# Chapter 7
# Harmonic Force Excitation Analysis of a Small-Body Asteroid/Satellite System

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**Abstract** A harmonic force excitation analysis is used to determine orbit stability of a small satellite around an asteroid with a complex gravitational field. Harmonic excitation phenomena occurs with both natural and man-made satellites. Jupiter influences asteroid distribution in the main asteroid belt through mean motion resonance where some regions are devoid of asteroids while other regions have an abundance. Simulations of a man-made satellite in orbit around the asteroid Itokawa, which was visited by the Hayabusa Space Mission, have also displayed harmonic excitation phenomena, including regions of high dynamic interactions. Specifically, the influence of the rate of spin of the asteroid and the stability of the orbit was investigated. The radial acceleration of the satellite is used to determine the frequency of gravitational perturbation from the asteroid on the satellite. It has been shown when the satellite is placed in an orbit away from its resonant frequency, the orbit remains stable. An early model was created to study this phenomenon and showed promise to predict regions of stability. From this initial study, further work using a more complex model and an updated harmonic force excitation analysis of the system was shown to be a more accurate predictor of orbital stability.

**Keywords** Harmonic analysis • Satellite control • Orbital stability • Itokawa • Space mission design

## 7.1 Introduction

The information in this paper stems from an ongoing collaborative research effort at the University of North Dakota. The Department of Mechanical Engineering and the Department of Space Studies has been working together in order to investigate some unique behaviors regarding the orbit of small-body asteroids. Orbiting Small Near-Earth Objects (NEOs) presents many challenges, but also many interesting opportunities of study. This paper concentrates on harmonic excitation behaviors observed while studying the complexities involved in the satellite-NEO system.

Previous work done by Church and Fevig looked at the feasibility of creating a highly detailed gravity map of small body asteroids (500 m and smaller). This allows for the determination of the internal structure of the asteroid by backing out the mass densities from the gravity model surrounding the asteroid. The process studied calls for two satellites in orbit around a body next to each other. Observations are recorded, from one satellite to the other, of the effects of the gravitational perturbations on the satellites' trajectories. This method has proven to be successful during the GRACE, and GRAIL missions around the Earth and moon, respectively. It was found, however, that two satellites put into orbit next to each other around a simulated model of asteroid 25143 Itokawa quickly diverge in their trajectories [1].

The asteroid Itokawa is not spherical, unlike most planets, which presents more complexity when attempting to maintain a stable orbit due to the changing magnitude of gravity as the satellite progresses through its trajectory. Early in this study it was observed that Itokawa has a highly erratic effect on a satellite's trajectory with minimal changes in initial conditions. Observations were made using AGI's Systems Tool Kit (STK) software package. Initially, two satellites were placed in an orbit next to each other, separated by 5° in true anomaly, the angle between their initial radius vectors. One quickly crashed into the asteroid while the other was ejected from the system. This indicated a significant difference in their behavior despite having similar initial conditions. Therefore, it was noted that the gravitational field surrounding Itokawa differs dramatically depending on the location of the satellite within its orbit.

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Later it was observed that retrograde orbits, orbits where the satellite's trajectory is opposite the direction of rotation of the asteroid, are much more stable than prograde orbits, orbits where the satellite's trajectory is in the same direction of the rotation of the asteroid. This observation led to the conclusion that harmonic excitation analysis can be used when studying the satellite-asteroid system. The gravitational perturbation has a much lower frequency of excitation on the satellite than the retrograde orbit, so the excitation becomes amplified as the excitation frequency approaches the natural frequency of the system [2].

## 7.2 Background

Asteroids or comets are considered Near-Earth Objects when their perihelion distance is less than 1.3 Astronomical Units (AU) [3]. The perihelion distance is the distance between the Sun and the orbiting body at its closest point to the Sun.

There is increasing interest in NEOs as time progresses. First, there is the constant threat of an asteroid striking the Earth. At the time of this writing, there are over 1500 Potentially Hazardous Asteroids (PHAs) that are being tracked and monitored by NASA [4]. The term potentially hazardous does not mean that an impact with Earth is imminent, only that the orbit and size of the object are cause for concern. With continuous study of these PHAs, scientists can better predict their orbit and asses the likelihood of a future close approach to Earth. Also, the resources on Earth are becoming more limited as the population continues to grow. There is a growing potential in mining NEOs for certain elements that can be either shipped back to Earth, or used in long-term space missions.

The satellite control problem in the complex NEO environment is being studied by researchers all around the world. NASA, ESA, and the Japanese Aerospace Exploration Agency (JAXA) have sent unmanned spacecraft that have operated in close-proximity to these objects [5, 6]. What makes the NEO environment so unique is that the gravity field surrounding an NEO is both tenuous and complex. Gravitational force is a function of the spacecraft's and NEO's mass that decays proportionally to the inverse square of the distance between these elements. This is defined by Newton's Law of Gravity. With an irregularly shaped asteroid being orbited by a satellite, the distance between the two bodies is constantly changing, and the magnitude of the gravity exerted on the satellite along with it.

