

Challenge to Scalability of Face Recognition Using Universal Eigenface

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Abstract. This paper approaches to the scalability problem of face recognition using the weight equations in a universal eigenface. Since the weight equations are linear equations, the optimal solution can be generated even when the number of registered faces exceeds the dimensionality of universal eigenface. Based on the characteristics of the underdetermined linear systems, this paper shows that effective preliminary elimination is possible with little loss by the parallel underdetermined systems. Finally, this paper proposes a preliminary elimination followed by a small-scale face recognition for a scalable face recognition.

Keywords: Face recognition · Eigenface · Weight equations · Underdetermined systems · Orthogonal partitions · Scalability

1 Introduction

Eigenspaces have been widely utilized in computer vision for various applications, including object recognition [6, 8]. Eigenspaces constructed from face images are often called eigenfaces [11] and widely used in face recognition [1, 7], tracking [2], and so on. On the other hand, face recognition has still been a hot topic in computer vision and discussed from various view points [10, 12], since this problem is a big problem for computer vision applications. Among them, Oka and Shakunaga [9] proposed an efficient method for real-time face tracking and recognition to cover pose and photometric changes. In their method, linear equations, called the weight equations, are used for both the face recognition (person identification) and the shape inference. Although the number of faces was at most 25 in their implementation, and the scalability seemed severe in this approach, Chugan et al. [3] showed that the weight equations also work in underdetermined systems, and they are effective for 100-face tracking and recognition when 140d eigenface is used.

This paper will show how much more faces can be recognized using the underdetermined systems. For the purpose, a framework of face recognition by weight equations is shown in Sect. 2. Section 3 discusses details of the parallel underdetermined approach. Section 4 shows final challenge to 2197-face recognition using 169d eigenface.

2 Framework of Face Recognition by Weight Equations

2.1 Universal and Individual Eigenfaces

Let \mathbf{V}_{kl} denote an n -d intensity vector of k -th person under l -th lighting condition. K and L indicate the number of persons, and the number of lighting conditions, respectively. The universal and individual eigenfaces are constructed and used as follows:

(1) Construction of universal eigenface. When a set of intensity vectors, $\{\mathbf{v}_{kl}\}$ are calculated by $\mathbf{v}_{kl} = \mathbf{V}_{kl}/\mathbf{1}^\top \mathbf{V}_{kl}$, the universal eigenface is constructed by average vector $\bar{\mathbf{v}}$ and m -principal eigenvectors Φ_m . Let it be described as $\langle \bar{\mathbf{v}}, \Phi_m \rangle$.

(2) Projection to universal eigenface. Let $\mathbf{P}\mathbf{V}$ denote a part of image, where \mathbf{P} is an $n \times n$ diagonal matrix, of which each diagonal element is 1 or 0. The projection \mathbf{s} of $\mathbf{P}\mathbf{V}$ is calculated by

$$\tilde{\mathbf{s}} = (\mathbf{P}\tilde{\Phi}_m)^+(\mathbf{P}\mathbf{V}) \quad (1)$$

when $\tilde{\Phi}_m = [\Phi_m \bar{\mathbf{v}}]$ and $\tilde{\mathbf{s}} = [\alpha \mathbf{s}^\top \alpha]^\top$, and $(\mathbf{P}\tilde{\Phi}_m)^+$ denotes (Moore-Penrose) pseudo inverse of $\mathbf{P}\tilde{\Phi}_m$. Once $\tilde{\mathbf{s}}$ is calculated from a given part of image $\mathbf{P}\mathbf{V}$, the normalized projection of $\tilde{\mathbf{s}}$ is given by $\hat{\mathbf{s}} = [\mathbf{s}^\top \ 1]^\top$.

(3) Construction of individual eigenfaces. In the learning stage, an individual eigenface is constructed in the universal eigenface, from a set of face images. When a set of \mathbf{s} -representations, $S_k = \{\mathbf{s}_{kl} \mid l = 1, \dots, L\}$, are calculated for person k by projection of a set of intensity vectors $\{\mathbf{V}_{kl} \mid l = 1, \dots, L\}$ to the universal eigenface, the k -th individual eigenface $\langle \bar{\mathbf{s}}_k, \eta_k \rangle$ is constructed from S_k in \mathbf{s} -domain, where $\bar{\mathbf{s}}_k$ and η_k denote the average and k -th individual eigenspace.

