Anti-synchronization of Hyperchaotic Systems via Novel Sliding Mode Control and Its Application to Vaidyanathan Hyperjerk System

Sundarapandian Vaidyanathan and Sivaperumal Sampath

Abstract Chaos in nonlinear dynamics occurs widely in physics, chemistry, biology, ecology, secure communications, cryptosystems and many scientific branches. Anti-synchronization of chaotic systems is an important research problem in chaos theory. Sliding mode control is an important method used to solve various problems in control systems engineering. In robust control systems, the sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as insensitivity to parameter uncertainties and disturbance. In this work, we derive a novel sliding mode control method for the anti-synchronization of identical chaotic or hyperchaotic systems. The general result derived using novel sliding mode control method is proved using Lyapunov stability theory. As an application of the general result, the problem of anti-synchronization of identical Vaidyanathan hyperjerk hyperchaotic systems (2015) is studied and a new sliding mode controller is derived. The Lyapunov exponents of the Vaidyanathan hyperjerk system are obtained as $L_1 = 0.1448$, $L_2 = 0.0328$, $L_3 = 0$ and $L_4 = -1.1294$. Since the Vaidyanathan hyperjerk system has two positive Lyapunov exponents, it is hyperchaotic. Also, the Kaplan–Yorke dimension of the Vaidyanathan hyperjerk system is obtained as $D_{KY} = 3.1573$. Numerical simulations using MATLAB have been shown to depict the phase portraits of the Vaidyanathan hyperjerk system and the sliding mode controller design for the anti-synchronization of identical Vaidyanathan hyperjerk systems.

Keywords Chaos · Chaotic systems · Hyperchaos · Hyperchaotic systems · Hyperjerk systems · Sliding mode control · Synchronization

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1 Introduction

Chaos theory describes the quantitative study of unstable aperiodic dynamic behaviour in deterministic nonlinear dynamical systems. For the motion of a dynamical system to be chaotic, the system variables should contain some nonlinear terms and the system must satisfy three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1].

A hyperchaotic system is defined as a chaotic system with at least two positive Lyapunov exponents [1]. Thus, the dynamics of a hyperchaotic system can expand in several different directions simultaneously. Thus, the hyperchaotic systems have more complex dynamical behaviour and they have miscellaneous applications in engineering such as secure communications [3, 9, 46], cryptosystems [5, 14, 50], fuzzy logic [19, 49], electrical circuits [44, 47], etc.

The minimum dimension of an autonomous, continuous-time, hyperchaotic system is four. The first 4-D hyperchaotic system was found by Rössler [15]. Many hyperchaotic systems have been reported in the chaos literature such as hyperchaotic Lorenz system [6], hyperchaotic Lü system [2], hyperchaotic Chen system [10], hyperchaotic Wang system [43], hyperchaotic Vaidyanathan systems [28, 30, 31, 38, 40, 42], hyperchaotic Pham system [11], etc.

The synchronization of chaotic systems aims to synchronize the states of master and slave systems asymptotically with time. There are many methods available for chaos synchronization such as active control [7, 16, 17, 33, 35], adaptive control [18, 20–22, 27, 32, 34], sliding mode control [23, 29, 37, 39], backstepping control [12, 13, 24, 36], etc.

The design goal of anti-synchronization of chaotic systems is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically [45, 48].

In this work, we use a novel sliding mode control method for deriving a general result for the anti-synchronization of chaotic or hyperchaotic systems using sliding mode control (SMC) theory. The sliding mode control method is an effective control tool which has the advantages of low sensitivity to parameter variations in the plant and disturbances affecting the plant.

