Chapter 46
New FRF Based Methods for Substructure Decoupling

Taner Kalaycıoğlu and H. Nevzat Özgüven

Abstract Substructuring methods are well known and are widely used in predicting dynamics of coupled structures. In theory, there is no reason why the same techniques could not be used in a reverse problem of predicting the dynamic behavior of a particular substructure from the knowledge of the dynamics of the coupled structure and of all the other substructures. However, the reverse problem, known as decoupling, usually requires matrix inversions, and therefore even small measurement errors may easily affect the accuracy of such methods. In this study two new FRF based approaches are presented for decoupling. The methods proposed require coupled system FRFs at coordinates that belong to the known subsystem as well as the measured or calculated FRFs of the known subsystem alone. Formulations are based on the reverse application of the structural coupling method proposed in a previous publication co-authored by one of the authors of this paper. The performances of the proposed methods are demonstrated and then compared with those of some well-known recent methods in the literature through a case study.

Keywords Decoupling • Uncoupling • Inverse substructuring • Subsystem identification • Subsystem subtraction

46.1 Introduction

The modal analysis and testing is widely used to analyze the dynamic characteristics of a whole machine or its components [1]. Since engineering structures are generally designed as an assembly of several components, a lot of effort has been devoted to structural coupling methods that predict the total dynamic behavior of a complex machine from those of its components in recent decades. Conversely, in some cases, the dynamic characteristics of a whole system may be known but that of its component may be hard to measure because of the difficulty of performing measurements or excitation on a subsystem individually under its normal operating conditions.

In this study, the decoupling problem, the identification of the dynamic behavior of a structural subsystem that is part of a larger system is addressed. One of the earliest studies on the substructure decoupling is performed by Okubo and Miyazaki [2] in 1986. In their solution, FRFs of the complete system and the known subsystem is used to obtain FRF of the unknown subsystem. After a long break, Gontier and Bensaibi [3] presented a decoupling technique based on time domain approach which still remains as the only technique in time domain. Silva et al. [4] presented a study regarding joint identification. They used decoupling methodology in order to eliminate difficulties in measurement and experimenting for joint identification. Later, they proposed a different technique for joint identification [5]. In this technique, they used coupling formulation of Jetmundsen et al. [6] and obtained a better formulation in terms of the number of matrix inversions. In this formulation, joint (i.e. connection) degrees of freedom (DoFs) are not taken into account. Ind and Ewins [7] presented an approach similar to that of Silva et al. [5]. Kalling et al. [8] studied the decoupling problem by performing state-space model identification. D'Ambrogio and Fregolent [9] presented a modal based approach for decoupling analyses which suffers from modal truncation problems.

D'Ambrogio and Fregolent [10] presented an FRF based decoupling technique similar to that of Okubo and Miyazaki [2]. In this work, they pointed out the problems due to unmeasured rotational DoFs as well. Afterwards, D'Ambrogio and

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Fregolent [11] proposed two decoupling procedures; namely, impedance based and mobility based approaches, which calculate the FRFs of the connection DoF on an unknown subsystem by using the FRFs of the coupled system and the residual subsystem at residual subsystem DoFs. The latter is equivalent to the approach presented by Sjövall and Abrahamsson [12]. A general framework for dynamic substructuring is provided in [13] and [14] in which the so called dual domain decomposition technique that allows retaining the full set of global DoFs by ensuring equilibrium at the interface between substructures is introduced. When performing substructuring by using the dual domain decomposition, the coupling problem can be directly formulated from [14], whereas a similar formulation for the decoupling problem is developed and discussed in [15] for collocated approach where DoFs used to enforce equilibrium are the same as DoFs used to enforce compatibility, and in [16] and [17] for non-collocated approach where DoFs used to enforce equilibrium are not the same as DoFs used to enforce compatibility.

Batista and Maia [18] proposed three different decoupling formulations based on the classical decoupling procedure of Jetmundsen et al. [6]. They consider the effects of including different sets of DoF of the coupled system: (1) exclusion of connection DoFs, (2) inclusion of connection DoFs only and (3) inclusion of connection DoFs and internal DoFs of the residual subsystem. Cloutier and Avitabile [19] presented inverse frequency based substructuring approach that requires measurements on the unknown substructure. Later, D'Ambrogio and Fregolent [20] proposed the so called hybrid assembly approach. They compared dual [15] and hybrid assembly approaches through an experimental case study and ended up with very similar results in terms of predicted FRFs of the unknown subsystem.

In this paper, two new FRF based decoupling approaches are developed which are based on the structural modification method suggested by Tahtalı and Özgüven [21] two decades ago. The approaches developed can predict the FRFs of an unknown subsystem from the measured FRFs of the coupled system and the measured or calculated FRFs of the other subsystem. The methods are tested on a simple lumped parameter system by using simulated experimental data. Results are compared with those obtained through some well-known decoupling methods.

46.2 Theory

In this section, the theory of the decoupling approaches proposed is given. In this approach, the structural modification method suggested by Tahtalı and Özgüven [21] is revisited and modified to be used for substructure decoupling. The notation used throughout the paper for all systems/subsystems and the coordinate sets are given in Fig. 46.1.

