Chapter 5
3D to 2D via Top-View Plans

Abstract Having become proficient at correlating 2D pictures with top plan numeric view representations learners used the Construction Box to create their own task card puzzles for peers to solve without the aid of the digital interface. These included 2D pictures of Soma assemblies and later, top plan view diagrams that they drew by hand. These tasks required adept mental transformation skills moving among the SOC representations. The Extended Construction Box module was created to allow users to construct digital cube figures with holes or overhangs within the first octant of a 3D coordinate space. Learners then developed a top plan view coding system that allowed for holes or overhangs. They also used their knowledge of top plan views to represent rectangular prisms made up of unit cubes.

5.1 Self-created Task Card Puzzles

Even though we had created hundreds of Soma figure assembly task cards over time, the children were motivated to work with task puzzles that their peers created. First each child selected two Soma figures to form an assembly figure; then, using the Construction Box number grid, the virtual conventional assembly figure was created and rotated to the user's satisfaction. At this point they learned to copy the virtual figure to the computer's clipboard and paste into a word processing document. They saved each day's work using their secret code names. These were designed for everyone to be able identify individuals but also to protect each child's identity in publication. Researcher Sack then adjusted the size of each child's figures to approximately 3 cm by 5 cm, and printed and laminated them for use as peer puzzles the following week. Examples are shown in Fig. 5.1. Using these puzzle cards, the children were challenged to find more than one pair of Soma figures that replicated the figure on the printed card even though the card creator most likely used only one pair of Soma figures.

In Fig. 5.2, from left to right: The first figure can be assembled using Soma figures 7 and 3, or Soma figures 5 and 4. The middle figure can be assembled using Soma figures 3 and 2, or Soma figures 5 and 7. The third figure can be assembled

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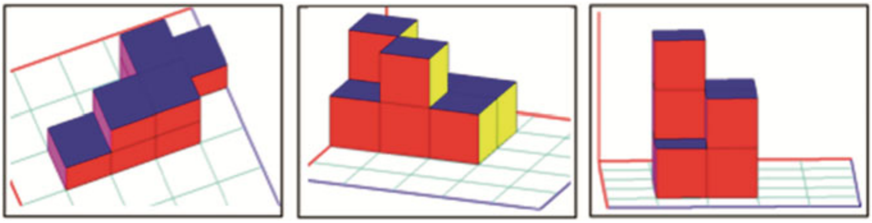


Fig. 5.1 Learner-created assembly task puzzles

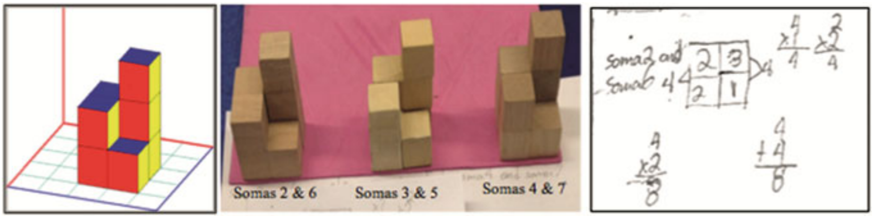


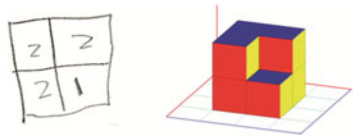
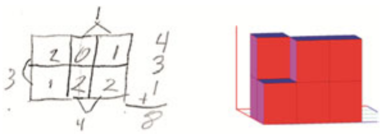
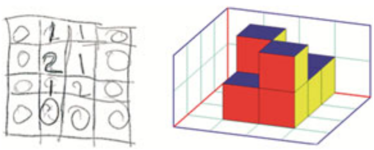
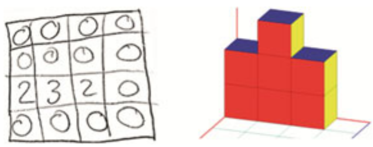
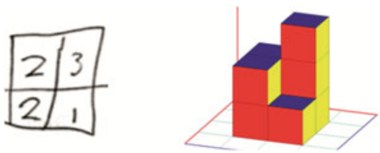
Fig. 5.2 Rosa's multiple solutions for one assembly puzzle using Soma figures 2, 3, 4, 5, 6 and 7

using Soma figures 2 and 7, or Soma figures 3 and 5. Other combinations may be possible.