This work is organized as follows. In Sect. 2, we discuss the problem statement for the anti-synchronization of identical chaotic or hyperchaotic systems. In Sect. 3, we derive a general result for the anti-synchronization of identical chaotic or hyperchaotic systems using novel sliding mode control. In Sect. 4, we describe the Vaidyanathan hyperjerk system [41] and its dynamic properties. The Lyapunov exponents of the Vaidyanathan hyperjerk system are obtained as $L_1 = 0.1448$, $L_2 = 0.0328$, $L_3 = 0$ and $L_4 = -1.1294$, which shows that the Vaidyanathan hyperjerk system is hyperchaotic. In Sect. 5, we describe the sliding mode controller design for the anti-synchronization of identical Vaidyanathan hyperjerk systems using novel sliding mode control and its numerical simulations using MATLAB. Section 6 contains the conclusions of this work.

2 Problem Statement

This section gives a problem statement for the anti-synchronization of identical chaotic or hyperchaotic systems.

As the *master* or *drive* system, we consider the chaotic or hyperchaotic system given by

$$\dot{\mathbf{x}} = A\mathbf{x} + f(\mathbf{x}) \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ denotes the state of the system, $A \in \mathbf{R}^{n\times n}$ denotes the matrix of system parameters and $f(\mathbf{x}) \in \mathbf{R}^n$ contains the nonlinear parts of the system.

As the *slave* or *response* system, we consider the controlled identical system given by

$$\dot{\mathbf{y}} = A\mathbf{y} + f(\mathbf{y}) + \mathbf{u} \tag{2}$$

where $\mathbf{y} \in \mathbf{R}^n$ denotes the state of the system and $\mathbf{u}$ is the control.

The anti-synchronization error between the systems (1) and (2) is defined as

$$\mathbf{e} = \mathbf{y} + \mathbf{x} \tag{3}$$

The error dynamics is easily obtained as

$$\dot{\mathbf{e}} = A\mathbf{e} + \psi(\mathbf{x}, \mathbf{y}) + \mathbf{u}, \tag{4}$$

where

$$\psi(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}) + f(\mathbf{y}) \tag{5}$$

Thus, the anti-synchronization problem between the systems (1) and (2) can be stated as follows: Find a controller $\mathbf{u}(\mathbf{x}, \mathbf{y})$ so as to render the anti-synchronization error $\mathbf{e}(t)$ to be globally asymptotically stable for all values of $\mathbf{e}(0) \in \mathbf{R}^n$, i.e.

$$\lim_{t\to\infty} \|\mathbf{e}(t)\| = 0 \ \text{ for all } \ \mathbf{e}(0) \in \mathbf{R}^n \tag{6}$$

3 A Novel Sliding Mode Control Method for Solving Anti-synchronization Problem

This section details the main results of this work, viz. novel sliding mode controller design for achieving anti-synchronization of chaotic or hyperchaotic systems.

First, we start the design by setting the control as

$$\mathbf{u}(t) = -\psi(\mathbf{x}, \mathbf{y}) + Bv(t) \tag{7}$$

In Eq. (7), $B \in \mathbf{R}^n$ is chosen such that (A, B) is completely controllable.

By substituting (7) into (4), we get the closed-loop error dynamics

$$\dot{\mathbf{e}} = A\mathbf{e} + Bv \tag{8}$$

The system (8) is a linear time-invariant control system with single input v.

Next, we start the sliding controller design by defining the sliding variable as

$$s(\mathbf{e}) = C\mathbf{e} = c_1 e_1 + c_2 e_2 + \cdots + c_n e_n, \tag{9}$$

where $C \in \mathbf{R}^{1\times n}$ is a constant vector to be determined.

The sliding manifold S is defined as the hyperplane

$$S = \{\mathbf{e} \in \mathbf{R}^n \ : \ s(\mathbf{e}) = C\mathbf{e} = 0\} \tag{10}$$

We shall assume that a sliding motion occurs on the hyperplane S.

