A Study of Argument Acceptability Dynamics Through Core and Remainder Sets

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Abstract. We analyze the acceptability dynamics of arguments through the proposal of two different kinds of minimal sets of arguments, core and remainder sets which are somehow responsible for the acceptability/rejection of a given argument. We develop a study of the consequences of breaking the construction of such sets towards the acceptance, and/or rejection, of an analyzed argument. This brings about the proposal of novel change operations for abstract argumentation first, and for logic-based argumentation, afterwards. The analysis upon logic-based argumentation shows some problems regarding the applicability of the standard semantics. In consequence, a reformulation of the notion of admissibility arises for accommodating the standard semantics upon logic-based argumentation. Finally, the proposed model is formalized in the light of the theory of belief revision by characterizing the corresponding operations through rationality postulates and representation theorems.

Keywords: Argumentation · Belief revision · Argumentation dynamics

1 Introduction

Argumentation theory [13] allows to reason over conflicting pieces of knowledge, *i.e.*, arguments. This is done by replacing the usual meaning of inference from classical logic by *acceptability* in argumentation: evaluation of arguments' interaction through conflict for deciding which arguments prevail. To that end, argumentation theory relies upon argumentation semantics and acceptance criteria. Semantics can be implemented through determination of extensions, *i.e.*, different kinds of conflict-free sets of arguments. For studying theoretic properties, like semantics, it is possible to abstract away from any particular representation of knowledge or structuring for building arguments. This is referred as abstract argumentation. On the other hand, the concretization of an argumentation framework (AF) to some specific logic and argument structure is called

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logic-based argumentation [2,6,15]. Investigations based upon abstract argumentation usually simplify the study of some specific problem, and may bring solid fundamentals for studying afterwards its application upon logic-based argumentation. However, adapting theories, and results, from abstract to logic-based argumentation may be not straightforward.

The classic theory of belief revision [1] studies the dynamics of knowledge, coping with the problem of how to change beliefs standing for the conceptualization of a modeled world, to reflect its evolution. Revisions, as the most important change operations, concentrate on the incorporation of new beliefs in a way that the resulting base ends up consistently. When considering AF*s* for modeling situations which are immersed in a naturally dynamic context, it is necessary to provide models for handling acceptability dynamics of arguments. That is, models for studying change in argumentation (for instance, [4,5,10]) for providing a rationalized handling of the dynamics of a set of arguments and their implications upon the acceptability condition of arguments [19]. This led us to investigate new approaches of belief revision, which operate over paraconsistent semantics –like argumentation semantics– avoiding consistency restoration.

Argumentation provides a theoretic framework for modeling paraconsistent reasoning, a subject of utmost relevance in areas of research like medicine and law. For instance, legal reasoning can be seen as the intellectual process by which judges draw conclusions ensuring the rationality of legal doctrines, legal codes, binding prior decisions like jurisprudence, and the particularities of a deciding case. This definition can be broaden to include the act of making laws. Observe that the evolution of a normative system –for modeling promulgation of laws– would imply the removal/incorporation of norms for ensuring some specific purpose but keeping most conflicts from the original AF unaffected.

Upon such motivation, we study new forms to handle acceptability dynamics of arguments, firstly on abstract argumentation, and afterwards upon logic-based argumentation. By relying upon extension semantics, we define two different sorts of sets for recognizing acceptance or rejection of arguments: core and remainder sets, respectively. Afterwards we propose a model of change towards the proposal of an acceptance revision operation which deals with the matter of incorporating a new argument while ensuring its acceptance. This is done first, from an abstract perspective, and afterwards upon logic-based argumentation. This unveils some specific problems regarding the applicability of standard semantics to this kind of argumentation. We propose then a reformulation on the notion of admissibility to overcome from such drawback by analyzing argumentation postulates from [2]. Finally, the rationality of the proposed change operators is provided through its axiomatic characterization and corresponding representation theorem according to classic belief revision and argument-based belief revision models like Argument Theory Change (ATC) [19].

2 Fundamentals for Abstract Frameworks

An *abstract argumentation framework* (AF) will be assumed as a pair $\langle \mathbf{A}, \mathbf{R_A} \rangle$, where $\mathbf{A}$ is a finite *set of arguments*, and the set $\mathbf{R_A} \subseteq \mathbf{A} \times \mathbf{A}$ identifies the finitary *defeat relation* between pairs of arguments $a \in \mathbf{A}$ and $b \in \mathbf{A}$, such that $(a, b) \in \mathbf{R_A}$ implies that argument a *defeats* argument b, or equivalently, a is a *defeater* of b. In this part of the article, arguments are deemed as abstract since we do not specify any concrete logic, nor inner-structure, for constructing them. Thus, arguments will be considered as indivisible elements of `AF`*s*. On the other hand, we will assume the defeat relation $\mathbf{R_A}$ to be obtained through a functional construction $\mathbf{R_A} : \wp(\mathbf{A}) \longrightarrow \wp(\mathbf{A} \times \mathbf{A})$[1]. This makes presumable the existence of a *defeating function* $\varepsilon : \mathbf{A} \times \mathbf{A} \longrightarrow \{\texttt{true}, \texttt{false}\}$, such that:

$$\text{for any pair of arguments } a, b \in \mathbf{A},\ \varepsilon(a, b) = \texttt{true} \textit{ iff } (a, b) \in \mathbf{R_A} \quad (1)$$

Definition 1 (Argumentation Framework Generator). *Let* $\mathbf{A}$ *be a finite set of arguments, an operator* $\mathbb{F}_\mathbf{A}$ *is an* ***argumentation framework generator from*** $\mathbf{A}$ *(or just,* `AF` ***generator****) iff* $\mathbb{F}_\mathbf{A}$ *is an* `AF` $\langle \mathbf{A}, \mathbf{R_A} \rangle$.

Our intention is to simplify `AF`*s* at the greatest possible level in order to concentrate firstly on specific matters for dealing with the acceptability dynamics of arguments, and afterwards, from Sect. 5, we will analyze the proposed theory for argumentation dynamics in the light of logic-based frameworks, where arguments will be constructed upon a specific logic $\mathcal{L}$. Consequently, when necessary, we will abstract away the construction of an `AF` $\mathbb{F}_\mathbf{A}$ from any set of arguments $\mathbf{A}$, by simply referring to an `AF` τ. In such a case, we will refer to the set of arguments of τ by writing $\mathbf{A}(\tau)$ and to the set of defeats of τ by writing $\mathbf{R}(\tau)$.

Next, we introduce some well known concepts from argumentation theory [13] that makes possible the acceptability analysis of arguments through the usage of argumentation semantics. Given an `AF` $\mathbb{F}_\mathbf{A}$, for any $\Theta \subseteq \mathbf{A}$ we say that:

- Θ **defeats** an argument $a \in \mathbf{A}$ *iff* there is some $b \in \Theta$ such that b defeats a.
- Θ **defends** an argument $a \in \mathbf{A}$ *iff* Θ defeats every defeater of a.
- Θ is **conflict-free** *iff* $\mathbf{R}_\Theta = \emptyset$.
- Θ is **admissible** *iff* it is conflict-free and defends all its members.

Given an `AF` $\mathbb{F}_\mathbf{A}$, for any set $\mathbf{E} \subseteq \mathbf{A}$ of arguments, we say that:

1. $\mathbf{E}$ is a **stable extension** if $\mathbf{E}$ is conflict-free and defeats any $a \in \mathbf{A} \setminus \mathbf{E}$
2. $\mathbf{E}$ is a **complete extension** if $\mathbf{E}$ is admissible and contains every argument it defends
3. $\mathbf{E}$ is a **preferred extension** if $\mathbf{E}$ is a maximal (wrt. set incl.) admissible set
4. $\mathbf{E}$ is the **grounded extension** if $\mathbf{E}$ is the minimal (wrt. set incl.) complete extension, *i.e.*, $\mathbf{E}$ is the least fixed point of $\mathcal{F}(X) = \{a \in \mathbf{A} | X \text{ defends } a\}$
5. $\mathbf{E}$ is a **semi-stable extension** if $\mathbf{E}$ is a complete extension and the set $\mathbf{E} \cup \{a \in \mathbf{A} | \mathbf{E} \text{ defeats } a\}$ is maximal wrt. set inclusion
6. $\mathbf{E}$ is the **ideal extension** if $\mathbf{E}$ is the maximal (wrt. set incl.) admissible set that is contained in every preferred extension

[1] Observe that we use the notation $\wp(\Theta)$ for referring to the powerset of Θ.

The above six notions are known as *extension semantics*. It is possible to have no stable extensions, and also that there may be more than a single stable, complete, preferred and semi-stable extensions, but only one grounded and one ideal extension. The set $\mathbb{E}_{\mathfrak{s}}(\tau)$ identifies the *set of* $\mathfrak{s}$*-extensions* $\mathbf{E}$ from the $\mathtt{AF}$ $\tau = \mathbb{F}_{\mathbf{A}}$, where an $\mathfrak{s}$*-extension* is an extension in τ according to some extension semantics $\mathfrak{s}$, and where $\mathfrak{s}$ adopts a value from $\{\mathtt{st}, \mathtt{co}, \mathtt{pr}, \mathtt{gr}, \mathtt{ss}, \mathtt{id}\}$ corresponding to the stable ($\mathtt{st}$), complete ($\mathtt{co}$), preferred ($\mathtt{pr}$), grounded ($\mathtt{gr}$), semi-stable ($\mathtt{ss}$), and ideal ($\mathtt{id}$) semantics. For instance, the set $\mathbb{E}_{\mathtt{pr}}(\tau)$ will contain all the preferred extensions in τ. Observe that any extension $\mathbf{E} \in \mathbb{E}_{\mathfrak{s}}(\tau)$ is an admissible set. The relation among extension semantics is shown as $\mathbb{E}_{\mathtt{st}}(\tau) \subseteq \mathbb{E}_{\mathtt{ss}}(\tau) \subseteq \mathbb{E}_{\mathtt{pr}}(\tau) \subseteq \mathbb{E}_{\mathtt{co}}(\tau)$, and also $\mathbb{E}_{\mathtt{gr}}(\tau) \subseteq \mathbb{E}_{\mathtt{co}}(\tau)$ and $\mathbb{E}_{\mathtt{id}}(\tau) \subseteq \mathbb{E}_{\mathtt{co}}(\tau)$.

