A No-Equilibrium Novel 4-D Highly Hyperchaotic System with Four Quadratic Nonlinearities and Its Adaptive Control

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Abstract In this work, we describe an eleven-term novel 4-D highly hyperchaotic system with four quadratic nonlinearities. The phase portraits of the eleven-term novel highly hyperchaotic system are depicted and the qualitative properties of the novel highly hyperchaotic system are discussed. We shall show that the novel hyperchaotic system does not have any equilibrium point. Hence, the novel 4-D hyperchaotic system exhibits hidden attractors. The Lyapunov exponents of the novel hyperchaotic system are obtained as $L_1 = 15.06593$, $L_2 = 0.03551$, $L_3 = 0$ and $L_4 = -42.42821$. The Maximal Lyapunov Exponent (MLE) of the novel hyperchaotic system is found as $L_1 = 15.06593$, which is large. Thus, the novel 4-D hyperchaotic system proposed in this work is highly hyperchaotic. Also, the Kaplan–Yorke dimension of the novel hyperchaotic system is derived as $D_{KY} = 3.3559$. Since the sum of the Lyapunov exponents is negative, the novel hyperchaotic system is dissipative. Next, an adaptive controller is designed to globally stabilize the novel highly hyperchaotic system with unknown parameters. Finally, an adaptive controller is also designed to achieve global chaos synchronization of the identical novel highly hyperchaotic systems with unknown parameters. MATLAB simulations are depicted to illustrate all the main results derived in this work.

Keywords Chaos · Chaotic systems · Hyperchaos · Hyperchaotic systems · Adaptive control · Chaos synchronization · Stability theory

1 Introduction

In the last few decades, Chaos theory has become a very important and active research field, employing many applications in different disciplines like physics, chemistry, biology, ecology, engineering and economics, among others.

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S. Vaidyanathan and C. Volos (eds.), *Advances and Applications in Nonlinear Control Systems*, Studies in Computational Intelligence 635,
DOI 10.1007/978-3-319-30169-3_12

Some classical paradigms of 3-D chaotic systems in the literature are Lorenz system [35], Rössler system [50], ACT system [1], Sprott systems [59], Chen system [15], Lü system [36], Cai system [13], Tigan system [70], etc.

Many new chaotic systems have been discovered in the recent years such as Zhou system [138], Zhu system [140], Li system [31], Wei–Yang system [131], Sundarapandian systems [63, 67], Vaidyanathan systems [78, 80, 82–85, 89, 96, 106, 107, 109, 115, 117, 120, 123, 124, 126], Pehlivan system [40], Sampath system [53], Pham system [43], etc.

The Lyapunov exponent is a measure of the divergence of phase points that are initially very close and can be used to quantify chaotic systems. It is common to refer to the largest Lyapunov exponent as the *Maximal Lyapunov Exponent* (MLE). A positive maximal Lyapunov exponent and phase space compactness are usually taken as defining conditions for a chaotic system.

A hyperchaotic system is defined as a chaotic system with at least two positive Lyapunov exponents. Thus, the dynamics of a hyperchaotic system can expand in several different directions simultaneously. Thus, the hyperchaotic systems have more complex dynamical behaviour and they have miscellaneous applications in engineering such as secure communications [18, 30, 133], cryptosystems [21, 48, 139], fuzzy logic [57, 136], electrical circuits [130, 134], etc.

The minimum dimension of an autonomous, continuous-time, hyperchaotic system is four. The first 4-D hyperchaotic system was found by Rössler [51]. Many hyperchaotic systems have been reported in the chaos literature such as hyperchaotic Lorenz system [23], hyperchaotic Lü system [14], hyperchaotic Chen system [32], hyperchaotic Wang system [129], hyperchaotic Vaidyanathan systems [52, 79, 87, 101, 105, 116, 118, 122, 125], hyperchaotic Pham system [42], etc.

Chaos theory and control systems have many important applications in science and engineering [2, 9–12, 141]. Some commonly known applications are oscillators [26, 58], chemical reactions [19, 41, 93, 94, 97, 99, 100, 104], biology [16, 28, 88, 90–92, 95, 98, 102, 103], ecology [20, 60], encryption [29, 137], cryptosystems [49, 71], mechanical systems [4–8], secure communications [17, 38, 135], robotics [37, 39, 127], cardiology [45, 132], intelligent control [3, 33], neural networks [22, 25, 34], memristors [44, 128], etc.

The control of a chaotic or hyperchaotic system aims to stabilize or regulate the system with the help of a feedback control. There are many methods available for controlling a chaotic system such as active control [61, 72, 73], adaptive control [62, 74, 81], sliding mode control [76, 77], backstepping control [119], etc.

The synchronization of chaotic systems aims to synchronize the states of master and slave systems asymptotically with time. There are many methods available for chaos synchronization such as active control [24, 54, 55, 110, 112], adaptive control [56, 64–66, 75, 108, 111], sliding mode control [68, 86, 114, 121], backstepping control [46, 47, 69, 113], etc.

