Sliding Mode Controller Design for the Global Stabilization of Chaotic Systems and Its Application to Vaidyanathan Jerk System

Sundarapandian Vaidyanathan

Abstract Chaos in nonlinear dynamics occurs widely in physics, chemistry, biology, ecology, secure communications, cryptosystems and many scientific branches. Control of chaotic systems is an important research problem in chaos theory. Sliding mode control is an important method used to solve various problems in control systems engineering. In robust control systems, the sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as insensitivity to parameter uncertainties and disturbance. In this work, we derive a novel sliding mode control method for the global stabilization of chaotic systems. The general control result derived using novel sliding mode control method is proved using Lyapunov stability theory. As an application of the general result, the problem of global stabilization of the Vaidyanathan jerk chaotic system (2015) is studied and a new sliding mode controller is derived. The Lyapunov exponents of the Vaidyanathan jerk system are obtained as $L_1 = 0.12476, L_2 = 0$ and $L_3 = -1.12451$. Since the Vaidyanathan jerk system has a positive Lyapunov exponent, it is chaotic. The Maximal Lyapunov Exponent (MLE) of the Vaidyanathan jerk system is given by $L_1 = 0.12476$. Also, the Kaplan–Yorke dimension of the Vaidyanathan jerk system is obtained as $D_{KY} = 2.11095$. Numerical simulations using MATLAB have been shown to depict the phase portraits of the Vaidyanathan jerk system and the sliding mode controller design for the global stabilization of the Vaidyanathan jerk system.

Keywords Chaos · Chaotic systems · Jerk systems · Sliding mode control · Stabilization

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1 Introduction

Chaos theory describes the quantitative study of unstable aperiodic dynamic behaviour in deterministic nonlinear dynamical systems. For the motion of a dynamical system to be chaotic, the system variables should contain some nonlinear terms and the system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [9].

The Lyapunov exponent is a measure of the divergence of phase points that are initially very close and can be used to quantify chaotic systems. It is common to refer to the largest Lyapunov exponent as the *Maximal Lyapunov Exponent* (MLE). A positive maximal Lyapunov exponent and phase space compactness are usually taken as defining conditions for a chaotic system.

Some classical paradigms of 3-D chaotic systems in the literature are Lorenz system [30], Rössler system [40], ACT system [1], Sprott systems [43], Chen system [15], Lü system [31], Liu system [29], Cai system [14], Chen–Lee system [16], Tigan system [51], etc.

Many new chaotic systems have been discovered in the recent years such as Zhou system [116], Zhu system [117], Li system [25], Wei–Yang system [111], Sundarapandian systems [48, 49], Vaidyanathan systems [60, 63, 65–68, 70, 81, 95–97, 99, 101, 104, 106–108], Pehlivan system [35], Sampath system [41], Pham system [36],etc.

Chaos theory and control systems have many important applications in science and engineering [2, 10–13, 118]. Some commonly known applications are oscillators [22, 42], lasers [26, 113], chemical reactors [71, 75, 77, 79, 82, 86–88], biological systems [69, 72–74, 76, 78, 83–85, 89, 91–93], ecology [18, 45], encryption [24, 115], cryptosystems [39, 52], mechanical systems [4–8], secure communications [17, 33, 114], robotics [32, 34, 109], cardiology [38, 112], intelligent control [3, 27], neural networks [20, 21, 28], finance [19, 44], memristors [37, 110], etc.

Control or regulation of a chaotic system deals with the design of a state feedback control law so as to stabilize or regulate the trajectories of the chaotic system. Many techniques have been devised for the global control of chaotic systems such as the active control method [46, 47, 55, 56, 102], adaptive control method [61, 62, 64, 90, 94, 100, 103, 105], sliding mode control method [50, 57–59, 98], etc.

In the sliding mode control theory, the control dynamics will have two sequential modes, viz. the reaching mode and the sliding mode. Basically, a sliding mode controller (SMC) design consists of two parts: hyperplane design and controller design. A hyperplane is first designed via the pole-placement approach in the modern control theory and a controller is then designed based on the sliding condition. The stability of the overall system is guaranteed by the sliding condition and by a stable hyperplane.