3 Preliminaries for Studying Dynamics of Arguments

We refer as *acceptance criterion* to the determination of acceptance of arguments in either a *sceptical* or *credulous* way. Several postures may appear. For instance, according to [15] a sceptical set is obtained by intersecting every $\mathfrak{s}$-extension (see Eq. 2), and a credulous set resulting from the union of every $\mathfrak{s}$-extension (Eq. 4). Since the latter posture may trigger non-conflict free sets, we suggest a different alternative for credulous acceptance, for instance, one may choose a single extension due to some specific preference, like selecting among those extensions of maximal cardinality, "the best representative" one according to some criterion upon ordering of arguments (Eq. 3). Assuming an abstract $\mathtt{AF}$ τ:

$$\bigcap_{\mathbf{E} \in \mathbb{E}_{\mathfrak{s}}(\tau)} \mathbf{E} \tag{2}$$

$$\begin{aligned} &\mathbf{E} \in \mathbb{E}_{\mathfrak{s}}(\tau) \text{ such that for any} \\ &\mathbf{E}' \in \mathbb{E}_{\mathfrak{s}}(\tau),\ |\mathbf{E}| \geq |\mathbf{E}'| \text{ holds} \end{aligned} \tag{3}$$

$$\bigcup_{\mathbf{E} \in \mathbb{E}_{\mathfrak{s}}(\tau)} \mathbf{E} \tag{4}$$

Definition 2 (Acceptance Function). *Given an* $\mathtt{AF}$ $\tau = \mathbb{F}_{\mathbf{A}}$ *and an extension semantics* $\mathfrak{s} \in \{\mathtt{st}, \mathtt{co}, \mathtt{pr}, \mathtt{gr}, \mathtt{ss}, \mathtt{id}\}$ *determining a set* $\mathbb{E}_{\mathfrak{s}}(\tau) \subseteq \wp(\mathbf{A})$ *of* $\mathfrak{s}$*-extensions, a function* $\delta : \wp(\wp(\mathbf{A})) \longrightarrow \wp(\mathbf{A})$ *is an* ***acceptance function*** *iff* $\delta(\mathbb{E}_{\mathfrak{s}}(\tau)) \subseteq \mathbf{A}$ *determines a conflict-free set of arguments from* $\mathbf{A}$.

The acceptance criterion can be applied through an *acceptance function* as defined above. Note that Eq. 4 does not fulfill the necessary conditions for an acceptance function given that it may trigger non-conflict-free sets. We will abstract away from a specific definition for an acceptance function and will only refer to δ when necessary. We refer as *(argumentation) semantics specification* $\mathcal{S}$ to a tuple $\langle \mathfrak{s}, \delta \rangle$, where $\mathfrak{s}$ stands for identifying some extension semantics and δ for an acceptance function implementing some acceptance criterion.

Definition 3 (Acceptable Set). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and a semantics specification* $\mathcal{S} = \langle \mathfrak{s}, \delta \rangle$*, the set* $\mathcal{A}_{\mathcal{S}}(\tau) \subseteq \mathbf{A}$ *is the* ***acceptable set*** *of* τ *according to* $\mathcal{S}$ *iff* $\mathcal{A}_{\mathcal{S}}(\tau) = \delta(\mathbb{E}_{\mathfrak{s}}(\tau))$*.*

For instance, adopting an acceptance function implementing Eq. 2, the set $\mathcal{A}_{\langle \mathtt{pr}, \delta \rangle}(\tau)$ identifies the sceptical acceptance set for a preferred semantics.

Definition 4 (Argument Acceptance/Rejection). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and a semantics specification* $\mathcal{S} = \langle \mathfrak{s}, \delta \rangle$*, an argument* $a \in \mathbf{A}$ *is* $\mathcal{S}$***-accepted*** *in* τ *iff* $a \in \mathcal{A}_{\mathcal{S}}(\tau)$*. Conversely,* $a \in \mathbf{A}$ *is* $\mathcal{S}$***-rejected*** *in* τ *iff* $a \notin \mathcal{A}_{\mathcal{S}}(\tau)$*.*

Admissible and *core sets* of an argument as the fundamental notions for recognizing the sources for the acceptability condition of a given argument.

Definition 5 (Admissible Sets of an Argument). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and an argument* $a \in \mathbf{A}$*; for any* $\Theta \subseteq \mathbf{A}$*, we say that:*

1. Θ *is an* a***-admissible set*** *in* τ *iff* Θ *is an admissible set such that* $a \in \Theta$*.*
2. Θ *is a* ***minimal*** a***-admissible set*** *in* τ *iff* Θ *is a-admissible and for any* $\Theta' \subset \Theta$*, it follows that* Θ' *is not a-admissible.*

Definition 6 (Core Sets). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and an argumentation semantics specification* $\mathcal{S}$*, for any* $\mathcal{C} \subseteq \mathbf{A}$*, we say that* $\mathcal{C}$ *is an* a***-core*** *in* τ*, noted as* a-$\mathtt{core}_{\mathcal{S}}$ *iff* $\mathcal{C}$ *is a minimal a-admissible set and* a *is* $\mathcal{S}$*-accepted in* τ*.*

Next we define *rejecting sets* of an argument a as the fundamental notion for studying and recognizing the basics for the rejecting condition of a. Intuitively, a rejecting set $\mathcal{R}$ for a should be that which ensures that a would end up $\mathcal{S}$-accepted in the AF $\mathbb{F}_{\mathbf{A} \setminus \mathcal{R}}$. Before formalizing rejecting sets through Definition 8, we propose the intermediate notions of *partially admissible* and *defeating sets.*

Definition 7 (Partially Admissible and Defeating Sets). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$*; for any* $\Theta \subseteq \mathbf{A}$ *and any argument* $a \in \mathbf{A}$*, we say that:*

1. b ***defeats*** Θ *iff* b *defeats some* $c \in \Theta$*.*
2. Θ *is* a***-partially admissible*** *iff* $a \in \Theta$*,* Θ *is conflict-free, and if* $c \in \Theta$*, with* $c \neq a$ *then there is some* $b \in \mathbf{A}$ *such that* c *defeats* b *and* b *defeats* $\Theta \setminus \{c\}$*.*
3. Θ *is* a***-defeating*** *iff there is some a-partially admissible set* Θ' *such that* $\Theta \supseteq \Upsilon \subseteq \{b \in \mathbf{A} | b \text{ defeats } \Theta'\}$*.*

The *partially admissible set* for a given argument a is an effort for constructing a set which would end up turning into an a-$\mathtt{core}_{\mathcal{S}}$ after removing an appropriate *a-defeating set* from the worked AF. The purpose of using a super-inclusion for constructing defeating sets is to capture particular situations when working with subargumentation. This will be clear in Sect. 5. Determining a correct defeating set depends on two sequential steps: firstly, it should ensure that its removal turns a into $\mathcal{S}$-accepted (see *rejecting sets* on Definition 8), and secondly, it should be minimal for such condition (*remainder sets* on Definition 9).

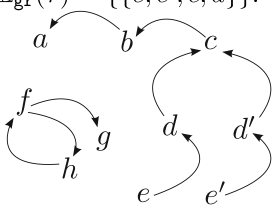
Definition 8 (Rejecting Sets). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*; for any* $\Theta \subseteq \mathbf{A}$*, we say that* Θ *is* $\mathcal{S}$*-*a***-rejecting*** *in* τ *iff* Θ *is* a*-defeating in* τ *and* a *is* $\mathcal{S}$*-accepted in* $\mathbb{F}_{\mathbf{A}\setminus\Theta}$*.*

Remainder sets state "responsibility" to arguments for the non-acceptability of an argument. Intuitively, an a-remainder is a minimal $\mathcal{S}$-a-rejecting set.

Definition 9 (Remainder Sets). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$ *and an argumentation semantics specification* $\mathcal{S}$*, for any* $\mathcal{R} \subseteq \mathbf{A}$*, we say that* $\mathcal{R}$ *is an* a***-remainder*** *in* τ*, noted as* a-$\texttt{remainder}_{\mathcal{S}}$ *iff* $\mathcal{R}$ *is a* $\mathcal{S}$*-*a*-rejecting set and for any* $\Theta \subset \mathcal{R}$*, it follows that* a *is* $\mathcal{S}$*-rejected in the* AF $\mathbb{F}_{\mathbf{A}\setminus\Theta}$*.*

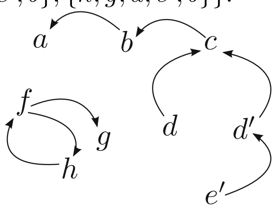
Example 10. Given the AF $\tau = \mathbb{F}_{\mathbf{A}}$, where $\mathbf{A} = \{a, b, c, d, d', e, e', f, g, h\}$ and $\mathbf{R}_{\mathbf{A}}$ renders the argumentation graph depicted below on the right. Argument b is not accepted by any semantics since there is no admissible set containing it. For instance, $\mathbb{E}_{\texttt{pr}}(\tau) = \{\{f, e, e', c, a\}, \{h, g, e, e', c, a\}\}$, and $\mathbb{E}_{\texttt{gr}}(\tau) = \{\{e, e', c, a\}\}$.

However, it is possible to propose different alternatives of change to move towards an epistemic state in which argument b turns to accepted in the resulting AF. For instance, let us consider a semantics specification $\mathcal{S} = \langle \mathfrak{s}, \delta \rangle$, where the acceptance function δ implements Eq. 2 and $\mathfrak{s} = \texttt{pr}$. In this case, the acceptable set would be $\mathcal{A}_{\mathcal{S}}(\tau) = \{e, e', c, a\}$. Note that $\{e\}$, $\{e'\}$, $\{c\}$ are b-$\texttt{remainder}_{\mathcal{S}}$ sets. This is so, given that $\{e\}$ is b-defeating for the b-partially admissible set $\{b, d\}$, in the same manner that $\{e'\}$ is for $\{b, d'\}$, and $\{c\}$ is for $\{b\}$. Note that $\{e, e', c\}$ is b-defeating for the b-partially admissible set $\{b, d, d'\}$, however while $\{e, e', c\}$ is a $\mathcal{S}$-b-rejecting set, it is not a b-$\texttt{remainder}_{\mathcal{S}}$ given that it is not minimal. Afterwards, considering the b-$\texttt{remainder}_{\mathcal{S}}$ $\{e\}$, we can build a new AF $\tau_1 = \mathbb{F}_{\mathbf{A}\setminus\{e\}}$ whose resulting acceptance set would be $\mathcal{A}_{\mathcal{S}}(\tau_1) = \{d, e', b\}$, since $\mathbb{E}_{\texttt{pr}}(\tau_1) = \{\{f, d, e', b\}, \{h, g, d, e', b\}\}$.