In order to capture the irregularities of the gravity model in three dimensions, a spherical harmonic model of the gravitational potential is used, as shown in Eq. 7.1.

$$U = \frac{\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left\{\left\{\left(\frac{R_{CB}}{r}\right)^{n} P_{nm} sin\theta \cos(m\lambda)\right\}C_{nm} + \left\{\left(\frac{R_{CB}}{r}\right)^{n} P_{nm} sin\theta \sin(m\lambda)\right\}S_{nm}\right\} \tag{7.1}$$

where U is gravitational potential. The coordinates $r$, $\theta$, and $\lambda$ are the radial distance, latitude, and longitude of the spacecraft in a coordinate system fixed to the object's center of mass. $R_{CB}$ is the mean radius for the body and $\mu$ is the object's gravitational parameter. The functions $P_{nm}$ are the normalized Legendre polynomials, and $C_{nm}$ and $S_{nm}$ are the gravity coefficients of degree $n$ and order $m$ [7].

Asteroid 25143 Itokawa has been studied extensively and a degree and order four spherical harmonic model was created based on the data collected by JAXA's Hayabusa satellite, which visited Itokawa in 2005 [8]. Studying the natural environment about Itokawa, with no control measures used, it was found that a stable prograde orbit can be achieved using a high-inclination orbit with an orbit radius between 1.0 and 1.5 km. Any closer to the surface of Itokawa and the orbit becomes disrupted by the spherical harmonic gravity model. Outside of this range the orbit becomes disrupted by other effects, such as solar radiation pressure (SRP).

For Itokawa's gravity model, the $C_{20}$, $C_{22}$, $C_{42}$, and $C_{44}$ coefficients are the most significant [9]. These values correspond with the effects of the asteroid's oblateness and its ellipticity. A body's oblateness and its effect on orbits has been characterized extensively. For example, this perturbation is observed in Earth-bound orbits. Due to the Earth's angular velocity as it spins about its polar axis, there is a bulge around equator. The effect this bulge has on Earth-bound orbits, known as the $J_2$ effect, causes a precession in the orbital plane. In other words, the orbital plane wobbles as it spins. More specifically, the orbit's right ascension of the ascending node (RAAN) ($\Omega$) rotates westward for prograde orbits around the Earth, and the argument of periapsis ($\omega$) rotates in the direction of the spacecraft's motion. Semi-major axis (a), eccentricity (e), and inclination (i) suffer no long-term perturbations from oblateness [10].

A body's ellipticity has more dramatic effects on the orbit and can cause the spacecraft to transition from a safe orbit into an impacting or ejecting orbit within a few periods. The ellipticity of the body causes changes in the orbit semi-major axis, eccentricity, and inclination while effecting both the orbit's energy $\varepsilon = -\mu/(2a)$ and angular momentum $h = [\mu a(1-e)]^{1/2}$ [11]. In previous studies, it has been observed that prograde orbits experience much larger changes in energy and angular momentum for each orbit, where retrograde orbits experience little, if any, changes per orbit [12].

This paper puts all of these ideas together into a harmonic excitation analysis of an asteroid/satellite system. Harmonic excitation occurs naturally in our Solar System. For example, in the main belt asteroid system, there is a mean-motion resonance (MMR) interaction with Jupiter. More specifically, there are clumps of asteroids in certain regions, such as the 3:2 MMR region, and other regions that are practically devoid of asteroids, such as the 3:1 MMR region [13]. With regards to this project, the satellite orbits the asteroid repeatedly. Therefore, the satellite experiences the same gravitational perturbations as it passes over the surface of the satellite over and over again, at a given frequency. It was found by using a model of the system including all of the complexities previously mentioned, there are certain orbital conditions that are more or less stable based on the frequency at which the satellite is subject to these orbital perturbations. This allows for optimal mission planning as the satellite's orbital trajectory can be manipulated in order to make use of the stable orbital excitation conditions.

## 7.3 System Model

As mentioned, this project focuses on asteroid Itokawa as the asteroid-satellite system's central body. Itokawa was visited in 2005 and a spherical harmonic gravity model has been constructed from the data collected. This allows for a computer simulation of a real-world environment. The shape model used in our simulation is a visual approximation as a triaxial ellipsoid with axial dimensions 0.2741 km × 0.1561 km × 0.1376 km. These were calculated from values of the overall dimensions of Itokawa, given as 0.5481 km × 0.3122 km × 0.2751 km [9]. The rotational period of Itokawa is 12.132 h according the data provided by JPL Horizons [14].

To conduct the study with a high degree of user manipulation and data capture, STK is used to run the simulation and real-time data analysis done in MATLAB. The satellite used in the simulation scenario has a mass of 55 kg. The initial Keplerian elements of the satellite's orbit around Itokawa used in this study are given in Table 7.1.

## 7.4 Analysis

This project looks exclusively at the effects of the spherical harmonic gravity model. Other effects, such as Solar Radiation Pressure (SRP) and third-body gravitation, are not considered. The equation of motion for a linear spring-mass system is defined as:

$$F = ma_x + kx \tag{7.2}$$

where m is the mass (kg), $a_x$ is the acceleration in the x-direction (m/s$^2$), k is the spring constant (N/m), and x is the displacement in the x-direction (m).

Newton's Law of Gravity is given as:

$$F_g = G\frac{m_1 m_2}{r^2} \tag{7.3}$$

where $F_g$ is the force of gravity (N), G is the universal gravitational constant ($\approx 6.674 \times 10^{-11}$ N (m/kg)$^2$), $m_1$ and $m_2$ are the mass of the two bodies interacting (kg), and r is the distance between the two bodies' centers of mass (m). The terms G and $m_1$ (the mass of the orbited body) are combined to make the value μ (km$^3$/s$^2$) which is called the body's gravitational parameter.