The sliding mode control method is an effective control tool which has the advantages of low sensitivity to parameter variations in the plant and disturbances affecting the plant.

In this work, we use a novel sliding mode control method for deriving a general result for the global stabilization of chaotic systems using sliding mode control (SMC) theory.

The general control result derived using novel sliding mode control method is proved using Lyapunov stability theory. As an application of the general result, the problem of global stabilization of the Vaidyanathan jerk chaotic system [80] is studied and a new sliding mode controller is derived.

This work is organized as follows. Section 2 contains the problem statement of global chaos control of chaotic systems. Section 3 describes the novel sliding mode controller design for globally stabilizing chaotic systems. The general control result derived using novel sliding mode control method is proved using Lyapunov stability theory.

Section 4 describes the Vaidyanathan jerk chaotic system [80] and its properties. The Lyapunov exponents of the Vaidyanathan jerk system are obtained as $L_1 = 0.12476$, $L_2 = 0$ and $L_3 = -1.12451$. Since the Vaidyanathan jerk system has a positive Lyapunov exponent, it is chaotic. The Maximal Lyapunov Exponent (MLE) of the Vaidyanathan jerk system is given by $L_1 = 0.12476$. Also, the Kaplan–Yorke dimension of the Vaidyanathan jerk system is obtained as $D_{KY} = 2.11095$.

Section 5 describes the application of the general result derived in Sect. 3 for the global chaos control of the Vaidyanathan jerk chaotic system [80]. Numerical simulations using MATLAB have been shown to depict the phase portraits of the Vaidyanathan jerk system and the sliding mode controller design for the global stabilization of the Vaidyanathan jerk system. Section 6 contains the conclusions of this work.

2 Problem Statement

This section gives a problem statement for the global chaos control of a given chaotic system.

To start with, we consider a chaotic system given by

$$\dot{\mathbf{x}} = A\mathbf{x} + f(\mathbf{x}) + \mathbf{u} \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ denotes the state of the system, $A \in \mathbf{R}^{n\times n}$ denotes the matrix of system parameters, $f(\mathbf{x}) \in \mathbf{R}^n$ contains the nonlinear parts of the system and $\mathbf{u}$ is the control.

Thus, the global chaos control for the given chaotic system (1) can be stated as follows: Find a feedback controller $\mathbf{u}(\mathbf{x})$ so as to render the state $\mathbf{x}(t)$ of the corresponding closed-loop system to be globally asymptotically stable for all values of $\mathbf{x}(0) \in \mathbf{R}^n$, i.e.

$$\lim_{t\to\infty} \|\mathbf{x}(t)\| = 0 \quad \text{for all} \;\; \mathbf{x}(0) \in \mathbf{R}^n \tag{2}$$

3 A Novel Sliding Mode Control Method for Global Stabilization of Chaotic Systems

This section details the main results of this work, viz. novel sliding mode controller design for achieving global asymptotic stabilization of a given chaotic system.

First, we start the design by setting the control as

$$\mathbf{u}(t) = -f(\mathbf{x}) + Bv(t) \tag{3}$$

In Eq. (3), $B \in \mathbf{R}^n$ is chosen such that (A, B) is completely controllable.

By substituting (3) into (1), we get the closed-loop plant dynamics

$$\dot{\mathbf{x}} = A\mathbf{x} + Bv \tag{4}$$

The system (4) is a linear time-invariant control system with single input v.

Next, we start the sliding controller design by defining the sliding variable as

$$s(\mathbf{x}) = C\mathbf{x} = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \tag{5}$$

where $C \in \mathbf{R}^{1\times n}$ is a constant vector to be determined.

The sliding manifold S is defined as the hyperplane

$$S = \{\mathbf{x} \in \mathbf{R}^n : \ s(\mathbf{x}) = C\mathbf{x} = 0\} \tag{6}$$

We shall assume that a sliding motion occurs on the hyperplane S.