Here, first, the basic equations of the Coupling Force Method suggested by Tahtalı and Özgüven [21] will be given. The displacement vectors for the unknown and the residual subsystems can be written, respectively, as:

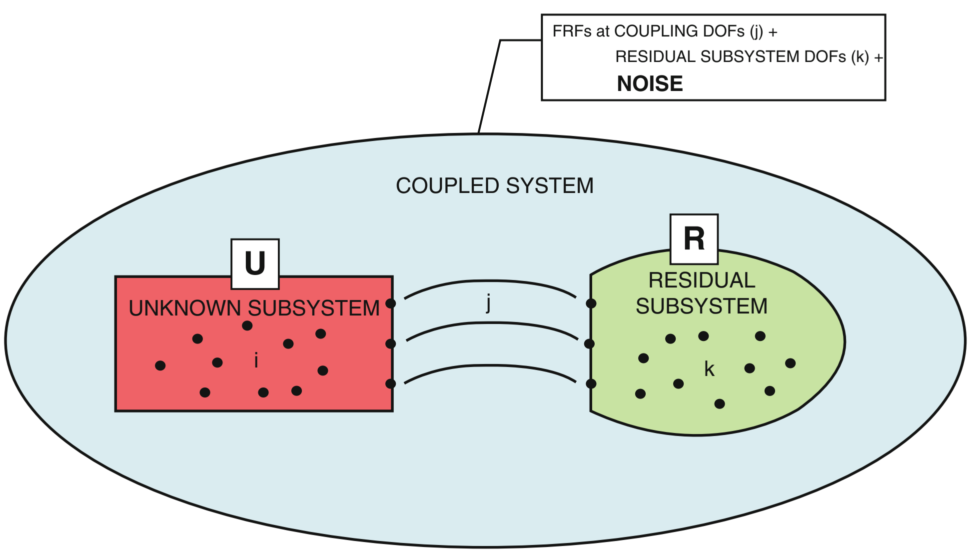

Fig. 46.1 Notation used for systems and sets of coordinates

$$\begin{Bmatrix} \mathbf{X}_i^U \\ \mathbf{X}_j^U \end{Bmatrix} = \begin{pmatrix} \mathbf{H}_{ii}^U & \mathbf{H}_{ij}^U \\ \mathbf{H}_{ji}^U & \mathbf{H}_{jj}^U \end{pmatrix} \begin{Bmatrix} \mathbf{F}_i^U \\ \mathbf{F}_j^U + \mathbf{f} \end{Bmatrix} \tag{46.1}$$

$$\begin{Bmatrix} \mathbf{X}_j^R \\ \mathbf{X}_k^R \end{Bmatrix} = \begin{pmatrix} \mathbf{H}_{jj}^R & \mathbf{H}_{jk}^R \\ \mathbf{H}_{kj}^R & \mathbf{H}_{kk}^R \end{pmatrix} \begin{Bmatrix} -\mathbf{f} \\ \mathbf{F}_k^R \end{Bmatrix} \tag{46.2}$$

where superscripts U and R refer to the vectors related to the unknown and the residual subsystems, respectively. Subscript i refers to the coordinates of the unknown subsystem only, subscript j refers to the coordinates that are common to the unknown and residual subsystem, and finally subscript k refers to the coordinates of the residual subsystem only (Fig. 46.1). Here, **f**, **F** and **H** represent coupling reaction force vector, external force vector and the FRF matrix, respectively. Expanding Eqs. (46.1) and (46.2) leads to following equations:

$$\mathbf{X}_i^U = \mathbf{H}_{ii}^U \mathbf{F}_i^U + \mathbf{H}_{ij}^U \left(\mathbf{F}_j^U + \mathbf{f}\right) \tag{46.3}$$

$$\mathbf{X}_j^U = \mathbf{H}_{ji}^U \mathbf{F}_i^U + \mathbf{H}_{jj}^U \left(\mathbf{F}_j^U + \mathbf{f}\right) \tag{46.4}$$

$$\mathbf{X}_j^R = -\mathbf{H}_{jj}^R \mathbf{f} + \mathbf{H}_{jk}^R \mathbf{F}_k^R \tag{46.5}$$

$$\mathbf{X}_k^R = -\mathbf{H}_{kj}^R \mathbf{f} + \mathbf{H}_{kk}^R \mathbf{F}_k^R \tag{46.6}$$

Note that, when two subsystems are rigidly coupled, $\mathbf{X}_j^U = \mathbf{X}_j^R$, which represents the displacement vector at the coupling coordinates. Therefore Eqs. (46.4) and (46.5) are equal to each other. Thus, by equating the right hand sides of these equations coupling reaction force can be written as follows:

$$\mathbf{f} = \left[\mathbf{H}_{jj}^U + \mathbf{H}_{jj}^R\right]^{-1} \left[\mathbf{H}_{jk}^R \mathbf{F}_k^R - \mathbf{H}_{ji}^U \mathbf{F}_i^U - \mathbf{H}_{jj}^U \mathbf{F}_j^U\right] \tag{46.7}$$

After having obtained the coupling reaction force **f**, the response of the coupled system can be obtained by substituting **f** in Eqs. (46.3), (46.5) and (46.6) as follows:

$$\mathbf{X}_i^U = \mathbf{H}_{ii}^U \mathbf{F}_i^U + \mathbf{H}_{ij}^U \left(\mathbf{F}_j^U + \left[\mathbf{H}_{jj}^U + \mathbf{H}_{jj}^R\right]^{-1} \left[\mathbf{H}_{jk}^R \mathbf{F}_k^R - \mathbf{H}_{ji}^U \mathbf{F}_i^U - \mathbf{H}_{jj}^U \mathbf{F}_j^U\right]\right) \tag{46.8}$$

$$\mathbf{X}_j^R = -\mathbf{H}_{jj}^R \left[\mathbf{H}_{jj}^U + \mathbf{H}_{jj}^R\right]^{-1} \left[\mathbf{H}_{jk}^R \mathbf{F}_k^R - \mathbf{H}_{ji}^U \mathbf{F}_i^U - \mathbf{H}_{jj}^U \mathbf{F}_j^U\right] + \mathbf{H}_{jk}^R \mathbf{F}_k^R \tag{46.9}$$

$$\mathbf{X}_k^R = -\mathbf{H}_{kj}^R \left[\mathbf{H}_{jj}^U + \mathbf{H}_{jj}^R\right]^{-1} \left[\mathbf{H}_{jk}^R \mathbf{F}_k^R - \mathbf{H}_{ji}^U \mathbf{F}_i^U - \mathbf{H}_{jj}^U \mathbf{F}_j^U\right] + \mathbf{H}_{kk}^R \mathbf{F}_k^R \tag{46.10}$$