In sliding mode, the following equations must be satisfied:

$$s = 0 \tag{11a}$$

$$\dot{s} = CA\mathbf{e} + CBv = 0 \tag{11b}$$

We assume that

$$CB \neq 0 \tag{12}$$

The sliding motion is influenced by the equivalent control derived from (11b) as

$$v_{\text{eq}}(t) = -(CB)^{-1}\, CA\mathbf{e}(t) \tag{13}$$

By substituting (13) into (8), we obtain the equivalent error dynamics in the sliding phase as

$$\dot{\mathbf{e}} = A\mathbf{e} - (CB)^{-1}CA\mathbf{e} = E\mathbf{e}, \tag{14}$$

where

$$E = \left[I - B(CB)^{-1}C\right]A \tag{15}$$

We note that E is independent of the control and has at most $(n - 1)$ non-zero eigenvalues, depending on the chosen switching surface, while the associated eigenvectors belong to $\ker(C)$.

Since (A, B) is controllable, we can use sliding control theory [25, 26] to choose B and C so that E has any desired $(n - 1)$ stable eigenvalues.

This shows that the dynamics (14) is globally asympotically stable.

Finally, for the sliding controller design, we apply a novel sliding control law, viz.

$$\dot{s} = -ks - qs^2 \operatorname{sgn}(s) \tag{16}$$

In (16), $\operatorname{sgn}(\cdot)$ denotes the *sign* function and the SMC constants $k > 0$, $q > 0$ are found in such a way that the sliding condition is satisfied and that the sliding motion will occur.

By combining Eqs. (11b), (13) and (16), we finally obtain the sliding mode controller $v(t)$ as

$$v(t) = -(CB)^{-1} \left[C(kI + A)\mathbf{e} + qs^2 \operatorname{sgn}(s) \right] \tag{17}$$

Next, we establish the main result of this section.

Theorem 1 *The sliding mode controller defined by (7) achieves anti-synchronization between the identical chaotic systems (1) and (2) for all initial conditions* $\mathbf{x}(0)$, $\mathbf{y}(0)$ *in* $\mathbf{R}^n$*, where v is defined by the novel sliding mode control law (17),* $B \in \mathbf{R}^{n\times 1}$ *is such that* (A, B) *is controllable,* $C \in \mathbf{R}^{1\times n}$ *is such that* $CB \neq 0$ *and the matrix E defined by (15) has* $(n-1)$ *stable eigenvalues.*

Proof Upon substitution of the control laws (7) and (17) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{\mathbf{e}} = A\mathbf{e} - B(CB)^{-1} \left[C(kI + A)\mathbf{e} + qs^2 \operatorname{sgn}(s) \right] \tag{18}$$

We shall show that the error dynamics (18) is globally asymptotically stable by considering the quadratic Lyapunov function

$$V(\mathbf{e}) = \frac{1}{2} s^2(\mathbf{e}) \tag{19}$$

The sliding mode motion is characterized by the equations

$$s(\mathbf{e}) = 0 \ \text{ and } \ \dot{s}(\mathbf{e}) = 0 \tag{20}$$

By the choice of E, the dynamics in the sliding mode given by Eq. (14) is globally asymptotically stable.

When $s(\mathbf{e}) \neq 0$, $V(\mathbf{e}) > 0$.

Also, when $s(\mathbf{e}) \neq 0$, differentiating V along the error dynamics (18) or the equivalent dynamics (16), we get

$$\dot{V}(\mathbf{e}) = s\dot{s} = -ks^2 - qs^3 \operatorname{sgn}(s) < 0 \tag{21}$$

Hence, by Lyapunov stability theory [8], the error dynamics (18) is globally asymptotically stable for all $\mathbf{e}(0) \in \mathbf{R}^n$. This completes the proof. ■

4 Vaidyanathan Hyperjerk System and Its Properties

In this section, we describe the Vaidyanathan hyperjerk system [41] and discuss its dynamic properties.

The Vaidyanathan hyperjerk system [41] is described by the 4-D dynamics

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= -x_1 - x_2 - bx_1^2 - ax_3 - cx_1^4 x_4\end{aligned} \tag{22}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c are constant, positive, parameters.

