Drive Selection of Multidirectional Mechanism with Excess Inputs

Alexander N. Evgrafov and Gennady N. Petrov

Abstract Mechanisms with excess inputs are mechanisms in which the number of engines is higher than the number of degrees of freedom. While the mechanism is in operation, some engines are disengaging, while other engines are switching on, whereas the number of active engines at each given moment equals the number of degrees of freedom. These mechanisms create better conditions for power transmission than mechanisms without excess inputs. However to realize full potential of these mechanisms it is necessary to solve the problem of operational drive selection. This article describes a mechanism of a spatial platform with excess inputs. For the purpose of this article, the following criteria were used to carry out a quality assessment of the configuration presented below: Jacobian determinant of equations in sequence for geometric analysis of the mechanism, sum of squares of the balancing forces and minimal natural frequency of the mechanism with fixed engines and flexible transmission device. This article also considers variations of motion of the movable operating element at pre-set conditions with an alternative engines activation under all three mentioned criteria, as well as variations for simultaneous operation of all engines.

Keywords Spatial mechanism · Multidirectional mechanism · Excess input · Drive selection

Introduction

Closed linkwork mechanisms are complex mechanical systems, and are widely used in mechanical engineering. Position functions of such mechanisms are non-linear and link output coordinates, defining positions of movable operating elements, with

A.N. Evgrafov (✉) · G.N. Petrov
Peter the Great Saint-Petersburg Polytechnic University, Saint Petersburg, Russia
e-mail: a.evgrafov@spbstu.ru

G.N. Petrov
e-mail: gnpet@mail.ru

© Springer International Publishing Switzerland 2016
A. Evgrafov (ed.), *Advances in Mechanical Engineering*,
Lecture Notes in Mechanical Engineering, DOI 10.1007/978-3-319-29579-4_4

input coordinates, setting positions of drive output links [1–6]. Yet a function of multidirectional mechanism position, which realizes its programmed motion through coordinated controlled motion of several engines, is even more complex [7–9]. In most cases, these nonlinear functions of position are multivalued. The same set of input coordinates corresponds to several positions (configurations) of the system.

Problem Statement

Multivaluedness of the above-mentioned function leads to occurrence of specific singular positions, which are bifurcation points in the area of system configuration. In those specific positions, mechanism acquires an additional degree of freedom, while simultaneously reaction forces in kinematic pairs increase without bound. It results in creation of “self-locking areas” in a vicinity of the specific positions, given the Coulomb friction force. Thereby problems of motion selection continuously arise. In the majority of cases those problems relate to positioning—repositioning the system from the given initial position to the pre-set target position. At the same time the need to ensure the choice of “best” sequence of configurations, while controlling the engines, arises. Systematic selection of the optimal path or the choice of optimal laws of motion along the trajectory could be carried through this present sequence of configurations. There is also a possibility to have trajectory and the law of motion pre-defined, thus, in such a case, it is only necessary to determine the optimal calculus of variation for driving forces and momentum.

Thus, we need the solution of practical problems resulting in the need of initially solving theoretical problems of quality assessment for various mechanism configurations with various trajectories. Criteria of quality configuration may be selected in various ways. This article discusses the criteria of quality, reflecting the system’s degree of proximity to specific positions. Some of these criteria were considered in the previous research [10], for instance, contact angle [11]. However, this criterion proved ineffective in movement optimization of a mechanism with multiple degrees of freedom.

Formation of other criteria could be based on the following properties of the specific positions:

1. The Jacobian determinant of equations in a sequence for geometric analysis of a mechanism is equal to zero, while in a specific position. Application of this criterion is discussed more in the previous research [12].
2. Driving forces and momentum, counterbalancing ultimate external load constant in magnitude, increase without bound in the vicinity of specific positions. Application of this criterion is discussed more in the previous research [13].
3. In case some additional local degree of freedom appears at the specific position, a mechanism’s natural frequency becomes zero. A detailed description of this criterion is discussed more in the previous research [14].

Theory

Figure 1 presents a mechanism—a platform where the desktop is set to eight legs. Platform legs are hydraulic cylinders, connected by spherical hinges with the platform itself and its base. The mechanism has six degrees of freedom and eight inputs. For standard operation it is sufficient for the mechanism to have six inputs, thus engines may operate in an alternative mode of activation, where at any given moment only six of eight engines (two engines are turned off), or all engines at once, are connected with the mechanism.

To determine which of the engines to activate at any given moment when of its position on the platform, it is suggested to introduce three criteria for the quality of the platform configuration (these are based on three properties of the specific positions described above). To gain deeper insight on the subject it is recommended to review the inverse problem of spatial platform geometric analysis.

