# Chapter 12
# Chaos Theory, Fractals and Scaling in the Radar: A Look from 2015

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**Abstract** Results of application of theory of fractal and chaos, scaling effects and fractional operators in the fundamental issues of the radio location and radio physic are presented in this report. The key point is detection and processing of super weak signals against the background of non-Gaussian intensive noises and strays. An alternative—the radar range is increased dramatically. The results of researches of spectrum fractal dimensions of lightning discharge in the middle atmosphere at attitudes from 20 to 100 km which are above the majority of clouds are presented. The author has been investigating these issues for exactly 35 years and has obtained results of the big scientific and practical worth. The reader is invited to look at the fundamental problems with the synergetic point of view of non-Markovian micro- and macro systems.

## 12.1 Introduction

The entire current radio engineering is based on the classical theory of an integer measure and an integer calculation. Thus an extensive area of mathematical analysis which name is the fractional calculation and which deals with derivatives and integrals of a random (real or complex) order as well as the fractal theory has been historically turned out “outboard” (!). At the moment the integer measures (integrals and derivatives with integer order), Gaussian statistics, Markov processes etc., are mainly and habitually used everywhere in the radio physics, radio electronics and processing of multidimensional signals. It is worth noting that the Markov processes theory has already reached its satiation and researches are conducted at the level of abrupt complication of synthesized algorithms. Radar systems should be considered with relation to open dynamical systems. Improvement of classical radar detectors

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of signals and its mathematical support basically reached its saturation and limit. It forces to look for fundamentally new ways of solving of problem of increasing of sensitivity or range of coverage for various radio systems.

At the same time I'd like to point out that it often occurs in science that the mathematical apparatus play a part of "Procrustean bed" for an idea. The complicated mathematical symbolism and its meanings may conceal an absolutely simple idea. In particular the author put forward one of such ideas *for the first time in the world* in the end of seventies of twentieth century. To be exact he suggested to introduce fractals, scaling and fractional calculation into the wide practice of radio physics, radio engineering and radio location. Now after long intellectual battles my idea has shown its advantages and has been positively perceived by the majority of the thoughtful scientific community. For the moment the list of the author's and pupils works counts more than 750 papers including 20 monographs on the given fundamental direction. Nowadays it is absolutely clear that the application of ideas of scale invariance—"scaling" along with the set theory, fractional measure theory, general topology, measure geometrical theory and dynamical systems theory reveals big opportunities and new prospects in processing of multidimensional signals in related scientific and engineering fields. In other words a full description of processes of modern signal and fields processing is impossible basing on formulas of the classical mathematics [1–11].

The work objective is to consider the use of the fractal theory and effects of physical scaling in development of new informational technologies using examples of solving of up-to-date basic radar problems. The author has been investigating these issues in V.A. Kotel'nikov IREE RAS for exactly 35 years.

## 12.2 On the Theory of Fractional Measure and Nonintegral Dimension

The main feature of fractals is the nonintegral value of its dimension. A development of the dimension theory began with the Poincare, Lebesgue, Brauer, Urysohn and Menger works. The sets which are negligibly small and indistinguishable in one way or another in the sense of Lebesgue measure arise in different fields of mathematics. To distinguish such sets with a pathologically complicated structure one should use unconventional characteristics of smallness—for example Hausdorff's capacity, potential, measures and dimension and so on. Application of the fractional Hausdorff's dimension which is associated with entropy conceptions, fractals and strange attractors has turned out to be most fruitful in the dynamical systems theory [1, 3–7, 9–11]. This fractional dimension is determined by the $p$—dimensional measure with an arbitrary real positive number $p$ proposed by Hausdorff in 1919. Generally the measure conception is related neither to metric nor to topology. However the Hausdorff measure can be built in an arbitrary metric space basing on its metric and the Hausdorff measure itself is related to the topological dimension. The Hausdorff–Besicovitch dimension is a metrical conception but there is its fundamental association with topological dimension dim $E$, which was established by L.S. Pontryagin and L.G. Shnirelman who introduced a conception of the metrical order in 1932: the greatest lower bound of the Hausdorff–Besicovitch dimension for all the metrics of compact $E$ is equal to its topological dimension $\dim E \leq \alpha(E)$. One of much used methods for estimation of sets Hausdorff dimension known as the mass distribution principle was proposed by Frostman in 1935.

Sets whose Hausdorff–Besicovitch dimension is a fractional number are called fractal sets or fractals. More strictly, set $E$ is called fractal (a fractal) in the wide sense (in the B. Mandelbrot sense) if its topological dimension is not equal to the Hausdorff–Besicovitch dimension, to be exact $\alpha_0(E) > \dim E$. For example, set $E$ of all the surd points [0; 1] is fractal in the wide sense since $\alpha_0(E) = 1$, $\dim E = 0$. Set $E$ is called fractal (a fractal) in the narrow sense if $\alpha_0(E)$ is not integer. A fractal set in the narrow sense is also fractal in the wide sense.

## 12.3 Measuring of Fractal Dimension and Fractal Signatures

Fractal methods can function on all signal levels: amplitude, frequency, phase and polarized. The absolute worth of Hausdorff–Besicovitch dimension is the possibility of experimental determining [3–10]. Let's consider some set of points $N_0$ in $d$—dimensional space. If there are $N(\varepsilon)$—dimensional sample bodies (cube, sphere) needed to cover that set with typical size $\varepsilon$, at that

$$N(\varepsilon) \approx 1/\varepsilon^D, \quad \varepsilon \to 0 \tag{12.1}$$

is determined by the self-similarity law.

The practical implementation of the method described above faces the difficulties related to the big volume of calculations. It is due to the fact that one must measure not just the ratio but the upper bound of that ratio to calculate the Hausdorff–Besicovitch dimension. Indeed, by choosing a finite scale which is larger than two discretes of the temporal series or one image element we make it possible to "miss" some peculiarities of the fractal. Building of *the fractal signature* [4–7] or estimates dependence (1) on the observation scale helps to solve this problem. Also the fractal signature describes the spatial *fractal cestrum* of the image. In IREE RAS we developed various original methods of measuring the fractal dimension including methods: dispersing, singularities accounting, on functionals, triad, basing on the Hausdorff metric, samplings subtraction, basing on the operation "Exclusive OR" and so on [4–7]. During the process of adjustment and algorithms mathematical modeling our own data were used: air photography (AP) and radar images (RI) on millimeter waves [9]. Season measurements of scattering characteristics of the earth coverings were already naturally conducted on wavelength 8.6 mm by the author in co-operation with representatives of Central Design Bureau "Almaz" from a helicopter MI-8 in the 1980s of twentieth century.

A significant advantage of dispersing dimension is its implementation simplicity, processing speed and calculations efficiency. In 2000 it was proposed to calculate a fractal dimension using the locally dispersing method (reference for example [4–7, 9–11]). In the developed algorithms they use two typical windows: scale and measuring. The scale window defines the necessary scale of measurements which the scaling is observed in. That is why the scale window serves for selection of the object to be recognized and its following description in the framework of fractal theory. An image brightness or image intensity local variance is determined by the measuring window. The locally dispersing method of the fractal dimension $D$ measurements is based on measuring a variance of the image fragments intensity/brightness for two spatial scales:

$$D \approx \frac{\ln \sigma_2^2 - \ln \sigma_1^2}{\ln \delta_2 - \ln \delta_1}. \tag{12.2}$$

In formula (2) $\sigma_1, \ \sigma_2$—root-mean-squares on the first $\delta_1$ and second $\delta_2$ scales of image fragment, respectively. Accuracy characteristics of the locally dispersing method were investigated in [4, 5, 7]. It is proved [7] that in the Gaussian case the dispersing dimension of a random sequence converges to the Hausdorff dimension of corresponding stochastic process. The essential problem is that any numerical method includes a discretization (or a discrete approximation) of the process or object under analysis and the discretization destroys fractal features. The development of special theory based on the methods of fractal interpolation and approximation is needed to fix this contradiction. Various topological and dimensional effects during the process of fractal and scaling detecting and multidimensional signals processing were studied in [4–11].