Note that the response of the coupled system can also be written as follows:

$$\begin{pmatrix} \mathbf{X}_i^U \\ \mathbf{X}_j^R \\ \mathbf{X}_k^R \end{pmatrix} = \begin{pmatrix} \mathbf{H}_{ii} & \mathbf{H}_{ij} & \mathbf{H}_{ik} \\ \mathbf{H}_{ji} & \mathbf{H}_{jj} & \mathbf{H}_{jk} \\ \mathbf{H}_{ki} & \mathbf{H}_{kj} & \mathbf{H}_{kk} \end{pmatrix} \begin{pmatrix} \mathbf{F}_i^U \\ \mathbf{F}_j^U \\ \mathbf{F}_k^R \end{pmatrix} \tag{46.11}$$

The above formulation gives the basic equations of the Coupling Force Method proposed by Tahtalı and Özgüven [21]. From now on, formulation to be given will be about the derivation of decoupling formulations. In the following section the use of these equations for decoupling will be given.

46.2.1 Formulation Using Equation (46.9)

Let us assume that external forcing is applied only to the kth coordinates of the coupled system and the rest of the external forcing is equal to zero;

$$\begin{pmatrix} \mathbf{F}_{\mathrm{i}}^{\mathrm{U}} \\ \mathbf{F}_{\mathrm{j}}^{\mathrm{U}} \\ \mathbf{F}_{\mathrm{k}}^{\mathrm{R}} \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{i\times 1} \\ \mathbf{0}_{j\times 1} \\ \mathbf{F}_{k\times 1} \end{pmatrix} \tag{46.12}$$

By using Eqs. (46.11) and (46.12), Eq. (46.9) can be rewritten as follows:

$$\mathbf{H}_{\mathrm{jk}}\mathbf{F}_{k\times 1} = -\mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} + \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1}\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}}\mathbf{F}_{k\times 1} + \mathbf{H}_{\mathrm{jk}}^{\mathrm{R}}\mathbf{F}_{k\times 1} \tag{46.13}$$

By multiplying both sides of Eq. (46.13) with $(\mathbf{F}_{k\times 1})^{-1}$ from right hand side, one can obtain

$$\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}} - \mathbf{H}_{\mathrm{jk}} = \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} + \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1}\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}} \tag{46.14}$$

Premultiplying all terms of Eq. (46.14) by $\left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1}$ and post multiplying them by $\left(\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}}\right)^{-1}$ yields

$$\left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1}\left(\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}} - \mathbf{H}_{\mathrm{jk}}\right)\left(\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}}\right)^{-1} = \left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} + \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1} \tag{46.15}$$

Taking inverse of the both sides of Eq. (46.15), the following equation can be written:

$$\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}}\left(\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}} - \mathbf{H}_{\mathrm{jk}}\right)^{-1}\mathbf{H}_{\mathrm{jj}}^{\mathrm{R}} = \mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} + \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}} \tag{46.16}$$

Rearranging Eq. (46.16) yields the final equation which gives the FRF matrix of the unknown subsystems in terms of those of coupled system and residual subsystem:

$$\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} = \mathbf{H}_{\mathrm{jk}}^{\mathrm{R}}\left(\mathbf{H}_{\mathrm{jk}}^{\mathrm{R}} - \mathbf{H}_{\mathrm{jk}}\right)^{-1}\mathbf{H}_{\mathrm{jj}}^{\mathrm{R}} - \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}} \tag{46.17}$$

It is interesting to note that if it is assumed that external forcing is applied only to the jth coordinates of the coupled system while the rest of the external forces are zero, that is

$$\begin{pmatrix} \mathbf{F}_{\mathrm{i}}^{\mathrm{U}} \\ \mathbf{F}_{\mathrm{j}}^{\mathrm{U}} \\ \mathbf{F}_{\mathrm{k}}^{\mathrm{R}} \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{i\times 1} \\ \mathbf{F}_{j\times 1} \\ \mathbf{0}_{k\times 1} \end{pmatrix} \tag{46.18}$$

one will end up with the decoupling formulation given by Batista and Maia [18]:

$$\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} = \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\left(\left(\mathbf{H}_{\mathrm{jj}}\right)^{-1}\mathbf{H}_{\mathrm{jj}}^{\mathrm{R}} - \mathbf{I}_{j\times j}\right)^{-1} \tag{46.19}$$

46.2.2 Formulation Using Equation (46.10)

Let us assume again that external forcing is applied only to the jth coordinates of the coupled system and the rest of the external forces are zero as given in Eq. (46.18), and let us use Eq. (46.10) rather than Eq. (46.9). Then by using Eqs. (46.11) and (46.18), one can rewrite Eq. (46.10) as follows:

$$\mathbf{H}_{\mathrm{kj}}\,\mathbf{F}_{j\times 1} = \mathbf{H}_{\mathrm{kj}}^{\mathrm{R}}\left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} + \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1}\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}}\,\mathbf{F}_{j\times 1} \tag{46.20}$$