In [41], it was shown that the system (22) is hyperchaotic when the parameters take the values

$$a = 3.7, \quad b = 0.2, \quad c = 1.5 \tag{23}$$

For numerical simulations, we take the initial values of the Vaidyanathan hyperjerk system (22) as

$$x_1(0) = 0.1, \quad x_2(0) = 0.1, \quad x_3(0) = 0.1, \quad x_4(0) = 0.1 \tag{24}$$

For the parameter values in (23) and the initial values in (24), the Lyapunov exponents of the Vaidyanathan hyperjerk system (22) are numerically obtained as

$$L_1 = 0.1448, \quad L_2 = 0.0328, \quad L_3 = 0, \quad L_4 = -1.1294 \tag{25}$$

Since there are two positive Lyapunov exponents in the LE spectrum given in (25), it follows that the Vaidyanathan hyperjerk system is *hyperchaotic*.

Since the sum of the Lyapunov exponents in (25) is negative, the Vaidyanathan hyperjerk system (22) is dissipative.

The Kaplan–Yorke dimension [4] of a chaotic system of order n is defined as

$$D_{KY} = j + \frac{L_1 + \cdots + L_j}{|L_{j+1}|} \tag{26}$$

where $L_1 \geq L_2 \geq \cdots \geq L_n$ are the Lyapunov exponents of the chaotic system and j is the largest integer for which $L_1 + L_2 + \cdots + L_j \geq 0$. (Kaplan–Yorke conjecture states that for typical chaotic systems, $D_{KY} \approx D_L$, the information dimension of the system.)

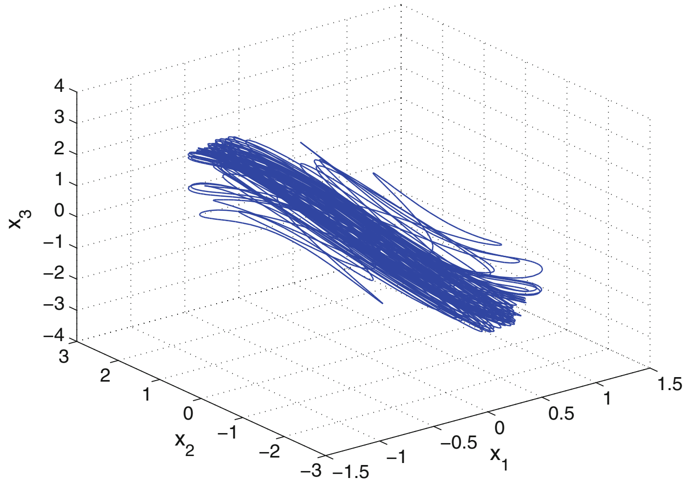

Fig. 1 3-D projection of the Vaidyanathan hyperjerk system on the (x_1, x_2, x_3) space

Thus, the Kaplan–Yorke dimension of the Vaidyanathan hyperjerk system (22) is calculated as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.1573, \tag{27}$$

which is fractional.

It is easy to show that the Vaidyanathan hyperjerk system (22) has two equilibrium points given by

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } E_1 = \begin{bmatrix} -5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{28}$$

In [41], it was shown that both E_0 and E_1 are saddle-focus points, and hence they are unstable.

For the initial conditions given in (24), phase portraits of the Vaidyanathan hyperjerk system (22) are plotted using MATLAB.

Figures 1, 2, 3 and 4 show the 3-D projections of the Vaidyanathan hyperjerk system (22) in (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

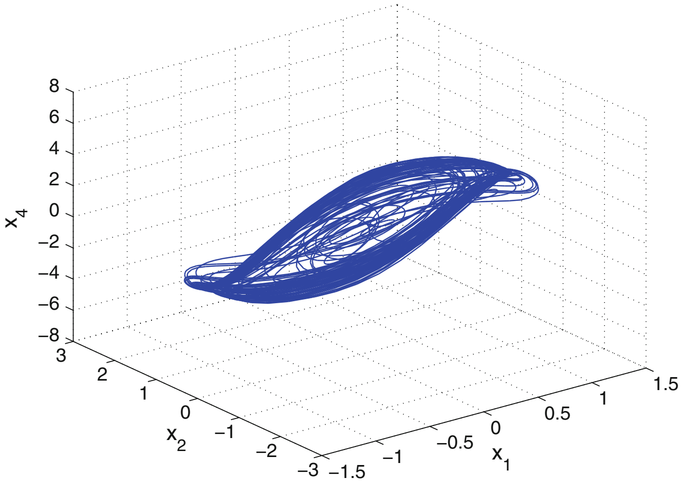

Fig. 2 3-D projection of the Vaidyanathan hyperjerk system on the (x_1, x_2, x_4) space