This work is organized as follows. Section 2 describes the dynamic equations and phase portraits of the eleven-term novel 4-D hyperchaotic system. Section 3 details the qualitative properties of the novel hyperchaotic system. In this section, we establish that the novel hyperchaotic system does not have any equilibrium point. Thus, it follows that the novel hyperchaotic system exhibits hidden attractors.

The Lyapunov exponents of the novel hyperchaotic system are obtained as $L_1 = 15.06593$, $L_2 = 0.03551$, $L_3 = 0$ and $L_4 = -42.42821$, while the Kaplan–Yorke dimension of the novel hyperchaotic system is obtained as $D_{KY} = 3.3559$. Since the Maximal Lyapunov Exponent (MLE) of the novel hyperchaotic system is $L_1 = 15.06593$, which is a large value, we conclude that the proposed novel hyperchaotic system is highly hyperchaotic. A novel contribution of this research work is the finding of a highly hyperchaotic 4-D system with hidden attractors.

In Sect. 4, we design an adaptive controller to globally stabilize the novel highly hyperchaotic system with unknown parameters. In Sect. 5, an adaptive controller is designed to achieve global chaos synchronization of the identical novel highly hyperchaotic systems with unknown parameters. MATLAB simulations have been shown to illustrate all the main results derived in this research work. Section 6 summarizes the main results of this research work.

2 A Novel 4-D Hyperchaotic System

In this section, we describe an eleven-term novel hyperchaotic system, which is given by the 4-D dynamics

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + x_4 \\ \dot{x}_2 = -c x_1 x_3 + 3x_2 + p x_3^2 \\ \dot{x}_3 = x_1 x_2 - b \\ \dot{x}_4 = -q(x_1 + x_2) \end{cases} \tag{1}$$

where x_1, x_2, x_3, x_4 are the states and a, b, c, p, q are constant positive parameters.

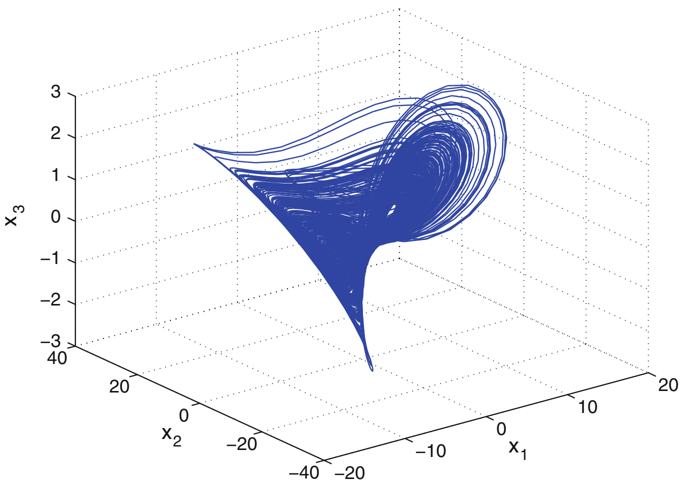

Fig. 1 3-D projection of the novel highly hyperchaotic system on the (x_1, x_2, x_3) space

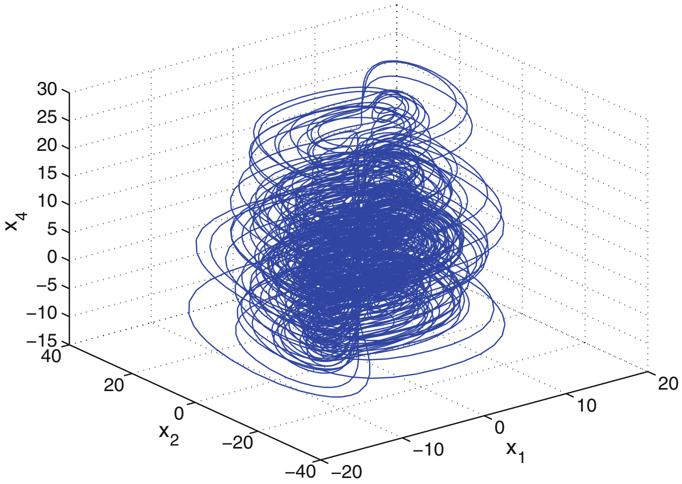

Fig. 2 3-D projection of the novel highly hyperchaotic system on the (x_1, x_2, x_4) space

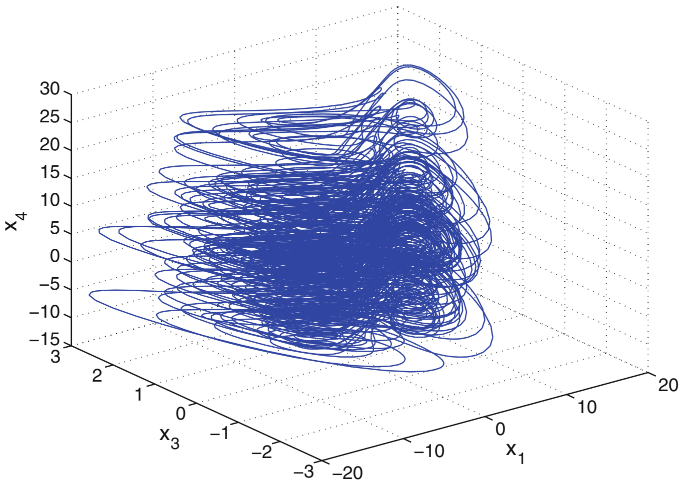

Fig. 3 3-D projection of the novel highly hyperchaotic system on the (x_1, x_3, x_4) space