## 12.4 Textural and Fractal Measures in Radio Location

During the process of radio location the useful signal from target is a part of the general wave field which is created by all reflecting elements of observed fragments of the target surrounding background, that is why in practice signals from these elements form the interfering component.

It is worthwhile to use the texture conception to create radio systems for the landscape real inhomogeneous images automatic detecting [4–6, 9]. A texture describes spatial properties of earth covering images regions with locally homogenous statistical characteristics. Target detecting and identification occurs in the case when the target shades the background region at that changing integral parameters of the texture. Many natural objects such as a soil, flora, clouds and so on reveal fractal properties in certain scales [4–6].

The fractal dimension $D$ or its signature $D(t, f, \overrightarrow{r})$ in different regions of the surface image is a measure of texture, i.e., properties of spatial correlation of radio waves scattering from the corresponding surface regions. At already far first steps the author initiated a detailed research of the texture conception during the process of radio location of the earth coverings and objects against its background. Further on a particular attention was paid to development of textural methods of objects detecting against the earth coverings background with low ratios of signal/background [4].

## 12.5 Fractal Signal and Image Processing in the Interference

The author was the first who shows that the fractal processing excellently does for solving modern problems when processing the low-contrast images and detecting super weak signals in high-intensity noise while the modern radars does not practically function [4–7, 9–11]. The author's developed fractal classification was approved by B. Mandelbrot during the personal meeting in USA in 2005. It is presented on Fig. 12.1 where the fractal properties are described, $D_0$—is a topological dimension of the space of embeddings.

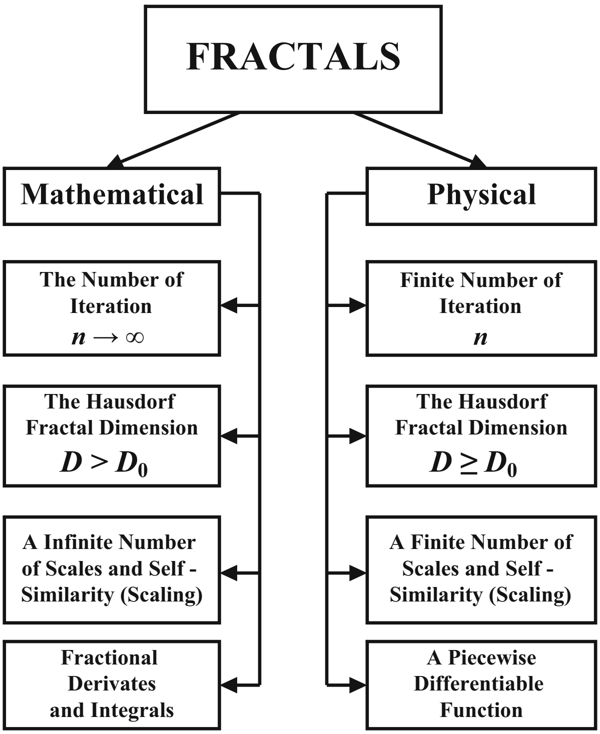


**Fig. 12.1** The author's classification of fractal sets and signatures

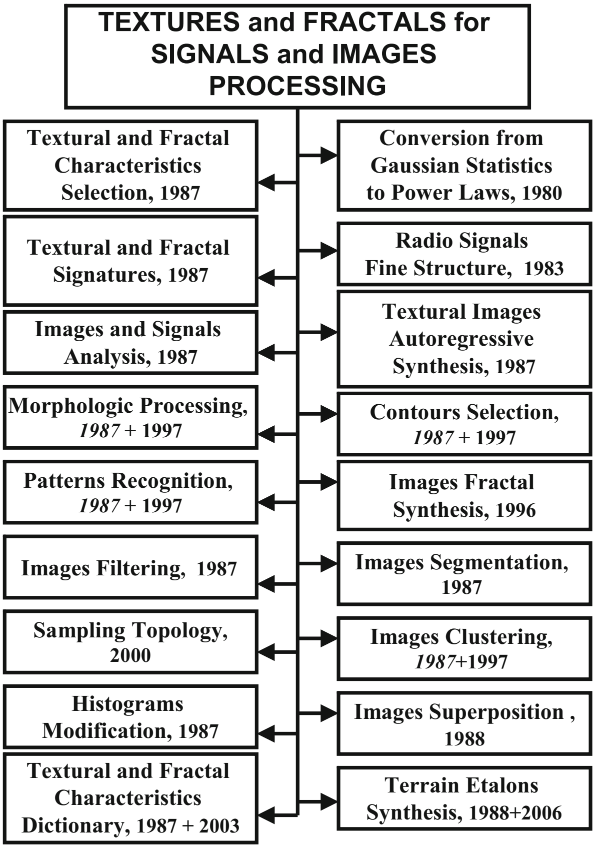


**Fig. 12.2** Textural and fractal methods of processing low-contrast images and super weak signals in high-intensity non-Gaussian noise

The textural and fractal digital methods under author's development (Fig. 12.2) allow to overcome a prior uncertainty in radar problems using *the sampling geometry or topology* (one- or multidimensional). At that topological peculiarities of the sampling and also the scaling hypothesis and stable laws with heavy "tails" get important as opposed to the average realizations which frequently have different behavior [4–7, 9–11].

## 12.6 Development of "Fractal Ideology" in Radio Physics

A critical distinction between the author's proposed fractal methods and classical ones is due to fundamentally different approach to the main components of a signal and a field. It allowed to switch over the new level of informational structure of the real non-Markov signals and fields. Thus this is *the fundamentally new* radio engineering. For 35 years of scientific researches my global fractal scaling method

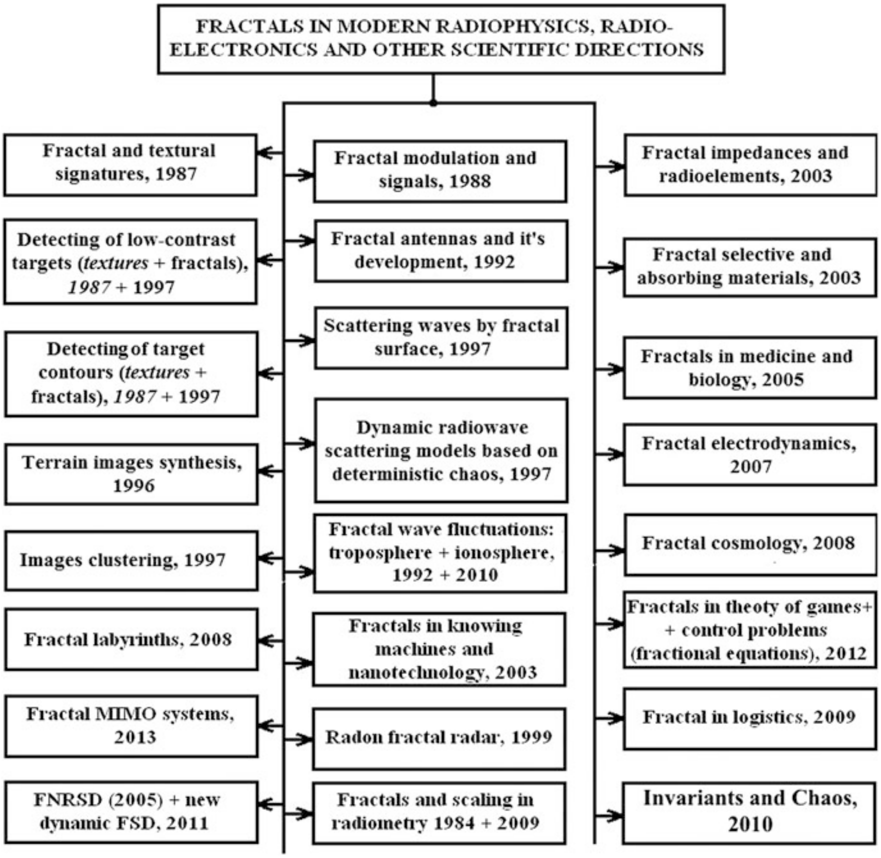


**Fig. 12.3** A sketch of author's new informational technologies development basing on fractals, fractional operators and scaling effects for nonlinear physics and radio electronics

has justified itself in many applications—Fig. 12.3. This is *a challenge to time* in a way. Here only the facts say! Slightly exaggerating one can say that the fractals formed a thin amalgam on the powerful framework of science of the end of twentieth century. In the modern situation attempts of underestimating its significance and basing only on the classical knowledge came to grief in an intellectual sense.