**Table 7.1** Initial conditions for satellite orbit used in the study

| Keplerian element | Value |
|---|---|
| Semi-major axis | 0.60 km |
| Eccentricity | 0 |
| Inclination | 0° (prograde) or 180° (retrograde) |
| Argument of perigee | 0° |
| Right-ascension of the ascending node | 0° |
| True anomaly | 0° |

This can be made to fit into Eq. 7.2 as the spring constant and becomes what is known in orbital mechanics as the simplified two-body problem:

$$F = ma_x + \frac{m\mu}{x^3}x \tag{7.4}$$

Now, some system parameters need to be defined in order to continue. From mechanical vibrations, the undamped natural frequency of a linear spring-mass system is defined as:

$$\omega_n = \sqrt{\frac{k}{m}} \tag{7.5}$$

This is the frequency at which an undamped system will naturally oscillate. This is an important parameter in vibrations because if a system is driven to oscillate at this resonant frequency, small excitations grow into large amplitudes of oscillation.

Replacing the spring constant (k) in Eq. 7.5 with the "non-linear spring" constant, it becomes:

$$\omega_n = \sqrt{\frac{\frac{m\mu}{x^3}}{m}} \tag{7.6}$$

which simplifies to:

$$\omega_n = \sqrt{\frac{\mu}{x^3}} \tag{7.7}$$

where μ is the gravitational parameter ($km^3/s^2$) of Itokawa and x is the radial distance from the satellite to Itokawa's center (km). This leaves us with the natural frequency of the system in radians per second (rad/s). Scheeres provides that μ for Itokawa is $2.36 \times 10^{-9}$ $km^3/s^2$ [9]. This gives a natural frequency of $1.05 \times 10^{-4}$ rad/s when evaluated at our chosen initial orbital radius of 0.60 km.

Using STK to run the simulations, we can manually adjust the rotational rate of Itokawa in order to study the harmonic excitation. With the spherical harmonic model, the relative frequency between the satellite and the asteroid can be expressed as:

$$\omega_{sat/asteroid} = (\omega_{sat} \pm \omega_{asteroid}) \tag{7.8}$$

where the frequencies add in the retrograde case, and subtract in the prograde case. This results in the excitation frequency:

$$\omega = m\omega_{sat/asteroid} \tag{7.9}$$

where m is based on the degree and order of the spherical gravity model coefficients significantly impacting the excitation. In the case of Itokowa, the $C_{20}$, $C_{22}$, $C_{42}$, and $C_{44}$ coefficients are the most significant. In looking at the shape of the corresponding Legendre polynomials these shapes primarily have two lobes, such that $m = 2$, in Eq. 7.9, for this asteroid.

The amplification ratio at which the oscillations grow when an undamped system is excited at it natural frequency is given as:

$$\frac{X}{\delta_{st}} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \tag{7.10}$$

where $X/\delta_{st}$ is the ratio of the dynamic to the static amplitude of motion, and $\omega/\omega_n$ is the ratio of the excitation frequency to the natural frequency of the system [15]. Figure 7.1 is a plot of $\omega/\omega_n$ versus $X/\delta_{st}$ for an undamped linear system. From this we expect to see greater instabilities in the spacecraft's orbit as the excitation frequency experienced by the spacecraft from the rotating asteroid approaches the natural frequency of the system.