2.2 Face Recognition by Weight Equations

In Oka and Shakunaga [9], linear equations, called the weight equations, are proposed for both the face recognition (person identification) and the shape inference. The weight equations are used for face recognition in this paper, as follows:

(1) Projection to universal eigenface. When an unknown image vector \mathbf{V} is given, the projection \mathbf{s} of $\mathbf{P}\mathbf{V}$ to the universal eigenface is calculated using Eq. (1) where \mathbf{P} is an appropriate part indicator matrix. We can set $\mathbf{P} = \mathbf{I}$, when a full projection is necessary. If a set of parts are required, a set of partial projections might be used. (Refer to Oka and Shakunaga [9].)

(2) Photometric adjustment. In \mathbf{s} -domain, for each k , a projection of \mathbf{s} to the k -th individual eigenface is calculated by

$$\mathbf{s}_k = \eta_k \eta_k^\top (\mathbf{s} - \bar{\mathbf{s}}_k) + \bar{\mathbf{s}}_k. \quad (2)$$

(3) Fundamental weight equations. After all \mathbf{s}_k are calculated from \mathbf{s} , the following linear equations are given, named the fundamental weight equations

$$\widehat{\mathbf{S}}_K \mathbf{w} = \widehat{\mathbf{s}} \quad (3)$$

where $\widehat{\mathbf{S}}_K = [\widehat{\mathbf{s}}_1 \cdots \widehat{\mathbf{s}}_K]$ and $\mathbf{w} = [w_1 \cdots w_K]^\top$. The optimal solution of Eq. (3) is given by

$$\mathbf{w} = \widehat{\mathbf{S}}_K^+ \widehat{\mathbf{s}} \quad (4)$$

and the optimal solution indicates the weights of individual persons.

(4) Face recognition. Face recognition is accomplished by selecting

$$k_{max} = \operatorname{argmax} w_k. \quad (5)$$

2.3 Scalability Problem with Face Recognition

In the face recognition scheme mentioned above, the weight equations serve an essential role. Let K and M denote the number of persons and $m + 1$, where m is the dimensionality of the universal eigenface. Then, the computational cost for solving the weight equations is $O(MK^2)(O(KM^2))$ in the overdetermined(underdetermined) system. In the overdetermined system, K should be sufficiently less than M for reliable solutions. On the other hand, M could not be so large because the dimensionality of the universal eigenface cannot be too big. To solve the dimensionality problem, Chugan et al. [3] has shown a solution in the underdetermined system. In [3], the case of $M = 141$ and $K = 249$ was solved by parallel underdetermined approach. This paper discusses, from now on, the parallel underdetermined approach for $K \gg M$, and how and why the approach is possible.

3 Parallel Underdetermined Approach

3.1 Solution of Underdetermined Weight Equations

Let us summarize characteristics and properties of the parallel underdetermined approach according to Chugan et al. [3]. When the fundamental weight equations are underdetermined, Eq. (4) provides \mathbf{w} so as to minimize $\mathbf{w}^\top \mathbf{w}$ in the solution space of Eq. (3).

When \mathbf{B} is a $K \times K$ diagonal matrix defined by

$$\mathbf{B} = \operatorname{diag} (d^{-1}(\mathbf{s}, \mathbf{s}_1) \cdots d^{-1}(\mathbf{s}, \mathbf{s}_K)) \quad (6)$$

$$\text{where } d(\mathbf{s}, \mathbf{s}_k) = \sqrt{(\mathbf{s} - \mathbf{s}_k)^\top (\mathbf{s} - \mathbf{s}_k)}, \quad (7)$$

diagonal terms of \mathbf{B} indicate inverse distances from \mathbf{s} to each \mathbf{s}_k . When $\mathbf{s}_k = \mathbf{s}$, a large number should be used for the k -th diagonal term instead of ∞ .

By substituting $\mathbf{w} = \mathbf{B}\mathbf{w}'$ to Eq. (3), the biased weight equations are provided in

$$\widehat{\mathbf{S}}_K \mathbf{B}\mathbf{w}' = \widehat{\mathbf{s}}. \quad (8)$$

From the optimal solution of Eq. (8), the weight vector is estimated in

$$\mathbf{w} = \mathbf{B}[\widehat{\mathbf{S}}_K \mathbf{B}]^+ \widehat{\mathbf{s}}. \quad (9)$$

Equation (9) still provides a solution of the fundamental weight equations, Eq. (3). Since $\mathbf{w}'^\top \mathbf{w}'$ is minimized in the biased weight equations, \mathbf{w} is optimized with considering distances between \mathbf{s} and each \mathbf{s}_k .