The system (1) exhibits a *strange hyperchaotic attractor* for the parameter values

$$a = 62, \quad b = 36, \quad c = 160, \quad p = 0.5, \quad q = 2.8 \tag{2}$$

For numerical simulations, we take the initial conditions as

$$x_1(0) = 0.2, \quad x_2(0) = 0.8, \quad x_3(0) = 0.6, \quad x_4(0) = 0.4 \tag{3}$$

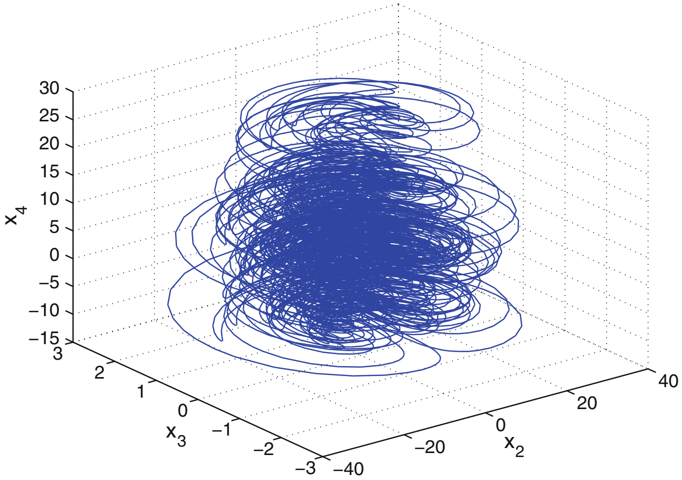

Fig. 4 3-D projection of the novel highly hyperchaotic system on the (x_2, x_3, x_4) space

Figures 1, 2, 3 and 4 show the 3-D projection of the novel hyperchaotic system (1) on the (x_1, x_2, x_3), (x_1, x_2, x_4), (x_1, x_3, x_4) and (x_2, x_3, x_4) spaces, respectively.

3 Analysis of the Novel 4-D Highly Hyperchaotic System

In this section, we study the qualitative properties of the novel 4-D highly hyperchaotic system (1). We take the parameter values as in the hyperchaotic case (2).

3.1 Dissipativity

In vector notation, the novel highly hyperchaotic system (1) can be expressed as

$$\dot{\mathbf{x}} = f(\mathbf{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3, x_4) \\ f_2(x_1, x_2, x_3, x_4) \\ f_3(x_1, x_2, x_3, x_4) \\ f_4(x_1, x_2, x_3, x_4) \end{bmatrix}, \tag{4}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3, x_4) = a(x_2 - x_1) + x_2 x_3 + x_4 \\ f_2(x_1, x_2, x_3, x_4) = -c x_1 x_3 + 3x_2 + p x_3^2 \\ f_3(x_1, x_2, x_3, x_4) = x_1 x_2 - b \\ f_4(x_1, x_2, x_3, x_4) = -q(x_1 + x_2) \end{cases} \tag{5}$$

Let Ω be any region in $\mathbf{R}^4$ with a smooth boundary and also, $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Furthermore, let $V(t)$ denote the hypervolume of $\Omega(t)$.

By Liouville's theorem, we know that

$$\dot{V}(t) = \int\limits_{\Omega(t)} (\nabla \cdot f)\, dx_1\, dx_2\, dx_3\, dx_4 \tag{6}$$

The divergence of the novel hyperchaotic system (4) is found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = -a + 3 = -\mu < 0 \tag{7}$$

since $\mu = a - 3 = 59 > 0$.

Inserting the value of $\nabla \cdot f$ from (7) into (6), we get

$$\dot{V}(t) = \int\limits_{\Omega(t)} (-\mu)\, dx_1\, dx_2\, dx_3\, dx_4 = -\mu V(t) \tag{8}$$

Integrating the first order linear differential equation (8), we get

$$V(t) = \exp(-\mu t) V(0) \tag{9}$$

Since $\mu > 0$, it follows from Eq. (9) that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the novel hyperchaotic system (1) is dissipative. Hence, the system limit sets are ultimately confined into a specific limit set of zero hypervolume, and the asymptotic motion of the novel hyperchaotic system (1) settles onto a strange attractor of the system.

3.2 Equilibrium Points

We take the parameter values as in the hyperchaotic case (2).

The equilibrium points of the 4-D system (1) are obtained by solving the system of equations

$$a(x_2 - x_1) + x_2 x_3 + x_4 = 0 \tag{10a}$$

$$-c x_1 x_3 + 3 x_2 + p x_3^2 = 0 \tag{10b}$$

$$x_1 x_2 - b = 0 \tag{10c}$$

$$-q(x_1 + x_2) = 0 \tag{10d}$$

Since $q \neq 0$, it is immediate from (10d) that

$$x_1 + x_2 = 0 \ \text{ or } \ x_1 = -x_2 \tag{11}$$

Substituting $x_1 = -x_2$ in (10c), we get

$$x_2^2 = -b \tag{12}$$

which has no solutions since $b > 0$.