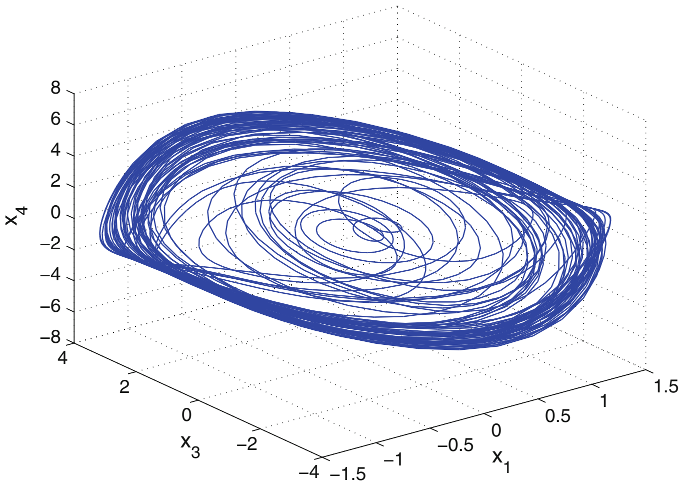

Fig. 3 3-D projection of the Vaidyanathan hyperjerk system on the (x_1, x_3, x_4) space

5 Sliding Mode Controller Design for the Anti-synchronization of Vaidyanathan Hyperjerk Systems

In this section, we describe the sliding mode controller design for the anti-synchronization of Vaidyanathan hyperjerk systems [41] by applying the novel method described by Theorem 1 in Sect. 3.

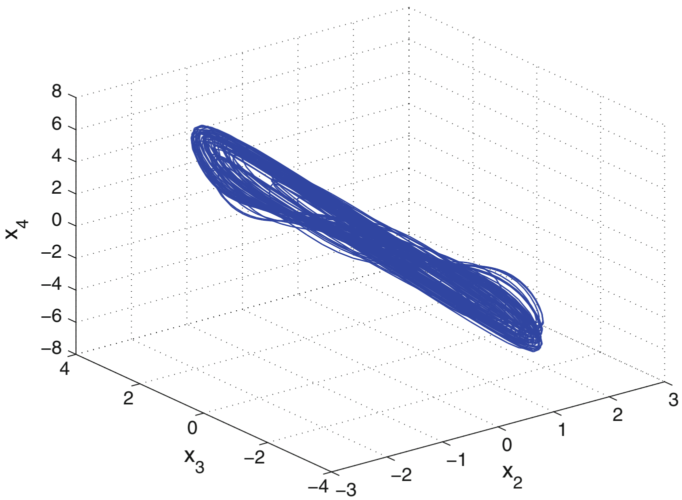

Fig. 4 3-D projection of the Vaidyanathan hyperjerk system on the (x_2, x_3, x_4) space

As the master system, we take the Vaidyanathan hyperjerk system given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -x_1 - x_2 - bx_1^2 - ax_3 - cx_1^4 x_4 \end{aligned} \tag{29}$$

where x_1, x_2, x_3, x_4 are the state variables and a, b, c are positive parameters.

As the slave system, we consider the controlled Vaidyanathan hyperjerk system given by

$$\begin{aligned} \dot{y}_1 &= y_2 + u_1 \\ \dot{y}_2 &= y_3 + u_2 \\ \dot{y}_3 &= y_4 + u_3 \\ \dot{y}_4 &= -y_1 - y_2 - by_1^2 - ay_3 - cy_1^4 y_4 + u_4 \end{aligned} \tag{30}$$

where y_1, y_2, y_3, y_4 are the state variables and u_1, u_2, u_3, u_4 are the controls.