In fractal researches I always rest upon my three global theses:

1. Processing of information distorted by non-Gaussian noise in the fractional measure space using scaling and stable non-Gaussian probabilistic distributions (1981)—Figs. 12.1, 12.2, and 12.3.
2. Application of continuous nondifferentiable functions (1990)—Fig. 12.1.
3. Fractal radio systems (2005)—Figs. 12.3 and 12.4 [4–7, 9–11].

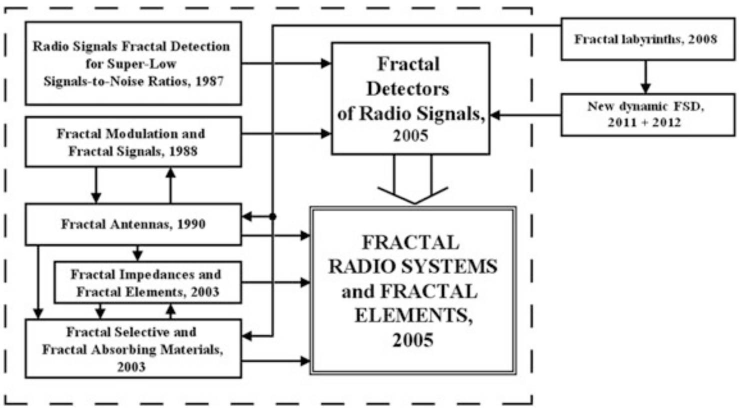


**Fig. 12.4** The author's conception of fractal radio systems, devices and radio elements

A logic aggregation of the problems triad described above into the general "fractal analysis and synthesis" creates a basis of *fractal scaling method* (2006) and a unified global idea of the fractal natural science and *fractal paradigm* (2011) which were proposed and are investigated by the author now [4–7, 9–11]. Basing on the matter reviewed above next we will proceed to description of *the fractal radar* conception and also issues of its scale-invariant principles application in other systems of radio monitoring. In fact the question is about a fundamentally new type of radio location: *fractal scale or scale-invariant radio location*.

## 12.7 Principles of Scale-Invariant or Fractal Scaling Radio Location and Its Applications

At the moment world investigations on fractal radio location are exclusively conducted in V.A. Kotel'nikov IREE RAS. Almost all the application points of hypothetic or currently projectable fractal algorithms, elements, nodes and processes which can be integrated into the classical radar scheme are represented on Fig. 12.5. The ideology of proceeding to the fractal radar is based on the fractal radio systems conception—Fig. 12.4.

In particular a multifrequency work mode is typical for the fractal MIMO-system [11–13] proposed by the author earlier since fractal antennas can radiate several waves lengths at the same time. Building of a tiny fractal radar with fractal elements and modern parametrons is possible for unmanned aerial vehicles (UAV).

At the same time the fractal processing at the point of control of UAV transmitted information will allow to improve sharply and automatize the processes of detecting, clustering and identification of targets and objects. Moreover UAV fractal coating will sharply reduce the probability of its detecting in flight.

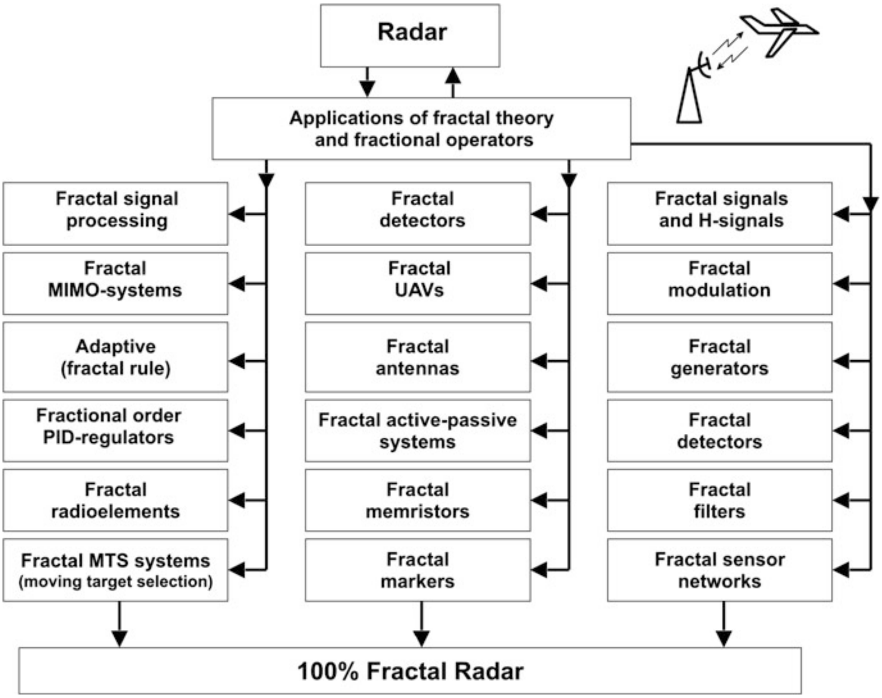


**Fig. 12.5** The points of application of fractals, scaling and fractional operators for proceeding to the fractal radar

## 12.8 Fractal Detection of Objects on Images from SAR and UAV

The base data for digital fractal processing of radar images were obtained by satellite radar with the synthetic aperture (SAR) PALSAR of *L*-range (Japan). PALSAR is a space SAR at wavelength 23 cm with spatial resolution of about 7 m which is developed by Japanese agency JAXA and which was successfully working on orbit from 2006 till 2011.

A radar image of Selenga estuary in Transbaikalia obtained in the FBS high resolution mode on the coherent horizontal polarization on 7 August 2006 is presented on Fig. 12.6 as an example.