Thus, we conclude that the novel highly hyperchaotic system (1) does not have any equilibrium points. Hence, the novel highly hyperchaotic system (1) exhibits hidden attractors.

3.3 Invariance

It is easy to see that the x_3-axis is invariant under the flow of the 4-D novel hyperchaotic system (1).

The invariant motion along the x_3-axis is characterized by the scalar dynamics

$$\dot{x}_3 = -b, \tag{13}$$

which is unstable.

3.4 Lyapunov Exponents and Kaplan–Yorke Dimension

We take the parameter values of the novel system (1) as in the hyperchaotic case (2), i.e.

$$a = 62, \quad b = 36, \quad c = 160, \quad p = 0.5, \quad q = 2.8 \tag{14}$$

We take the initial state of the novel system (1) as given in (3), i.e.

$$x_1(0) = 0.2, \quad x_2(0) = 0.8, \quad x_3(0) = 0.6, \quad x_4(0) = 0.4 \tag{15}$$

Then the Lyapunov exponents of the system (1) are numerically obtained using MATLAB as

$$L_1 = 15.06593, \quad L_2 = 0.03551, \quad L_3 = 0, \quad L_4 = -42.42821 \tag{16}$$

Since there are two positive Lyapunov exponents in (16), the novel system (1) exhibits *hyperchaotic* behavior.

From the LE spectrum (16), we see that the maximal Lyapunov exponent of the novel hyperchaotic system (1) is $L_1 = 15.06593$, which is large.

We find that

$$L_1 + L_2 + L_3 + L_4 = - - 27.32677 < 0 \tag{17}$$

Thus, it follows that the novel highly hyperchaotic system (1) is dissipative.

Also, the Kaplan–Yorke dimension of the novel hyperchaotic system (1) is calculated as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.3559, \tag{18}$$

which is fractional.

Since the Kaplan–Yorke dimension of the novel highly hyperchaotic system (1) has a large value, it follows that the 4-D system (1) exhibits highly complex dynamics and hence, it is suitable for engineering applications like secure communication and cryptosystems.

4 Adaptive Control of the Novel Highly Hyperchaotic System

In this section, we apply adaptive control method to derive an adaptive feedback control law for globally stabilizing the novel 4-D highly hyperchaotic system with unknown parameters. We use parameter estimates in lieu of the unknown system parameters. The main control result in this section is established using Lyapunov stability theory.

Thus, we consider the controlled novel 4-D highly hyperchaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + x_4 + u_1 \\ \dot{x}_2 = -c x_1 x_3 + 3x_2 + p x_3^2 + u_2 \\ \dot{x}_3 = x_1 x_2 - b + u_3 \\ \dot{x}_4 = -q(x_1 + x_2) + u_4 \end{cases} \tag{19}$$

In (19), x_1, x_2, x_3, x_4 are the states and u_1, u_2, u_3, u_4 are the adaptive controls to be determined using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ for the unknown parameters a, b, c, p, q, respectively.

We consider the adaptive feedback control law

$$\begin{cases} u_1 = -\hat{a}(t)(x_2 - x_1) - x_2 x_3 - x_4 - k_1 x_1 \\ u_2 = \hat{c}(t) x_1 x_3 - 3x_2 - \hat{p}(t) x_3^2 - k_2 x_2 \\ u_3 = -x_1 x_2 + \hat{b}(t) - k_3 x_3 \\ u_4 = \hat{q}(t)(x_1 + x_2) - k_4 x_4 \end{cases} \tag{20}$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (20) into (19), we get the closed-loop plant dynamics as

$$\begin{cases} \dot{x}_1 = [a - \hat{a}(t)](x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 = -[c - \hat{c}(t)]x_1 x_3 + [p - \hat{p}(t)]x_3^2 - k_2 x_2 \\ \dot{x}_3 = -[b - \hat{b}(t)] - k_3 x_3 \\ \dot{x}_4 = -[q - \hat{q}(t)](x_1 + x_2) - k_4 x_4 \end{cases} \tag{21}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{22}$$

In view of (22), we can simplify the plant dynamics (21) as

$$\begin{cases} \dot{x}_1 = e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 = -e_c x_1 x_3 + e_p x_3^2 - k_2 x_2 \\ \dot{x}_3 = -e_b - k_3 x_3 \\ \dot{x}_4 = -e_q(x_1 + x_2) - k_4 x_4 \end{cases} \tag{23}$$

Differentiating (22) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{24}$$

We consider the quadratic candidate Lyapunov function defined by

$$V(\mathbf{x}, e_a, e_b, e_c, e_p, e_q) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right) + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2 + e_p^2 + e_q^2\right) \tag{25}$$

Differentiating V along the trajectories of (23) and (24), we obtain

$$\begin{aligned} \dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 + e_a\left[x_1(x_2 - x_1) - \dot{\hat{a}}\right] + e_b\left[-x_3 - \dot{\hat{b}}\right] \\ + e_c\left[-x_1 x_2 x_3 - \dot{\hat{c}}\right] + e_p\left[x_2 x_3^2 - \dot{\hat{p}}\right] + e_q\left[-x_4(x_1 + x_2) - \dot{\hat{q}}\right] \end{aligned} \tag{26}$$

In view of (26), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}}(t) = x_1(x_2 - x_1) \\ \dot{\hat{b}}(t) = -x_3 \\ \dot{\hat{c}}(t) = -x_1 x_2 x_3 \\ \dot{\hat{p}}(t) = x_2 x_3^2 \\ \dot{\hat{q}}(t) = -x_4(x_1 + x_2) \end{cases} \tag{27}$$

Next, we state and prove the main result of this section.