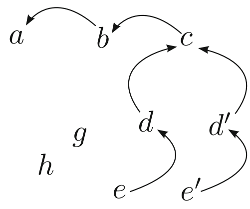


Graph of AF τ

Once again, considering the AF τ under the same semantic specification, note that g is not $\mathcal{S}$-accepted despite there is an extension $\{h, g, e, e', c, a\} \in \mathbb{E}_{\texttt{pr}}(\tau)$ which contains g. The situation here arises from the acceptance function δ which requires intersecting every extension in $\mathbb{E}_{\texttt{pr}}(\tau)$. Note also that there is a g-admissible set $\{g, h\}$. However, it is possible to propose an alternative of change to move towards an epistemic state in which argument g turns to accepted in the resulting AF. To that end, we can construct two g-partially admissible sets $\{g\}$ and $\{g, h\}$. Note that, for any of them, it appears a g-defeating set $\{f\}$ which ends up being a $\mathcal{S}$-g-rejecting set and also a g-$\texttt{remainder}_{\mathcal{S}}$ in the resulting AF $\tau_2 = \mathbb{F}_{\mathbf{A}\setminus\{f\}}$ whose acceptance set would be $\mathcal{A}_{\mathcal{S}}(\tau_2) = \{g, h, e, e', c, a\}$, since it ends up being the unique preferred extension in $\mathbb{E}_{\texttt{pr}}(\tau_2)$.



Graph of AF τ_1



Graph of AF τ_2

Observe that by considering an acceptance function δ implementing Eq. 3 under the preferred semantics, the acceptable set would be $\mathcal{A}_{\mathcal{S}}(\tau) = \{h, g, e, e', c, a\}$. Thus, it is natural to have that the unique g-$\texttt{remainder}_{\mathcal{S}}$ ends up being the empty set. This is so, given that although both $\emptyset$ and $\{f\}$ are g-defeating sets, and even both of them are also $\mathcal{S}$-g-rejecting sets, $\{f\}$ does not fulfill the requirements for being a g-$\texttt{remainder}_{\mathcal{S}}$ given that is is not minimal. This will be of utmost relevance for pursuing the verification of the well known principle of minimal change.

Definition 11 (Set of Cores and Set of Remainders). *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*, we say that:*

1. $\top_{\mathcal{S}}(\tau, a)$ *is the* ***set of cores of*** a *iff* $\top_{\mathcal{S}}(\tau, a)$ *contains every* a-$\texttt{core}_{\mathcal{S}}$ $\mathcal{C} \subseteq \mathbf{A}$.
2. $\bot_{\mathcal{S}}(\tau, a)$ *is the* ***set of remainders of*** a *iff* $\bot_{\mathcal{S}}(\tau, a)$ *contains every* a-$\texttt{remainder}_{\mathcal{S}}$ $\mathcal{R} \subseteq \mathbf{A}$.

Example 12 (Continues from Example 10). Considering the acceptance function implementing Eq. 2 over the preferred semantics, the set of cores for argument a ends up being $\top_{\mathcal{S}}(\tau, a) = \{\{a, c, e, e'\}\}$. Also, the corresponding set of remainders for argument b is $\bot_{\mathcal{S}}(\tau, b) = \{\{e\}, \{e'\}, \{c\}\}$. On the other hand, if we consider the b-$\texttt{remainder}_{\mathcal{S}}$ $\{e\}$ for analyzing the AF $\tau_1 = \mathbb{F}_{\mathbf{A}\setminus\{e\}}$, argument b turns out being $\mathcal{S}$-accepted since it is possible to identify a b-$\texttt{core}_{\mathcal{S}}$. In such a case, the resulting set of cores for b would be $\top_{\mathcal{S}}(\tau_1, b) = \{\{b, d\}\}$.

Proposition 13. *Given an* AF $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*; the following properties hold: (1)* $\top_{\mathcal{S}}(\tau, a) = \emptyset$ *iff* $\bot_{\mathcal{S}}(\tau, a) \neq \emptyset$*, (2)* $a \in \mathcal{A}_{\mathcal{S}}(\tau)$ *iff* $\top_{\mathcal{S}}(\tau, a) \neq \emptyset$*, and (3)* $a \notin \mathcal{A}_{\mathcal{S}}(\tau)$ *iff* $\bot_{\mathcal{S}}(\tau, a) \neq \emptyset$*.*

Proposition 13 states the interrelation between the sets of cores and remainders and how they relate with an argument's $\mathcal{S}$-acceptance.

4 Argumentation Dynamics Through Retractive Methods

For a rational handling of the acceptability dynamics of arguments, a change operation applied to an AF τ should provoke a controlled alteration of the acceptable set $\mathcal{A}_{\mathcal{S}}(\tau)$ towards achieving a specific purpose. For instance, a contraction operation may modify the acceptable set in order to *contract the acceptance condition* of a specific argument. The *acceptance contraction* of an argument can be achieved through the removal of arguments from the set $\mathbf{A}(\tau)$. However, observe that the acceptable set $\mathcal{A}_{\mathcal{S}}(\tau)$ has a non-monotonic construction from τ. This means that removing/incorporating arguments from/to the argumentation framework does not imply that the resulting acceptable set would be de/increased regarding the original one. Consequently, it is also possible to consider the addition of new arguments to the framework, in order to ensure an argument a to be rejected in the resulting framework. The former alternative could be achieved by breaking all a-$\texttt{core}_{\mathcal{S}}$ sets, whereas for the latter alternative, the idea would be to incorporate new arguments towards the construction of a-$\texttt{remainder}_{\mathcal{S}}$ sets. On the other hand, a contraction operation may modify the acceptable set in order to *contract the rejection condition* of a specific argument. The *rejection contraction* of an argument a ensures that a ends up accepted. We can achieve acceptance of an argument a either by removing arguments from $\mathbf{A}$ to break the existence of a-$\texttt{remainder}_{\mathcal{S}}$ sets, or also by incorporating arguments to $\mathbf{A}$ to construct a-$\texttt{core}_{\mathcal{S}}$ sets. It is possible to establish an analogy between classical belief revision, where a contraction by a formula α (resp. of, $\neg\alpha$) ensures α's truth (resp. of, falsity) is not inferred and, belief revision in argumentation, where an acceptance contraction (resp. of, rejection contraction) by an argument a ensures a is not accepted (resp. of, not rejected).

Revisions and contractions are usually defined independently with the intention to interrelate them afterwards by setting up a duality. A philosophic discussion is sustained on the matter of the nature of such independence. Some researchers assert that there is really no contraction whose existence could be justified without a revision. In fact, they state that a contraction conforms an intermediate state towards the full specification of the revision. Such an intuition fits quite well our approach. For instance, if we think the argumentation stands for a normative system, it is natural to assume that a new norm is intended to be incorporated –through a revision– for ensuring afterwards its acceptance –through some intermediate contraction. Another alternative is to assume a derogative norm, whose purpose is to enter the system –through a revision– for ensuring afterwards the rejection of an elder norm –through some intermediate contraction for ensuring the acceptance of the derogative norm. In this paper we focus on an *acceptance revision operation* obtained through the removal of arguments from the set $\mathbf{A}$, *i.e.*, a sort of *retractive* acceptance revision. Such a revision operation retracts from the $\texttt{AF}$ some a-$\texttt{remainder}_{\mathcal{S}}$ set –for ensuring the acceptance of a new argument a– through the usage of a rejection contraction. Thus, with a retractive acceptance revision, we assume the idea of provoking change to the $\texttt{AF}$ for altering the acceptable set with the intention to pursue acceptability of an argument a, which can be external to the original $\texttt{AF}$.

An operator '$\circledast$' ensures that given an $\texttt{AF}$ τ and a new argument a, the acceptance revision of τ by a ends up in a new $\texttt{AF}$ $\tau \circledast a$ in which a is $\mathcal{S}$-accepted. We refer to an early contribution by Levi [17] to belief revision, where he related revisions to contractions. He suggested that a revision ('$*$') of a base Σ by a new information α should be achieved through two stages. Firstly, by contracting ('$-$') all possibility of deriving $\neg\alpha$ for obtaining a new base which would be consistent with α. Afterwards, it could be added ('$+$') the new information α ensuring that this stage would end up consistently. This intuition was formalized in an equivalence referred to as the *Levi identity*: $\Sigma * \alpha = (\Sigma - \neg\alpha) + \alpha$. In argumentation, it is natural to think that the new argument a should be incorporated to the $\texttt{AF}$ τ through an *expansion* operator '$+$', and ensuring afterwards its acceptability through a contraction operation for breaking the rejection of a, *i.e.*, a *rejection contraction* '$\ominus_{\perp}$'. Note that it is mandatory to invert the two stages of the original Levi identity[2] since it is necessary for the new argument to be recognized by the framework in order to analyze its acceptability condition. This renders an equivalence between *acceptance revision* and *rejection contraction* through the generalization of the Levi identity: $\tau \circledast a = (\tau + a) \ominus_{\perp} a$.

We will analyze the construction of two sub-operations for achieving the acceptance revision. Firstly, we need to recognize new arguments to be incorporated to the framework. For such matter, let us assume a *domain of abstract arguments* $\mathbb{A}$, such that for any abstract `AF` $\mathbb{F}_{\mathbf{A}}$, it follows that $\mathbf{A} \subseteq \mathbb{A}$. Next we formalize the concept of *external argument*, and afterwards we define a simple *expansion operation* for incorporating an external argument to a framework.