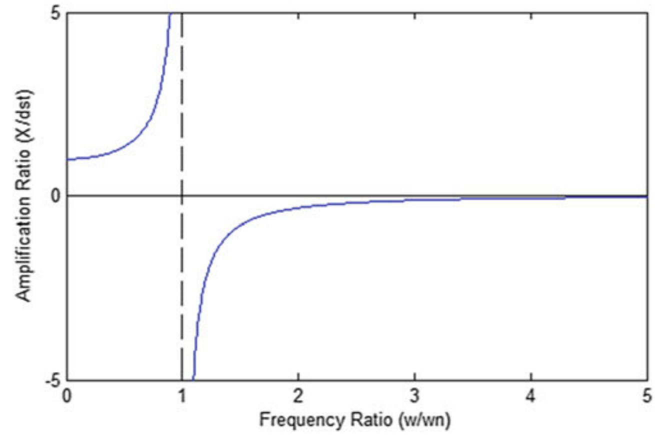


**Fig. 7.1** Amplification ratio of an undamped linear system

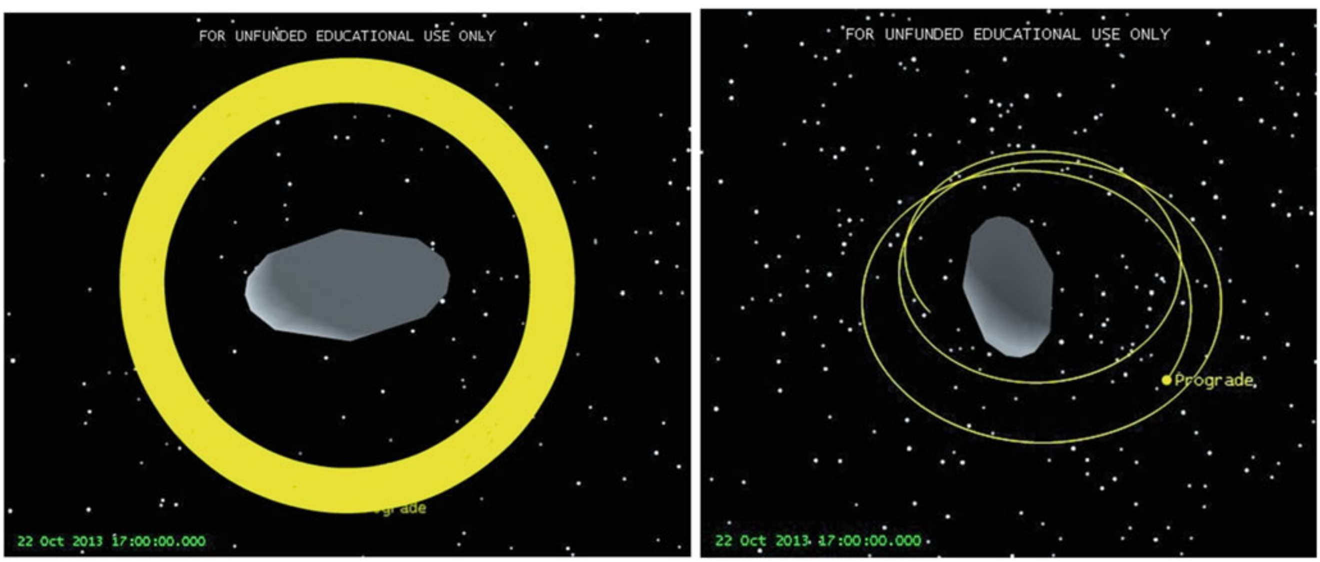


**Fig. 7.2** (*Left*) A stable retrograde orbit that maintains a mostly circular orbit, but experiences precession around the central body for upwards of 6 months. (*Right*) An unstable prograde orbit that is perturbed enough to cause the orbit to diverge from its intended circular orbit and crash into the asteroid

Multiple scenarios were simulated in STK varying the rate of rotation of the asteroid and the direction of the orbit. This allows for the study of the interaction between the excitation frequency (ω), which changes according to Eqs. 7.8 and 7.9, and the natural frequency ($\omega_n$), which remains constant for a given spacecraft and altitude. These allowed for the assessment of the validity of the simplified model in a complex environment.

Using the models described and the STK simulations, five spin rates multiples of the natural Itokawa spin rate were investigated; ⅓, ½, 1, 2, & 3. Figure 7.2 shows representative examples of the orbital path for a stable and an unstable orbit over time. Note that even the stable orbit changes over time due to a slight eccentricity in the orbital path. Figures 7.3, 7.4, 7.5, 7.6, 7.7, 7.8, 7.9, 7.10, 7.11 and 7.12 show the time history plots of the radial acceleration indicating the excitation levels and FFTs of the data to identify the significant driving frequencies.

In the FFT plots there are multiple excitation frequencies observed, but in general two frequencies stand out as having a greater influence than the others. Table 7.2 shows the frequencies for select scenarios with the most significant frequencies highlighted. Some prograde scenarios had no significant frequencies due to the presence of orbital instability from the start of the orbit. In other words, the satellite did not complete enough revolutions before crashing in order to observe any excitation frequencies. In these scenarios the radial acceleration grows exponentially larger as the satellite approaches the asteroid prior to impact or the satellite escaped the stable orbit and the radial acceleration goes to zero.

Now it's important to make note of something about each of the above data sets from the scenarios. The first significant frequency in all of these scenarios is quite similar. This can be explained by a brief discussion of orbital mechanics.