The shooting zone of about $60 \times 50$ km includes the forest covered mountainous area Hamar-Daban (at the bottom, it is reproduced by a brighter tone with the typical "crumpled" structure), the flat area of Selenga estuary (in the middle of the top image part, it is reproduced by darker tones) and the smooth water surface of the lake Baikal (the black segment in the left upper corner of the image). The banded structures are seen in the flat part of the image, these are the bounds of agricultural fields. Also the clusters of bright objects are seen, these are the strongly reflecting

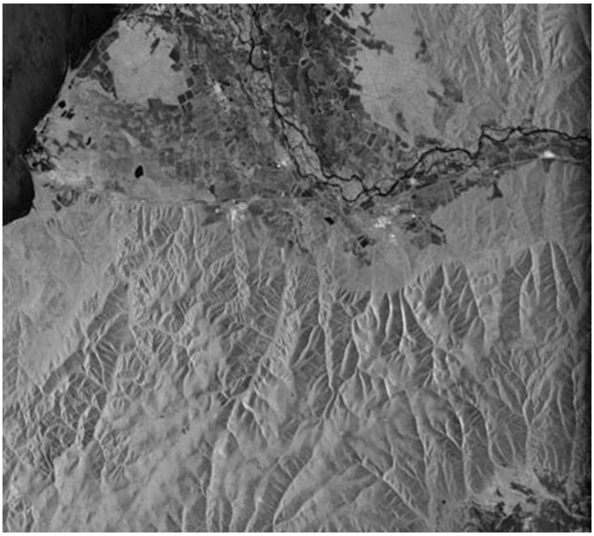

**Fig. 12.6** Selenga estuary on the PCA PALSAR photo from 7 August 2006

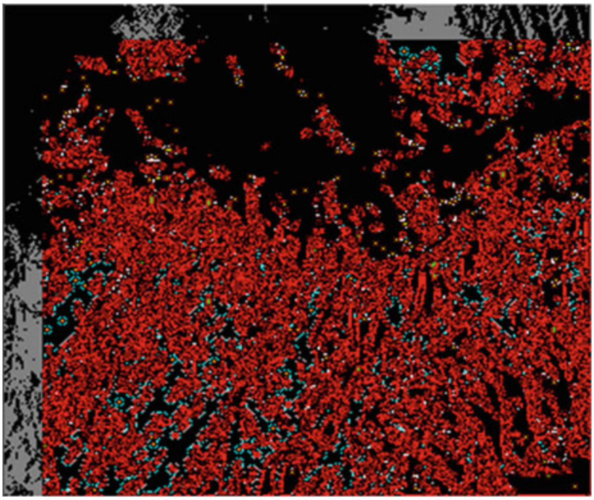

**Fig. 12.7** The result of fractal processing of the PCA PALSAR

elements of buildings and other constructions in the range of settlements. The long twisting dark lines on the plain are the multiple arms of Selenga.

The fields of local values of dispersing fractal dimension $D$ were measured at the first stage of radar images fractal processing by a SAR (Fig. 12.7). Next the empiric distribution of values of the instant fractal dimension $D$ was obtained Fig. 12.8.

Below the examples of fractal clustering over $D$ are presented (Figs. 12.9 and 12.10). The selected image fragment with fractal dimension $D \approx 2.2$ nearby the first big peak (Fig. 12.8) is presented on Fig. 12.9. The selected image fragment with fractal dimension $D \approx 2.5$ ($\approx$ Brownian surface) nearby the third and fourth big peak (Fig. 12.8) is shown on Fig. 12.10.

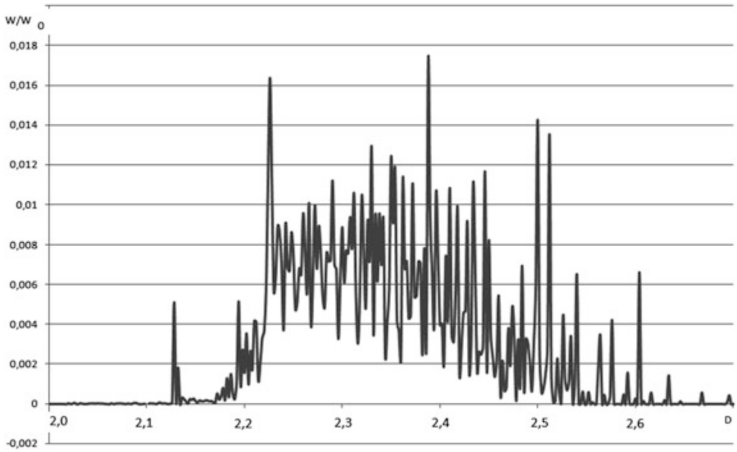


**Fig. 12.8** An empiric distribution of values of the instant fractal dimension $D$

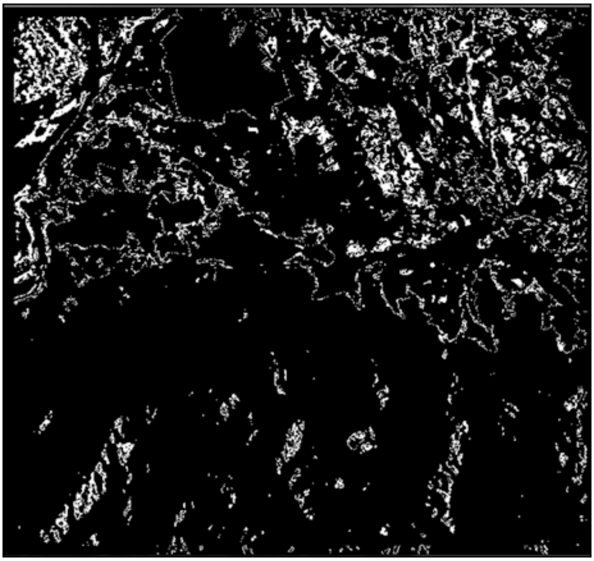

**Fig. 12.9** A fragment with $D \approx 2.2$

Previously *invisible* (*hidden*) peculiarities (for example earth coverings distant probing clustering data [4–6]) along with a stable distribution by earth coverings types are registered after fractal processing of surface images. It allows speaking of application of fractal recognition methods for the identification of image parts which are "invisible" when using classical methods of clusterization over the brightness field.

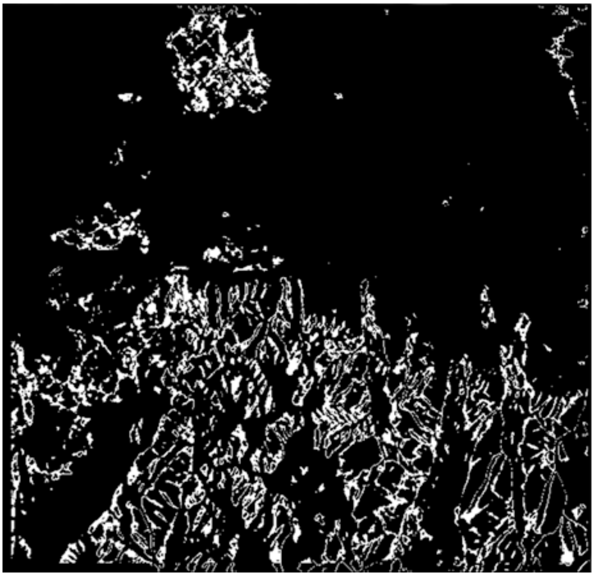

**Fig. 12.10** A fragment with $D \approx 2.5$

## 12.9 Fractal Characteristics of the High-Altitude Discharges in Ionosphere

Four million lightnings draw the sky every 24 h and about 50 lightnings draw the sky every second. And over the lead thunderheads, a light show of "unreal lightnings" is developing in the upper atmosphere: azure jets, red-purple sprites, red rings of highly soaring elves. These are discharges of very high energy which do strike the ionosphere and not the ground! Thus high-altitude electrical discharges (20–100 km) subdivide into several basic types: elves, jets, sprites, halo and so on—Fig. 12.11 (This is the first colour image captured of one by NASA aircraft in 1994). A history brief: a significant event occurred in the Earth study history in the night of 5 to 6 July 1989. Retired professor and 73 years old NASA veteran John Randolph Winkler pointed an extremely sensitive camera recorder to thunderstorm clouds and then he detected two bright blazes during inspecting the record frame by frame. The blazes go up to the ionosphere in contrast to lightning's which should go down to the ground. This way the sprites were discovered. The sprites are the biggest high-altitude discharges in the Earth atmosphere. After these publications NASA had not already been able to disregard the potential threat to space vehicles and they started a comprehensive research of high-altitude discharges.