The anti-synchronization error between the Vaidyanathan hyperjerk systems is defined by

$$\begin{aligned} e_1 &= y_1 + x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 + x_3 \\ e_4 &= y_4 + x_4 \end{aligned} \tag{31}$$

Then the error dynamics is obtained as

$$
\begin{aligned}
\dot{e}_1 &= e_2 + u_1 \\
\dot{e}_2 &= e_3 + u_2 \\
\dot{e}_3 &= e_4 + u_3 \\
\dot{e}_4 &= -e_1 - e_2 - ae_3 - b\left(y_1^2 + x_1^2\right) - c\left(y_1^4 y_4 + x_1^4 x_4\right) + u_4
\end{aligned}
\tag{32}
$$

In matrix form, we can write the error dynamics (32) as

$$\dot{\mathbf{e}} = A\mathbf{e} + \psi(\mathbf{x}, \mathbf{y}) + \mathbf{u} \tag{33}$$

The matrices in (33) are given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -1 & -a & 0 \end{bmatrix} \text{ and } \psi(\mathbf{x}, \mathbf{y}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -b(y_1^2 + x_1^2) - c(y_1^4 y_4 + x_1^4 x_4) \end{bmatrix} \tag{34}$$

We follow the procedure given in Sect. 3 for the construction of the novel sliding controller to achieve anti-synchronization of the identical Vaidyanathan hyperjerk systems (29) and (30).

First, we set $\mathbf{u}$ as

$$\mathbf{u}(t) = -\psi(\mathbf{x}, \mathbf{y}) + Bv(t) \tag{35}$$

where B is selected such that (A, B) is completely controllable.

A simple choice of B is

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{36}$$

It can be easily checked that (A, B) is completely controllable.

The Vaidyanathan hyperjerk system displays a strange attractor when the parameter values are selected as

$$a = 3.7, \quad b = 0.2, \quad c = 1.5 \tag{37}$$

Next, we take the sliding variable as

$$s(\mathbf{e}) = C\mathbf{e} = \begin{bmatrix} 15 & 8 & -9 & -13 \end{bmatrix} \mathbf{e} = 15e_1 + 8e_2 - 9e_3 - 13e_4 \tag{38}$$

Next, we take the sliding mode gains as

$$k = 5, \quad q = 0.2 \tag{39}$$

From Eq. (17) in Sect. 3, we obtain the novel sliding control v as

$$v(t) = -88e_1 - 68e_2 - 11.1e_3 + 74e_4 - 0.2s^2\,\text{sgn}(s) \tag{40}$$

As an application of Theorem 1 to the identical Vaidyanathan hyperjerk systems, we obtain the following main result of this section.

Theorem 2 *The identical Vaidyanathan hyperjerk systems (29) and (30) are globally and asymptotically anti-synchronized for all initial conditions* $\mathbf{x}(0), \mathbf{y}(0) \in \mathbf{R}^4$ *with the sliding controller* $\mathbf{u}$ *defined by (35), where* $\psi(\mathbf{x}, \mathbf{y})$ *is defined by (34), B is defined by (36) and v is defined by (40).* ■

For numerical simulations, we use MATLAB for solving the systems of differential equations using the classical fourth-order Runge–Kutta method with step size $h = 10^{-8}$.

The parameter values of the Vaidyanathan hyperjerk systems are taken as in the hyperchaotic case, viz. $a = 3.7$, $b = 0.2$ and $c = 1.5$.

The sliding mode gains are taken as $k = 5$ and $q = 0.2$.