During Year 7, one particular child, Rosa (pseudonym) had been retained to repeat 3rd grade due to low reading and numeration ability. She was encouraged to join the Geocadabra class to develop her visualization skills that would in turn support her numeration skills. She was a relatively reserved child who liked to engage in the tasks quietly. She progressed very well demonstrating good visualization skills but then stunned us by solving one particular task puzzle using all of Soma figures 2 through 7 (those with 4 cubes) in pairs. The puzzle card and her solutions are shown in Fig. 5.2. Rosa's superior visualization skills were never put to the test during the school day. Individuals who serve on committees to define formal curriculum standards should balance analytical skills that depend on symbolic decoding (numbers or variables) with visualization skills. By attaching contextual situations to analytical skill development one may integrate visualization skills since learners can mentally make sense of a given situation and imagine the objects of the problem.

In a follow-up activity the children drew the top-plan views of their own task card figures. The top-plan views became the new puzzles for others to solve. This required mental imaging of the actual figure while manipulating the seven Soma figures to create the assembly figure without the aid of the Construction Box interface. Table 5.1 shows some examples with peer solutions. Only the top plan codes were provided. Note that some of the children drew 4-by-4 grids like those on the Construction Box' default grid even though their figures did not need all of the rows or columns. However, they did not all coordinate their zeros in their drawings with

Table 5.1 Examples of top plan view puzzles of two-Soma assemblies and peer solutions

Puzzle	Top plan coding and task card	Peer solutions
A		Soma #1 and Soma #5 OR Soma #1 and Soma #6
B		Soma #6 and Soma #7
C		Soma #5 and Soma #6 OR Soma #2 and Soma #3
D		Soma #4 and Soma #1 OR Soma #2 and Soma #1
E		Soma #7 and Soma #4 Soma #3 and Soma #5 OR Soma #6 and Soma #2

the zero positions on their printed task cards. Two different children solved puzzle E using three different Soma figure combination.

During Year 1, we developed activities based on our perceptions of natural extensions to activities just experienced. We debriefed together after each session, taking detailed notes of our observations. These were supported by video-recordings and subsequent transcriptions to help us triangulate observation data. Sometimes the children posed interesting questions and we allowed them to explore these to see what would evolve. We did not have a set trajectory in mind other than having the children develop proficiency moving among 3D models, 2-D conventional pictures (usually assemblies of Soma figures) and top plan views using the Construction Box. When they began to construct assembly task puzzles, some became frustrated that they had used the Soma figures to assemble interesting figures with holes or overhangs that could not be constructed using the bottom-up process on the Construction Box. We contacted Lecluse, the developer of the Geocadabra platform to ask about the possibility of constructing such figures. Within three days he sent us a new module, the Extended Construction box.

5.2 Extended Construction Box

The new module allowed the user to build block figures within a 3D octant space. When *New Block* is pressed, a new block appears in the (1, 1, 1) position as shown in Fig. 5.3. By clicking on the arrows (up, down; left, right; forward, backward) the block moves one space for each click. If the block lies against one of the walls or on the floor of the space it will not move through that wall. By selecting *Shadowing* on the bottom, back wall and side wall, the exact position of each cube can be seen. If several blocks are connected, then by selecting the button that looks like Soma figure #2, one can translate or rotate the entire figure using the arrows. Using the *Control View Line* option one can turn the figure using the mouse to show sides that are hidden.