In sliding mode, the following equations must be satisfied:

$$s = 0 \tag{7a}$$

$$\dot{s} = CA\mathbf{x} + CBv = 0 \tag{7b}$$

We assume that

$$CB \neq 0 \tag{8}$$

The sliding motion is influenced by the equivalent control derived from (7b) as

$$v_{\text{eq}}(t) = -(CB)^{-1}\,CA\mathbf{x}(t) \tag{9}$$

By substituting (9) into (4), we obtain the equivalent system dynamics in the sliding phase as

$$\dot{\mathbf{x}} = A\mathbf{x} - (CB)^{-1}CA\mathbf{x} = E\mathbf{x}, \tag{10}$$

where

$$E = \left[I - B(CB)^{-1}C\right]A \tag{11}$$

We note that E is independent of the control and has at most $(n-1)$ non-zero eigenvalues, depending on the chosen switching surface, while the associated eigenvectors belong to $\ker(C)$.

Since (A, B) is controllable, we can use sliding control theory [53, 54] to choose B and C so that E has any desired $(n-1)$ stable eigenvalues.

This shows that the dynamics (10) is globally asympotically stable.

Finally, for the sliding controller design, we apply a novel sliding control law, viz.

$$\dot{s} = -ks - qs^2 \operatorname{sgn}(s) \tag{12}$$

In (12), $\operatorname{sgn}(\cdot)$ denotes the *sign* function and the SMC constants $k > 0$, $q > 0$ are found in such a way that the sliding condition is satisfied and that the sliding motion will occur.

By combining Eqs. (7b), (9) and (12), we finally obtain the sliding mode controller $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)\mathbf{x} + qs^2 \operatorname{sgn}(s)\right] \tag{13}$$

Next, we establish the main result of this section.

Theorem 1 *The chaotic system (1) is globally asymptotically stabilized for all initial conditions* $\mathbf{x}(0)$ *in* $\mathbf{R}^n$*, where* v *is defined by the novel sliding mode control law (13),* $B \in \mathbf{R}^{n\times 1}$ *is such that* (A, B) *is controllable,* $C \in \mathbf{R}^{1\times n}$ *is such that* $CB \neq 0$ *and the matrix* E *defined by (11) has* $(n-1)$ *stable eigenvalues.*

Proof Upon substitution of the control laws (3) and (13) into the system dynamics (1), we obtain the closed-loop system as

$$\dot{\mathbf{x}} = A\mathbf{x} - B(CB)^{-1}\left[C(kI + A)\mathbf{x} + qs^2 \operatorname{sgn}(s)\right] \tag{14}$$

We shall show that the error dynamics (14) is globally asymptotically stable by considering the quadratic Lyapunov function

$$V(\mathbf{x}) = \frac{1}{2} s^2(\mathbf{x}) \tag{15}$$

The sliding mode motion is characterized by the equations

$$s(\mathbf{x}) = 0 \quad \text{and} \quad \dot{s}(\mathbf{x}) = 0 \tag{16}$$

By the choice of E, the dynamics in the sliding mode given by Eq. (10) is globally asymptotically stable.

When $s(\mathbf{x}) \neq 0$, $V(\mathbf{x}) > 0$.

Also, when $s(\mathbf{x}) \neq 0$, differentiating V along the error dynamics (14) or the equivalent dynamics (12), we get

$$\dot{V}(\mathbf{x}) = s\dot{s} = -ks^2 - qs^3 \operatorname{sgn}(s) < 0 \tag{17}$$

Hence, by Lyapunov stability theory [23], the error dynamics (14) is globally asymptotically stable for all $\mathbf{x}(0) \in \mathbf{R}^n$.

This completes the proof. ■

4 Vaidyanathan Jerk Chaotic System and Its Properties

In this section, we describe the Vaidyanathan jerk chaotic system [80] and discuss its dynamic properties.

The Vaidyanathan jerk chaotic system [80] is described by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 - x_2^2 \end{aligned} \tag{18}$$

where x_1, x_2, x_3 are the states and a, b are constant, positive, parameters.