Definition 14 (External Argument). *Given an* `AF` $\tau = \mathbb{F}_{\mathbf{A}}$*, an argument* a *is* ***external to*** τ *(or just,* ***external****) iff* $a \in \mathbb{A}$ *but* $a \notin \mathbf{A}$*.*

Definition 15 (Expansion). *Given an* `AF` $\tau = \mathbb{F}_{\mathbf{A}}$ *and an external argument* $a \in \mathbb{A}$*. The operator* $+$ *stands for an* ***expansion*** *iff* $\tau + a = \mathbb{F}_{\mathbf{A} \cup \{a\}}$*.*

From Proposition 13, we know that an argument a is $\mathcal{S}$-accepted *iff* there is no a-$\texttt{remainder}_{\mathcal{S}}$ set. Therefore, it is sufficient to break one single a-$\texttt{remainder}_{\mathcal{S}}$ $\mathcal{R} \in \perp_{\mathcal{S}}(\tau, a)$ in order to obtain a new `AF` in which we could construct a-$\texttt{core}_{\mathcal{S}}$ sets, implying the acceptance of a. For such purpose, we define a *remainder selection*, as a function by which it is possible to select the best option among the several a-$\texttt{remainder}_{\mathcal{S}}$ sets from $\perp_{\mathcal{S}}(\tau, a)$.

Definition 16 (Remainder Selection). *Given an* `AF` $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*. A* ***remainder selection*** *is obtained by a* ***selection function*** $\gamma : \wp(\wp(\mathbf{A})) \longrightarrow \wp(\mathbf{A})$ *applied over the set* $\perp_{\mathcal{S}}(\tau, a)$ *for selecting some* a-$\texttt{remainder}_{\mathcal{S}}$*, where* $\gamma(\perp_{\mathcal{S}}(\tau, a)) \in \perp_{\mathcal{S}}(\tau, a)$ *is such that for every* $\mathcal{R} \in \perp_{\mathcal{S}}(\tau, a)$ *it holds* $\gamma(\perp_{\mathcal{S}}(\tau, a)) \preccurlyeq_{\gamma} \mathcal{R}$*, where* $\preccurlyeq_{\gamma}$ *is a* ***selection criterion*** *by which it is possible to select the best representative* a-$\texttt{remainder}_{\mathcal{S}}$ *set.*

The selection criterion can be any method for ordering sets of arguments. In the sequel, we will abstract away from any specific selection criterion. Now it is easy to define the *rejection contraction* by relying upon a selection function.

Definition 17 (Rejection Contraction). *Given an* `AF` $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an argument* $a \in \mathbf{A}$*. The operator* $\ominus_{\perp}$ *stands for a* ***rejection contraction*** *iff* $\tau \ominus_{\perp} a = \mathbb{F}_{\mathbf{A} \setminus \mathcal{R}}$*, where* $\mathcal{R} = \gamma(\perp_{\mathcal{S}}(\tau, a))$*.*

The *acceptance revision* may be formally given by relying upon an expansion operation and a rejection contraction determined by a selection function.

Definition 18 (Acceptance Revision). *Given an* `AF` $\tau = \mathbb{F}_{\mathbf{A}}$*, a semantics specification* $\mathcal{S}$*, and an external argument* $a \in \mathbb{A}$*. The operator* $\circledast$ *stands for an* ***acceptance revision*** *(or just,* ***revision****) iff* $\tau \circledast a = \mathbb{F}_{\mathbf{A}'}$*, where* $\mathbf{A}' = \mathbf{A}(\tau + a) \setminus \gamma(\perp_{\mathcal{S}}(\tau + a, a))$*. When necessary, we will write* $\tau \circledast_{\gamma} a$ *to identify the remainder selection function* γ *by which the revision* $\tau \circledast a$ *is obtained.*

[2] Inverting the Levi identity leads to an inconsistent intermediate state. This is not an issue in argumentation since we only incorporate new pairs to the defeat relation.

The axiomatization of the acceptance revision is achieved by analyzing the different characters of revisions from classical belief revision [1,16] and from ATC revision [19], for adapting the classical postulates to argumentation.

(success) a is $\mathcal{S}$-accepted in $\tau \circledast a$
(consistency) $\mathcal{A}_{\mathcal{S}}(\tau \circledast a)$ is conflict-free
(inclusion) $\tau \circledast a \subseteq \tau + a$
(vacuity) If a is $\mathcal{S}$-accepted in $\tau + a$ then $\mathbf{A}(\tau + a) \subseteq \mathbf{A}(\tau \circledast a)$
(core-retainment) If $b \in \mathbf{A}(\tau) \setminus \mathbf{A}(\tau \circledast a)$ then exists an $\texttt{AF}$ τ' such that $\mathbf{A}(\tau') \subseteq \mathbf{A}(\tau)$ and a is $\mathcal{S}$-accepted in $\tau' + a$ but $\mathcal{S}$-rejected in $(\tau' + b) + a$.

In classic belief revision, the *success* postulate states that the new information should be satisfied by the revised knowledge base. From the argumentation standpoint, this may be interpreted as the requirement of acceptability of the new argument. Through *consistency* a classic revision operation ensures that the new revised base ends up consistently always that the new belief to be incorporated is so. From the argumentation standpoint, there should be no need for ensuring a consistent (or conflict-free) set of arguments since the essence of such theory is to deal with inconsistencies. However, this requirement makes sense when thinking about the acceptable set of the framework for ensuring that the argumentation semantics allows a consistent reasoning methodology. The consistency postulate for extension semantics has been studied before in [2], among others. *Inclusion* aims at guaranteeing that the only new information to be incorporated is the object by which the base is revised. The restatement to argumentation may be seen as the sole inclusion of the external argument. *Vacuity* captures the conditions under which the revision operation has nothing to do but the sole incorporation of the new information. Its restatement to argumentation may be seen as the fact of a being $\mathcal{S}$-accepted straightforwardly, with no need to remove any argument. That is, the simple expansion of the external argument would end up forming a new framework in which it is possible to construct a-$\texttt{core}_{\mathcal{S}}$ sets. The vacuity postulate is usually referred as complementary to the inclusion postulate, thus, a change operation satisfying both postulates ends up verifying the equality $\tau \circledast a = \tau + a$ whenever the external argument is straightforwardly $\mathcal{S}$-accepted in the expanded framework. Through *core-retainment* the amount of change is controlled by avoiding removals that are not related to the revision operation, *i.e.*, every belief that is lost serves to make room for the new one. In argumentation dynamics we care on the changes perpetrated to the framework in order to achieve acceptability for the external argument. Hence, any argument that is removed should be necessary for such purpose. The rational behavior of the acceptance revision operation is ensured through the following *representation theorem.*

Theorem 19. *Given an* $\texttt{AF}$ τ*, a semantics specification* $\mathcal{S}$*, and an external argument* $a \in \mathbb{A}$*;* $\tau \circledast a$ *is an acceptance revision iff '*$\circledast$*' satisfies success, consistency, inclusion, vacuity, and core-retainment.*

5 Fundamentals for Logic-Based Frameworks

We will assume a logic $\mathcal{L}$ to which the represented knowledge will correspond. In addition, we will assume an *argument domain set* referred as $\mathbb{A}_{\mathcal{L}}$ to which *(logic-based) arguments* containing $\mathcal{L}$ formulae will conform. Arguments will be defined upon $\mathcal{L}$ knowledge through a set of premises and a claim such that an argument $a \in \mathbb{A}_{\mathcal{L}}$ can be expressed through a pair, namely the *argument interface*, $\langle S, \vartheta \rangle \in \mathbb{A}_{\mathcal{L}}$, where $S \subseteq \mathcal{L}$ is referred as the *support* and $\vartheta \in \mathcal{L}$ as the *claim*. The logic $\mathcal{L}$ will be considered along with its corresponding inference operator $\models$, constituting a complete *deductive system* $\langle \mathcal{L}, \models \rangle$. Therefore, according to the classic notion of argument, we can assume that given an argument $\langle S, \vartheta \rangle \in \mathbb{A}_{\mathcal{L}}$, the basic three principles are satisfied: *(deduction)* $S \models \vartheta$, *(minimality)* there is no subset $S' \subset S$ such that $S' \models \vartheta$, and *(consistency)* S is consistent according to $\mathcal{L}$, *i.e.*, $S \not\models \bot$. Finally, we will eventually say that an argument *a supports* ϑ *from* S to informally specify that the argument claim is the formula $\vartheta \in \mathcal{L}$ and similarly that its support is given by the set $S \subseteq \mathcal{L}$, or formally that the argument $a = \langle S, \vartheta \rangle \in \mathbb{A}_{\mathcal{L}}$. We will rely upon two functions $\mathfrak{cl} : \mathbb{A}_{\mathcal{L}} \longrightarrow \mathcal{L}$ and $\mathfrak{sp} : \mathbb{A}_{\mathcal{L}} \longrightarrow \wp(\mathcal{L})$ to identify both the claim and support set of $\mathbb{A}_{\mathcal{L}}$-arguments. Hence, given an argument $a \in \mathbb{A}_{\mathcal{L}}$, we can refer to the claim and support set as $\mathfrak{cl}(a) \in \mathcal{L}$ and $\mathfrak{sp}(a) \subseteq \mathcal{L}$, respectively. Moreover, the function $\mathfrak{sp}$ will be overloaded as $\mathfrak{sp} : \wp(\mathbb{A}_{\mathcal{L}}) \longrightarrow \wp(\mathcal{L})$ in order to be applied over sets of arguments such that given a set $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$, $\mathfrak{sp}(\Theta) = \bigcup_{a \in \Theta} \mathfrak{sp}(a)$ will identify the base determined by the set of supports of arguments contained in Θ.