The most short-lived high-altitude discharges are *elves*. They arise in the lower ionosphere at altitudes 80–100 km. The luminescence arise in the center and expands to 300–400 km for less than a millisecond and then it goes out. The elves are born in 300 μs after a strong lightning stroke from a thunderstorm cloud to the ground. It gets altitude 100 km for 300 μs where it "arouse" a red

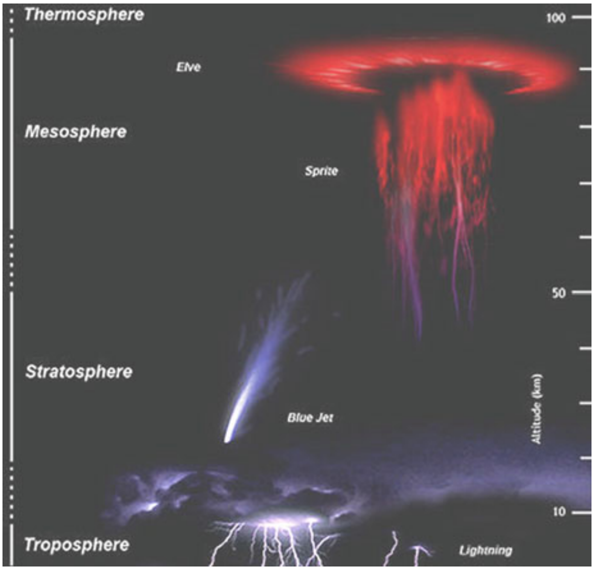


**Fig. 12.11** Dynamical fractal structures in the atmosphere (copyright: Abestrobi (Wikipedia))

luminescence of nitrogen molecules. The most enigmatic high-altitude discharges are azure *jets*. These are also a luminescence of nitrogen molecules in the ultraviolet-blue band. They look like an azure narrow inverse cone which "starts" from the upper edge of a thunderstorm cloud. Sometimes jets reach altitude 40 km. Their propagation speed varies from 10 up to 100 km/s. Their occurrence is not always due to lightning discharges. Besides azure jets they mark out "azure starters" (they propagate up to altitudes $\leq$25 km) and "giant jets" (they propagate up to altitudes of the lower ionosphere about 70 km). *Sprites* are very bright three-dimensional blazes with duration around milliseconds. They arise at altitude 70–90 km and descend down 30–40 km. Their width reaches tens of kilometers in the upper part. Sprites blaze up in the mesosphere in about 100th part of a second after the discharge of powerful lightnings "cloud–ground." Sometimes it occurs at a distance of several tens kilometers horizontally from the lightning channel. The red-purple colour of sprites as well as elves is due to the atmosphere nitrogen. The frequency of sprites occurrence is about several 1000 events per 24 h over the entire globe. The fine structure of the lower sprites part is characterized by dozens of luminous channels with cross sectional dimensions from tens to hundreds meters. Sprites occurrence is related with formation of high electrical dipole moment of uncompensated charge after especially powerful lightning discharges cloud–ground with usually positive polarity.

Dynamical spatial-temporal singularities and morphology of sprites can be particularly explained by the discharges fractal geometry and percolation [14]. Here we have one more example of a self-organized criticality when the system (a high-altitude discharge in this case) dynamics is determined by reaching the threshold of the so called directed percolation which characterizes a formation of branchy

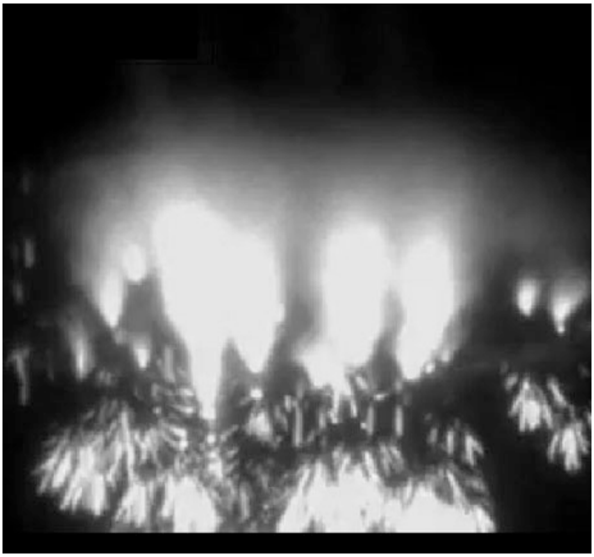

**Fig. 12.12** The original sprite image (USA, NASA http://science.compulenta.ru/701264/)

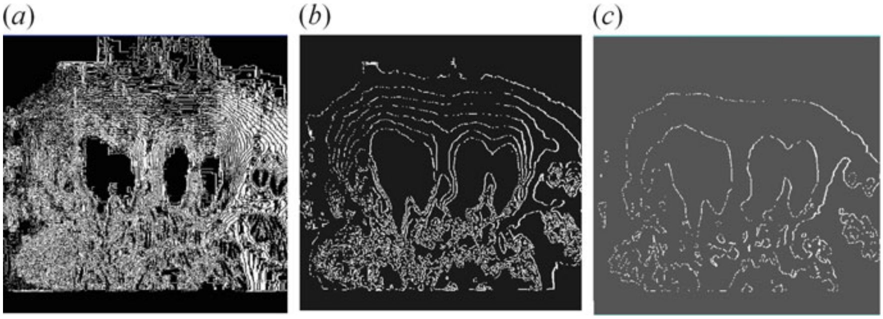


**Fig. 12.13** Results of fractal filtering of a sprite image: (**a**) a pattern of fractal dimension with the mean value $D = 2.3$; (**b**) 2.8; (**c**) 3.0

(fractal) conductive channels overlapping all the sprite length. A different situation arises with issues of data statistical processing.

Here the classical methods are used by tradition. It does not allow to extract all the information about such newest atmospherically structures. Selected examples of our fractal processing of sprite profiles (Fig. 12.12) are presented on Fig. 12.13a–c. Examples of fractal processing of a jet (Fig. 12.14a) are presented on Fig. 12.14b, c.

The fractal-scaling methodology which was used for describing the morphology of jets, sprites and elves can be successfully used to estimate their parameters and dynamics of their evolution [14]. Then the mathematical physics problems are solved.

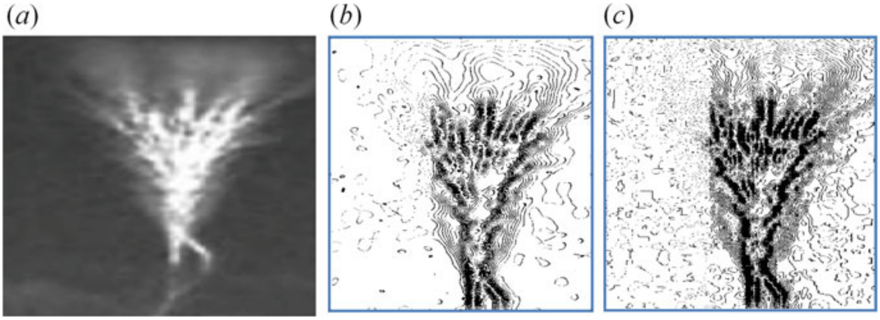


**Fig. 12.14** Results of fractal filtering of a giant jet image (the photos were taken in China August 12, 2010) (**a**) the jet image [15], (**b**) and (**c**) profiles of *D* estimates

## 12.10 Fractal Signal Detectors in Radiolocation

Classical detectors and their mathematical supply have virtually reached its saturation and limit. It causes searching principally new ways of solving the problem. Principally, fractals and fractional operators are not possible one without the other. We showed for the first time that fractal processing is suitable as well as possible for solving modern problems of the low-contrast images identification and ultra weak signal detection in the presence of intensive non-Gaussian noises, when modern radars can not operate. One of our main conclusions is that working on the pointed evaluation of the fractal dimension *D* leads to absurd results. At the same time almost all the authors who begins using the fractal signal processing give absolutely accurate meanings even with the RMS deviation! In our works we introduced fractal signatures and fractal kepsters [4–7, 9, 16]. Therefore the accuracy problems in digital fractal processing in real-time mode are solved.