In [80], it was shown that the system (18) exhibits a *strange chaotic attractor*, when the parameters take the values

$$a = 7.5 \;\; b = 4 \tag{19}$$

For numerical simulations, we take the initial values of the Vaidyanathan jerk chaotic system (18) as

$$x_1(0) = 0.2, \;\; x_2(0) = 0.6, \;\; x_3(0) = 0.4 \tag{20}$$

For the parameter values in (19) and the initial values in (20), the Lyapunov exponents of the Vaidyanathan jerk system (18) are numerically obtained as

$$L_1 = 0.12476, \;\; L_2 = 0, \;\; L_3 = -1.12451 \tag{21}$$

Since the sum of the Lyapunov exponents in (21) is negative, the Vaidyanathan jerk system (18) is dissipative.

The Kaplan–Yorke dimension of the Vaidyanathan jerk system (18) is calculated as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.11095, \tag{22}$$

which is fractional.

It is easy to show that the Vaidyanathan hyperjerk system (18) has two equilibrium points given by

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad E_1 = \begin{bmatrix} 7.5 \\ 0 \\ 0 \end{bmatrix} \tag{23}$$

In [80], it was shown that both E_0 and E_1 are saddle-focus points, and hence they are unstable.

For the initial conditions given in (20), phase portraits of the Vaidyanathan jerk system (18) are plotted using MATLAB.

Figures 1 shows the strange chaotic attractor of the Vaidyanathan jerk system (18). Figures 2, 3 and 4 show the 2-D projection of the Vaidyanathan jerk system (18) on the (x_1, x_2), (x_2, x_3) and (x_1, x_3) planes, respectively.