Theorem 1 *The novel 4-D highly hyperchaotic system (19) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (20) and the parameter update law (27), where* k_1, k_2, k_3, k_4 *are positive gain constants.*

Proof We prove this result by applying Lyapunov stability theory [27].

We consider the quadratic Lyapunov function defined by (25), which is clearly a positive definite function on $\mathbf{R}^9$.

By substituting the parameter update law (27) into (26), we obtain the time-derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 \tag{28}$$

From (28), it is clear that $\dot{V}$ is a negative semi-definite function on $\mathbf{R}^9$.

Thus, we can conclude that the state vector $\mathbf{x}(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[x_1(t)\ x_2(t)\ x_3(t)\ x_4(t)\ e_a(t)\ e_b(t)\ e_c(t)\ e_p(t)\ e_q(t) \right]^T \in \mathbf{L}_\infty.$$

We define $k = \min\{k_1, k_2, k_3, k_4\}$.

Then it follows from (28) that

$$\dot{V} \leq -k \|\mathbf{x}(t)\|^2 \tag{29}$$

Thus, we have

$$k \|\mathbf{x}(t)\|^2 \leq -\dot{V} \tag{30}$$

Integrating the inequality (30) from 0 to t, we get

$$k \int_0^t \|\mathbf{x}(\tau)\|^2 \, d\tau \ \leq \ V(0) - V(t) \tag{31}$$

From (31), it follows that $\mathbf{x} \in \mathbf{L}_2$.

Using (23), we can conclude that $\dot{\mathbf{x}} \in \mathbf{L}_\infty$.

Using Barbalat's lemma [27], we conclude that $\mathbf{x}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{x}(0) \in \mathbf{R}^4$.

Thus, the novel 4-D highly hyperchaotic system (19) with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law (20) and the parameter update law (27).

This completes the proof. ■

For the numerical simulations, the classical fourth-order Runge–Kutta method with step size $h = 10^{-8}$ is used to solve the systems (19) and (27), when the adaptive control law (20) is applied.

The parameter values of the novel 4-D hyperchaotic system (19) are taken as in the hyperchaotic case (2), i.e.

$$a = 62, \quad b = 36, \quad c = 160, \quad p = 0.5, \quad q = 2.8 \tag{32}$$

We take the positive gain constants as

$$k_1 = 8, \quad k_2 = 8, \quad k_3 = 8, \quad k_4 = 8 \tag{33}$$

Furthermore, as initial conditions of the novel 4-D highly hyperchaotic system (19), we take

$$x_1(0) = 18.5, \quad x_2(0) = -14.7, \quad x_3(0) = 24.8, \quad x_4(0) = -12.3 \tag{34}$$

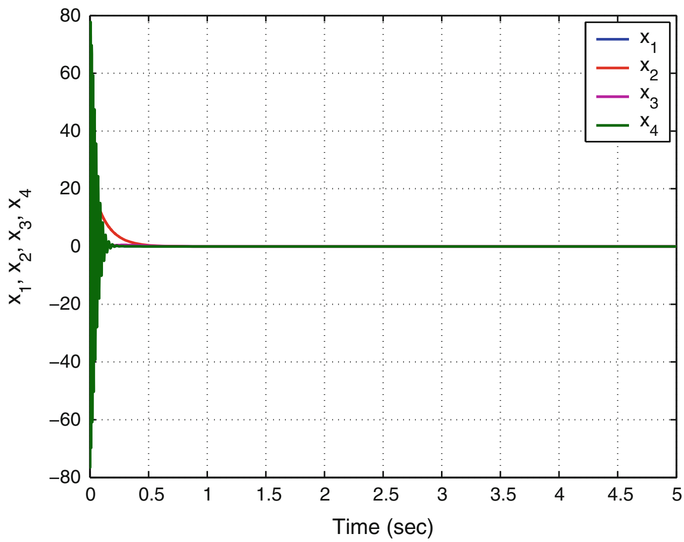

Fig. 5 Time-history of the controlled states x_1, x_2, x_3, x_4

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 15.6, \quad \hat{b}(0) = 12.4, \quad \hat{c}(0) = 22.7, \quad \hat{p}(0) = 4.8, \quad \hat{q}(0) = 19.4 \tag{35}$$

In Fig. 5, the exponential convergence of the controlled states of the novel 4-D hyperchaotic system (19) is shown.

5 Adaptive Synchronization of the Identical Novel Hyperchaotic Systems

In this section, we use adaptive control method to derive an adaptive feedback control law for globally synchronizing identical novel 4-D highly hyperchaotic systems with unknown parameters.

