Chapter 13
Development of Models for the Estimation of Global Solar Radiation Over Selected Stations in India

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Introduction

Solar radiation, passing through the atmosphere, reaching the earth on a horizontal surface can be classified into two components: beam radiation and diffuse radiation. Beam radiation is the solar radiation propagating along the line joining the receiving surface and the center of the sun. It is also referred to as direct radiation. Diffuse radiation on the other hand, is the solar radiation scattered by aerosols, dust and molecules in the atmosphere, it does not have a unique direction. The total radiation is the sum of the beam and diffuses radiation and is sometimes referred to as the global radiation. When the amount of diffuse radiation reaching the earth's surface is less than or equal to 25 % of global radiation, the sky is termed as clear sky (Duffie and Beckman 1991).

In many applications of solar energy, the solar irradiance incident on the surface of the earth at the location of interest is an important input parameter. The temporal and spatial fluctuations of such irradiance necessitate a method to predict them.

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The systematic variation of solar irradiance outside the earth's atmosphere makes it possible to introduce many models for such prediction (Munroe 1980).

Knowledge of global solar irradiance at a site is essential for the proper design and assessment of solar energy conversion systems. Some of the systems such as concentrating systems require information on direct beam component whereas in the case of tilted plain surfaces the diffuse component of solar irradiance is also important for the computation of system performance (Khogali et al. 1983). However, at locations on the Earth's surface, the solar radiation is also a function of variables such as the nature and extent of cloud cover, the aerosol and water vapor content of atmosphere, etc. Good prediction of the actual value of solar irradiance for a given location requires, in principle, long-term, average meteorological data, which are still scarce for many developing countries like India (Leung 1980; Ezekwe and Ezeilo 1981; Khogali 1983). It is, therefore, not always possible to predict the actual value of solar irradiance for a location of interest.

There are several formulae available in literature on solar radiation modelling that relate global radiation to climatological parameters such as sunshine hours, relative humidity, maximum temperature, and average temperature. The first such correlation proposed for estimating the monthly average daily global irradiation is due Angstrom (1924). The original Angstrom-type regression equation relates monthly average daily radiation to clear day radiation at the location in question and average fraction of possible sunshine hours:

$$H/H_c = a + b\left(\frac{S}{S_o}\right) \tag{13.1}$$

A basic difficulty with Eq. (13.1) lies in the ambiguity of the terms S/S_o and H_o. Page (1961) modified the method to base it on extraterrestrial radiation on a horizontal surface rather than on clear sky day radiation.

$$H/H_0 = a' + b'\left(\frac{S}{S_o}\right) \tag{13.2}$$

Inspite of having complication in Hc calculations, better results were obtained using Hc instead of H_0 (Toğrul 1999). The major objective of this article to investigate usability of clear sky radiation to predict and express the average measured values of solar irradiance on a horizontal surface by using various regression analyses for selected locations in India.

Clear Sky Solar Radiation

Hottel (1976) has presented a method for estimating the beam radiation transmitted through clear atmospheres which takes into account zenith angle and altitude for a standard atmosphere and four climate types. The atmospheric transmittance for beam radiation is given in the form:

Table 13.1 Correction factors for different climates

Climate type	r_0	r_1	r_k
Tropical	0.95	0.98	1.02
Midlatitude summer	0.97	0.99	1.02
Subarctic summer	0.99	0.99	1.01
Midlatitude winter	1.03	1.01	1.00

$$\tau_b = a_o + a_1 \exp(-k \cos \theta_z) \tag{13.3}$$

The constant and k for the standard atmosphere with 23 km visibility are found from a_o*, a_1* and K* which are given for altitudes less than 2.5 km by

$$a_o^* = 0.4237 - 0.00821(6 - A)^2 \tag{13.4}$$

$$a_1^* = 0.5055 + 0.00595(6.5 - A)^2 \tag{13.5}$$

$$K^* = 0.2711 + 0.01858(2.5 - A)^2 \tag{13.6}$$

where, A is the altitude of the observer in kilometers.

The correction factors are applied to a_o*, a_1* and $K*$ to allow for changes in climate types.

The correction factors $r_o = a_o/a_o*$, $r_1 = a_1/a_1*$ and $r_k = k/K*$ are given in Table 13.1.