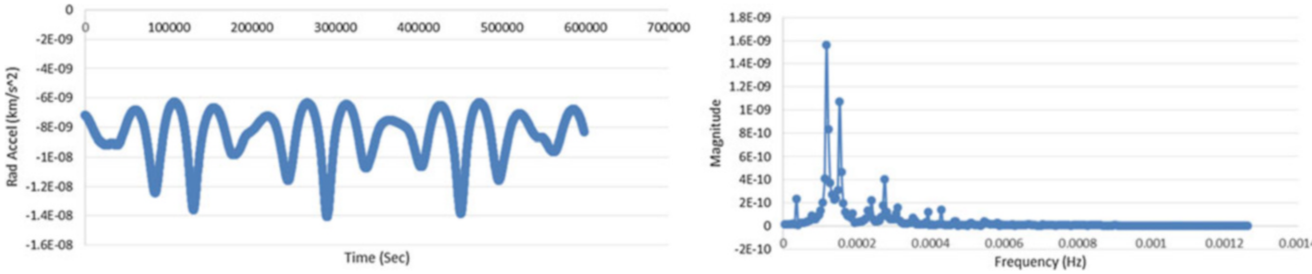


**Fig. 7.3** ⅓× natural spin rate in prograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

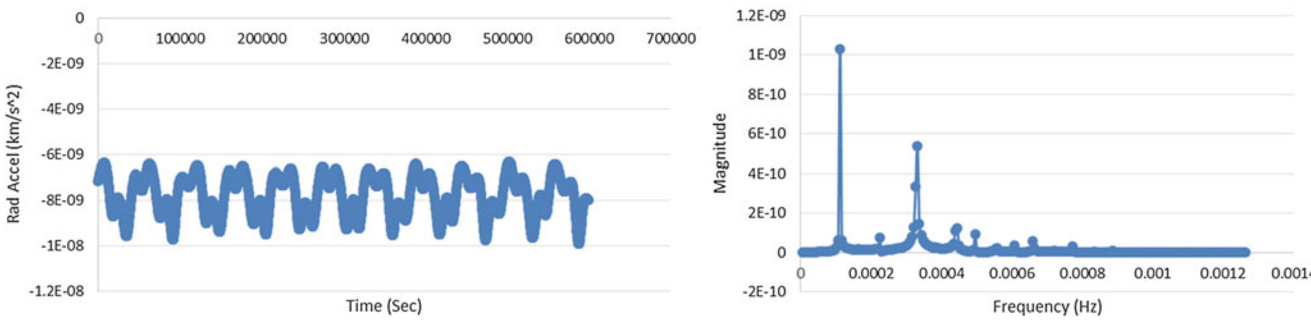


**Fig. 7.4** ⅓× natural spin rate in retrograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

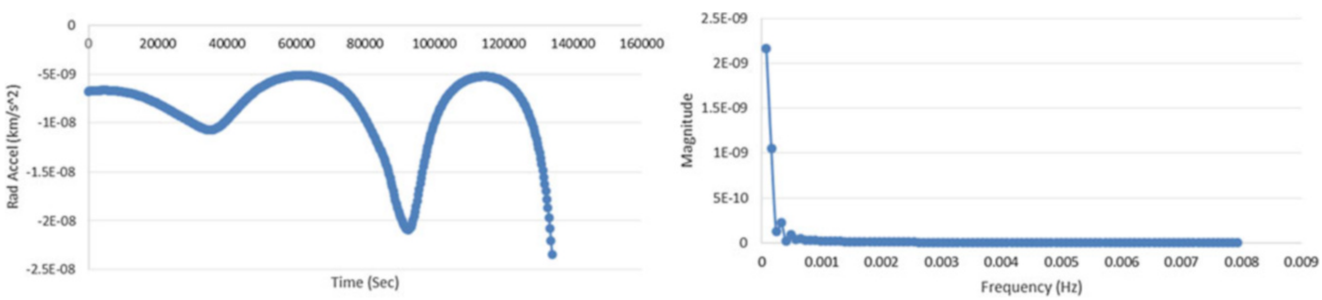


**Fig. 7.5** ½× natural spin rate in prograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

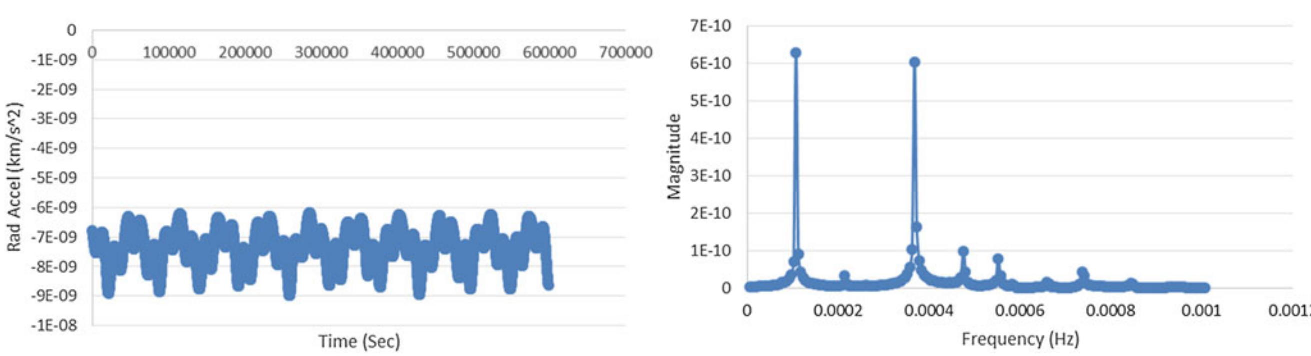


**Fig. 7.6** ½× natural spin rate in retrograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

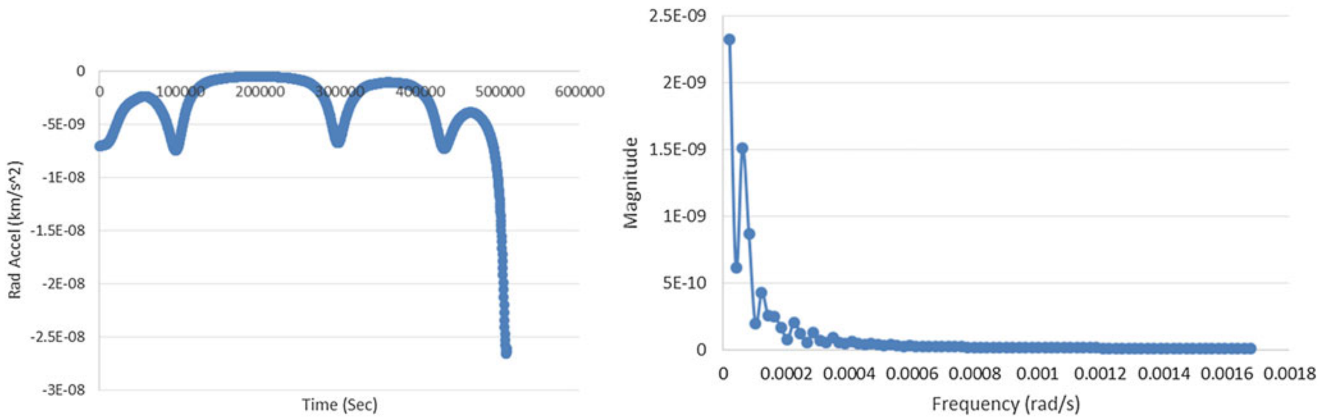


**Fig. 7.7** 1× natural spin rate in prograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

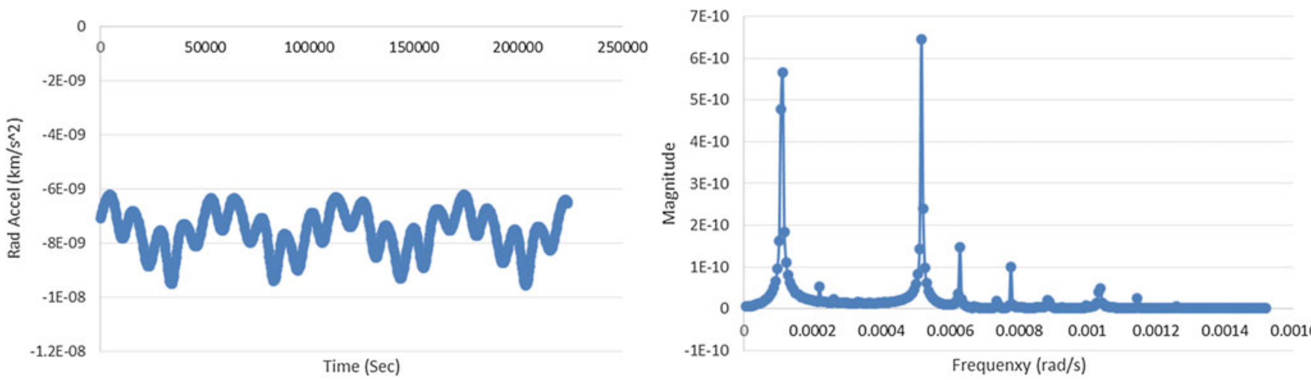


**Fig. 7.8** 1× natural spin rate in retrograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

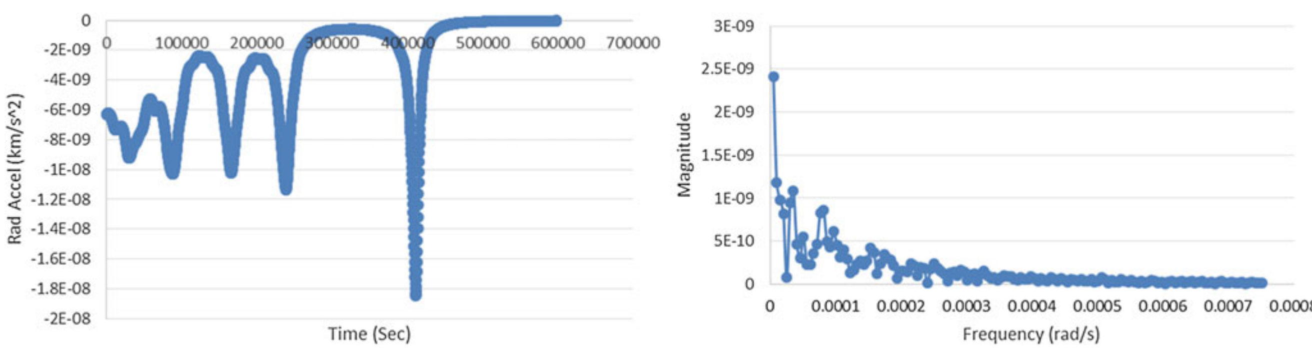


**Fig. 7.9** 2× natural spin rate in prograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

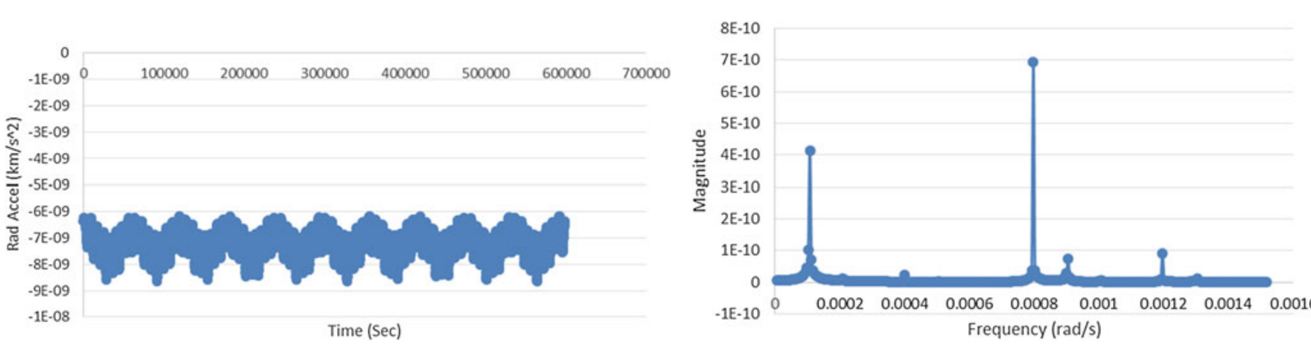


**Fig. 7.10** 2× natural spin rate in retrograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

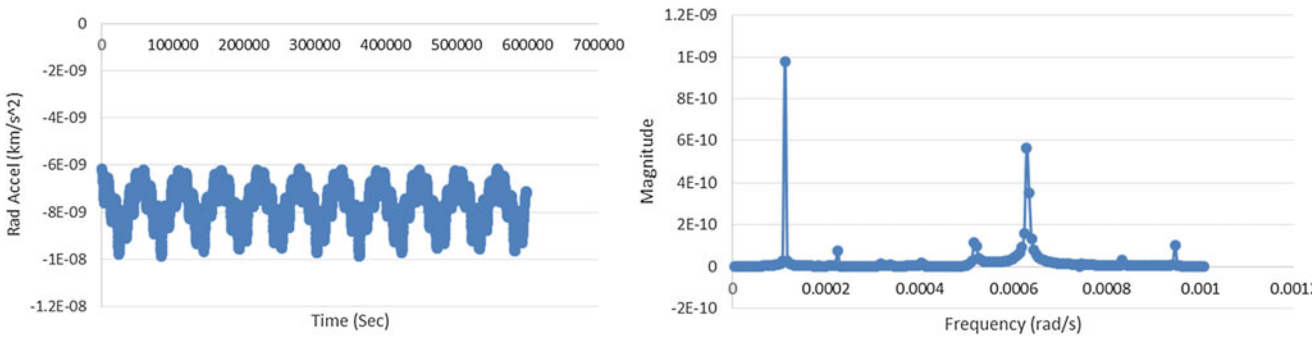


**Fig. 7.11** 3× natural spin rate in prograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

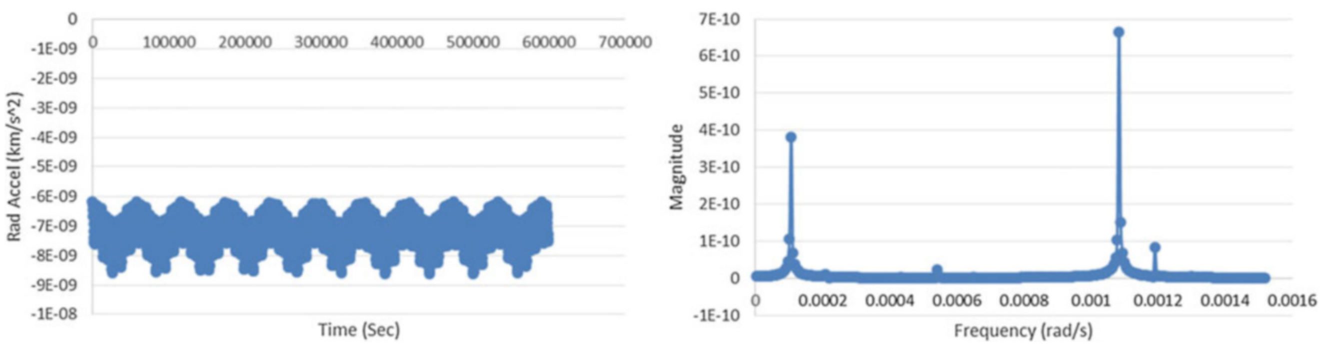