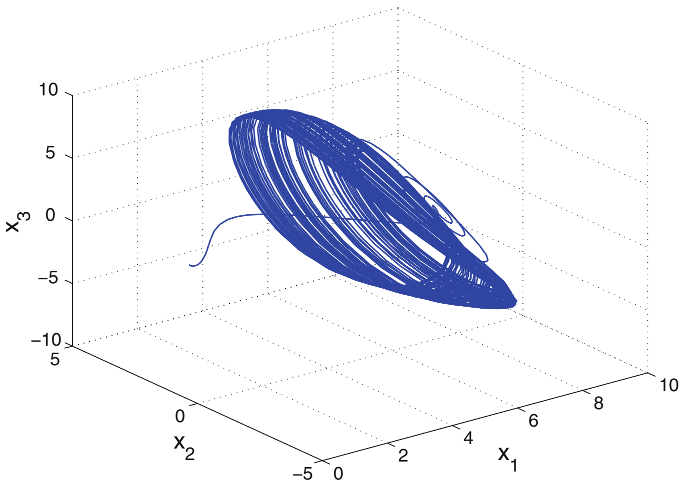

Fig. 1 Strange chaotic attractor of the Vaidyanathan jerk system

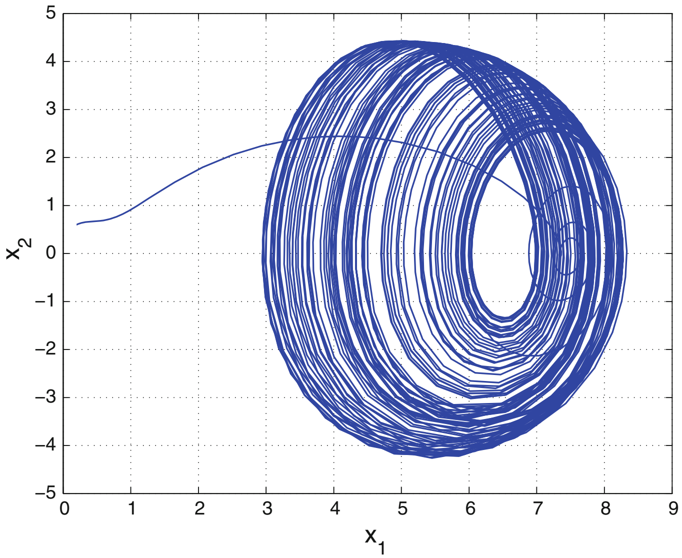

Fig. 2 2-D projection of the Vaidyanathan jerk system on the (x_1, x_2) plane

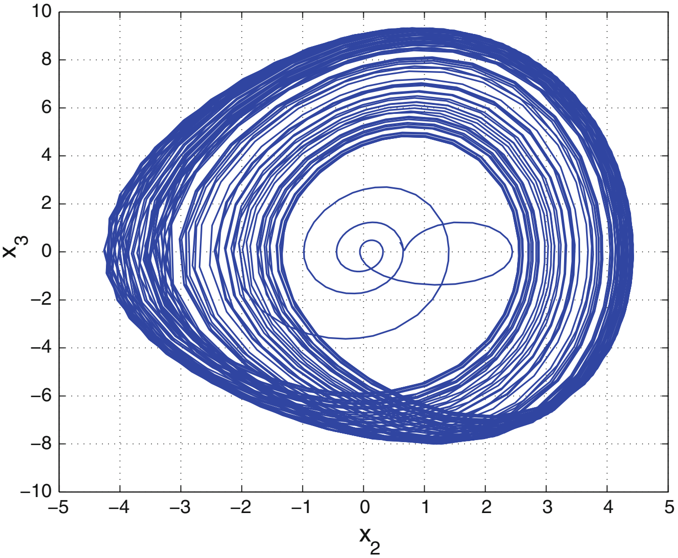

Fig. 3 2-D projection of the Vaidyanathan jerk system on the (x_2, x_3) plane

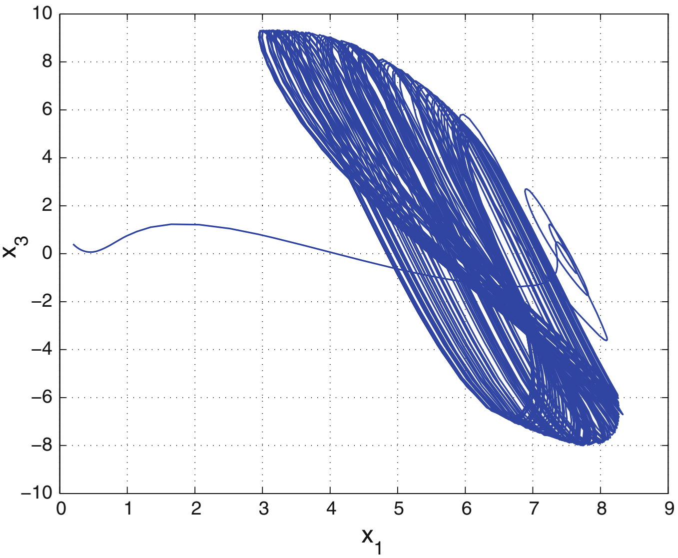

Fig. 4 2-D projection of the Vaidyanathan jerk system on the (x_1, x_3) plane

5 Sliding Mode Controller Design for the Global Stabilization of Vaidyanathan Jerk System

In this section, we describe the sliding mode controller design for the global stabilization of Vaidyanathan jerk system [80] by applying the novel method described by Theorem 1 in Sect. 3.

Thus, we consider the controlled Vaidyanathan jerk system given by

$$\begin{aligned} \dot{x}_1 &= x_2 + u_1 \\ \dot{x}_2 &= x_3 + u_2 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 - x_2^2 + u_3 \end{aligned} \tag{24}$$

In matrix form, we can write the error dynamics (24) as

$$\dot{\mathbf{x}} = A\mathbf{x} + \psi(\mathbf{x}) + \mathbf{u} \tag{25}$$

The matrices A and ψ in (25) are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & -b & -1 \end{bmatrix}, \quad \psi(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ -x_1^2 - x_2^2 \end{bmatrix} \tag{26}$$

We follow the procedure given in Sect. 3 for the construction of the novel sliding controller to achieve global stabilization of the Vaidyanathan jerk system (24).

We take the parameter values of a and b as in the chaotic case, i.e.

$$a = 7.5, \quad b = 4 \tag{27}$$

First, we set $\mathbf{u}$ as

$$\mathbf{u}(t) = -\psi(\mathbf{x}) + Bv(t) \tag{28}$$

where B is selected such that (A, B) is completely controllable.

A simple choice of B is

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{29}$$

It can be easily checked that (A, B) is completely controllable.