We consider the position of point K of the platform (x_K, y_K, z_K) and Euler angles (ψ, θ, ϕ) that define the orientation of the moving coordinate system x', y', z' in relation to the fixed coordinate system x, y, z as given values. We assume that leg drives with lengths $\ell_i, i = 1, \ldots, 6$ are activated at the current moment. We introduce matrix columns $\eta = (x_K; y_K; z_K; \psi; \theta; \phi)^{\mathrm{T}}$ and $\ell = (\ell_1; \ldots; \ell_6)^{\mathrm{T}}$, so that the calculation of the lengths based on the known column η becomes a simple task:

$$\ell_i = \|\mathbf{R}_i(\eta) - \mathbf{R}_{0i}\|, \quad i = 1, \ldots, 6$$

where $\mathbf{R}_i(\eta)$—radius-vector of fixed point of attachment of the i leg to the platform, and $\mathbf{R}_{0i}$—radius-vector of fixed point of attachment of the i leg to the pillar.

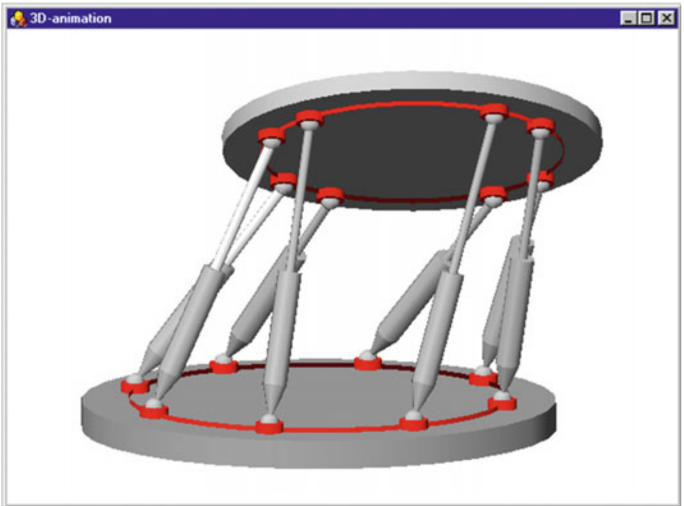

Fig. 1 The spatial platform

The direct problem of geometric analysis is more complex. Defining a column η by known lengths $\ell = (\ell_1; \ldots; \ell_6)^{\mathrm{T}}$ requires a solution for the system of six equations with six unknowns:

$$\Phi_i(\eta, \ell_i) = \|\mathbf{R}_i(\eta) - \mathbf{R}_{0i}\| - \ell_i = 0, \quad i = 1, \ldots, 6.$$

We may define the Jacobian of this system of equations as follows:

$$J = \det\left(\frac{\partial \Phi}{\partial \eta}\right), \quad \text{where } \Phi = (\Phi_1; \ldots; \Phi_6)^{\mathrm{T}}, \frac{\partial \Phi}{\partial \eta} \text{—Jacobian matrix.}$$

It is known that Jacobian equals to zero, while in specific position. However, it is possible to avoid the vicinity of specific positions by alternating inputs. Thus, we may define a Jacobian module of equations in sequence for geometric analysis as the first criterion of quality of the mechanism configuration.

We may present the angular velocity of the platform in the following form:

$$\boldsymbol{\omega} = \dot{\psi} \cdot \mathbf{k} + \dot{\theta} \cdot \mathbf{n} + \dot{\phi} \cdot \mathbf{k}',$$

where $\mathbf{k}, \mathbf{n}, \mathbf{k}'$—corresponding unit vectors.

We assume that the force $\mathbf{F}$ and momentum $\mathbf{M}$ are applied to the platform at the point K.

Let us introduce matrix-column $P = \left(F_x; F_y; F_z; M_1; M_2; M_3\right)^{\mathrm{T}}$, where F_x, F_y, F_z—projections of the vector $\mathbf{F}$ on the fixed coordinate system axis; M_1, M_2, M_3—projections of the vector $\mathbf{M}$ in the direction of the unit vectors $\mathbf{k}, \mathbf{n}, \mathbf{k}'$:

$$\mathbf{M} = M_1 \cdot \mathbf{k} + M_2 \cdot \mathbf{n} + M_3 \cdot \mathbf{k}'.$$

Evidently, drive forces $F_D = (F_{D1}; \ldots; F_{D6})^{\mathrm{T}}$ may be determined from the formula

$$F_D = -\left(\frac{\partial \eta}{\partial \ell}\right)^{\mathrm{T}} \cdot P.$$

When in the vicinity of specific positions of the mechanism, the absolute value of at least one of the driving forces increases without bound. Thus, it is considered reasonable to define the sum of squares of the driving forces as a second criterion of quality of the mechanism configuration.