The children were now able to construct 2D assembly task card puzzles with holes or overhangs as shown in Fig. 5.4. They set their own levels of challenge according to their confidence with the new interface and with their ability to move from the 3D assembly figures to the virtual 2D image on the interface.

As soon as they had begun to construct these task cards, they immediately started to make coding puzzles as they had for the bottom-up task figures they had previously created on the regular Construction Box interface. This posed a new problem for the research team since they were not aware of a coding system for figures with holes or overhangs that would uniquely determine each 3D figure. They knew that a set of face views, shown as shadows on the bottom and two walls could be produced from a variety of 3D figures. The children enthusiastically set out to invent a new coding system. They created several interesting but very complicated ways of showing holes or overhangs. However, for each coding puzzle, the coder would write a key for peers to read and use to make sense of the puzzle. Most of these were very complicated and the decoder would have to interrupt the coder to clarify. We led them to the challenge of selecting the “best” coding system that all would use.

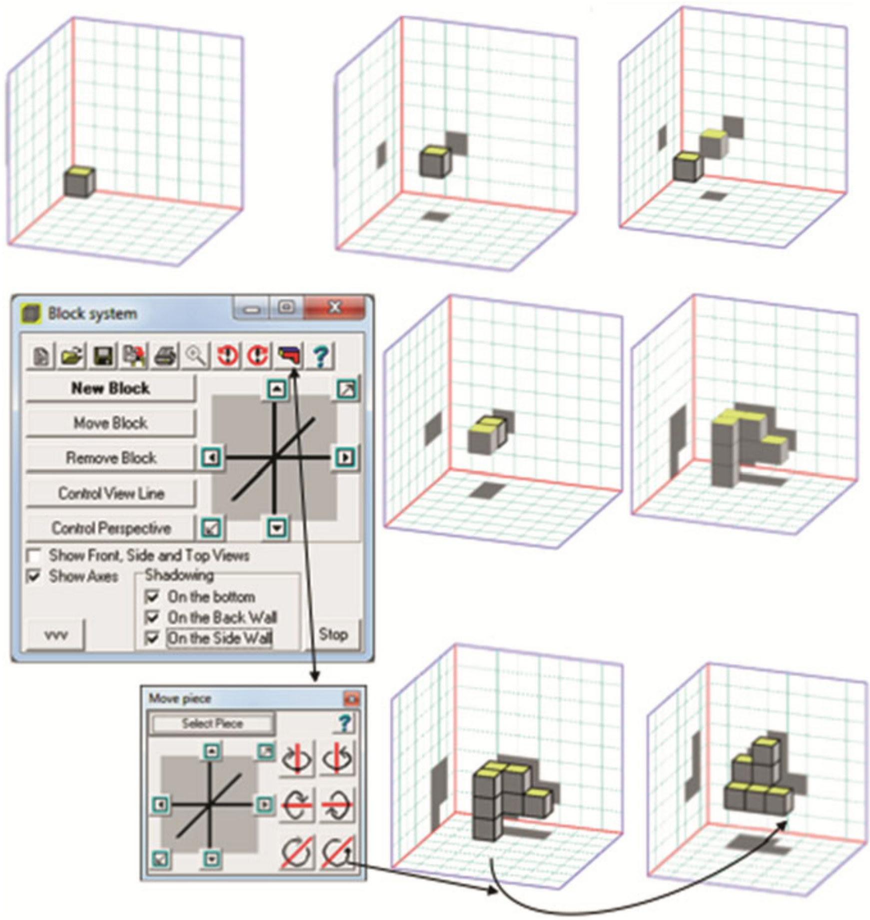


Fig. 5.3 The extended construction box

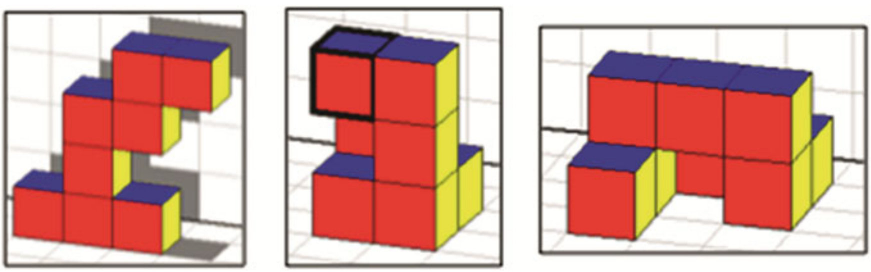


Fig. 5.4 Two-Soma assembly task card puzzles with holes or overhangs

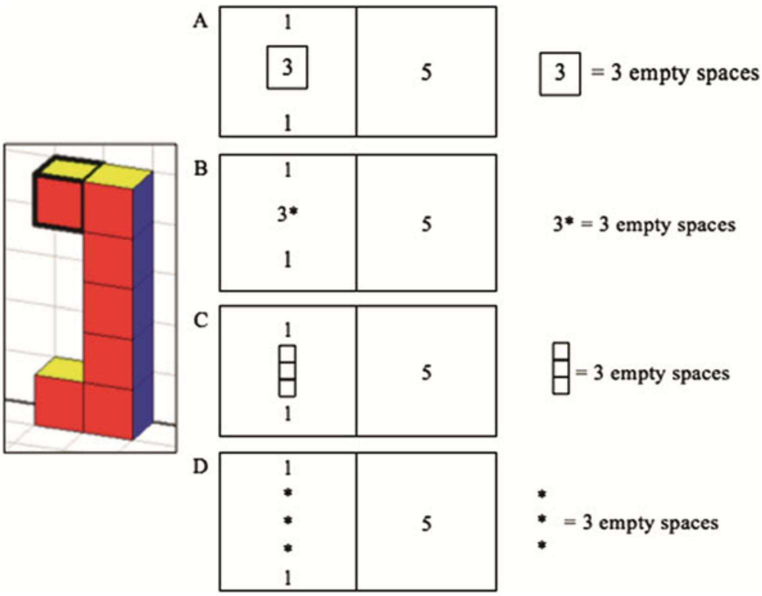


Fig. 5.5 Third graders' invented coding for figures with holes or overhangs