Next, we take the sliding variable as

$$s(\mathbf{x}) = C\mathbf{x} = \begin{bmatrix} 1 & -1 & -20 \end{bmatrix} \mathbf{e} = x_1 - x_2 - 20x_3 \tag{30}$$

If we define $E = [I - B(CB)^{-1}C]A$, then the matrix E has the eigenvalues

$$\text{eig}(E) = \{0, -1.7500 \pm 2.1243i\} \tag{31}$$

which shows that the motion along the sliding manifold is globally asymptotically stable.

Next, we take the sliding mode gains as

$$k = 5, \quad q = 0.2 \tag{32}$$

From Eq. (13) in Sect. 3, we obtain the novel sliding control v as

$$v(t) = -7.25x_1 + 3.8x_2 - 4.05x_3 + 0.01s^2 \,\text{sgn}(s) \tag{33}$$

As an application of Theorem 1 to the identical Vaidyanathan jerk chaotic system, we obtain the following main result of this section.

Theorem 2 *Vaidyanathan jerk chaotic system (24) is globally and asymptotically stabilized for all initial conditions* $\mathbf{x}(0) \in \mathbf{R}^3$ *with the sliding controller* $\mathbf{u}$ *defined by (28), where* $\psi(\mathbf{x})$ *is defined by (26), B is defined by (29) and v is defined by (33).* ∎

For numerical simulations, we use MATLAB for solving the systems of differential equations using the classical fourth-order Runge–Kutta method with step size $h = 10^{-8}$.

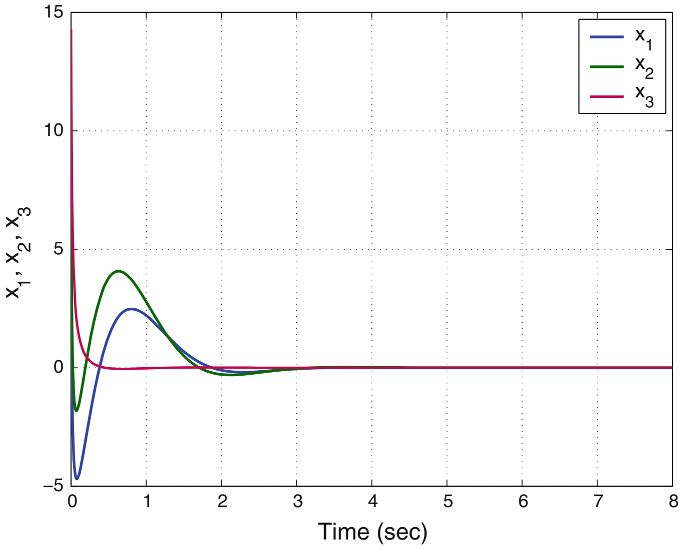

Fig. 5 Time-history of the controlled states x_1, x_2, x_3

The parameter values of the Vaidyanathan jerk system are taken as in the chaotic case, viz. $a = 7.5$ and $b = 4$.

The sliding mode gains are taken as $k = 5$ and $q = 0.2$.

As an initial condition for the Vaidyanathan jerk system (24), we take

$$x_1(0) = 5.7, \quad x_2(0) = 8.2, \quad x_3(0) = 14.3 \tag{34}$$

Figure 5 shows the time-history of the controlled states x_1, x_2, x_3.

6 Conclusions

In this work, we derived a novel sliding mode control method for the global stabilization of chaotic systems. The general control result derived using novel sliding mode control method was proved using Lyapunov stability theory. As an application of the general result, the problem of global stabilization of the Vaidyanathan jerk chaotic system (2015) was studied and a new sliding mode controller has been derived. The Lyapunov exponents of the Vaidyanathan jerk system have been obtained as $L_1 = 0.12476$, $L_2 = 0$ and $L_3 = -1.12451$. Since the Vaidyanathan jerk system has a positive Lyapunov exponent, it is chaotic. The Maximal Lyapunov Exponent (MLE) of the Vaidyanathan jerk system is seen as $L_1 = 0.12476$. Also, the Kaplan–Yorke dimension of the Vaidyanathan jerk system has been derived as $D_{KY} = 2.11095$. Numerical simulations using MATLAB were shown to depict the phase portraits of the Vaidyanathan jerk system and the sliding mode controller design for the global stabilization of the Vaidyanathan jerk system.

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