As the master system, we consider the novel 4-D hyperchaotic system given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 + x_4 \\ \dot{x}_2 = -c x_1 x_3 + 3x_2 + p x_3^2 \\ \dot{x}_3 = x_1 x_2 - b \\ \dot{x}_4 = -q(x_1 + x_2) \end{cases} \tag{36}$$

In (36), x_1, x_2, x_3, x_4 are the states and a, b, c, p, q are unknown system parameters.

As the slave system, we consider the 4-D novel hyperchaotic system given by

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + y_2 y_3 + y_4 + u_1 \\ \dot{y}_2 = -c y_1 y_3 + 3y_2 + p y_3^2 \\ \dot{y}_3 = y_1 y_2 - b + u_3 \\ \dot{y}_4 = -q(y_1 + y_2) + u_4 \end{cases} \tag{37}$$

In (37), y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the adaptive controls to be determined using estimates $\hat{a}(t), \hat{c}(t), \hat{p}(t), \hat{q}(t)$ for the unknown parameters a, c, p, q, respectively.

The synchronization error between the novel hyperchaotic systems (36) and (37) is defined by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \\ e_4 = y_4 - x_4 \end{cases} \tag{38}$$

Then the error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 = 3e_2 - c(y_1 y_3 - x_1 x_3) + p(y_3^2 - x_3^2) + u_2 \\ \dot{e}_3 = y_1 y_2 - x_1 x_2 + u_3 \\ \dot{e}_4 = -q(e_1 + e_2) + u_4 \end{cases} \tag{39}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) - e_4 - y_2 y_3 + x_2 x_3 - k_1 e_1 \\ u_2 = -3e_2 + \hat{c}(t)(y_1 y_3 - x_1 x_3) - \hat{p}(t)(y_3^2 - x_3^2) - k_2 e_2 \\ u_3 = -y_1 y_2 + x_1 x_2 - k_3 e_3 \\ u_4 = \hat{q}(t)(e_1 + e_2) - k_4 e_4 \end{cases} \tag{40}$$

where k_1, k_2, k_3, k_4 are positive gain constants.

Substituting (40) into (39), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 = \left[a - \hat{a}(t)\right](e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = -\left[c - \hat{c}(t)\right](y_1 y_3 - x_1 x_3) + \left[p - \hat{p}(t)\right](y_3^2 - x_3^2) - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \\ \dot{e}_4 = -\left[q - \hat{q}(t)\right](e_1 + e_2) - k_4 e_4 \end{cases} \tag{41}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \\ e_q(t) = q - \hat{q}(t) \end{cases} \tag{42}$$

In view of (42), we can simplify the error dynamics (41) as

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) - k_1 e_1 \\ \dot{e}_2 = -e_c(y_1 y_3 - x_1 x_3) + e_p(y_3^2 - x_3^2) - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \\ \dot{e}_4 = -e_q(e_1 + e_2) - k_4 e_4 \end{cases} \tag{43}$$

Differentiating (42) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \\ \dot{e}_q(t) = -\dot{\hat{q}}(t) \end{cases} \tag{44}$$

We use adaptive control theory to find an update law for the parameter estimates.

We consider the quadratic candidate Lyapunov function defined by

$$V(\mathbf{e}, e_a, e_c, e_p, e_q) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) + \frac{1}{2}\left(e_a^2 + e_c^2 + e_p^2 + e_q^2\right) \tag{45}$$

Differentiating V along the trajectories of (43) and (44), we obtain

$$\begin{aligned}\dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[e_1(e_2 - e_1) - \dot{\hat{a}}\right] \\ &+ e_c\left[-e_2(y_1 y_3 - x_1 x_3) - \dot{\hat{c}}\right] + e_p\left[e_2(y_3^2 - x_3^2) - \dot{\hat{p}}\right] \\ &+ e_q\left[-e_4(e_1 + e_2) - \dot{\hat{q}}\right]\end{aligned} \tag{46}$$

In view of (46), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}}(t) = e_1(e_2 - e_1) \\ \dot{\hat{c}}(t) = -e_2(y_1 y_3 - x_1 x_3) \\ \dot{\hat{p}}(t) = e_2(y_3^2 - x_3^2) \\ \dot{\hat{q}}(t) = -e_4(e_1 + e_2) \end{cases} \tag{47}$$

Next, we state and prove the main result of this section.

Theorem 2 *The novel hyperchaotic systems (36) and (37) with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (40) and the parameter update law (47), where k_1, k_2, k_3, k_4 are positive gain constants.*

Proof We prove this result by applying Lyapunov stability theory [27].

We consider the quadratic Lyapunov function defined by (45), which is clearly a positive definite function on $\mathbf{R}^8$.

By substituting the parameter update law (47) into (46), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{48}$$

From (48), it is clear that $\dot{V}$ is a negative semi-definite function on $\mathbf{R}^8$.

Thus, we can conclude that the error vector $\mathbf{e}(t)$ and the parameter estimation error are globally bounded, i.e.

$$\left[\, e_1(t)\ e_2(t)\ e_3(t)\ e_4(t)\ e_a(t)\ e_c(t)\ e_p(t)\ e_q(t) \,\right]^T \in \mathbf{L}_\infty. \tag{49}$$

We define $k = \min\{k_1, k_2, k_3, k_4\}$.