As shown in [3], there is a simple relation between the biased weight equations and nearest neighbor criterion. When the weight equations are specified in 0d space, Eq. (9) becomes

$$\mathbf{w} = \mathbf{B}[\mathbf{1}^\top \mathbf{B}]^+ = \frac{1}{\sum d^{-2}(\mathbf{s}, \mathbf{s}_k)} \begin{bmatrix} d^{-2}(\mathbf{s}, \mathbf{s}_1) \\ \vdots \\ d^{-2}(\mathbf{s}, \mathbf{s}_K) \end{bmatrix}. \quad (10)$$

Therefore, the heaviest person indicated by Eq. (5) becomes equivalent to the nearest person.

3.2 Parallel Underdetermined Systems

A simple parallelism is implemented by partitioning the universal eigenface into subspaces. When m' denotes the dimensionality of each subspace in m -d eigenspace, and $m = Jm'$ holds, J -parallel underdetermined systems are implemented.

In the parallel implementation, the j -th fundamental weight equations are represented as

$$\begin{bmatrix} \mathbf{S}_K^{(j)} \\ \mathbf{1}^\top \end{bmatrix} \mathbf{w}^{(j)} = \begin{bmatrix} \mathbf{s}^{(j)} \\ 1 \end{bmatrix}, \quad (11)$$

$$\text{where } \mathbf{w}^{(j)} = [w_1^{(j)} \dots w_K^{(j)}]^\top, \quad (12)$$

$$\mathbf{S}_K = \begin{bmatrix} \mathbf{S}_K^{(1)\top} & \dots & \mathbf{S}_K^{(J)\top} \end{bmatrix}^\top, \quad (13)$$

$$\text{and } \mathbf{s} = [\mathbf{s}^{(1)\top} \dots \mathbf{s}^{(J)\top}]^\top. \quad (14)$$

After solving all the equations, average over all the solutions

$$\mathbf{w} = \frac{1}{J} \sum_{j=1}^J \mathbf{w}^{(j)} \quad (15)$$

provides a final weight vector. In the parallel implementation, each underdetermined system could be transformed to the biased weight equations with using the same distances measured in the universal eigenface.

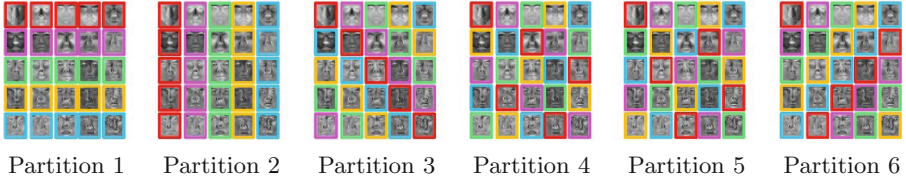


Fig. 1. Example of orthogonal subspace partitions

(1) Concurrency by orthogonal subspace partitions. In the parallel underdetermined approach, the entire eigenfaces should be decomposed into exclusive subspaces. Let us call the decomposition a *subspace partition*. This paper proposes a parallel implementation of the underdetermined weight equations using a set of subspace partitions to improve recognition performance.

In the parallel implementation, a set of subspace partitions should satisfy the following requirement: Between a subspace in a partition and a subspace in another partition, intersection of them is at most 1d subspace.

When a set of subspace partitions satisfy it, let these partitions called *orthogonal subspace partitions*. An example of orthogonal subspace partitions is as shown in Fig. 1 when 25 eigenbases are arranged in a 5×5 matrix. In each partition, 5 colors indicate 5d subspaces, respectively. Since any two partitions in the figure satisfy the above requirement, they are orthogonal subspace partitions. For example, Partitions 1 and 2 are row-wise and column-wise partitions and they obviously satisfy the requirement. Since the other partitions are complete Latin squares [5], they are mutually orthogonal and orthogonal to Partitions 1 and 2, too. Note that when K is a prime number, there are exactly $K + 1$ orthogonal subspace partitions for K^2 -d eigenface, since $K - 1$ complete Latin squares and row-wise and column-wise partitions are orthogonal to each other.

(2) Face recognition by orthogonal partitions. Let C and $\mathbf{w}_{(c)}^{(j)}$ denote the number of orthogonal subspace partitions and the optimal weight vector of the j -th subspace of the c -th partition. Then, average over all the optimal vectors is represented in

$$\mathbf{w} = \frac{1}{CJ} \sum_{c=1}^C \sum_{j=1}^J \mathbf{w}_{(c)}^{(j)}, \quad (16)$$

and used for face recognition.

(3) Utilization of parallel partial projections. It is widely known that parallel partial projections are also useful for robust face recognition [9]. When partial projections are combined with weight equations, partial sub images are more precisely approximated by weighted averages of dictionary images. In the cases, weights are calculated in each sub image in each underdetermined system. In this paper, face recognition is performed by combining the orthogonal subspace partitions and the parallel partial projections.