For our 3rd grade group, we selected the task card shown in Fig. 5.5, constructed with Somas #1 and #2, since it was quite simple and would not be challenging for those who may still be developing proficiency with standard top plan view coding. We asked the children to code this figure so that we could compare and contrast the different codings and, as a group, select one that everyone would then easily use. Some children drew the picture as a front view with 1s in each square. We showed them how exchanging Soma #1 with Soma #5 would be very problematic with that coding. We also suggested that they find ways to adapt the conventional top plan view format from the basic Construction Box to depict empty spaces within a structure. After some time, four different coding systems emerged also shown in Fig. 5.5.

During whole class discussion, where the children who had coded the figure as shown in Fig. 5.5A–D explained their work, the class agreed that codings A and B were more efficient than codings C or D, especially when you have to consider say, 10 empty spaces. They selected coding A and also decided that instead of placing the number for empty spaces in a square, they would circle the number, to represent "zero" cubes.

We provided a slightly more challenging figure, shown in Fig. 5.6, with a gap and an overhang for the 4th grade class. Again, the children produced very complicated coding systems that the class felt would be difficult to remember. One child produced a coding system similar to some of the 3rd graders' (Fig. 5.6E). We shared two of the 3rd grade codes, Fig. 5.5A, B, and the 4th grade class also agreed to use a circle

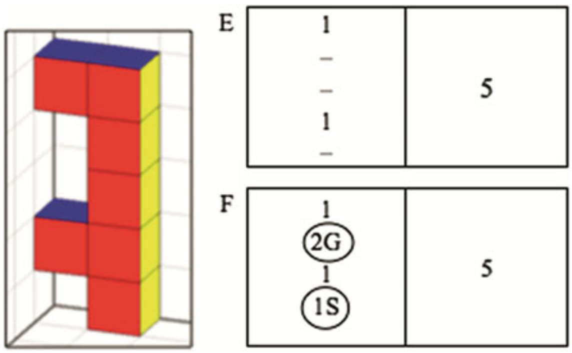


Fig. 5.6 Fourth graders' invented coding for figures with holes or overhangs

to show the number of empty spaces, with the letters S (for spaces) or G (for gaps) shown in Fig. 5.6F.