A *(logic-based) argumentation framework* (`AF`) will be assumed as a pair $\langle \mathbf{A}, \mathbf{R_A} \rangle$, where $\mathbf{A} \subseteq \mathbb{A}_{\mathcal{L}}$ is a finite *set of arguments*, and the set $\mathbf{R_A} \subseteq \mathbb{A}_{\mathcal{L}} \times \mathbb{A}_{\mathcal{L}}$ identifies the finitary *defeat relation* between pairs of arguments such that:

$$\mathbf{R_A} = \{(a, b) | a, b \in \mathbf{A}, \mathfrak{sp}(a) \cup \mathfrak{sp}(b) \models \bot, \text{ and } a \not\prec b\} \tag{5}$$

A pair $(a, b) \in \mathbf{R_A}$ implies that $a \in \mathbf{A}$ *defeats* $b \in \mathbf{A}$, or equivalently, a is a *defeater* of b, meaning that the supports of both arguments a and b cannot be simultaneously assumed in a consistent manner, and also that a is preferred over b, according to some abstract *preference relation* $\not\prec$. We will keep the defeating function ε abstract, assuming that it is *valid iff* condition (1) in p. 5 is satisfied. Different instantiations of such a function has been widely studied in [15].

Since any logic-based argument is built from a set of formulae –standing for its support set– it is natural to think that any subset of the support set can be used to build another argument. This intuition describes the concepts of *sub-arguments* (and *super-arguments*). We will identify a sub-argument relation by writing $a \sqsubseteq b$ for expressing that an argument $a \in \mathbb{A}_{\mathcal{L}}$ is a sub-argument of argument $b \in \mathbb{A}_{\mathcal{L}}$ (and also that b is a super-argument of a), implying that $\mathfrak{sp}(a) \subseteq \mathfrak{sp}(b)$ holds. We will also identify the set of all sub-arguments of an argument $a \in \mathbb{A}_{\mathcal{L}}$ through the function $\mathfrak{subs} : \mathbb{A}_{\mathcal{L}} \longrightarrow \wp(\mathbb{A}_{\mathcal{L}})$ such that $\mathfrak{subs}(a) = \{b \in \mathbb{A}_{\mathcal{L}} | b \sqsubseteq a\}$, for any argument $a \in \mathbb{A}_{\mathcal{L}}$.

Logic-based argumentation may unveil some problems with regards to the conflict recognition between pairs of arguments. Consider the following example where arguments are constructed upon a propositional logic $\mathcal{L}$.

Example 20. Assuming $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$ such that $\Theta = \{a, b, c\}$ where $a = \langle\{p\}, p\rangle$, $b = \langle\{q\}, q\rangle$, and $c = \langle\{\neg p \vee \neg q\}, \neg p \vee \neg q\rangle$. The AF generator $\mathbb{F}_{\Theta}$ will construct an AF with an empty set of defeats $\mathbf{R}_{\Theta}$. Note that Θ is admissible given that it is conflict-free and that it has no defeaters. However, $\mathfrak{sp}(\Theta) \models \bot$ holds.

The problem presented in Example 20 relies on the construction of logic-based AF*s* from arbitrary sets of arguments. It is necessary to build all possible arguments, including sub and super arguments, in order to ensure that the resulting AF will deliver rational responses through an argumentation semantics. We say that a set of arguments is closed whenever it contains all the sub- and super-arguments that can be constructed from its arguments. This ensures an exhaustive construction of arguments from an initial base of arguments. We provide such implementation through an *argumentation closure operator* $\mathbb{C}$.

Definition 21 (Argumentation Closure). *An operator* $\mathbb{C}$ *is an* ***argumentation closure*** *iff for any* $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$*, it holds* $\mathbb{C}(\Theta) = \{a \in \mathbb{A}_{\mathcal{L}} | a \sqsubseteq b$*, for any* $b \in \Theta\} \cup \{a \in \mathbb{A}_{\mathcal{L}} | \mathfrak{subs}(a) \subseteq \Theta\}$*. We say that* Θ *is* ***closed*** *iff it holds* $\Theta = \mathbb{C}(\Theta)$*.*

The following proposition shows that the closure of a set Θ of arguments triggers the complete set of arguments that can be constructed using the formulae involved in arguments contained in Θ.

Proposition 22. *Given a set of arguments* $\Theta \subseteq \mathbb{A}_{\mathcal{L}}$*, the underlying knowledge base* $\Sigma = \mathfrak{sp}(\Theta)$*, and the set* $\mathbf{A}_{\Sigma} \subseteq \mathbb{A}_{\mathcal{L}}$ *of all the possible arguments constructed from* Σ*. The set* Θ *is closed iff* $\Theta = \mathbf{A}_{\Sigma}$*.*

We refer to a structure $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$ as a *closed* AF *iff* it is constructed through a closed set of arguments $\mathbf{A} \subseteq \mathbb{A}_{\mathcal{L}}$, *i.e.*, $\mathbf{A} = \mathbb{C}(\mathbf{A})$. Depending on the specification of the language $\mathcal{L}$, the argumentation closure may trigger multiple different arguments with a unique support and even more, it could result infinitary, triggering an infinite set of arguments if the closure is achieved in an uncontrolled manner. Several alternatives may arise to keep a finite, and still closed, set of arguments. For instance, it is possible to restrict the claim of arguments to some specific form in order to avoid constructing several arguments with logically equivalent claims and a same support set. A nice alternative for doing this is to restrict the construction of arguments to their *canonical form* [6], in which for any argument a, its claim has the form $\mathfrak{cl}(a) = \bigwedge \mathfrak{sp}(a)$. In the sequel, and just for simplicity, we will abstract away from such specific matters involving the construction of arguments, by simply referring to a domain $\mathbb{A}^{*}_{\mathcal{L}} \subseteq \mathbb{A}_{\mathcal{L}}$, where $\mathbb{A}^{*}_{\mathcal{L}}$ is the domain of arguments of a unique representation: for any pair of arguments $a, b \in \mathbb{A}^{*}_{\mathcal{L}}$, it follows that if $\mathfrak{sp}(a) = \mathfrak{sp}(b)$ then $\mathfrak{cl}(a) = \mathfrak{cl}(b)$, and thus it holds $a = b$. This restriction ensures that any set $\Theta \subseteq \mathbb{A}^{*}_{\mathcal{L}}$ of arguments ends up in a finite closed set $\mathbb{C}^{*}(\Theta) = \mathbf{A}$ independently of the method used for ensuring it, where $\mathbb{C}^{*}(\Theta)$ is the closed set of $\mathbb{A}^{*}_{\mathcal{L}}$-arguments such that $\mathbb{C}^{*}(\Theta) = \mathbb{C}(\Theta) \cap \mathbb{A}^{*}_{\mathcal{L}}$. From now on, we will write $\mathbf{A}$ (or $\mathbf{A}'$) for referring only to closed sets of $\mathbb{A}^{*}_{\mathcal{L}}$-arguments.

In what follows, we will write $\mathbb{F}_{\mathbf{A}}$ for referring to the AF $\langle \mathbf{A}, \mathbf{R}_{\mathbf{A}} \rangle$, where $\mathbf{A} \subseteq \mathbb{A}^{*}_{\mathcal{L}}$ is a closed set, *i.e.*, $\mathbb{C}^{*}(\mathbf{A}) = \mathbf{A}$. In such a case, we say that $\mathbb{F}_{\mathbf{A}}$ is a *closed* AF. This will also allow us to refer to any *sub-framework* $\mathbb{F}_{\Theta} = \langle \Theta, \mathbf{R}_{\Theta} \rangle$, where $\Theta \subseteq \mathbf{A}$ is a not necessarily closed set of arguments. In such a case, we will overload the sub-argument operator '$\sqsubseteq$' by also using it for identifying sub-frameworks, writing $\mathbb{F}_\Theta \sqsubseteq \mathbb{F}_\mathbf{A}$. Observe that, if $\mathbb{C}^*(\Theta) = \mathbf{A}'$ and $\mathbf{A}' \subset \mathbf{A}$, then $\mathbb{F}_{\mathbf{A}'}$ is a closed *strict sub-framework* of $\mathbb{F}_\mathbf{A}$, *i.e.*, $\mathbb{F}_{\mathbf{A}'} \sqsubset \mathbb{F}_\mathbf{A}$.

By relying upon closed argumentation frameworks we ensure that the acceptable set $\mathcal{A}_\mathcal{S}(\mathbb{F}_\mathbf{A})$ will trigger rational results. A closed AF $\mathbb{F}_\mathbf{A}$ will be necessary for satisfying *closure under sub-arguments* and *exhaustiveness* postulates from to [2]. On the other hand, a set $\mathbf{R}_\mathbf{A}$ as defined in Eq. 5 describes a general defeat relation which is *conflict-dependent* and *conflict-sensitive* according to [2]. This means that any minimal inconsistent set of formulae implies the construction of a pair of arguments which will necessarily be conflicting, and that any pair of conflicting arguments implies a minimal source of inconsistency. This property guarantees that the framework will satisfy the postulate referred as *closure under sub-arguments* under any of the extension semantics reviewed before. This postulate is necessary to ensure a rational framework independently of the semantics adopted given that, for any $\mathfrak{s}$-extension $\mathbf{E}$ we will ensure that if $a \in \mathbf{E}$ then for any sub-argument $b \sqsubseteq a$ it holds $b \in \mathbf{E}$. The *closure under CN* postulate [2] will not be verified given that we prevent the construction of several claims for a same argument's body through a unique representation like canonical arguments. However, it holds in a "semantic sense": closed AF*s* ensure drawing all such possible claims.

Example 23 (Continues from Example 20). By assuming $\mathbb{A}^*_\mathcal{L}$ as the domain of canonical arguments, the argumentation closure renders the closed set of arguments: $\mathbf{A} = \mathbb{C}^*(\Theta) = \{a, b, c, d, e, f\}$, where $d = \langle\{p, q\}, p \wedge q\rangle$, $e = \langle\{p, \neg p \vee \neg q\}, p \wedge (\neg p \vee \neg q)\rangle$, and $f = \langle\{q, \neg p \vee \neg q\}, q \wedge (\neg p \vee \neg q)\rangle$. For the construction of the set of defeats, we will assume that any argument in Θ is preferred over any other argument which is not in Θ, whereas when considering a pair of arguments where both are either Θ insiders or outsiders, the preference relation will be symmetric. Thus, we obtain the following pairs of defeats: $\mathbf{R}_\mathbf{A} = \{(a, f), (b, e), (c, d), (d, e), (d, f), (e, d), (f, d)\}$. Observe however that although $\mathfrak{sp}(\Theta)$ is inconsistent, Θ is still admissible.