The series of principally new fractal signal detectors (FSD) not mentioned by me in press is shown below as an example of effective operation of the global fractal methodology and the conception of radio systems and devices created by the author. The main principles of fractal detection were proposed by us for the first time as early as in 1989 works. At the same time a working model of the fractal non-parametric radar signals detector (FNRSD—Fig. 12.3) was created. The high accuracy of fractal detecting was proved. The main kinds of FSD proposed by us during 2011–2012 are shown at Fig. 12.15.

Figures 12.16, 12.17, and 12.18 show selected results of fractal nonparametric filtering of low-contrast objects. Aircraft images were masked by an additive Gaussian noise. In this case, the signal/noise ratio (SNR) $q_0^2 = -3$ dB. It is seen in the figures that all desired information is hidden in the noise.

The optimum mode of filtering of necessary contours or objects is chosen by the operator using the spatial distribution of fractal dimensions *D* of a scene. This distribution is determined automatically and is shown in the right panel of the computer display [4–7, 9].

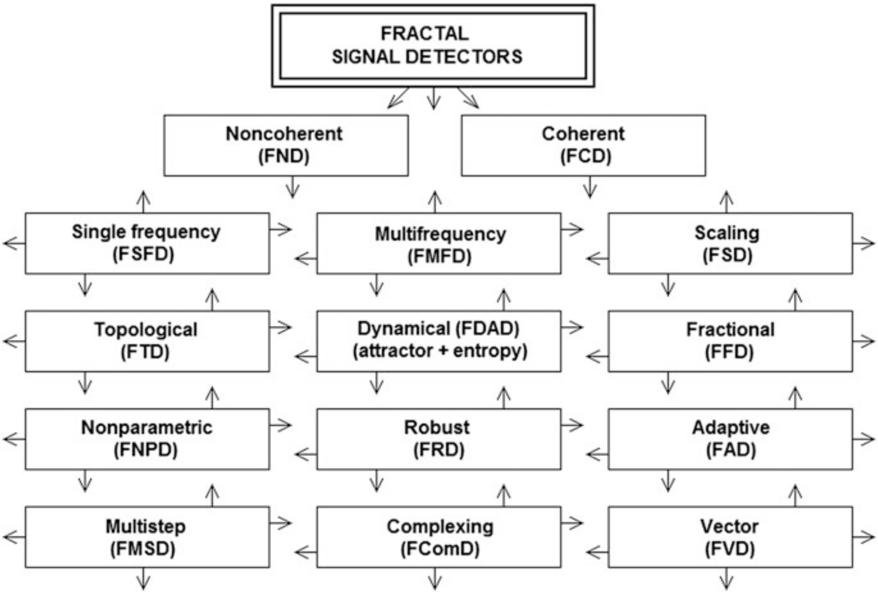


**Fig. 12.15** The main kinds of new dynamical FSD proposed by author

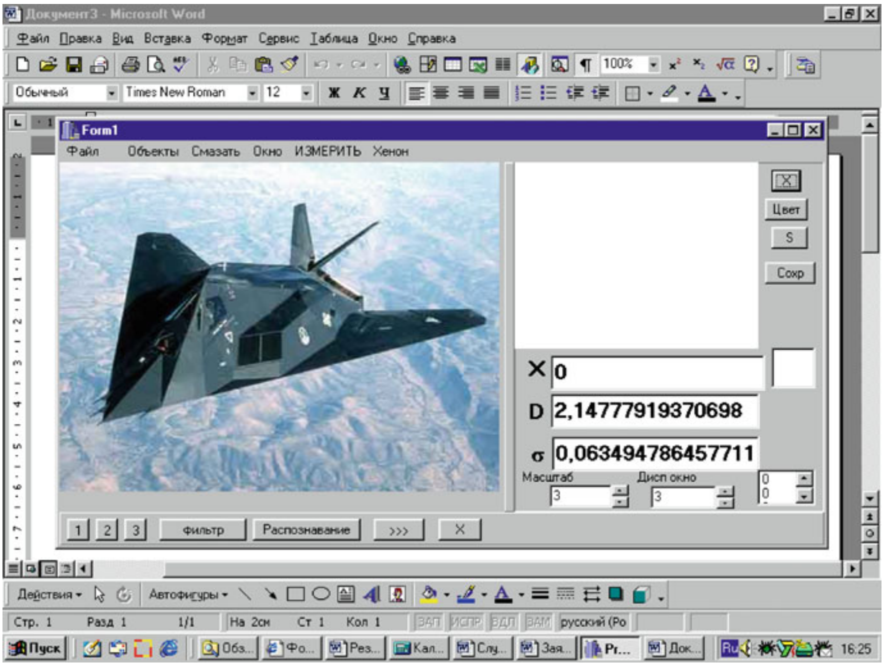


**Fig. 12.16** Real image

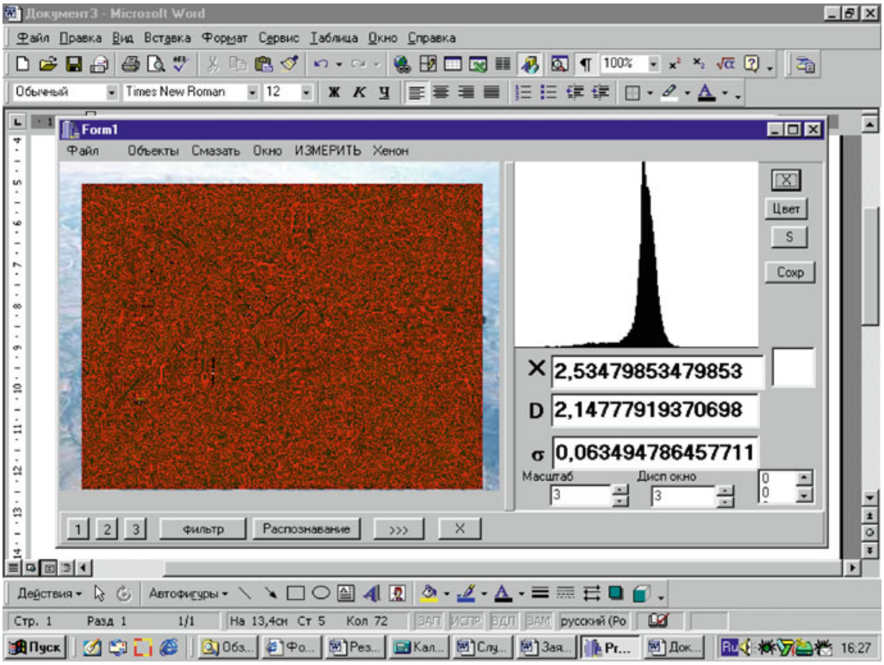


Fig. 12.17 Source image and noise $q_0^2$–3 dB

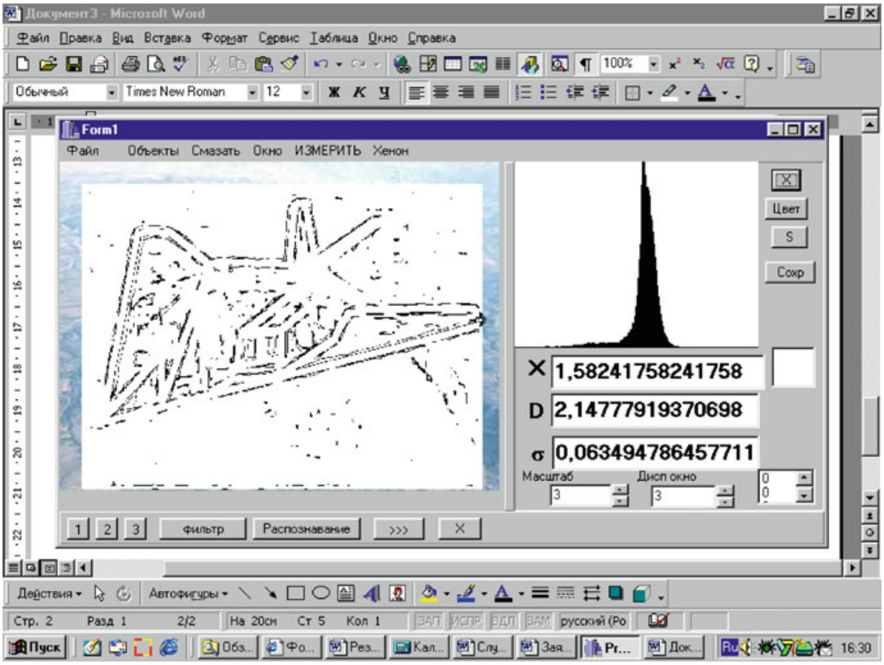


Fig. 12.18 Results of fractal filtration Fig. 12.17

## 12.11 Wave Scattering by Fractal Surface

In many works it has been shown that diffraction by fractal surfaces fundamentally differs from diffraction by conventional random surfaces and some of classical statistical parameters like correlation length and root-mean-square deviation go to infinity. This fact is result of self-similarity of fractal surface. In our work band-limited Weierstrass function was used. For the scattered field analysis we use Kirchhoff approach [17].

The most convenient function which both describes fractals well and is easy for using in calculations is the modified 2D band-limited Weierstrass function. It has a view:

$$W(x, y) = c_w \sum_{n=0}^{N-1} q^{(D-3)n} \sum_{m=1}^{M} \sin\left\{ Kq^n \left[ x \cdot \cos\left(\frac{2\pi m}{M}\right) + \ y \cdot \sin\left(\frac{2\pi m}{M}\right) \right] + \varphi_{nm} \right\} \tag{12.3}$$

where $c_w$—the constant, that provides unit normalization; $q > 1$—the fundamental spatial frequency; $D$—the fractal dimension ($2 < D < 3$); $K$—is the fundamental wave number; $N$ and $M$—number of tones; $\varphi_{\mathrm{nm}}$—an arbitrary phase that has a uniform distribution over the interval $[-\pi,\ \pi]$.