Let B and $\mathbf{w}_{(bc)}^{(j)}$ denote the number of partial projections and the optimal weight vector for the b -th partial projection of the j -th subspace in the c -th partition. Then, average over all the optimal vectors is represented in

$$\mathbf{w} = \frac{1}{BCJ} \sum_{b=1}^B \sum_{c=1}^C \sum_{j=1}^J \mathbf{w}_{(bc)}^{(j)}, \quad (17)$$

3.3 Preliminary Elimination by Parallel Underdetermined System

In conventional pattern recognition, supervised learning is performed before recognition. When a big number of classes should be learned, however, clustering is widely used for efficient selection of candidate classes from all classes. In these approaches, if clustering (or unsupervised learning) is accomplished for all the classes a priori, a smaller number of clusters are considered in the first stage of recognition. However, the clustering problems often suffer from computational cost and quality of clustering result. These problems are often serious for scalability of recognition algorithm.

In unsupervised learning, all the classes should be considered between each other without knowing what input is provided. Although the rough optimization seems valid for a wide variety of unknown inputs, only a small part of the rough optimization works for each particular input in the recognition stage.

In the parallel underdetermined approach, the number of classes has no limit in principle, and no clustering is necessary before the recognition stage. However, an average of a set of the weight equations can provide a weight ranking list of all the classes for each particular input. The result is regarded as a local but precise clustering result just around the particular input.

4 Experimental Challenge to Scalability

4.1 Construction of Scalable Database

In this paper, a subset of CMU Multi-PIE [4] is used for construction of the universal eigenface. The subset is composed of 141 faces (included in 249 neutral faces in session 1) of CMU Multi-PIE. The subset, called Data-1, is composed of 141 faces taken under 20 lighting conditions. The other 108 faces are excluded from the subset because they include glasses, mustache or teeth in his/her images.

Face shapes are different from person to person. The fact means that a geometric normalization is required for making a meaningful eigenface used in recognition of faces in monocular images. For this purpose, three points, which are located in centers of right and left eyes, and a center of lips on each face surface, are transposed to standard positions by affine transform in this paper, as shown in Fig. 2. After the geometric normalization, a normalized face image is made up by cropping a face region. In the current implementation, the face region is fixed as shown in Fig. 2(b).

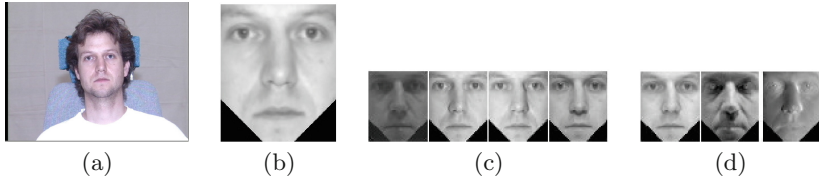


Fig. 2. Example of geometric normalization: (a) and (b) show a face image before/after geometric normalization. (c) and (d) show 4 images for an individual person and constructed individual eigenface, respectively.

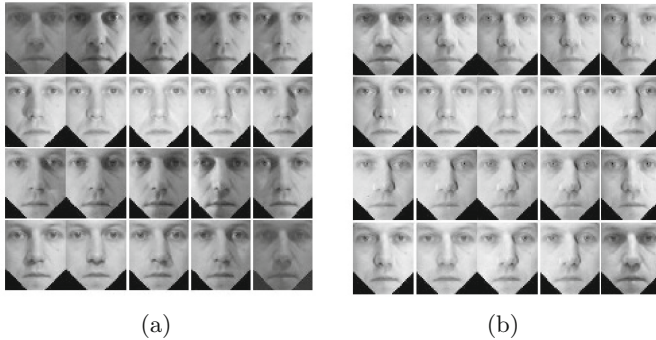


Fig. 3. Example of 20 images for an individual; (a) and (b) show a face image before/after image correction.

4.2 Refinement of Universal Eigenface

After the geometric normalization, 169d universal eigenface, called EF-1, was constructed by PCA on all (141x20) images in Data-1. In this process, universal eigenface was refined by the method described as follows:

Since the original image set includes a lot of kinds of noise including reflections and shadows, the universal eigenface is affected by these noise factors if the original images are directly used for the eigenface construction.

In order to suppress these noise factors in the original images, the following image correction is applied to the original images. Then, the universal eigenface is reconstructed from the corrected images.

For the image correction, 2d individual eigenface, as shown in Fig. 2(d), is constructed from 4 images (No. 0, 6, 8, 16) of 20 original images, as shown in Fig. 2(c).