By multiplying both sides of Eq. (46.20) with $\left(\mathbf{F}_{j\times 1}\right)^{-1}$ from right hand side, one can obtain:

$$\mathbf{H}_{\mathrm{kj}} = \mathbf{H}_{\mathrm{kj}}^{\mathrm{R}}\left(\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} + \mathbf{H}_{\mathrm{jj}}^{\mathrm{R}}\right)^{-1}\mathbf{H}_{\mathrm{jj}}^{\mathrm{U}} \tag{46.21}$$

Premultiplying all terms of Eq. (46.21) by $\left(\mathbf{H}_{jj}^{R}\right)^{-1}$ and post multiplying them by $\left(\mathbf{H}_{jj}^{U}\right)^{-1}$ gives

$$\left(\mathbf{H}_{kj}^{R}\right)^{-1}\mathbf{H}_{kj}\left(\mathbf{H}_{jj}^{U}\right)^{-1} = \left(\mathbf{H}_{jj}^{U} + \mathbf{H}_{jj}^{R}\right)^{-1} \tag{46.22}$$

Taking inverse of both sides of Eq. (46.22), the following equation can be obtained

$$\mathbf{H}_{jj}^{U}\left(\mathbf{H}_{kj}\right)^{-1}\mathbf{H}_{kj}^{R} = \mathbf{H}_{jj}^{U} + \mathbf{H}_{jj}^{R} \tag{46.23}$$

Then, the final equation which gives the FRF matrix of the unknown subsystems in terms of those of coupled system and residual subsystem can be obtained as follows:

$$\mathbf{H}_{jj}^{U} = \mathbf{H}_{jj}^{R}\left(\left(\mathbf{H}_{kj}\right)^{-1}\mathbf{H}_{kj}^{R} - \mathbf{I}_{j\times j}\right)^{-1} \tag{46.24}$$

Again it is interesting to note that if it is assumed that external forcing is applied only to the k$^{\text{th}}$ coordinates of the coupled system as shown in Eq. (46.12), the formulation given by Maia et al. [5] can be easily obtained:

$$\mathbf{H}_{jj}^{U} = \mathbf{H}_{jk}^{R}\left(\mathbf{H}_{kk}^{R} - \mathbf{H}_{kk}\right)^{-1}\mathbf{H}_{kj}^{R} - \mathbf{H}_{jj}^{R} \tag{46.25}$$

46.3 Case Studies

In this section, application of the proposed decoupling formulations to a lumped parameter system is presented. Furthermore, performances of the proposed formulations are compared with those of some well-known techniques by using the same case study.

46.3.1 *Application of the Approaches to a Lumped Parameter System*

The coupled system considered in this application is composed of two subsystems rigidly connected to each other as shown in Fig. 46.2. Physical parameters of the residual and the unknown subsystem are given in Table 46.1. Note that k, m and c represent stiffness, mass and viscous damping parameters, respectively.

Here, it is assumed that the FRFs of the coupled system are experimentally measured and the physical model of the residual subsystem is available. It is aimed to determine the FRF of the unknown subsystem at its connection coordinate.

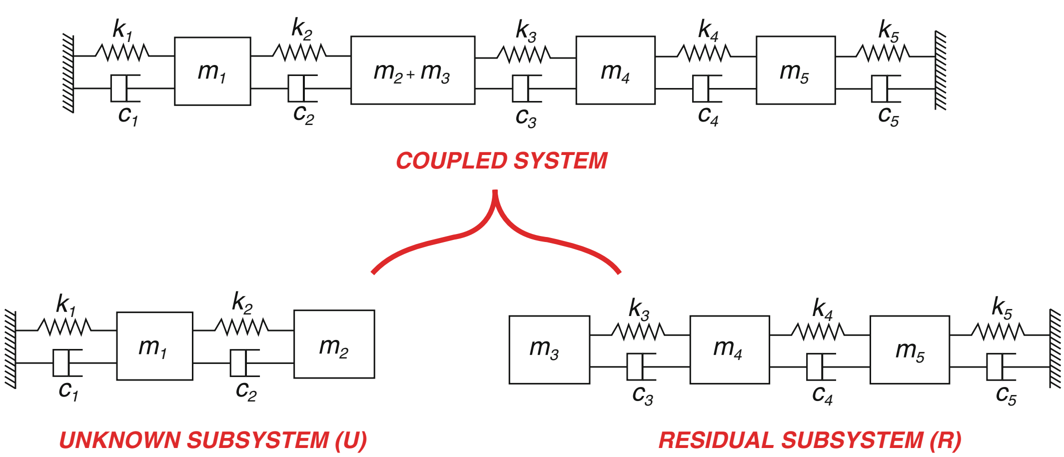

Fig. 46.2 Lumped parameter system model

Table 46.1 Physical parameters

Element Number (i)	m_i [kg]	k_i [N/m]	c_i [Ns/m]
1	2.5	1500	0.15
2	3	2000	0.20
3	2	2100	0.21
4	3	1900	0.19
5	2.5	2200	0.22