Through the argumentation closure, we have provided a method for ensuring that a closed AF is complete given that we have all the possible arguments that can be constructed from the set of arguments and therefore all the sources of conflict will be identified through the defeat relation. However, we still have a problem: as is shown in Example 23, $\Theta \subseteq \mathbf{A}$ keeps being admissible given that it is conflict-free. Thus, it is necessary to reformulate the abstract notion for admissible sets by requiring their closure.

Definition 24 (Logic-based Admissibility). *Given an* AF $\mathbb{F}_\mathbf{A}$*, for any* $\Theta \subseteq \mathbf{A}$ *we say that* Θ *is* ***admissible*** *iff* Θ *is closed (i.e.,* $\Theta = \mathbb{C}^*(\Theta)$*), conflict-free, and defends all its members.*

Once again, regarding postulates in [2], working with closed AF*s* and taking in consideration the reformulated notion of admissibility in logic-based frameworks, guarantees the *consistency* postulate which ensures that every $\mathfrak{s}$-extension contains a consistent support base, *i.e.*, for any closed $\mathtt{AF}$ τ, $\mathfrak{sp}(\mathbb{E}_{\mathfrak{s}}(\tau)) \not\models \bot$ holds.

Example 25 (Continues from Example 23). Under the new definition of admissibility, we have that Θ cannot be admissible since it is not closed. The following admissible sets appear: $\{a\}, \{b\}$, and $\{c\}$. Note that the sets $\{a, b, d\}, \{a, c, e\}$, and $\{b, c, f\}$, are not admissible given that although they are closed and conflict-free, none of them defends all its members.

Definition 24 for admissibility in logic-based frameworks makes core sets end up closed without inconvenient. However, the case of remainder sets is different. Having a set $\Theta \subseteq \mathbf{A}$, the problem is that we only can ensure that an argument a is accepted in the sub-framework $\mathbb{F}_{\mathbf{A}\setminus\Theta}$ if we can ensure that $\mathbf{A}\setminus\Theta$ is a closed set (see Example 20). This ends up conditioning Definition 9. Hence, it is necessary to provide some constructive definition for remainder sets. This allows determining which property should satisfy a set $\Theta \subseteq \mathbf{A}$ for ensuring that if $\mathbf{A}$ is a closed set then the operation $\mathbf{A} \setminus \Theta$ also determines a closed set. In Definition 27, we propose an *expansive closure* which will rely upon the identification of a set of *atomic arguments*: arguments that have no strict sub-arguments inside. That is, given an argument $a \in \mathbb{A}^*_{\mathcal{L}}$, a is *atomic iff* $|\mathfrak{sp}(a)| = 1$.

Definition 26 (Set of Atomic Arguments). *Given an* $\mathtt{AF}$ $\mathbb{F}_{\mathbf{A}}$ *and an argument* $a \in \mathbf{A}$*, a function* $\mathfrak{at} : \mathbb{A}^*_{\mathcal{L}} \longrightarrow \wp(\mathbb{A}^*_{\mathcal{L}})$ *is an* ***atoms function*** *iff it renders the* ***set of atomic arguments*** $\mathfrak{at}(a) \subseteq \mathbf{A}$ *of* a *such that* $\mathfrak{at}(a) = \{b \in \mathbf{A} | b \sqsubseteq a$ *and there is no* $c \in \mathbf{A}$ *such that* $c \sqsubset b\}$*.*

We will overload the atoms function as $\mathfrak{at} : \wp(\mathbb{A}^*_{\mathcal{L}}) \longrightarrow \wp(\mathbb{A}^*_{\mathcal{L}})$ to be applied over sets of arguments such that $\mathfrak{at}(\Theta) = \bigcup_{a \in \Theta} \mathfrak{at}(a)$.

Definition 27 (Expansive Closure). *Given an* $\mathtt{AF}$ $\mathbb{F}_{\mathbf{A}}$ *and a set* $\Theta \subseteq \mathbf{A}$*, an operator* $\mathbb{P}$ *is an* ***expansive closure*** *iff* $\mathbb{P}(\Theta) = \{a \in \mathbf{A} | b \sqsubseteq a$*, for every* $b \in \mathfrak{at}(\mathbb{P}_0(\Theta))\}$*, where* $\mathbb{P}_0(\Theta) = \{a \in \Theta |$ *there is no* $b \in \Theta$ *such that* $b \sqsubset a\}$*. We say that* Θ *is* ***expanded*** *iff it holds* $\Theta = \mathbb{P}(\Theta)$*.*

Note that $\mathbb{P}_0(\Theta)$ contains all the arguments from Θ having no sub-arguments in Θ, while $\mathbb{P}(\Theta)$ contains all the arguments from $\mathbf{A}$ having some atomic sub-argument of some argument in $\mathbb{P}_0(\Theta)$. The expansive closure is a sort of super-argument closure in the sense that it contains all the arguments that should disappear by removing Θ from $\mathbf{A}$. Proposition 28 verifies that if we remove from a closed set another set which is expanded then we obtain a new closed set.

Proposition 28. *Given two sets* $\mathbf{A} \subseteq \mathbb{A}^*_{\mathcal{L}}$ *and* $\Theta \subseteq \mathbb{A}^*_{\mathcal{L}}$*, where* $\mathbf{A}$ *is closed; if* $\Theta \subseteq \mathbf{A}$ *then* $\mathbf{A}' = \mathbf{A} \setminus \mathbb{P}(\Theta)$ *is a closed set, i.e.,* $\mathbf{A}' = \mathbb{C}^*(\mathbf{A}')$*.*

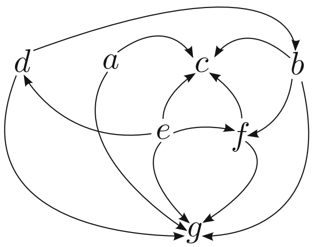
Definition 29 (Logic-based Remainder Sets). *Given an* $\mathtt{AF}$ $\mathbb{F}_{\mathbf{A}}$ *and a semantics specification* $\mathcal{S}$*, for any* $\Theta \subseteq \mathbf{A}$*, we say that* Θ *is an* a-***remainder*** *in* $\mathbb{F}_{\mathbf{A}}$*, noted as* a-$\mathtt{remainder}_{\mathcal{S}}$ *iff* Θ *is a minimal expanded* $\mathcal{S}$-a*-rejecting set:*

1. Θ is a $\mathcal{S}$-a-rejecting set,
2. $\Theta = \mathbb{P}(\Theta)$, and
3. for any set $\Theta' \subset \Theta$ such that $\Theta' = \mathbb{P}(\Theta')$, it holds a is $\mathcal{S}$-rejected in $\mathbb{F}_{\mathbf{A}\setminus\Theta'}$.

The following example shows how a propositional logic $\mathcal{L}$ for constructing logic-based frameworks affects the notions of core and remainder sets.

Example 30. We will assume $\mathcal{L}$ as the propositional logic and $\mathbb{A}^*_{\mathcal{L}}$ as the domain of canonical arguments. Let $\Theta \subseteq \mathbb{A}^*_{\mathcal{L}}$ be a set of canonical arguments such that $\Theta = \{a, b, c, d\}$, where $a = \langle\{p \wedge q_1\}, p \wedge q_1\rangle$, $b = \langle\{p \wedge q_2\}, p \wedge q_2\rangle, c = \langle\{\neg p\}, \neg p\rangle$, and $d = \langle\{\neg q_2\}, \neg q_2\rangle$. The argumentation closure renders the closed set of arguments $\mathbf{A} = \mathbb{C}^*(\Theta) = \{a, b, c, d, e, f, g\}$, where:

$e = \langle\{p \wedge q_1, p \wedge q_2\}, p \wedge q_1 \wedge q_2\rangle$, where $\mathfrak{subs}(e) = \{a, b\}$
$f = \langle\{p \wedge q_1, \neg q_2\}, p \wedge q_1 \wedge \neg q_2\rangle$, where $\mathfrak{subs}(f) = \{a, d\}$
$g = \langle\{\neg p, \neg q_2\}, \neg p \wedge \neg q_2\rangle$, where $\mathfrak{subs}(g) = \{c, d\}$



$\mathbb{F}_{\mathbf{A}}$

Then, the AF $\mathbb{F}_{\mathbf{A}}$ is closed and through a preference relation $\mathbf{R}_{\mathbf{A}} = \{(a, c), (b, c), (d, g), (d, b), (e, c), (e, d), (b, f), (f, c), (a, g), (b, g), (e, f), (e, g), (f, g)\}$. Assuming $\mathcal{S} = \langle \texttt{co}, \delta\rangle$, where δ implements Eq. 3, observe that a b-$\texttt{core}_{\mathcal{S}}$ $\mathcal{C}_b = \{a, b, e\}$ is constructed by the closure $\mathbb{C}^*(\{b, e\})$. Since c and d are $\mathcal{S}$-rejected, we have remainder sets for both of them: a c-$\texttt{remainder}_{\mathcal{S}}$ $\mathcal{R}_c = \{a, e, f\}$ and two d-$\texttt{remainder}_{\mathcal{S}}$ sets $\mathcal{R}_d = \{a, e, f\}$ and $\mathcal{R}'_d = \{b, e\}$. Observe that $\Upsilon = \{a, b, e, f\}$ is the result of expanding the $\mathcal{S}$-d-rejecting set $\{e\}$, *i.e.*, $\Upsilon = \mathbb{P}(\{e\})$. However Υ is not a d-$\texttt{remainder}_{\mathcal{S}}$ since it is not minimal: there are two d-defeating sets $\{a, e\}$ and $\{b, e\}$ whose respective expansions are $\mathbb{P}(\{a, e\}) = \mathcal{R}_d$ and $\mathbb{P}(\{b, e\}) = \mathcal{R}'_d$. Note that, although $\{e\}$ is a d-defeating set, the superinclusion in Definition 7, item 3, allows the consideration of some additional argument/s. Clearly, the only alternative for that is to incorporate some atom/s of some argument/s included in the defeating set.