Thus, the transmittance of this standard atmosphere for beam radiation can be determined for any zenith angle and any altitude up to 2.5 km. The clear sky beam radiation (G_{cb}, W/m^2) is than

$$G_{cb} = G_{on}\tau_b \tag{13.7}$$

where, G_{on} is the extraterrestrial radiation, measured on the plane normal to the radiation on the nth day of the year and given in the following form (W/m^2).

$$G_{on} = G_{sc}\left(1 + 0.33 \cos\left(\frac{360n}{365}\right)\right) \tag{13.8}$$

where, G_{sc} is the solar constant equal to 1367 W/m^2.

The clear sky horizontal beam radiation is

$$G_{cb} = G_{on}\tau_b \cos \theta_z \tag{13.9}$$

It is also necessary to estimate the clear sky diffuse radiation on a horizontal surface to get the total radiation as suggested by Liu and Jordan (1960). They developed an empirical relationship between the transmission coefficients for beam and diffuse radiation for clear days.

$$\tau_d = 0.271 - 0.294\tau_b \tag{13.10}$$

where τ_d is the ratio of diffuse radiation to the extraterrestrial radiation on the horizontal plane. The clear sky diffuse radiation G_{cd} (W/m^2).

$$G_{cd} = G_{on}\tau_d \cos\theta_z \tag{13.11}$$

Thus, the clear sky global solar radiation is given by

$$G_c = G_{cb} + G_{cd} \tag{13.12}$$

Meteorological Data

In the present study, the monthly average global solar radiation, have been calculated, using sunshine hour data on horizontal surfaces. Six models have been developed which include the effect of latitude and altitude of a location. These six stations have been selected for different geographical locations covering most part of India (Table 13.2).

The measured values of the monthly average global solar radiation (G) and the S/S_o ratio for six locations are obtained from Chandel et al. (2005) and are provided in Table 13.3. G_c values calculated by Hottle's model for six cities are given in Table 13.4.

Development of Models

The following equations were obtained by investigating the relation between G/G_c and S/S_o by different regression analysis. The scatter of monthly mean values between G/G_c and S/S_o are given in Fig. 13.1.

Table 13.2 Geographical location of six Indian cities (Chandel et al. 2005)

Station (State)	Longitude (°E)	Latitude (°N)	Altitude (m)
Palampur (Himachal Pradesh)	76.30	32.60	1270
Amritsar (Punjab)	74.87	31.63	234
Kodaikanal (Tamil Nadu)	77.47	10.23	2339
Nandi Hill (Karnatka)	77.68	13.37	1479
New Delhi (Delhi)	77.20	28.58	216
Shillong (Meghalaya)	91.88	25.57	1600

Table 13.3 Measured values of monthly average global solar radiation (G) and S/S_o for six cities (Chandel et al. 2005)

	Palampur		Amritsar		Kodaikanal		Nandi Hills		Delhi		Shillong	
Month	G	S/S_o	G	S/S_o	G	S/S_o	G	S/S_o	G	S/S_o	G	S/S_o
January	2.80	0.57	3.10	0.66	6.28	0.62	6.37	0.86	3.99	0.73	4.01	0.52
February	3.37	0.55	3.99	0.69	6.71	0.67	6.26	0.81	5.00	0.81	5.08	0.65
March	4.31	0.58	4.88	0.63	6.87	0.63	7.49	0.84	6.14	0.69	5.63	0.68
April	5.24	0.65	6.26	0.73	6.43	0.55	7.33	0.80	6.94	0.68	5.70	0.62
May	6.15	0.69	6.23	0.65	5.84	0.41	7.29	0.62	7.29	0.60	5.23	0.53
June	5.33	0.60	6.60	0.74	5.28	0.27	6.27	0.33	6.54	0.43	4.12	0.33
July	3.92	0.29	6.32	0.63	4.54	0.20	4.92	0.29	5.33	0.43	4.26	0.39
August	4.34	0.33	5.30	0.65	4.70	0.32	5.17	0.32	5.05	0.43	4.21	0.35
September	3.94	0.53	5.45	0.71	4.80	0.33	4.78	0.36	5.60	0.57	3.87	0.41
October	5.08	0.85	4.47	0.76	4.44	0.30	5.66	0.55	5.36	0.78	4.02	0.54
November	3.61	0.82	3.71	0.78	4.74	0.36	3.47	0.39	4.52	0.87	3.94	0.55
December	2.76	0.66	3.01	0.74	5.22	0.53	4.38	0.57	3.84	0.78	4.15	0.57