Since the natural surfaces are neither purely random nor periodical and are often anisotropic [2, 4] then function that was proposed above is a good candidate for characterizing of natural surfaces. Figure 12.19 shows us examples of band-limited Weierstrass function for different scales. It is also important that function (12.3) describes the mathematical fractals only if $M$ and $N$ go to infinity. It is clear from Fig. 12.19 that the function proposed possesses the self-similarity and multi-scale.

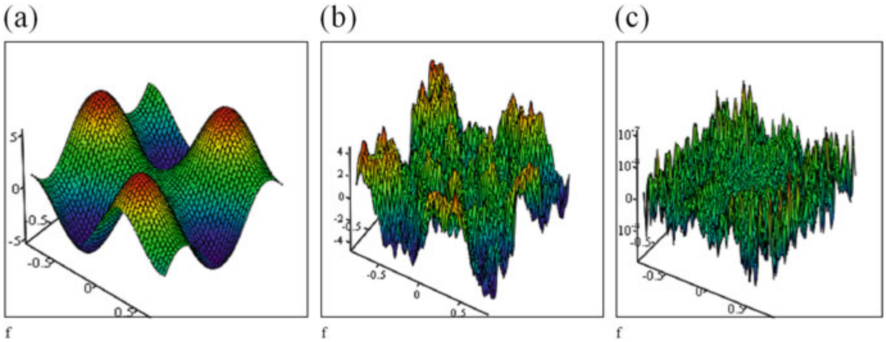


**Fig. 12.19** $W(x,y)$ for (a)—$N = 2$, $M = 3$, $D = 2.01$, $q = 1.01$; (b)—$N = 5$, $M = 5$, $D = 2.5$, $q = 3$; (c)—$N = 10$, $M = 10$, $D = 2.99$, $q = 7$

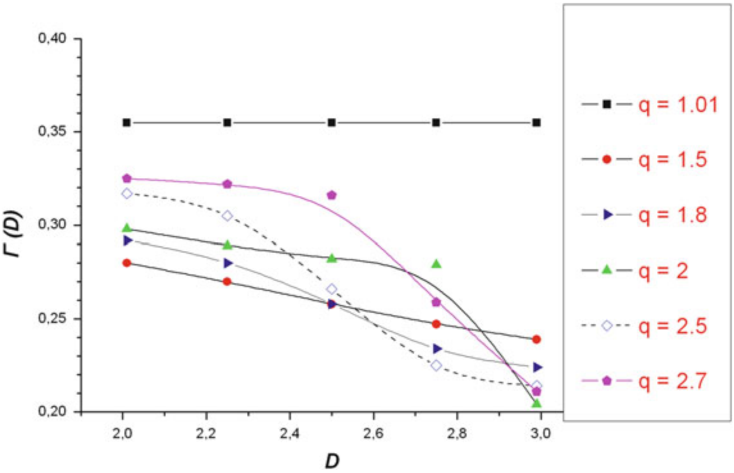


**Fig. 12.20** Function $\tilde{\Gamma}$ dependence on $D$ for various values of $q$

In this section of our work a statistical parameter is introduced for estimation of the fractal dimension $D$ influence and other fractal parameters influence on the surface roughness. Such parameter as the correlation length $\Gamma$ is conventionally used for numerical characterization of rough surface [4, 18, 19]:

$$\tilde{\Gamma}(\tau) = \langle \rho(\tau) \rangle_s = \left[ \frac{\left(1 - q^{2(D-3)}\right)}{\left(1 - q^{2(D-3)N}\right)} \right] \sum_{n=0}^{N-1} q^{2(D-3)n} J_0\left(K q^n \tau\right) \tag{12.4}$$

There are $\tilde{\Gamma}$ dependences on $q$ and $D$ in Fig. 12.20 and Fig. 12.21 respectively. It is shown that with increased value of $D$, $\tilde{\Gamma}$ decreases more rapidly for the same variation of $q$. It is shown in Fig. 12.20 that value of $\tilde{\Gamma}$ reduces steadily with the increase of $D$ value. However $\tilde{\Gamma}$ does not change when $q = 1.01$.

As mentioned above the Kirchhoff approach has been already used for analysis of wave scattering by fractal surfaces [18, 19]. Conventional conditions of the Kirchhoff approach applicability are the following: irregularities are large-scale; irregularities are smooth and flat. In the following calculations we assume that observation is carried out from Fraunhofer zone, an incident wave is plane and monochromatic, there are no points with infinite gradient on the surface, the Fresnel coefficient $V_0$ is constant for this surface, surface large scales are much greater than incident wave length. Shading effects will be taken into account in the following our investigations and studies.