Then it follows from (48) that

$$\dot{V} \le -k\|\mathbf{e}(t)\|^2 \tag{50}$$

Thus, we have

$$k\|\mathbf{e}(t)\|^2 \leq -\dot{V} \tag{51}$$

Integrating the inequality (51) from 0 to t, we get

$$k\int_0^t \|\mathbf{e}(\tau)\|^2 \, d\tau \ \leq \ V(0) - V(t) \tag{52}$$

From (52), it follows that $\mathbf{e} \in \mathbf{L}_2$.

Using (43), we can conclude that $\dot{\mathbf{e}} \in \mathbf{L}_\infty$.

Using Barbalat's lemma [27], we conclude that $\mathbf{e}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{e}(0) \in \mathbf{R}^4$.

This completes the proof. ■

For the numerical simulations, the classical fourth-order Runge–Kutta method with step size $h = 10^{-8}$ is used to solve the systems (36), (37) and (47), when the adaptive control law (40) is applied.

The parameter values of the novel hyperchaotic systems are taken as in the hyperchaotic case (2), i.e.

$$a = 62, \quad b = 36, \quad c = 160, \quad p = 0.5, \quad q = 2.8 \tag{53}$$

We take the positive gain constants as

$$k_1 = 8, \quad k_2 = 8, \quad k_3 = 8, \quad k_4 = 8 \tag{54}$$

Furthermore, as initial conditions of the master system (36), we take

$$x_1(0) = 12.3, \quad x_2(0) = 6.4, \quad x_3(0) = -9.7, \quad x_4(0) = -22.8 \tag{55}$$

As initial conditions of the slave system (37), we take

$$y_1(0) = 5.1, \quad y_2(0) = -18.5, \quad y_3(0) = 24.8, \quad y_4(0) = 3.7 \tag{56}$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 12.6, \quad \hat{c}(0) = 5.4, \quad \hat{p}(0) = 17.9, \quad \hat{q}(0) = 25.8 \tag{57}$$

Figures 6, 7, 8 and 9 describe the complete synchronization of the novel hyperchaotic systems (36) and (37), while Fig. 10 describes the time-history of the synchronization errors e_1, e_2, e_3, e_4.