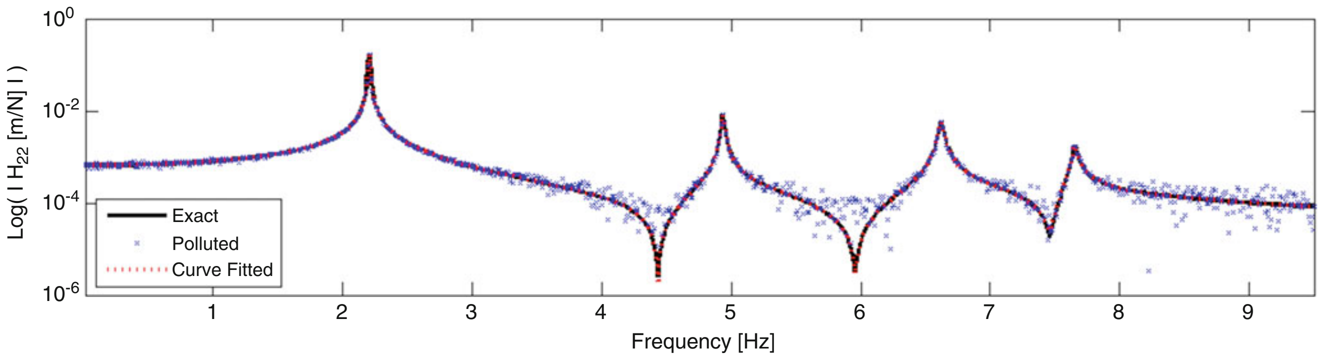

Fig. 46.3 Driving point FRF of the coupled system at the 2nd DoF: true (*solid black line*), polluted (*blue asterisks*) and curve fitted (*red dashed lines*)

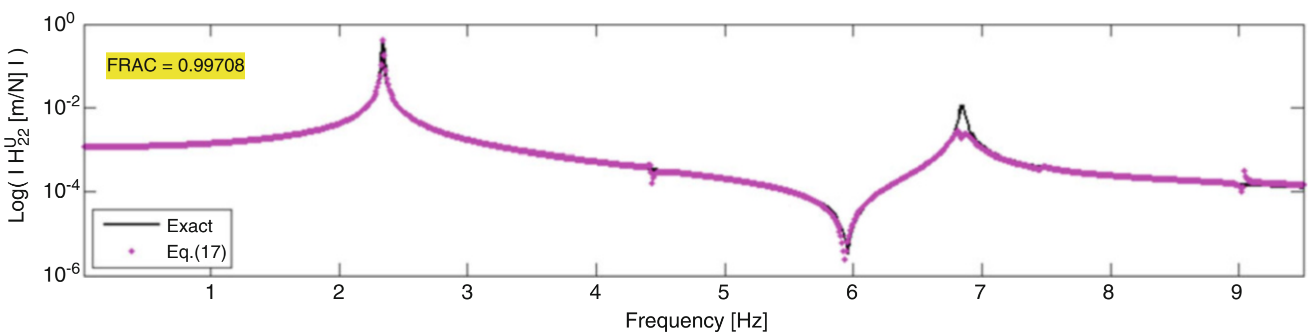

Fig. 46.4 Driving point FRF at the 2nd DoF of the unknown subsystem: true (*solid black line*), predicted using Eq. (46.17) (*magenta asterisks*)

In order to simulate the measured FRFs of the coupled system, first the exact FRFs of the coupled system ($\hat{\mathbf{H}}$) are calculated by using the physical parameters given in Table 46.1 and then they are polluted by simply adding complex random variables as shown below:

$$\mathbf{H}_{\mathrm{ij}}\left(\omega_{\mathrm{k}}\right) = \widehat{\mathbf{H}}_{\mathrm{ij}}\left(\omega_{\mathrm{k}}\right) + p_{\mathrm{ij,k}} + i\, q_{\mathrm{ij,k}} \tag{46.26}$$

Here, $p_{\mathrm{ij,k}}$ and $q_{\mathrm{ij,k}}$ are independent random variables with Gaussian distribution, zero mean and a standard deviation of 5×10^{-5} m/N. The effect of such a pollution on the driving point FRF at the 2nd DoF (the coupling DoF) of the coupled system is shown in Fig. 46.3 together with the FRF obtained after curve fitting.

Then, by using the curves fitted to the polluted FRFs of the coupled structure, driving point FRF at the coupling DoF of the unknown subsystem is calculated using the proposed formulations, and the results are given in Figs. 46.4 and 46.5.

Figures 46.4 and 46.5 show that both approaches predict the unknown subsystem FRF satisfactorily. If the performances of both approaches are compared with each other around resonances, the predicted FRF via Eq. (46.24) seem to fit better to the true FRF by visual inspection.

However, in order to make a reliable and sound comparison, it is required to use a metric rather than making visual inspection. For that purpose, the Frequency Response Assurance Criterion (FRAC) [22] is used. The FRAC values calculated for FRFs calculated by using Eqs. (46.17) and (46.24) are 0.99708 and 0.99791, respectively. So, it can be said again that, at least for the example case given here, both equations can successfully be used for decoupling, and Eq. (46.24) gives slightly better results compared to Eq. (46.17).

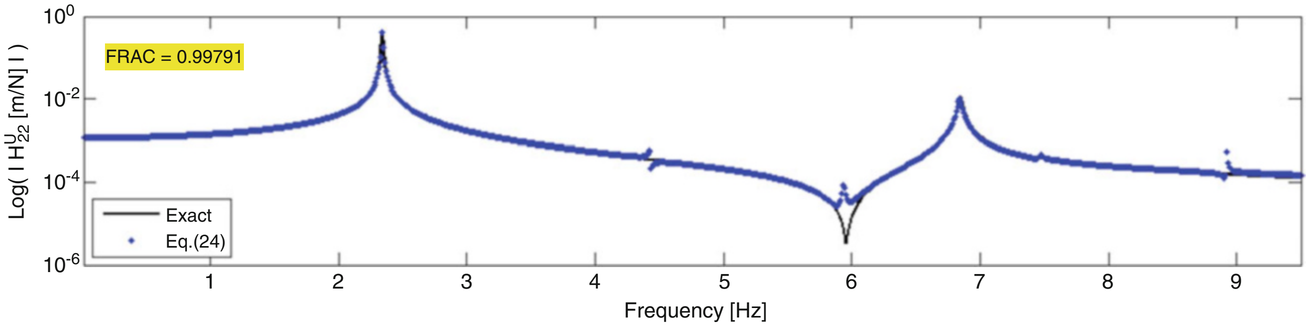

Fig. 46.5 Driving point FRF at the 2nd DoF of the unknown subsystem: true (*solid black line*), predicted using Eq. (46.24) (*blue asterisks*)