6 Argumentation Dynamics in Logic-Based Frameworks

We need to consider closed logic-based frameworks which provokes a necessary reformulation of the expansion operation. This ensures a closed resulting framework after the incorporation of an external argument $a \in \mathbb{A}^*_{\mathcal{L}}$.

Definition 31 (Expansion). *Given an* AF $\mathbb{F}_{\mathbf{A}}$ *and an external argument* $a \in \mathbb{A}^*_{\mathcal{L}}$. *The operator + stands for an* ***expansion*** *iff* $\mathbb{F}_{\mathbf{A}} + a = \mathbb{F}_{\mathbb{C}^*(\mathbf{A}\cup\{a\})}$.

Definitions for change operations proposed in Sect. 4 will perfectly apply for logic-based frameworks if the references to expansion operations are interpreted as logic-based expansions, according to Definition 31. Thus, a revision $\tau \circledast a$ will refer to an operation $(\tau + a) \ominus_{\perp} a$, where $+$ is a logic-based expansion. Change operations for logic-based frameworks incorporated the necessary consideration of $\mathcal{L}$-formulae. This brings about the necessity to discuss additional postulates for the complete rationalization of closed frameworks. In classic belief revision, the *closure* postulate states that if a base Σ is a closed set (referred as belief set) then the result of the revision should also be ensured to be closed. In this case, by closure they refer to a closure under logical consequences, obtaining, in general, infinite closed sets. This kind of closure is different from the proposed argumentation closure. As being explained before, the argumentation closure – applied over singleton construction of arguments, *i.e.*, arguments from a domain $\mathbb{A}^*_{\mathcal{L}}$– ensures a finite closed set of arguments. However, the purpose of the argumentation closure also differs from the closure under logical consequences in that $\mathbb{C}$ ensures the presence of all the constructible arguments (see Proposition 22) from a common knowledge respecting a specific construction $\mathbb{A}^*_{\mathcal{L}}$, but not the construction of all the equivalent arguments. This subject makes rationality of acceptance revision for logic-based frameworks more similar to revision of bases than belief sets. Finally, from the argumentation standpoint, we should ensure that the revision of a closed AF ends up in a new closed AF.

$$(\textbf{closure}) \text{ if } \mathbf{A}(\tau) = \mathbb{C}^*(\mathbf{A}(\tau)) \text{ then } \mathbf{A}(\tau \circledast a) = \mathbb{C}^*(\mathbf{A}(\tau \circledast a))$$

In belief revision, it is natural to assume that revisions applied to a base by logically equivalent formulae, have necessarily identical outcomes. The choice of which elements of the base to retain should depend on their logical relations to the new information. Therefore, if two sentences are inconsistent with the same subsets of the base, they should push out the same elements from the base. This is known as *uniformity*. Since we are considering arguments built from $\mathcal{L}$-formulae, it is natural to analyze the existing relations between two different arguments which coincide in their conflict-relations, supports and claims. For this matter it is necessary to specify an *equivalence relation* for arguments in order to ensure that the revisions $\tau \circledast a$ and $\tau \circledast b$ have equivalent outcomes (see [19]).

Definition 32 (Equivalence [19]). *For any pair of arguments $a, b \in \mathbb{A}^*_{\mathcal{L}}$, we say that a and b are equivalent arguments, noted as $a \equiv b$ iff $\mathfrak{cl}(a) \models \mathfrak{cl}(b)$ and $\mathfrak{cl}(b) \models \mathfrak{cl}(a)$ and for any $a' \sqsubset a$ there is $b' \sqsubset b$ such that $a' \equiv b'$.*

$$(\textbf{uniformity}) \text{ if } a \equiv b \text{ then } \mathbf{A}(\tau) \cap \mathbf{A}(\tau \circledast a) = \mathbf{A}(\tau) \cap \mathbf{A}(\tau \circledast b)$$

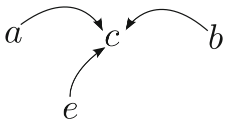
Inspired by smooth incisions in Hansson's Kernel Contractions [16], we introduce an additional condition on remainder selection functions for guaranteeing uniformity. Under the consideration of two equivalent arguments a and b, the idea is to ensure that a remainder selection function will trigger a same remainder $\mathcal{R}$ which is common to both sets of remainders $\perp_{\mathcal{S}}(\tau + a, a)$ and $\perp_{\mathcal{S}}(\tau + b, b)$.

Definition 33 (Smooth Remainder Selection). *Given an AF τ and two external arguments $a, b \in \mathbb{A}^*_{\mathcal{L}}$. If $a \equiv b$ then $\gamma(\perp_{\mathcal{S}}(\tau + a, a)) = \gamma(\perp_{\mathcal{S}}(\tau + b, b))$.*

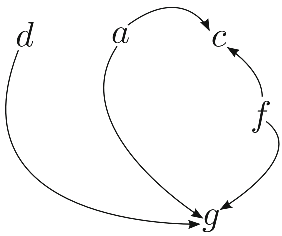
Given an AF τ and an external argument $a \in \mathbb{A}^*_{\mathcal{L}}$, we will refer to any operation $\tau \circledast_\gamma a$ as *smooth acceptance revision iff* $\tau \circledast_\gamma a$ is an acceptance revision obtained through a smooth remainder selection 'γ'. Now we are able to formalize the representation theorem for smooth acceptance revisions.

Theorem 34. *Given an* AF τ*, a semantics specification* $\mathcal{S}$*, and an external argument* $a \in \mathbb{A}^*_{\mathcal{L}}$*;* $\tau \circledast a$ *is a smooth acceptance revision iff '*$\circledast$*' satisfies closure, success, consistency, inclusion, vacuity, core-retainment, and uniformity.*

Example 35 [Continues from Example 30] Suppose, we have $\Theta' = \{a, b, c\}$, where $a = \langle \{p \wedge q_1\}, p \wedge q_1 \rangle$, $b = \langle \{p \wedge q_2\}, p \wedge q_2 \rangle$, and $c = \langle \{\neg p\}, \neg p \rangle$. The argumentation closure renders the set $\mathbf{A}' = \mathbb{C}^*(\Theta') = \{a, b, c, e\}$, such that $e = \langle \{p \wedge q_1, p \wedge q_2\}, p \wedge q_1 \wedge q_2 \rangle$, where $\mathfrak{subs}(e) = \{a, b\}$. Then, for the closed AF $\mathbb{F}_{\mathbf{A}'}$, the attack relation is $\mathbf{R}_{\mathbf{A}'} = \{(a, c), (b, c), (e, c)\}$. We need to revise $\tau = \mathbb{F}_{\mathbf{A}'}$ by the external argument $d = \langle \{\neg q_2\}, \neg q_2 \rangle$.



$\mathbb{F}_{\mathbf{A}'}$



$\tau \circledast d$

Note $\tau \circledast d = \mathbb{F}_{\mathbf{A}''}$, where $\mathbf{A}'' = \mathbf{A}(\tau + d) \setminus \gamma(\perp_{\mathcal{S}}(\tau + d, d))$, is equivalent to $\tau \circledast d = (\tau + d) \ominus_{\perp} d$ through the generalization of the Levi identity (p. 11). Note that $\tau + d = \mathbb{F}_{\mathbf{A}}$ (see Example 30). We know there are two d-$\texttt{remainder}_{\mathcal{S}}$ sets $\mathcal{R}_d = \{a, e, f\}$ and $\mathcal{R}'_d = \{b, e\}$. Assuming a selection criterion $\mathcal{R}'_d \preccurlyeq_\gamma \mathcal{R}_d$, we have $\gamma(\perp_{\mathcal{S}}(\tau + d, d)) = \mathcal{R}'_d$ and also $\mathbf{A}'' = \mathbf{A} \setminus \mathcal{R}'_d = \{a, c, d, f, g\}$. Finally, the resulting revised framework ends up as $\tau \circledast d = \mathbb{F}_{\mathbf{A}''}$, where $\mathbf{R}_{\mathbf{A}} = \{(a, c), (d, g), (f, c), (a, g), (f, g)\}$.

Note that the acceptance revision can be seen as $\mathfrak{sp}(\mathbf{A}(\tau + d)) \setminus \mathfrak{sp}(\Theta'')$, where $\Theta'' = \mathbb{P}_0(\gamma(\perp_{\mathcal{S}}(\tau + d, d))) = \mathbb{P}_0(\mathcal{R}'_d) = \{b\}$ (see Definition 27). Hence, $\mathfrak{sp}(\mathbf{A}(\tau \circledast d)) = \mathfrak{sp}(\mathbf{A}(\tau + d)) \setminus \mathfrak{sp}(\Theta'') = \mathfrak{sp}(\{a, b, c, d, e, f, g\}) \setminus \mathfrak{sp}(\{b\}) = \{p \wedge q_1, p \wedge q_2, \neg p, \neg q_2\} \setminus \{p \wedge q_2\} = \{p \wedge q_1, \neg p, \neg q_2\}$, corresponding to the set of arguments $\{a, c, d\}$, whose closure is $\mathbb{C}^*(\{a, c, d\}) = \{a, c, d, f, g\} = \mathbf{A}''$.

The previous example shows the relation between an acceptance revision applied directly over the set of arguments, regarding a related operation upon the underlying knowledge base from which the logic-based AF is constructed.