5.3 Rectangular Prisms and Their Volumes

During Year 2, five children who had attended during Year 1 returned to seek help with making sense of rectangular prisms. During their regular mathematics class they were expected to understand and use the Volume = length × width × height formula. They struggled to make sense of these measurements and the notion of volume being the number of cubic units enclosed by the figure. We provided them with a bag of loose cubes and asked them to build prisms with 24 cubes, since the large number of factors could produce a variety of different rectangular prisms. As soon as we had a nice collection of prisms, standing on different size bases, we asked them to record the length, width and height dimensions. This appeared to be confusing to all of them and so we asked how they could represent their buildings. We were stunned when every single child drew a rectangle, gridded in according to the number of rows and columns for the base of each figure, and then represented the prism's height as a number in each square. They had recalled the top plan view representation from the previous year and used this with ease. They now had no difficulty understanding that the number of squares in each row and the number of rows in their grids represented the length and the width of the figure; and that the number in each square, which was the same for every square on the base represented the height, or the number of layers in each figure. We were very careful to show them how the linear measurements translated into rows and columns on the base.

As a result, we added and retained this activity in our 3rd grade program to ensure success for every participant as they made sense of geometric measurement formulas. We provided each group a large bag of loose cubes and asked them to build as many 24-cube rectangular prisms as they could. Then, we asked them to sketch all of the top plan view representations for these prisms. When everyone had

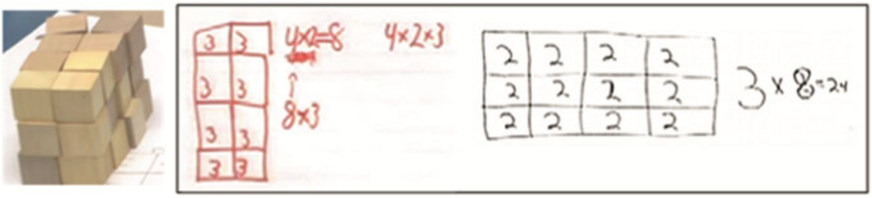


Fig. 5.7 Representing rectangular prism volume

completed at least one of these, we called them all together to share their drawings. From these we asked what mathematical expression they could give to a friend as a puzzle to see if he or she could reproduce the top plan view grid for that particular prism. Examples of grids and multiplication expressions for total number of cubes that children drew on the board are shown in Fig. 5.7.

All of the children in the project were able to make strong conceptual connections between the top plan view and the volume of a rectangular prism. They were able to mentally picture the figure directly from the top plan view. Furthermore, they now had the means to represent the 3D figure, albeit abstract, compared to representing a conventional 2D rectangular prism figure. This was a significant result of the project, as other researchers have commented about the difficulties in enumerating hidden cubes (Battista 1999) and also in representing 3D prisms on paper (Outhred et al. 2003). This activity integrates visualization with numeration, the topic of Chap. 6, in which we describe additional numeration activities linked to our visualization program.

References

Battista, M. (1999). Fifth graders enumeration of cubes in 3D arrays: Conceptual progress in an inquiry-based classroom. *Journal for Research in Mathematics Education, 30*, 417–448.

Outhred, L., Mitchelmore, M., McPhail, D., & Gould, P. (2003). Count me into measurement: A program for the early elementary school. In Douglas Clements & George Bright (Eds.), *Learning and teaching measurement: 2003 NCTM yearbook* (pp. 81–99). Reston: National Council of Teachers of Mathematics.