Table 46.2 List of most recent decoupling methods

Ref.	Final equation	Needs (residual)	Needs (coupled)	Equation
[18]	$\mathbf{H}^U_{jj} = \mathbf{H}^R_{jk}\mathbf{H}^R_{kj}\left(\mathbf{H}^R_{jk}\left(\mathbf{H}^R_{kk}\text{-}\mathbf{H}_{kk}\right)\mathbf{H}^R_{kj}\right)^{-1}\mathbf{H}^R_{jk}\mathbf{H}^R_{kj}\text{-}\mathbf{H}^R_{jj}$	$\begin{pmatrix}\mathbf{H}^R_{jj} & \mathbf{H}^R_{jk}\\ \mathbf{H}^R_{kj} & \mathbf{H}^R_{kk}\end{pmatrix}$	$\mathbf{H}_{kk}$	(46.27)
[18]	$\mathbf{H}^U_{jj} = \left(\mathbf{H}^R_{jj}\left(\mathbf{H}^R_{jj}\text{-}\mathbf{H}_{jj}\right)^{-1} - \mathbf{I}_{jj}\right)\mathbf{H}^R_{jj}$	$\mathbf{H}^R_{jj}$	$\mathbf{H}_{jj}$	(46.28)
[18]	$\mathbf{H}^U_{jj} = \mathbf{H}^R_{jj}\mathbf{H}^R_{jj}\left(\mathbf{H}^R_{jk}\left(\mathbf{H}^R_{kj}\text{-}\mathbf{H}_{kj}\right)\mathbf{H}^R_{jj}\right)^{-1}\mathbf{H}^R_{jk}\mathbf{H}^R_{kj}\text{-}\mathbf{H}^R_{jj}$	$\begin{pmatrix}\mathbf{H}^R_{jj} & \mathbf{H}^R_{jk}\\ \mathbf{H}^R_{kj} & \mathbf{H}^R_{kk}\end{pmatrix}$	$\mathbf{H}_{kj}$	(46.29)
[5]	$\mathbf{H}^U_{jj} = \mathbf{H}^R_{jk}\left(\mathbf{H}^R_{kk}\text{-}\mathbf{H}_{kk}\right)^{-1}\mathbf{H}^R_{kj}\text{-}\mathbf{H}^R_{jj}$	$\begin{pmatrix}\mathbf{H}^R_{jj} & \mathbf{H}^R_{jk}\\ \mathbf{H}^R_{kj} & \mathbf{H}^R_{kk}\end{pmatrix}$	$\mathbf{H}_{kk}$	(46.30)
[2]	$\mathbf{H}^U_{jj} = \left(\mathbf{I}\text{-}\mathbf{H}_{jj}\left[\mathbf{H}^R_{jj}\right]^{-1}\right)^{-1}\mathbf{H}_{jj}$	$\mathbf{H}^R_{jj}$	$\mathbf{H}_{jj}$	(46.31)
[11]	$\mathbf{H}^U_{jj} = \begin{pmatrix}\mathbf{I}\text{-}\mathbf{H}_{jj}\,\mathbf{Z}^R_{jj} - \mathbf{H}_{jk}\,\mathbf{Z}^R_{kj}\\ -\mathbf{H}_{kj}\,\mathbf{Z}^R_{jj} - \mathbf{H}_{kk}\,\mathbf{Z}^R_{kj}\end{pmatrix}^{+}\begin{pmatrix}\mathbf{H}_{jj}\\ \mathbf{H}_{kj}\end{pmatrix}$	$\begin{pmatrix}\mathbf{Z}^R_{jj} & -\\ \mathbf{Z}^R_{kj} & -\end{pmatrix}$	$\begin{pmatrix}\mathbf{H}_{jj} & \mathbf{H}_{jk}\\ \mathbf{H}_{kj} & \mathbf{H}_{kk}\end{pmatrix}$	(46.32)
[11]	$\mathbf{H}^U_{jj} = \begin{pmatrix}\mathbf{H}_{jj}\\ \mathbf{H}_{jk}\end{pmatrix}^{T}\left(\begin{pmatrix}\mathbf{I}_{jj} & \mathbf{0}_{jk}\end{pmatrix} - \begin{pmatrix}\mathbf{H}^R_{jj}\\ \mathbf{H}^R_{kj}\end{pmatrix}^{+}\begin{pmatrix}\mathbf{H}_{jj} & \mathbf{H}_{jk} - \mathbf{H}^R_{jk}\\ \mathbf{H}_{kj} & \mathbf{H}_{kk} - \mathbf{H}^R_{kk}\end{pmatrix}\right)^{+}$	$\begin{pmatrix}\mathbf{H}^R_{jj} & \mathbf{H}^R_{jk}\\ \mathbf{H}^R_{kj} & \mathbf{H}^R_{kk}\end{pmatrix}$	$\begin{pmatrix}\mathbf{H}_{jj} & \mathbf{H}_{jk}\\ \mathbf{H}_{kj} & \mathbf{H}_{kk}\end{pmatrix}$	(46.33)
[15]	$\mathbf{H}^U = \begin{pmatrix}\mathbf{H} & \mathbf{0}\\ \mathbf{0} & -\mathbf{H}^R\end{pmatrix} - \begin{pmatrix}\mathbf{H} & \mathbf{0}\\ \mathbf{0} & -\mathbf{H}^R\end{pmatrix}\begin{pmatrix}\mathbf{B}^T\\ (\mathbf{B}^R)^T\end{pmatrix}\cdots$ $\left(\begin{pmatrix}\mathbf{B} & \mathbf{B}^R\end{pmatrix}\begin{pmatrix}\mathbf{H} & \mathbf{0}\\ \mathbf{0} & -\mathbf{H}^R\end{pmatrix}\begin{pmatrix}\mathbf{B}^T\\ (\mathbf{B}^R)^T\end{pmatrix}\right)^{-1}\begin{pmatrix}\mathbf{B} & \mathbf{B}^R\end{pmatrix}\begin{pmatrix}\mathbf{H} & \mathbf{0}\\ \mathbf{0} & -\mathbf{H}^R\end{pmatrix}$	$\begin{pmatrix}\mathbf{H}^R_{jj} & \mathbf{H}^R_{jk}\\ \mathbf{H}^R_{kj} & \mathbf{H}^R_{kk}\end{pmatrix}$	$\begin{pmatrix}\mathbf{H}_{jj} & \mathbf{H}_{jk}\\ \mathbf{H}_{kj} & \mathbf{H}_{kk}\end{pmatrix}$	(46.34)