As an initial condition for the master system (29), we take

$$x_1(0) = 1.7,\quad x_2(0) = 0.5,\quad x_3(0) = 1.8,\quad x_4(0) = 1.2 \tag{41}$$

As an initial condition for the slave system (30), we take

$$y_1(0) = 3.1,\quad y_2(0) = 2.4,\quad y_3(0) = 0.3,\quad y_4(0) = 0.5 \tag{42}$$

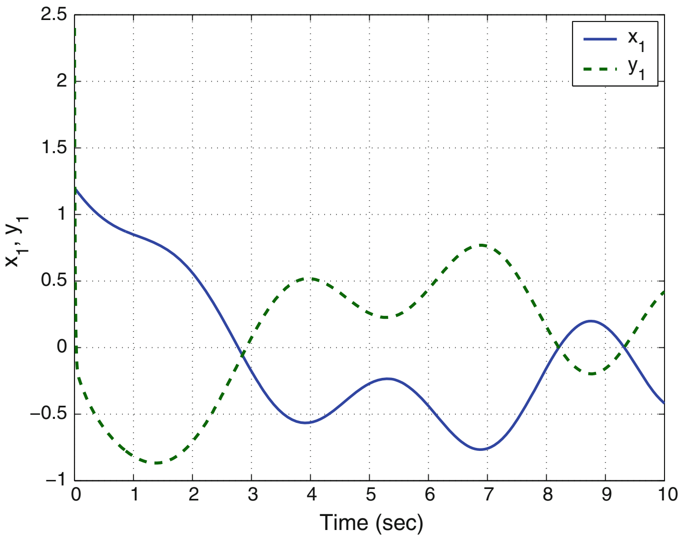

Fig. 5 Anti-synchronization of the states x_1 and y_1

Figures 5, 6, 7 and 8 show the anti-synchronization of the states of the identical Vaidyanathan hyperjerk systems (29) and (30).

Figure 9 shows the time-history of the anti-synchronization errors e_1, e_2, e_3, e_4.