**Fig. 7.12** 3× natural spin rate in retrograde orbit. (*Left*) Radial acceleration vs. time. (*Right*) FFT of time history data

**Table 7.2** Observed prograde frequencies for various Itokawa rotation rates

| | Asteroid spin rate multiplier | Observed prograde frequencies (rad/s) | | Observed retrograde frequencies (rad/s) | |
|---|---|---|---|---|---|
| | 1/3 | 1.18E-04 | 1.53E-04 | 1.1249E-04 | 3.3236E-04 |
| | 1/2 | – | – | 1.0738E-04 | 3.6816E-04 |
| Natural rate | 1 | – | – | 1.1249E-04 | 5.1644E-04 |
| | 2 | – | – | 1.0738E-04 | 7.9767E-04 |
| | 3 | 1.12E-04 | 6.29E-04 | 1.0738E-04 | 1.0840E-03 |

When the scenario is created in STK, an initial velocity is calculated within the program for a point-mass, two-body model. These scenarios are not a point-mass, two-body scenario and have a complex gravity model. So the initial velocity that is given from STK is the initial velocity needed to maintain a circular orbit around a sphere, not the initial velocity required to maintain a circular orbit around an oblate and eccentric body like Itokawa. This discrepancy in initial velocities results in a slight eccentricity in the satellite's orbit from the beginning of the scenario. This slight eccentricity is what is causing the first significant excitation frequency, and explains why it is similar in all scenarios despite the fact that the rotation rate of the central body is changing significantly.

With this in mind, we can look at the FFT in order to filter out this frequency caused by the eccentric orbit. This will give excitation frequencies caused solely by the significant complex gravity model coefficients, and not by the eccentricity of the orbit. Using the filtered frequencies from the STK scenarios, the results are in Table 7.3

In all the retrograde orbit cases shown the resulting orbit quality is stable and the frequency ratios are all nearly three or higher. These orbit are predicted to be stable and the simulations proved this to be valid. In the prograde orbit cases below six show signs of instability. Interestingly, in the 2× case the satellite escaped from the orbit and flew off into space. The time history plot shows that the satellite began a fairly stable orbit, but then the eccentricity got larger and larger until it was ejected. In the ½× and 1× cases, the orbital radial acceleration levels grew until the satellite flew nearer the asteroid and eventually crashed into the surface. The predicted prograde frequency ratios show negative values when the satellite is