To gain deeper insight on the subject we consider a machine with a flexible mechanism, taking into consideration stiffness of the leg drives. We assume that six inputs are active while two are disabled (thus we are not taking into consideration flexibility properties of those two). We assume that the mechanism performs natural oscillations in a vicinity of the specific position.

We introduce matrix columns: $\theta = (\theta_1; \ldots; \theta_6)^{\mathrm{T}}$—column, representing a small strain of resilient members and $\rho_S = \left(\Delta x_S; \Delta y_S; \Delta z_S; \Delta\phi_{x'}; \Delta\phi_{y'}; \Delta\phi_{z'}\right)^{\mathrm{T}}$—column, representing platform deviations from the equilibrium position, where $\Delta x_S, \Delta y_S, \Delta z_S$—deviation of the Platform's center of mass towards corresponding axes; $\Delta\phi_{x'}, \Delta\phi_{y'}, \Delta\phi_{z'}$—deviation of the Platform's alignment towards corresponding axes of the moving coordinate system x', y', z'.

Based on the previous work done by Loitsiansky and Lurie [15] we know the relation of deviations of the platform's orientation to deviations of Euler angles:

$$\begin{aligned}
\Delta\phi_{x'} &= \Delta\psi\ \sin\theta \sin\phi + \Delta\theta\ \cos\phi, \\
\Delta\phi_{y'} &= \Delta\psi\ \sin\theta \cos\phi - \Delta\theta\ \sin\phi, \\
\Delta\phi_{z'} &= \Delta\psi\ \cos\theta + \Delta\phi.
\end{aligned}$$

We introduce the square diagonal matrix:

$$A = diag\left(m; m; m; I_{x'}; I_{y'}; I_{z'}\right), \quad C = diag(C_1; \ldots; C_6),$$

where m—mass of platform, $I_{x'}, I_{y'}, I_{z'}$—primary central axial moments of inertia, and $C_1, \ldots, C_6$—stiffness of the active drives.

For the purpose of this article, we disregard mass of platform legs, thus we may present the equation of the resilient members' natural oscillations in the following form

$$\left(\frac{\partial\rho_S}{\partial\ell}\right)^{\mathrm{T}} A \frac{\partial\rho_S}{\partial\ell}\ddot{\theta} + C\theta = 0.$$

Accordingly, we introduce symbol $H = C^{-1}\left(\frac{\partial\rho_S}{\partial\ell}\right)^{\mathrm{T}} A \frac{\partial\rho_S}{\partial\ell}$.

The natural frequencies of a flexible mechanism are equal to reciprocals of the matrix H's eigenvalue roots. One of the natural frequencies tends to zero, once the mechanism is in a vicinity of the specific position.

In consequence, we may define the value of the fundamental frequency of a flexible mechanism as the third criterion of quality of the mechanism configuration. However, it should be emphasized that, the mechanism flexibility is introduced conditionally, for the purpose of this work, while mechanism's natural frequencies are defined within a range of static positions along the pre-set trajectory.

In case of simultaneous operation of all engines, the selection of driving forces on pre-determined movement and at a given load becomes ambiguous. As a result, we may outline a system of six equations with eight unknowns and an infinite set of solutions, which in turn allows us to set the problem to minimize the sum of squares of the driving forces. Detailed description of the solution is discussed more in the previous research [14].

The analytical solution of the problem is rather laborious, thus, the authors decided to use the numerical calculation method. Typically software tools such as

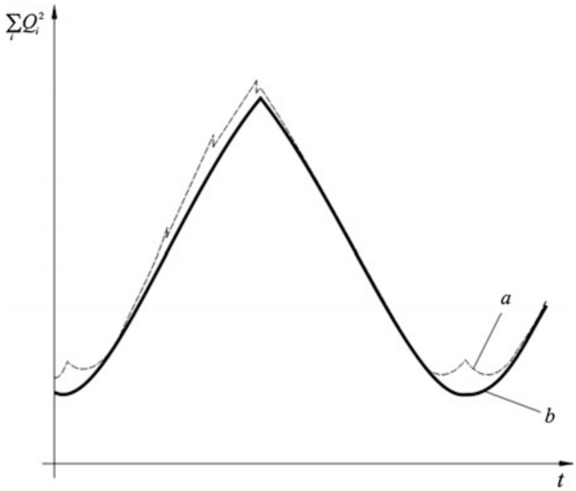

Fig. 2 Diagrams represent sum of squares of the platform's driving forces, given that six engines are operating according to the criterion of minimum sum of squares of the driving forces (*a*) and at simultaneous operation of all eight engines (*b*)

Mathcad and Matlab are used in such cases (Both Mathcad and Matlab usage is discussed in the previous research: [16–18] and [19] respectively). For convenience, obtained results are visualized through animation of mechanisms kinematics (as discussed in the previous research [20]). However, for the purpose of this work, and to be able to obtain an interactive visualization of the calculation results of a complex spatial mechanism, authors selected "Model Vision" software (discussed in the previous research [21]).