Table 13.4 Calculated G_c values for each city (using Hottle's Model) in $kW/m^2/day$

Months	Palampur	Amritsar	Kodaikanal	Nandi Hill	Delhi	Shillong
January	3.6401	3.3702	6.6410	6.0672	3.7228	4.6388
February	4.6831	4.3351	7.2561	6.7572	4.6764	5.6139
March	5.9138	5.4802	7.7983	7.4148	5.7955	6.7156
April	6.9983	6.4898	8.0260	7.7831	6.7738	7.6232
May	7.5867	7.0321	7.9153	7.7856	7.2861	8.0549
June	7.7606	7.1892	7.7717	7.6932	7.4271	8.1534
July	7.6540	7.0924	7.8044	7.7038	7.3364	8.0782
August	7.2133	6.6886	7.9330	7.7408	6.9578	7.7667
September	6.3071	5.8471	7.8448	7.5193	6.1472	7.0345
October	5.0445	4.6710	7.3853	6.9265	5.0007	5.9287
November	3.8828	3.5941	6.7671	6.2146	3.9439	4.8615
December	3.3433	3.0965	6.4305	5.8408	3.4459	4.3497

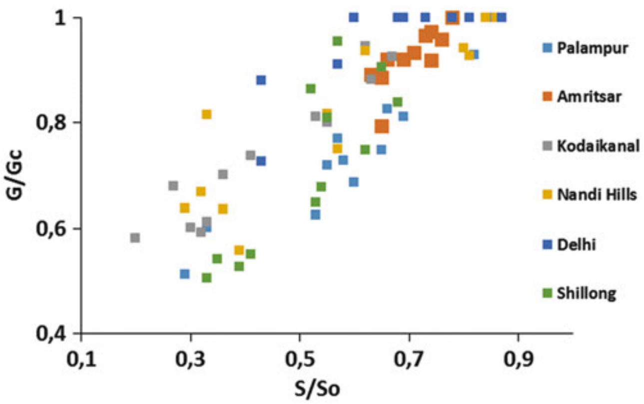

Fig. 13.1 G/G_c vs S/S_o for six cities in India

$$\frac{G}{G_c} = 0.7604\left(\frac{S}{S_o}\right) + 0.3750 \quad \text{(linear)} \tag{13.13}$$

$$\frac{G}{G_c} = 0.0163\left(\frac{S}{S_o}\right)^2 - 0.7424\left(\frac{S}{S_o}\right) + 0.3794 \quad \text{(quadratic)} \tag{13.14}$$

$$\frac{G}{G_c} = -3.2664\left(\frac{S}{S_o}\right)^3 + 5.5108\left(\frac{S}{S_o}\right)^2 - 2.1434\left(\frac{S}{S_o}\right) + 0.8428 \quad \text{(cubic)} \tag{13.15}$$

$$\frac{G}{G_c} = 1.0742\left(\frac{S}{S_o}\right)^{0.4933} \quad \text{(power)} \tag{13.16}$$

$$\frac{G}{G_c} = 0.3761\ln\left(\frac{S}{S_o}\right) + 1.0401 \quad \text{(logarithmic)} \tag{13.17}$$

$$\frac{G}{G_c} = 0.4511\exp^{0.989\left(\frac{S}{S_o}\right)} \quad \text{(exponential)} \tag{13.18}$$

Various statistical methods are available in solar energy literature, which deal with the assessment and comparison of solar radiation estimation models (Stone 1993; Toğrul 1999; Toğrul and Onat 1999; Walpole and Mayers 1989; Şahin 2007). Presently, statistical tests, root mean square error (RMSE), mean bias error (MBE) and t-statistics are utilized to evaluate the accuracy of the correlations developed.

Methods of Comparison

Statistical tests, root mean square error (RMSE) and mean bias error (MBE) were used to evaluate the accuracy of the correlations developed. Also, t-statistics was applied to the developed models to illustrate the statistically significant results.