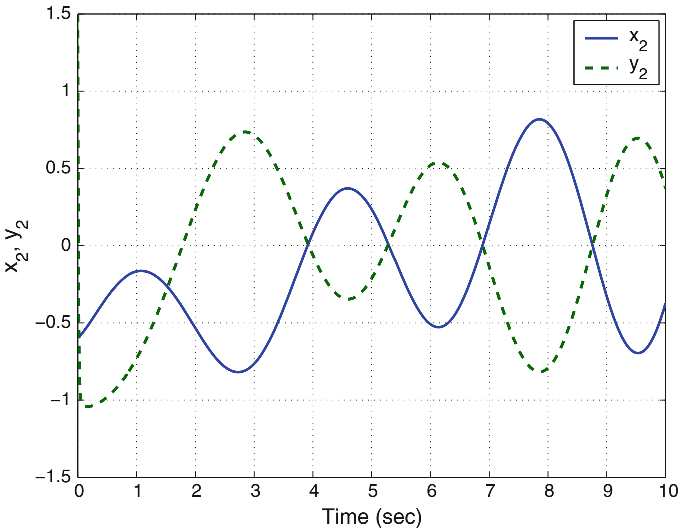

Fig. 6 Anti-synchronization of the states x_2 and y_2

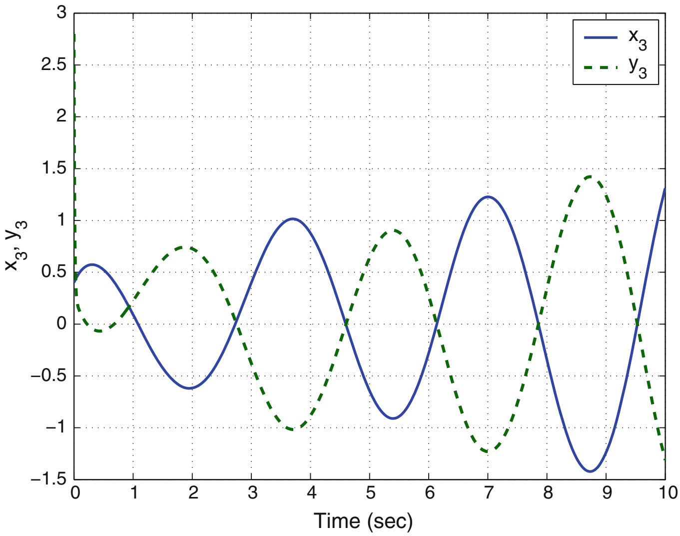

Fig. 7 Anti-synchronization of the states x_3 and y_3

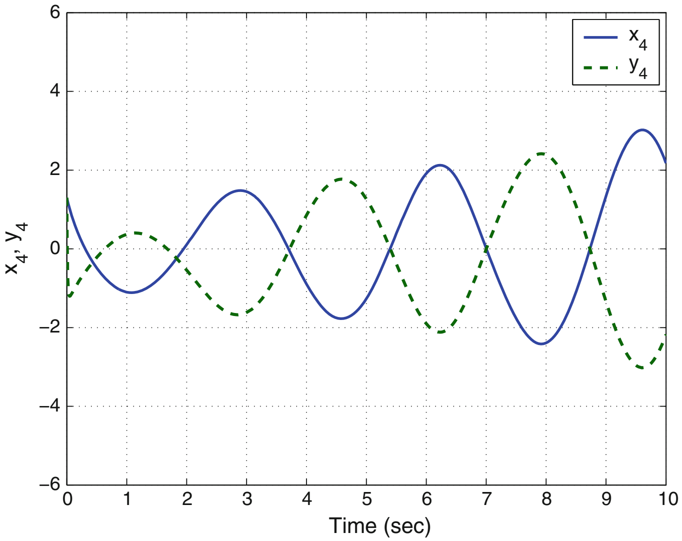

Fig. 8 Anti-synchronization of the states x_4 and y_4

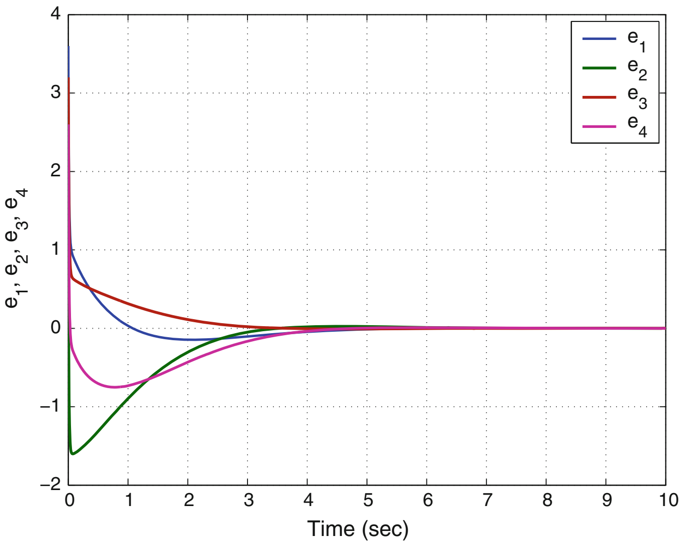

Fig. 9 Time-history of the anti-synchronization errors e_1, e_2, e_3, e_4

6 Conclusions

Chaos and hyperchaos have important applications in science and engineering. Hyperchaotic systems have more complex behaviour than chaotic systems and they have miscellaneous applications in areas like secure communications, cryptosystems, etc. In robust control systems, the sliding mode control is commonly used due to its inherent advantages of easy realization, fast response and good transient performance as well as insensitivity to parameter uncertainties and disturbance. In this work, we derived a novel sliding mode control method for the anti-synchronization of identical chaotic or hyperchaotic systems. We proved the main result using Lyapunov stability theory. As an application of the general result, the problem of anti-synchronization of identical Vaidyanathan hyperjerk hyperchaotic systems (2015) was studied and a new sliding mode controller has been derived. Numerical simulations using MATLAB were shown to depict the phase portraits of the Vaidyanathan hyperjerk system and the sliding mode controller design for the anti-synchronization of identical Vaidyanathan hyperjerk systems.

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