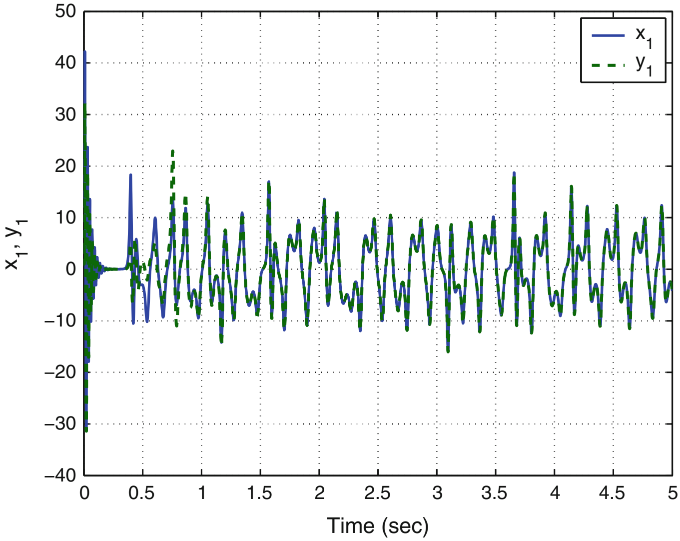

Fig. 6 Synchronization of the states x_1 and y_1

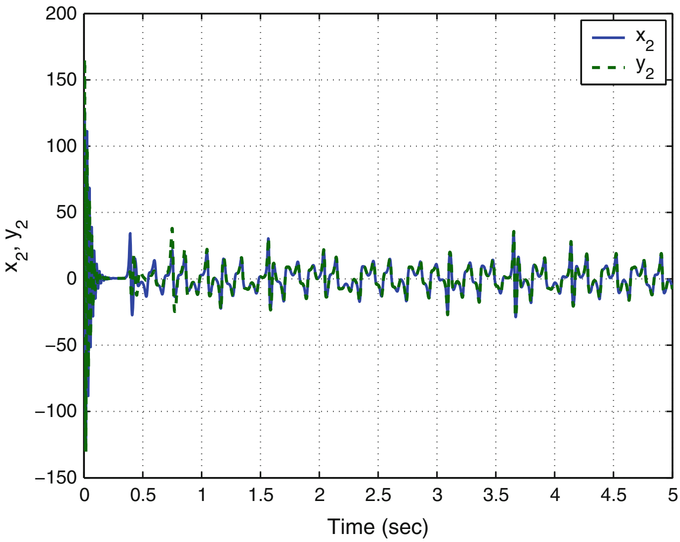

Fig. 7 Synchronization of the states x_2 and y_2

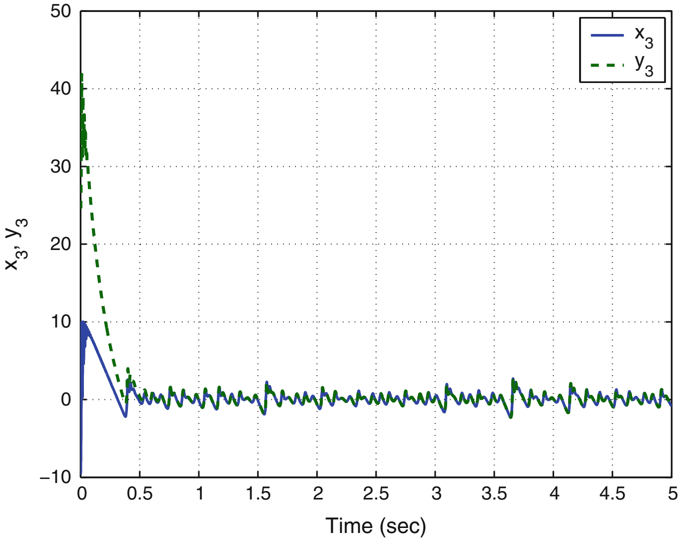

Fig. 8 Synchronization of the states x_3 and y_3

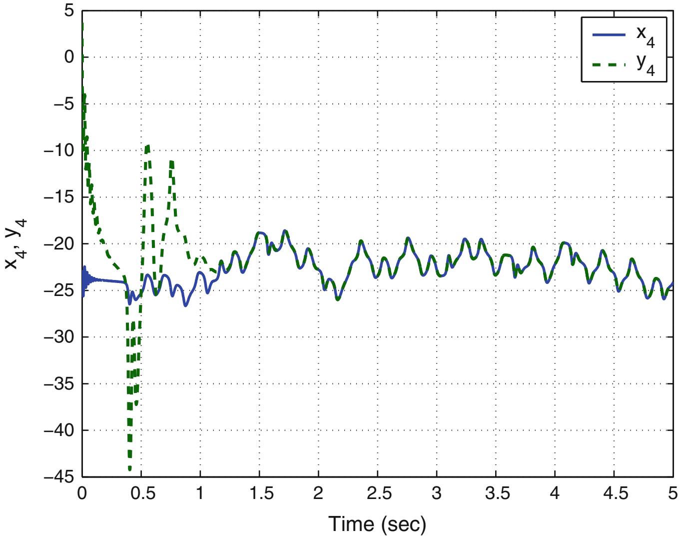

Fig. 9 Synchronization of the states x_4 and y_4

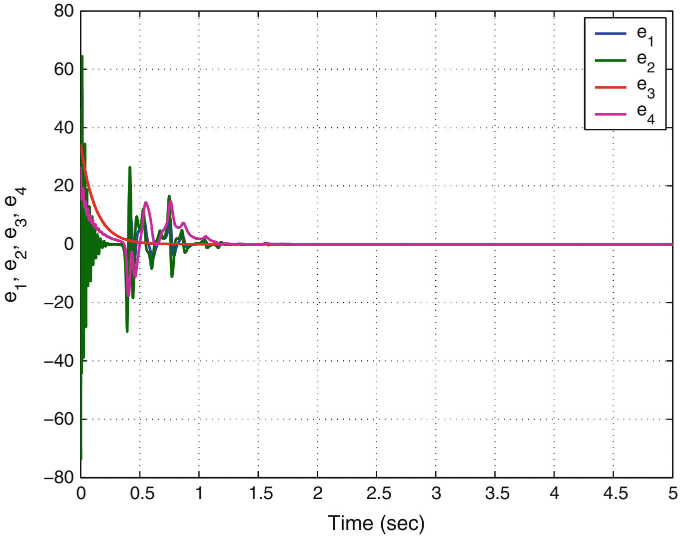

Fig. 10 Time-history of the synchronization errors e_1, e_2, e_3, e_4

6 Conclusions

In this work, we described an eleven-term novel 4-D highly hyperchaotic system with four quadratic nonlinearities. The qualitative properties of the novel highly hyperchaotic system were discussed in detail. We showed that the novel hyperchaotic system does not have any equilibrium point. Hence, the novel 4-D hyperchaotic system exhibits hidden attractors. The Lyapunov exponents of the novel hyperchaotic system have been obtained as $L_1 = 15.06593, L_2 = 0.03551, L_3 = 0$ and $L_4 = -42.42821$. The Maximal Lyapunov Exponent (MLE) of the novel hyperchaotic system is found as $L_1 = 15.06593$, which is large. Thus, the novel 4-D hyperchaotic system proposed in this work is highly hyperchaotic. Also, the Kaplan–Yorke dimension of the novel hyperchaotic system has been derived as $D_{KY} = 3.3559$. Since the sum of the Lyapunov exponents is negative, the novel hyperchaotic system is dissipative. Next, an adaptive controller was designed to globally stabilize the novel highly hyperchaotic system with unknown parameters. Finally, an adaptive controller was also designed to achieve global chaos synchronization of the identical novel highly hyperchaotic systems with unknown parameters. MATLAB simulations were shown to illustrate all the main results derived in this work.

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