46.3.2 A Comparison of the Approaches with Well-Known Existing Methods

In this section, the performances of proposed methods are compared with those of well-known recent methods. The final equations for these methods, their references and the input data required for each of them are summarized in Table 46.2.

Among the first three formulations given in the table, Eq. (46.28) was shown to be the one that produces the smallest error throughout the frequency range [18]. Moreover, among the last three formulations given in the table, Eq. (46.34) was shown to give better results [15]. Note also that Eq. (46.31) is a special case of the Eq. (46.32) as also mentioned in reference [11]. Consequently, it will be more to the point to compare the proposed formulations with Eqs. (46.28), (46.30) and (46.34) in Table 46.2.

So, the problem given in Sect. 46.3.1 is also solved by using Eqs. (46.28), (46.30) and (46.34) in addition to employing the proposed formulations, i.e., Eqs. (46.23) and (46.24). Moreover in order to see the effect of increasing noise level in measured FRFs on the performances of different methods, coupled system FRFs are polluted by five different sets of random variables, $p_{ij,k}$ and $q_{ij,k}$ in Eq. (46.26), with Gaussian distribution, zero mean and standard deviations ranging from 5×10^{-5} to 25×10^{-5} m/N. Results obtained for the standard deviation of 15×10^{-5} m/N are given in Fig. 46.6.

It is observed that using different pollution sets with the same standard deviation may give slightly different results. Therefore, in order to compare the performances of different formulations and to study the effect of increasing measurement errors (increasing standard deviation of pollution), calculations with each method are repeated 100 times for each standard deviation of pollution, and the averages of the FRAC values are compared in Table 46.3.