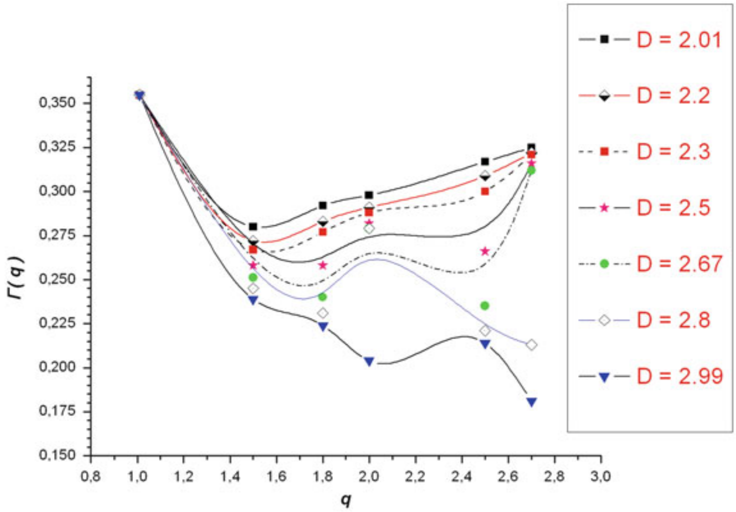


**Fig. 12.21** Function $\tilde{\Gamma}$ dependence on $q$ for various values of $D$

Scattering indicatrix for average field intensity and two-dimensional surface [18, 19]:

$$g \approx \frac{F^2(\theta_1, \theta_2, \theta_3)}{\cos^2\theta_1} \left\{ \left[ 1 - \frac{1}{2}(kC\sigma)^2 \right] \cdot \mathrm{sinc}^2\ (kAL_x) \cdot \mathrm{sinc}^2\ \left(kBL_y\right) + \frac{1}{4} C_f^2 \sum_{n=0}^{N-1} \sum_{m=1}^{M} q^{2(D-3)n} \cdot \right.$$
$$\left. \cdot\, \mathrm{sinc}^2 \left[ \left( kA + Kq^n \cos\frac{2\pi \cdot m}{M} \right) L_x \right] \cdot\ \mathrm{sinc}^2 \left[ \left( kB + Kq^n \sin\frac{2\pi \cdot m}{M} \right) L_y \right] \right\} \tag{12.5}$$

We have got a data base of scattering indicatrixes for various fractal scattering surfaces [19–21]. Also in terms of Weierstrass function (12.1) for one-dimensional fractal scattering surface we obtained scattering field absolute value dependences on incident angle and surface fractal dimension $D$. In subsequent computer calculations, we used the above expression for the coherence function

$$\Psi_k = \langle E_s(k_1)E_s(k_2) \rangle \tag{12.6}$$

of the fields scattered by the fractal surface [19–21].

We can show that the tail intensity of signals reflected by a fractal surface is described by power functions:

$$I(t) \sim 1/\left(t'\right)^{3-D} \tag{12.7}$$

Result (12.7) is very important because, for standard cases, the intensity of a reflected quasi-monochromatic signal decreases exponentially. Thus, the shape of a signal scattered by a fractal statistically rough surface substantially differs from the shape of a scattered signal obtained with allowance for classical effects of diffraction by smoothed surfaces [20, 22].

The results obtained can be widely applied for designing various modern radio systems in the microwave, optical, and acoustic bands.

## 12.12 Personal Meetings with Benois Mandelbrot

As it's seen from above the author uses the "fractal" term almost everywhere. In conclusion I'd like to share my impressions about the meeting with B. Mandelbrot with readers. The meeting occurred in his house near New York in December 2005. At that time I was responsible for the international project. I had to visit America frequently. My personal meeting with the founder of the fractal geometry B. Mandelbrot occurred on Friday 16 December 2005. Before this the intensive correspondence was going on both when I was still in Moscow and when I flew across USA from south to north with my lectures on the results of 5 years international project. Mandelbrot himself was in an extensive trip but his secretary phoned and told that the maitre would come back home ad hoc. He was extremely interested to meet at home and talk with the Russian physicist who dealt with various "fractal" experiments and applications of the fractal theory in radio physics and radio electronics.

I and my translator got to the New York Central railroad terminal by taxi and then we got to an electric train leaving at 9.30 local time. After a while we went down on a small station and went to the B. Mandelbrot's house by taxi. As approaching to the house we saw a silhouette of a high strong grayish man with glasses on a lace appeared behind the door. He dressed in home clothing. While we are getting off the car he's already opened the front glassed-in door. Mandelbrot's looking at me, smiling, holding out a hand first. Then he's suggesting us to undress and all of us are going to his room. He's asking me to seat down in front of him explaining that this way is better for him to talk and there is a more comfortable arm chair for me. There is his world-famous fundamental book "*Mandelbrot B.B.* The Fractal Geometry of Nature" on the table. I am taking out my monograph on fractals and presenting to Mandelbrot. At that I am telling about scientific work and my results on fractal applications in radio physics, answering his questions. Mandelbrot is listening with a keen interest and very attentively. It is a surprise for him that there is such a success on fractals' applications in Russia and there is already the direct approach to the fractal technologies. He became very interested in my proposed conception of fractal radio systems and in designing an essentially new fractal elemental base.

Hi is well familiar with fractal antennas. Suddenly he’s spoken that sometime the matter would be in producing a fractal capacitor! I reply with enthusiasm that I am already “caught” with this idea for a lot of years and a big paper about physical modeling of fractal impedances, fractional operators and production of fractal capacitors is ready for the press. This is incredibly: we are thinking about realizing the same idea on the opposite sides of the world! Mathematical questions are less interesting for Mandelbrot. He is getting more and more interested in disciplines created in IREE RAS: the fractal radio physics and the fractal radio electronics, its development.

Human simplicity, openness, interest in the surrounding world and wisdom—these particular properties are peculiar to B. Mandelbrot. Sometime they tell about B. Mandelbrot’s arrogance. I can assert only the reverse. He did not make me feel the difference between our statuses during all the conversation. He first inquired about all fractal developments.

Forty minutes later Mandelbrot stands up and after apologizing and going out to other room he comes back with a pile of his books. He asks me if I already have some of these books. Mandelbrot says that he likes my works. He inquires when and how my book was written. I reply that I prepared the first version as early as in the beginning of nineties of XX. Then the search for publishers began and at the same time improving and significant rework of the monograph text was going on. Mandelbrot says that now he has two books on the go: the one is in Italy and the other is in America. With a smile he admits that he writes slowly, thoroughly using all his old works. Our conversation’s been lasting for almost 2 h. Tempus fugit. At a certain moment he is called by the phone. He suggests to give us a lift to the railway station on his car. Mandelbrot drives the car on his own. We are at the railway terminal already at noon. I tell goodbye to B. Mandelbrot and we are waving to each other. This is the unforgettable meeting. There are all the minutest details of the meeting with the great scientist in my memory.

## 12.13 Conclusions

The fractal problem in radio location, radio physics and radio engineering is indeed immense. Here I illustrate only fundamental initial issues. It is always hard and even impossible to recede from habitual standards… But the author has good reasons to think that the extensive and valuable material he already obtained and the results of further researches will be used in advanced radio systems. The fractal radio physics, fractal radio engineering and fractal radio location are peculiar radio sciences. They are suffused with a spirit and ideas of the classical radio physics and radio engineering but at the same time they are fundamentally new areas of focus. The results of conducted researches oriented to enhancing the interference immunity of work of radio systems on a radio channel with high-intensity noise and distortion showed opportunities of the approach on the basis of using textural and fractal-scaling methods of detecting and processing random signals and fields.

The author raised these questions back in 1980, and for 35 years has been successfully working on their resolution [4–6]. Fractal methods similar to ones presented in this work can be applied when considering wave and oscillatory processes in optics, acoustics and mechanics. Results and conclusions obtained by the author and his pupils have great innovative potential. We think that its realization will resolve a number of current problems of radio physics, radio engineering, radio location, communication and operation and also will allow to provide a new quality for detecting and recognition systems and also development of the new informational technologies.

Many important stages in fractal directions development including the stage of this science field formation have been already passed. However many problems are still to be solved. Results and specific solutions are not of so greatest value like *the solution method* and its approach are. The method is created by the author [4–14, 16, 19–21, 23–25]. It is necessary to put it all into practice!

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