Root Mean Square Error (RMSE)

The root mean square error is defined as

$$RMSE = \left(\frac{1}{N}\sum_{i=1}^{N}\left(G_{i,pre} - G_{i,meas}\right)^2\right)^{\frac{1}{2}} \tag{13.19}$$

where, $G_{i,pre}$ is *i*th predicted value, $G_{i,meas}$ is the *i*th measured value, and *N* is the total number of observations. The RMSE is always positive, a zero value is ideal. This test provides information on short-terms performance of the correlation by arranging a term by term comparison of the actual deviation between the calculated value and the measured value. The smaller the value, the better the model's performance, however, a few large errors in the sum can produce a significant increase in RMSE.

Mean Bias Error (MBE)

The mean bias error is defined as

$$MBE = \frac{1}{N}\sum_{i=1}^{N}\left(G_{i,pre} - G_{i,meas}\right) \tag{13.20}$$

This test provides information on the long term performance. A low value is desired. Ideally a zero value of MBE should be obtained. A positive value gives the average amount of over-estimation in the calculated value and vice versa. A drawback of this test is that over estimation of an individual observation will cancel under estimation in a separate observation.

It is obvious that each test by itself may not be an adequate indicator of a model's performance. It is possible to have a large RMSE value and at the same time a small MBE (a large scatter about the line of perfect estimation). On the other hand, it is also possible to have a relatively small RMSE and a relatively large MBE (a consistently small over or under estimation).

However, these statistical indicators generally provide a reasonable procedure to compare models, they do not objectively indicate whether a model's estimates are statistically significant, i.e. not significantly different from their measured counterparts. Therefore, an additional statistical indicator, the t-statistic is used. This statistical indicator allows models to be compared and at the same time indicate whether or not a model's estimates are statistically significant at a particular confidence level. It was seen that the t-statistic used in addition to the RMSE and MBE gave more reliable and explanatory results (Walpole and Mayers 1989).

t-Statistics

$$t = \left[\frac{(N-1)MBE^2}{RMSE^2 - MBE^2}\right]^{\frac{1}{2}} \tag{13.21}$$

The smaller the value of t, the better is the model's performance. To determine whether a model's estimates are statistically significant, one simply has to determine a critical t-value obtainable from standard statistical tables, at a particular confidence level, i.e. $t_{\alpha/2}$ at an α-level of significance and (N-1) degrees of freedom. For the model's estimates to be judged statistically significant at the (1-α) confidence level, the calculated t-value must be less than the critical t-value.

Results and Discussion

As observed from Table 13.5, good results were not seen in the short term (RMSE) but relatively good results were observed in the long term performance (MBE). Equation (13.17) has the best result among the equations developed.

When investigations were compared among all the equations the best MBE value was seen in Eq. (13.17). The best RMSE value was obtained with Eq. (13.15) which is a cubic equation.

Table 13.5 RMSE and MBE for the whole country (India)

Equation number	RMSE	MBE
13.13	0.5726	−0.0222
13.14	0.5723	−0.0227
13.15	***0.5662***	−0.0233
13.16	0.5869	−0.0496
13.17	0.6096	***−0.0192***
13.18	0.5711	−0.0511

Also, it is observed that considering country as a whole and each city individually, the performance of developed equations is different. Therefore, the MBE and RMSE values of the developed equations separately for each city were calculated. The results of this statistical comparison are given in Table 13.6.

At the first view, it is seen that the MBE values of Table 13.4 are higher than tabulated in Table 13.5 RMSE and MBE for the whole country (India). Each equation developed for the city was compared with the equations in its group and the results obtained were setup in order below (Table 13.7).

Although the models give overall good results for the whole country (Table 13.5) but the errors were higher for individual cities (Table 13.6). These tables did not include adequate information about on usability of the developed equation. In view of this, t-statistics is applied to the developed models to investigate the usability of each model. The critical *t*-values are shown in Table 13.8, *t*-values higher than the critical *t*-values show that the equation has no statistical significance.

Equations which have *t*-values lower than the critical *t*-value exhibited good and logical results. In case of Palampur, Eqs. (13.16) and (13.17) are significant. In case of Amritsar and Kodaikan almost all of the equations showed good and logical results. In case of Nandi Hills and Delhi Eqs. (13.13), (13.14) and (13.18) are significant. In case of Shillong, Eqs. (13.15) and (13.18) are significant.