Figure 2 shows two diagrams, representing the sum of squares of the platform's driving forces, given that six engines are operating according to the criterion of minimum sum of squares of the driving forces (a) and at simultaneous operation of all eight engines (b).

Conclusion

The resulting driving forces provide a minimum sum of squares of the driving forces at any given point of the trajectory.

It is possible to evaluate the energy efficiency of the operational mode where all eight engines function simultaneously by comparing the sum of squares of driving forces with the minimum value of such a sum attained when only six engines are functioning. Taking into consideration criterion of the sum of squares of the driving forces and criterion of the fundamental frequency, authors may conclude that the operational mode where all eight engines function simultaneously is considered more beneficial. Additionally, ease of management may be considered as an advantage of the alternative inputs selection.

References

1. Dimentberg FM (1982) The theory of spatial hinge mechanisms. Moscow, Nauka, 336 p. (In Russian)
2. Zinoviev VA (1952) Spatial mechanisms with lower pairs. Moscow, Gostehizdat, 432 p. (In Russian)
3. Mertsalov NI (1951) The theory of spatial mechanisms. Moscow, Gostehizdat, 206 p. (In Russian)
4. Uicker JJ (1963) Velocity and acceleration analysis of spatial mechanisms using 4 × 4 matrices. Northwestern University, The Technological Institute, Evanston, Illinois
5. Mudrov PG (1976) Spatial mechanisms with rotational pairs. Publishing House of Kazan University, Kazan, 264 p. (In Russian)
6. Hrostitsky AA, Evgrafov AN, Tereshin VA (2011) The geometry and kinematics of spatial six-bar mechanisms with excess links. In: Scientific and technical news of SPbPU. Publishing House of the Polytechnic University, St-Petersburg, vol 2 (123), pp 170–176. (In Russian)
7. Kolovsky MZ, Evgrafov AN, Semenov YA, Slousch AV (2000) Advanced theory of mechanisms and machines. Springer, Berlin, 394 p
8. Kolovsky MZ, Evgrafov AN, Semenov YA, Slousch AV (2013) Theory of mechanisms and machines: textbook for high school students, 4th revised edn. Publishing Center, Moscow, Akademia, 560 p. (In Russian)
9. Evgrafov AN (2015) Theory of mechanisms and machines: textbook/A.N. Evgrafov, M.Z. Kolovsky, G.N. Petrov. Publishing House of the Polytechnic University, St.-Petersburg, 248 p. (In Russian)
10. Evgrafov AN, Petrov GN Quality criteria used in computer aided mechanism synthesis. In: 2nd international scientific and practical conference “modern engineering: science and education”. Publishing House of the Polytechnic University, St.-Petersburg, pp 48–52. (In Russian)
11. Evgrafov AN (1997) Contact angle as a quality criteria of the force transmission. Mashinovedenie. Collection of scientific papers. Publishing house of the Polytechnic University, St.-Petersburg, pp 84–90. (In Russian)
12. Kolovsky MZ (1997) Quality criterion of multidirectional lever mechanisms positions, no 2. (In Russian)
13. Kolovsky MZ, Slousch AV (1998) Movement control of the closed lever mechanisms with excess inputs. Mat. of XXV summer school “Nonlinear oscillations–97”. St.-Petersburg Institute of Problems of Mechanical Engineering of RAS. (In Russian)
14. Kolovsky MZ, Petrov GN, Slousch AV (2000) Movement control of the closed lever mechanisms with several degrees of freedom. J Mach Manuf Reliab, no 4. (In Russian)
15. Loitsiansky LG, Lurie AI (1982) Theoretical mechanics course: in 2 volumes, vol I. Statics and kinematics. Nauka, Moscow, 352 p. (In Russian)
16. Petrov GN (1993) Computer aided kinetostatic algorithm for closed lever mechanisms. J Mach Manuf Reliab (3). (In Russian)
17. Evgrafov AN, Petrov GN Geometric and kinetostatic analysis of plain lever mechanisms of the second class. Theor Mech Mach 2:50–63. (In Russian)
18. Ziborov KA, Matsyuk IN, Shlyahov EM (2010) Power analysis of mechanisms using Mathcad. Theor Mech Mach 8(1):83–88. (in Russian)
19. Mkrtychev OV (2013) Computer simulations in the plain mechanisms power calculation. Theor Mech Mach 11(1):77–83. (In Russian)
20. Evgrafov AN, Petrov GN (2008) Computer animation of kinematic schemes in excel and mathcad. Theor Mech Mach 6(1):71–80. (In Russian)
21. Petrov GN (2004) Computer simulation of mechanical systems in the “ModelVision”. Theor Mech Mach 2(1):75–79. (In Russian)