**Table 7.3** Orbit scenario observations with varying asteroid spin rates and changing orbital direction (prograde and retrograde)

| | Asteroid spin rate multiplier | Prograde simulated (ω/ωn) | Prograde predicted (ω/ωn) | Observed orbit quality | Retrograde simulated (ω/ωn) | Retrograde predicted (ω/ωn) | Observed orbit quality |
|---|---|---|---|---|---|---|---|
| | 1/3 | 1.45 | −1.08 | *Stable* | 3.18 | 2.91 | *Stable* |
| | 1/2 | – | −0.63 | **Unstable** | 3.52 | 3.37 | *Stable* |
| Natural rate | 1 | – | 0.75 | **Unstable** | 4.94 | 4.75 | *Stable* |
| | 2 | – | 3.5 | ***Semi-stable*** | 7.63 | 7.50 | *Stable* |
| | 3 | 5.99 | 6.25 | *Stable* | 10.37 | 10.25 | *Stable* |

moving faster that the asteroid, however the magnitude of the ratio is still the defining parameter. It was observed that low values of the frequency ratios result in the unstable behavior. This corresponds to the regions where the amplification ratio is greater than one, as shown in Fig. 7.1. The simulated frequency ratios for the retrograde cases match very well and the behavior is well predicted. In the prograde cases the values show more difference, but still are fairly close. However, the correlation with the amplification ratio is not as well behaved. This behavior is not well understood by the authors, but may have to do with the relatively low frequency ratios and the complex shape of the asteroid. Other Legendre polynomial terms may be playing a larger role in these orbits. However, the general trend still is captured and the relative speed of the satellite and rotational speed of the asteroid can provide good correlation to areas where greater orbit stability can be expected.

## 7.5 Conclusion

Harmonic excitation analysis can be used as a tool when planning out a space mission as long as some information is known about the target body. This project used a linear approach in analyzing a nonlinear system and produced promising results. The FFT allowed us to filter out different excitation frequencies in order to study their effects on the system. When considering the complex gravity model by itself, a harmonic excitation analysis allowed for a fairly accurate prediction of the satellite's behavior in a complex system. If the mathematical model of the dynamic system is precisely known, accurate results may be obtained with STK simulations. However, without accurate information about unexplored asteroids, a retrograde orbit with a high excitation frequency produces a more stable obit.

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## References

1. Church, C., Fevig, R.: A feasibility study on the characterization of the internal structure of small NEOs with small spacecraft. In: Lunar and Planetary Science Conference (LPSC XLIV), Houston, March 2013
2. Johnson, J., Semke, W., Zimmer, M., Fevig, R.: Orbit stability determination of satellites using harmonic force excitation analysis. In: Proceedings of the International Modal Analysis Conference (IMAC) XXXIII: A Conference and Exposition on Structural Dynamics, 2015
3. Yeomans, D.: NEO groups, from http://neo.jpl.nasa.gov/neo/groups.html, 2014
4. Yeomans, D. Orbit diagrams, from http://neo.jpl.nasa.gov/orbits/, 2014
5. Netting, R.: NEAR-shoemaker, from http://science.nasa.gov/missions/near/, 2014
6. Kawaguchi, J., Fujiwara, A., Uesugi, T.: Hayabusa (MUSES-C) – rendezvous and proximity operation, IAC-05-A3.5.A.01. In: International Astronautical Congress, International Astronautical Federation, Fouduoka, 2005
7. Montenbruck, O., Gill, E.: Satellite Orbits: Models, Methods, and Applications, pp. 53–116. Springer, Heidelberg (2000)
8. Church, C.: A feasibility study on the implementation of satellite-to-satellite tracking around a small near-earth object. M.S. Thesis, Department of Space Studies, University of North Dakota (2014)
9. Scheeres, D.J., Gaskell, R., Abe, S., Barnouin-Jha, O., Hashimoto, T., Kawaguchi, J., Kubota, T., Saito, J., Yoshikawa, M., Hirata, N., Mukai, T., Ishiguro, M., Kominato, T., Shirakawa, K., Uo, M.: The actual dynamical environment about Itokawa, AIAA 2006-6661. In: AIAA/AAS Astrodynamics Specialist Conference and Exhibit, American Institute of Aeronautics and Astronautics, Inc., Keystone, 2006
10. Scheeres, D.J., Ostro, S.J., Hudson, R.S., Werner, R.A.: Orbits close to asteroid 4769 Castalia. Icarus **121**, 67–87 (1996). Elsevier Limited
11. Scheeres, D.J., Marzari, F., Tomasella, L., Vanzani, V.: Rosetta mission: satellite orbits around a cometary nucleus. Planet. Space Sci. **36**(6/7), 649–671 (1998). Elsevier Limited

12. Scheeres, D.J., Williams, B.G., Miller, J.K.: Evaluation of the dynamic environment of an asteroid: applications to 433 eros. J. Guid. Control Dyn. **23**(3), 466–475 (2000)
13. Nesvorny, D., Ferraz-Mello, S., Holman, M., Morbidelli, A.: Regular and chaotic dynamics in the mean-motion resonances: implications for the structure and evolution of the asteroid belt. Asteroids III **1**, 379–394 (2002)
14. JPL HORIZON System, telnet://horizons.jpl.nasa.gov:6775
15. Rao, S.S.: Mechanical Vibrations, 5th edn, pp. 259–362. Upper Saddle River, Prentice Hall (2011)