Conclusions

It was observed that the clear sky solar radiation and sunshine hour can be used to estimate the global radiation in India. It was further observed that the cubic equation gave the lowest RMSE error, and the lowest MBE error was obtained by the logarithmic equation developed for the whole year (Table 13.5). It is also observed that the performance of the equations is different for all the stations taking together and for the cities individually. Cubic and logarithmic models gave the best results among all the developed equations for the country as a whole. While for individual cities cubic, logarithmic and exponentials yield better results.

Finally these results clearly indicate that reliance on the RMSE and MBE used separately can lead to a wrong decision in selecting the best model suited from the candidate models and that the use of the RMSE and MBE in isolation is not an adequate indicator of model performance. Therefore, the t-statistics should be used in conjunction with these two indicators to evaluate a model's performance in a better way.

Table 13.6 The RMSE and MBE values of the equation developed for each city

	Palampur		Amritsar		Kodaikanal		Nandi Hills		Delhi		Shillong	
Equation	RMSE	MBE	RMSE	MBE	RMSE	MBE	RMSE	MBE	RMSE	MBE	RMSE	MBE
13.13	0.5992	0.4945	0.2162	−0.0699	0.3796	−0.2552	0.5645	−0.1028	0.7373	−0.6424	0.7448	0.4428
13.14	0.5977	0.4938	0.2164	−0.0711	0.379	−0.2547	0.5643	−0.1016	0.7385	−0.6431	0.744	0.4408
13.15	0.6213	0.1173	**0.2137**	**−0.0067**	**0.3537**	**−0.2075**	**0.5634**	−0.1548	0.7543	−0.6709	**0.6931**	0.4031
13.16	0.5988	0.1115	0.2301	−0.1023	0.433	−0.2941	0.5757	−0.1673	0.7527	−0.666	0.7549	0.4594
13.17	0.6495	0.1194	0.2275	−0.0729	0.4984	−0.2906	0.5864	−0.1608	**0.7171**	**−0.6277**	0.806	0.5307
13.18	**0.5515**	**0.1091**	0.2273	−0.1011	0.3998	−0.2826	0.5823	**−0.0897**	0.7765	−0.6686	0.7064	**0.3729**

Table 13.7 Equations with significant results for selected stations

Station	Equation number
Palampur	13.18
Amritsar	13.15
Kodaikanal	13.15
Nandi Hills	13.15 and 13.18
Delhi	13.17
Shillong	13.15 and 13.18

Table 13.8 Critical *t*-values and the results of *t*-statistics analyses for each city

Equation	Palampur	Amritsar	Kodaikanal	Nandi Hills	Delhi	Shillong
13.13	4.8466	**1.1332**	**3.012**	**0.6143**	**5.8881**	2.4523
13.14	4.8633	**1.1538**	**3.0099**	**0.6071**	**5.875**	2.4392
13.15	4.4236	**0.104**	**2.4026**	0.9478	5.4541	**2.3712**
13.16	**4.2603**	1.6462	**3.0693**	1.0073	6.2982	2.5436
13.17	**4.1172**	**1.1219**	**2.3803**	0.9457	6.0041	2.9015
13.18	5.1024	1.6471	3.3142	**0.5171**	**5.6157**	**2.0614**
Critical *t*	**4.2658**	**1.2702**	**3.1036**	**0.7687**	**5.979**	**2.4175**

Nomenclature

A	Altitude (km)
a, *b*, *a*', *b*'	Empirical constants
G_c	Clear sky global solar radiation (W/m^2)
G_{cb}	Clear sky beam radiation (W/m^2)
G_{cd}	Clear sky diffuse radiation (W/m^2)
G_{on}	Extraterrestrial radiation (W/m^2)
G_{sc}	Solar constant (=1367 (W/m^2))
H	Monthly mean daily global radiation on a horizontal surface (MJ/m^2)
H_c	Clear sky monthly mean daily global radiation on a horizontal surface (MJ/m^2)
H_o	Monthly mean daily extraterrestrial radiation (MJ/m^2)
n	Day of the year
N	Number of observations
S	Monthly average daily hours of bright sunshine
S_o	Monthly average of maximum possible daily hours of bright sunshine (i.e. day length of average day of the month)

Greek Symbols

α	Level of significance
θ_z	Zenith angle (°)
τ_b	Atmospheric transmittance for beam radiation

τ_d Atmospheric transmittance for diffuse radiation
ω The sunset hour angle (°)
ϕ Latitude (°)
δ Declination angle (°)

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