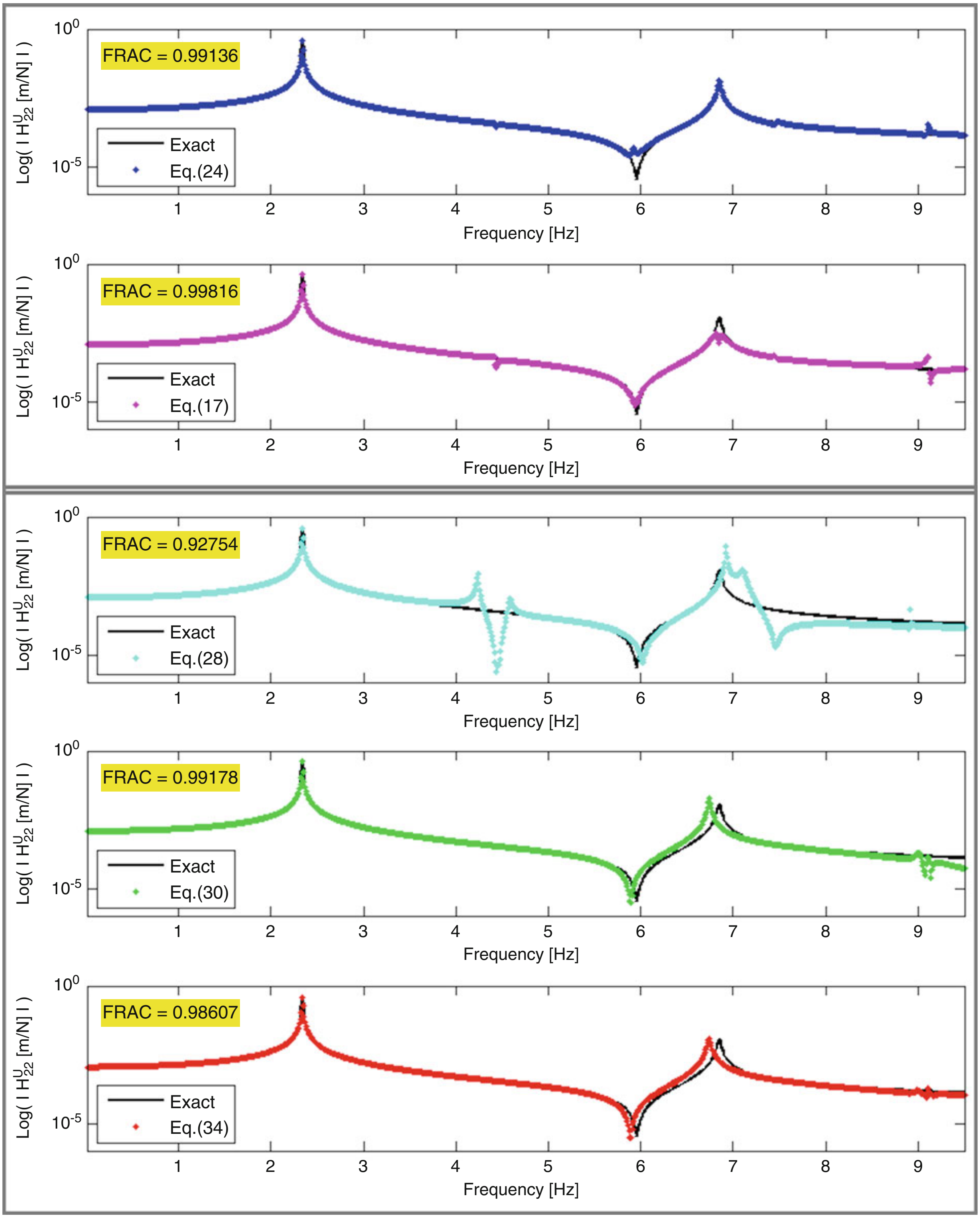

Fig. 46.6 Driving point FRF at the 2nd DoF of the unknown subsystem: true (*solid black line*), predicted using standard deviation of 15×10^{-5} m/N via proposed formulations: Eq. (46.24) (*blue asterisks*), Eq. (46.17) (*magenta asterisks*) and via formulations given in literature: Eq. (46.28) (*cyan asterisks*), Eq. (46.30) (*green asterisks*), Eq. (46.34) (*red asterisks*)

When mean FRAC values given in the Table 46.3 are compared to each other, the overall performances of the proposed formulations (i.e., Eqs. (46.24) and (46.17)) are found to be better. Especially, Eq. (46.24) proved to be statistically the best performer among all formulations.

Table 46.3 Mean (M) and Standard deviation (SD) of FRAC values obtained for each method

Method	FRAC values (M ± SD values calculated for 100 runs)				
	SD of 5×10^{-5} m/N	SD of 1×10^{-4} m/N	SD of 15×10^{-5} m/N	SD of 2×10^{-4} m/N	SD of 25×10^{-5} m/N
Eq. (46.24)	0.9979 ± 0.0019	0.9970 ± 0.0024	0.9900 ± 0.0234	0.9869 ± 0.0107	0.9792 ± 0.0184
Eq. (46.17)	0.9948 ± 0.0218	0.9947 ± 0.0077	0.9912 ± 0.0096	0.9796 ± 0.0189	0.9744 ± 0.0238
Eq. (46.28)	0.9971 ± 0.0215	0.9759 ± 0.0274	0.9701 ± 0.0307	0.9601 ± 0.0404	0.9522 ± 0.0639
Eq. (46.34)	0.9928 ± 0.0192	0.9794 ± 0.0358	0.9767 ± 0.0529	0.9615 ± 0.1067	0.9778 ± 0.0340
Eq. (46.30)	0.9921 ± 0.0050	0.9859 ± 0.0183	0.9822 ± 0.0211	0.9736 ± 0.0435	0.9696 ± 0.0562

46.4 Discussion and Conclusions

In this paper the decoupling problem, i.e. the prediction of the dynamic behavior of a structural subsystem, starting from information about the remaining subsystems (residual subsystems) and from the known dynamic behavior of the complete system, is considered. The dynamic behavior of the whole structure is assumed to be known from experiments, together with the experimentally measured or theoretically calculated dynamics of the residual substructure.

In this work, two different decoupling formulations derived from the work of Tahtalı and Özgüven [21] are presented and thus two new methods are proposed. Both methods give exact results, as it is the case in most of the decoupling methods, when exact FRFs are used in all equations. However, the problem in all of such methods is the sensitivity of the formulations to even very slight errors which are inevitable due to the use of measured data. All formulations usually include matrix inversions, and depending on the nature of the equations, some methods are more sensitive to measurement errors and therefore do not perform well. Hence, it is important to test the performance of any new decoupling technique, and compare its performance with existing best ones. Application of the proposed decoupling formulations is presented on a lumped parameter system. In this case study simulated experimental results are used, and in order to simulate experimentally measured FRFs of the coupled system, theoretically calculated exact FRFs are polluted. In studying the performances of the proposed methods, Frequency Response Assurance Criteria, which shows the correlation between the predicted FRFs and the true FRFs of the unknown subsystem, are used.

Furthermore, performances of the proposed formulations together with some of those available in the literature are investigated through the same case study. Also, effect of noise on the performance of the decoupling methods is examined by polluting the true FRFs of the coupled system using different sets of independent random variables with same mean but gradually increased standard deviations. However, decoupling methods in question do not yield FRAC values in regular trend to the increasing level of noise. Thus, an appropriate statistical comparison becomes essential to establish comparability between the decoupling methods investigated. It is observed from the results of the statistical comparison that proposed methods come up with the most correlated results for each of five standard deviation of pollution.

Consequently, it can be said that proposed methods can be used as alternative approaches results of which should be taken into consideration during decoupling studies. The applicability and accuracy of the methods proposed are demonstrated only on a simple lumped parameter system and additionally they need to be tested on real structures.

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