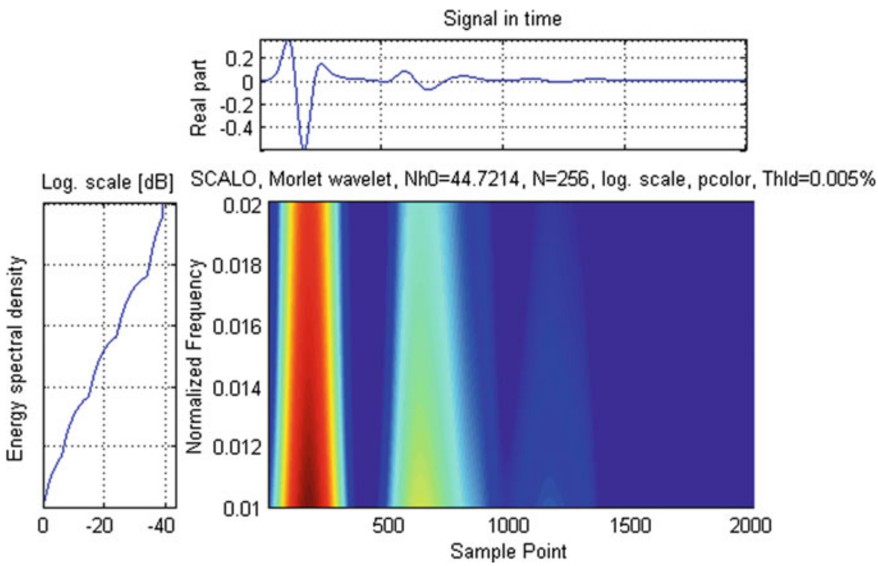

Fig. 2 Time-frequency representation using continuous wavelet transform

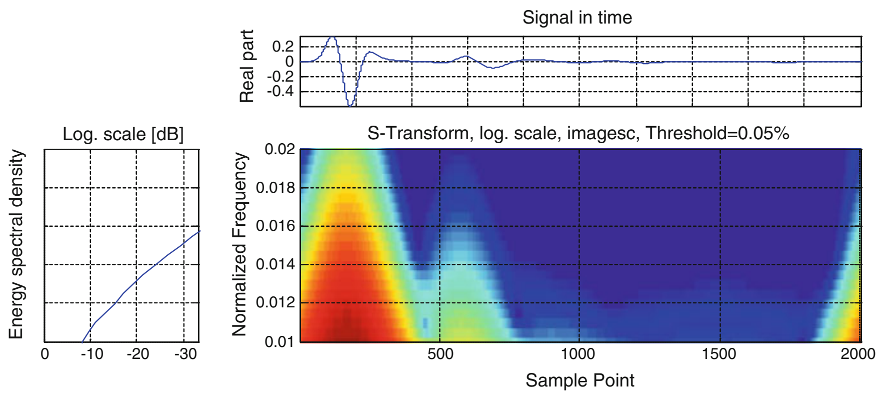

Fig. 3 Time-frequency representation using Stockwell transform

resolution for all frequency components. In fact, the S-transform suffers from inherently poor frequency resolution, particularly at the high frequencies, in addition to the misleading interference terms.

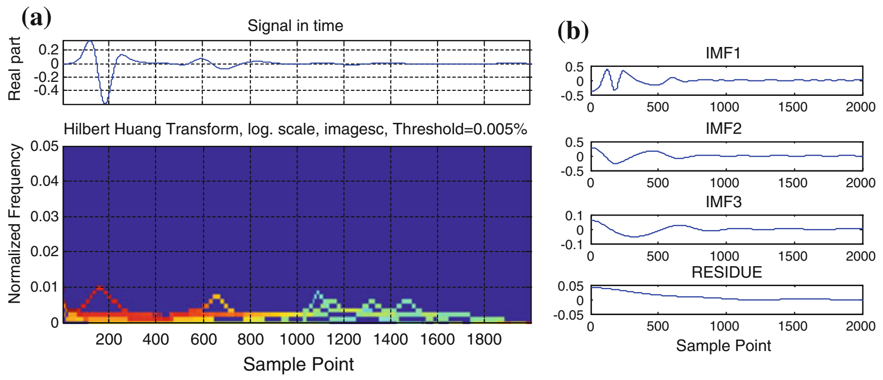

Fig. 4 Time-frequency representation using Hilbert Huang transform: **a** Time-frequency spectrum, **b** EMD decomposition which show the three IMF and the residue

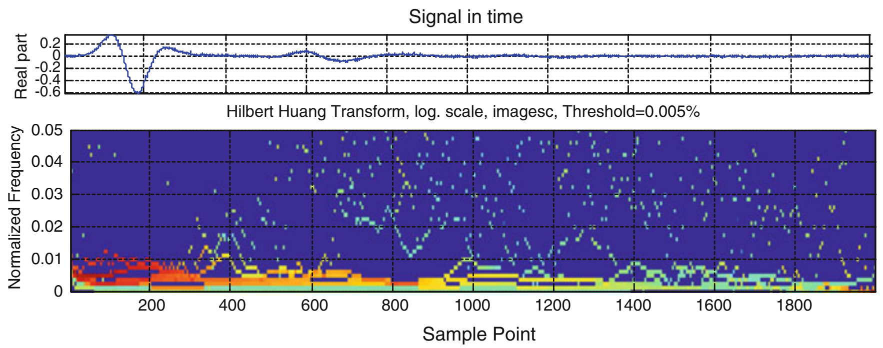

Fig. 5 Time-frequency representation using Hilbert Huang Transform for noisy data: SNR = −46.05 dB

4 Conclusion

This research aims to calculate the travel time inside a cavity, i.e. the time between two transition that are highly attenuated, in order to identify the nature of the dielectric inside this cavity, which become impossible due to the attenuation phenomena. Accordingly to the comparison study done between the three time-frequency representations algorithms explained, it was clear that the CWT gives good time-frequency resolution of the events (not affected by noise or misleading terms).

However the Hilbert Huang technique could be modified in the phase of EMD decomposition and IMF selection to perform the desired task. Future works will be concentrated on this points.

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