Proposition 36. *Given an* AF τ*, a semantics specification* $\mathcal{S}$*, an external argument* $a \in \mathbb{A}^*_{\mathcal{L}}$*, and a smooth acceptance revision '*$\circledast$*'; assuming* $\mathbf{A}^*_{\Sigma} \subseteq \mathbb{A}^*_{\mathcal{L}}$ *as the set of all canonical arguments* $\mathbb{A}^*_{\mathcal{L}}$ *constructible from a knowledge base* $\Sigma \subseteq \mathcal{L}$*, it holds* $\mathbf{A}(\tau \circledast a) = \mathbf{A}^*_{\Sigma}$*, where* $\Sigma = \mathfrak{sp}(\mathbf{A}(\tau + a)) \setminus \mathfrak{sp}(\mathbb{P}_0(\gamma(\perp_{\mathcal{S}}(\tau + a, a))))$*.*

7 Conclusions

Related and Future Work. The expansion proposed here can be seen as a *normal expansion* [4] since we do not restrict the directionality of the new attacks which appear after a new argument is incorporated to the framework. Authors there propose some general properties for ensuring the (im)possibility of *enforcing* a set of abstract arguments, which refers to the modification of the abstract framework for achieving a specific result through some standard semantics. Logic-based argumentation is out of the scope of the article. There are two other differences with our work. Firstly, they pursue a kind of multiple expansion since they consider the addition of an entire set of arguments and the interaction through attack with the existing ones. In our work, this is possible but only from the perspective of several subarguments which are part of a single superargument. Secondly, they only consider expansions. In a subsequent work [3], authors incorporate deletion of attacks. However, only minimal change is considered which renders no possibility for a complete characterization of change through representation theorems. There, the formalization of the minimal change principle is achieved through the introduction of numerical measures for indicating how far two argumentation frameworks are. Another revision approach in an AGM spirit is presented in [10] through revision formulæ that express how the acceptability of some arguments should be changed. As a result, they derive argumentation systems which satisfy the given revision formula, and are such that the corresponding extensions are as close as possible to the extensions of the input system. The revision presented is divided in two subsequent levels: firstly, revising the extensions produced by the standard semantics. This is done without considering the attack relation. Secondly, the generation of argumentation systems fulfilling the outcome delivered by the first level. Minimal change is pursued in two different levels, firstly, by ensuring as less change as possible regarding the arguments contained in each extension, and secondly, procuring as less change as possible on the argumentation graph. The methods they provide do not provoke change upon the set of arguments, but only upon the attack relations. Similar to [3], their operator is more related to a distance based-revision which measures the differences from the actual extensions with respect to the ones obtained for verifying the revision formula. They give a basic set of rationality postulates in the very spirit of AGM, but more closed to the perspective given in [14]. They only show that the model presented satisfies the postulates without giving the complete representation theorem for which the way back of the proof, *i.e.*, from postulates to the construction, is missing. However, the very recent work [12], which is in general a refinement of [9,10], proposes a generic solution to the revision of argumentation frameworks by relying upon complete representation theorems. In addition, the revision from the perspective of argumentation frameworks is also considered. Other distance based approaches in this direction are the works by Booth *et al.* [7,8], were authors develop a general AGM-like approach for modeling the dynamics of argumentation frameworks based on the distance between conflict-free labellings for the complete semantics only. They propose the notion of *fall back beliefs* for representing the rational outcome of an $\texttt{AF}$ from a constraint. A different approach, but still in an AGM spirit was presented in [5], where authors propose expansion and revision operators for Dung's abstract argumentation frameworks (AFs) based on a novel proposal called *Dung* *logics* with the particularity that equivalence in such logics coincides with strong equivalence for the respective argumentation semantics. The approach presents a reformulation of the AGM postulates in terms of monotonic consequence relations for AF*s*. They finally state that standard approaches based on measuring distance between models are not appropriate for AF*s*.

In general, the aforementioned works differ from ours in the perspective of dealing with the argumentation dynamics. This also renders different directions to follow for achieving rationality. To our knowledge, [19] was the first work to propose AGM postulates for rationalizing argumentation dynamics, providing also complete representation theorems for the proposed revision operations built upon logic-based argumentation. The rationalization done here is mainly inspired by such results, however change methods are pursued upon standard semantics in contrast to dialectical trees as done in [19]. Similar to the notion of remainders, in [19] and other ATC approaches like [18,20], authors recognize from the dialectical trees some sets of arguments which are identified as "responsible" for the rejection of arguments. In this paper we follow a similar intuition, however, core and remainder sets are more general notions for identifying the sources of acceptance/rejection of specific arguments upon standard semantics.

The problem of revising a framework by a set of arguments has been shown in [11] to suffer from failures regarding enforcement as originally defined in [4]. This is an interesting problem to which the theory here proposed may bring different solutions. To that end, it would be interesting to extend the acceptance revision operator for revising a framework by an unrestricted set of arguments rather than a single one, or a superargument including several subarguments. Such an operation seems to fit better as an argumentation *merge*.

Discussion. We proposed a model of change for argumentation based on the novel concepts of core and remainder sets. Core sets can be thought as minimal sets which are necessary for ensuring the acceptability of a given argument whereas remainder sets can be understood as minimal sets which are somehow responsible for the rejection of a given argument. The proposed model of change was firstly studied upon abstract argumentation and afterwards, upon logic-based argumentation. The resulting acceptance revision operation was characterized through the proposal of rationality postulates *á la* belief revision, and afterwards, the through corresponding representation theorems.

Another aspect that we wanted to demonstrate is that abstract argumentation can be counterproductive when the research is not immersed in the appropriate context of applicability. When the model, firstly proposed for abstract argumentation, was observed in the context of logic-based argumentation, several new inconveniences appeared requiring special attention, showing that abstraction can also be a path to trivialization. A conclusion that we draw is that standard semantics may not apply correctly to a logic-based argumentation system (AS). The usage of argumentation postulates [2,15] facilitates the analysis for understanding how rational a set of extensions can be. Such rationality can be achieved from two standpoints. Either from the construction of the framework, by putting special attention on how to model conflicts, or on the other hand, by tackling the problem straightforwardly from the construction of the extensions. In this sense, we proposed a new perspective for enriching the concept of admissibility for being applied over logic-based arguments through the notion of argumentation closure. We have shown that standard semantics relying on logic-based admissibility can make things easier for verifying argumentation postulates (see discussion in p. 15).

Regarding argumentation dynamics, we focus minimal change from the perspective of the knowledge base at first, and from the set of arguments, afterwards. We believe this is an appropriate manner to tackle such principle, since logic-based argumentation stands for reasoning upon inconsistencies of an underlying knowledge base. Another way to observe minimal change –which was not attended here– is from the perspective of the outcomes of the framework. A final conclusion that we draw is that although dynamics of abstract arguments can also be studied by proposing models of change affecting the set of attacks, it is not an appropriate perspective for logic-based argumentation. These sort of problems are really interesting, however they do not seem to fit well to such context of application considering that attacks are finally adjudicated in terms of logical contradictions.

References

1. Alchourrón, C., Gärdenfors, P., Makinson, D.: On the logic of theory change: partial meet contraction and revision functions. J. Symbolic Logic **50**, 510–530 (1985)
2. Amgoud, L.: Postulates for logic-based argumentation systems. Int. J. Approximate Reasoning **55**(9), 2028–2048 (2014)
3. Baumann, R.: What does it take to enforce an argument? Minimal change in abstract argumentation. In: Frontiers in Artificial Intelligence and Applications, ECAI 2012, Montpellier, France, vol. 242, pp. 127–132. IOS Press (2012)
4. Baumann, R., Brewka, G.: Expanding argumentation frameworks: enforcing and monotonicity results. In: Frontiers in Artificial Intelligence and Applications, COMMA 2010, D. del Garda, Italy, vol. 216, pp. 75–86. IOS Press (2010)
5. Baumann, R., Brewka, G.: AGM meets abstract argumentation: expansion and revision for dung frameworks. In: Yang and Wooldridge [21], pp. 2734–2740
6. Besnard, P., Hunter, A.: Elements of Argumentation. The MIT Press, Cambridge (2008)
7. Booth, R., Caminada, M., Podlaszewski, M., Rahwan, I.: Quantifying disagreement in argument-based reasoning. In: AAMAS 2012, IFAAMAS, pp. 493–500, Valencia, Spain (2012)
8. Booth, R., Kaci, S., Rienstra, T., van der Torre, L.: A logical theory about dynamics in abstract argumentation. In: Liu, W., Subrahmanian, V.S., Wijsen, J. (eds.) SUM 2013. LNCS, vol. 8078, pp. 148–161. Springer, Heidelberg (2013)
9. Coste-Marquis, S., Konieczny, S., Mailly, J.-G., Marquis, P.: A translation-based approach for revision of argumentation frameworks. In: Fermé, E., Leite, J. (eds.) JELIA 2014. LNCS, vol. 8761, pp. 397–411. Springer, Heidelberg (2014)
10. Coste-Marquis, S., Konieczny, S., Mailly, J., Marquis, P.: On the revision of argumentation systems: minimal change of arguments statuses. In: KR 2014, Vienna, Austria. AAAI Press (2014)

11. Coste-Marquis, S., Konieczny, S., Mailly, J., Marquis, P.: Extension enforcement in abstract argumentation as an optimization problem. In: Yang and Wooldridge [21], pp. 2876–2882
12. Diller, M., Haret, A., Linsbichler, T., Rümmele, S., Woltran, S.: An extension-based approach to belief revision in abstract argumentation. In: Yang and Wooldridge [21], pp. 2926–2932
13. Dung, P.M.: On the acceptability of arguments and its fundamental role in non-monotonic reasoning and logic programming and n-person games. Artif. Intell. **77**, 321–357 (1995)
14. Gärdenfors, P.: Knowledge in Flux: Modelling the Dynamics of Epistemic States. The MIT Press, Bradford Books, Cambridge (1988)
15. Gorogiannis, N., Hunter, A.: Instantiating abstract argumentation with classical logic arguments: postulates and properties. Artif. Intell. **175**(9–10), 1479–1497 (2011)
16. Hansson, S.O.: A Textbook of Belief Dynamics. Theory Change and Database Updating. Kluwer Academic, London (1999)
17. Levi, I.: Subjunctives, dispositions and chances. Synthese **34**(4), 423–455 (1977)
18. Moguillansky, M.O., Rotstein, N.D., Falappa, M.A., García, A.J., Simari, G.R.: Argument theory change applied to defeasible logic programming. In: Fox, D., Gomes, C.P. (eds.) AAAI, pp. 132–137. AAAI Press (2008)
19. Moguillansky, M.O., Wassermann, R., Falappa, M.A.: Inconsistent-tolerant base revision through argument theory change. Logic J. IGPL **20**(1), 154–186 (2012)
20. Rotstein, N.D., Moguillansky, M.O., Falappa, M.A., García, A.J., Simari, G.R.: Argument theory change: revision upon warrant. In: Proceedings of COMMA, pp. 336–347 (2008)
21. Yang, Q., Wooldridge, M. (eds.): Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015. AAAI Press, Buenos Aires, Argentina, 25–31 July 2015