Then, each original image is projected to the 2d individual eigenface, denoted by $\tilde{\Psi}$, and the following equation corrects each original image, \mathbf{V} , to

$$\mathbf{V}' = \tilde{\Psi} \tilde{\Psi}^+ \mathbf{V}. \quad (18)$$

Figure 3 (a) shows an example of 20 original images, and (b) shows images after the image correction.

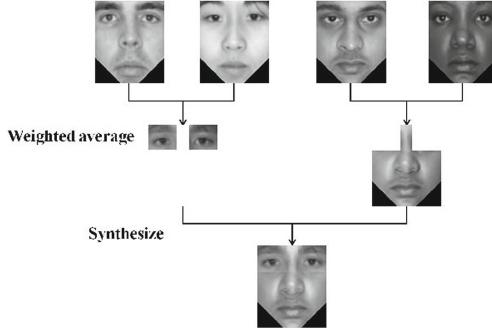


Fig. 4. Example of image composition.

When all the original images are corrected by the image correction algorithm, refined universal eigenface can be constructed by the method shown in Sect. 2.1.

4.3 Composition of Face Data

From Data-1, Extended-Data-1 is constructed by repetitive composition of faces from original 141 persons, using projection coefficients. In the face composition, at first, 4 different persons, q, r, q', r' , are selected randomly from 141 persons. Then, the whole face image is composed of a weighted average of q and r , and the eye-part image is composed of a weighted average of q' and r' . In these compositions, 2 parameters, β_1 and β_2 , randomly selected between 0 and 1, are also used for weight control between two persons. The following equations show how to compose the whole and eye images, when \mathbf{P}_e denotes a part indicator matrix for eyes region.

$$\mathbf{v}_{kl} = (\mathbf{I} - \mathbf{P}_e)\mathbf{v}'_{kl} + \frac{\mathbf{1}^\top \mathbf{P}_e \mathbf{v}'_{kl}}{\mathbf{1}^\top \mathbf{P}_e \mathbf{v}''_{kl}} \mathbf{P}_e \mathbf{v}''_{kl} \quad (19)$$

$$\text{where } \mathbf{v}'_{kl} = \beta_1 \mathbf{v}_{ql} + (1 - \beta_1) \mathbf{v}_{rl}, \quad (20)$$

$$\text{and } \mathbf{v}''_{kl} = \beta_2 \mathbf{v}_{q'l} + (1 - \beta_2) \mathbf{v}_{r'l}. \quad (21)$$

Figure 4 shows how a virtual image is composed from four faces according to Eq. (19).

We can make an arbitrary number of virtual faces by Eq. (19). For any virtual face, 20 virtual images are synthesized that were taken under 20 lighting conditions.

4.4 Specifications of Face Recognition Experiments

On Extended-Data-1, 4 of 20 images (No. 1, 13, 14, 18 of 20 images) were used for training data, to construct 2d individual eigenface in EF-1. The other 16 images were used for test. The training data and the test data are exclusive to each other.



Fig. 5. Example of noise suppression for training data: (a) and (b) show a face image before/after noise suppression.

In the learning stage, an input image often includes some reflections and shadows. These noise factors directly affect the face model synthesis. In order to suppress these noise factors in the input image, another image correction algorithm is used. In the image correction, the universal eigenface Φ_m is used. The image correction algorithm is specified using a residue r_i that is calculated for the i -th pixel of \mathbf{v} by

$$r_i = \mathbf{e}_i^T (\mathbf{v} - \tilde{\Phi}_m \hat{\mathbf{s}}), \quad (22)$$

where \mathbf{e}_i is a unit vector which has 1 in the i -th element and 0s in the others. When $|r_i|$ is more than 3σ , where σ is a standard deviation of the absolute residuals over the input image, the i -th pixel of \mathbf{v} should be replaced by $\mathbf{e}_i^T (\tilde{\Phi}_m \hat{\mathbf{s}})$. The image correction makes an intensity value to be consistent with the projection. For example, shadows and reflection regions are corrected. It is noted that the normality of the image doesn't hold after the correction. Therefore, the corrected image should be re-normalized when all the pixels are checked and corrected (Fig. 5).

4.5 Fundamental Experiments of Face Recognition

As fundamental experiments of face recognition, face recognition rates were compared between the nearest neighbor (NN) method and the parallel underdetermined systems (PUS). In both methods, full projection (FP) and parallel partial projections (PPP) were also compared. In the experiments, the nearest neighbor method was implemented by Eq. (10) since it provides equivalent results.

When K^2 or K^3 persons were registered, the recognition rates were shown in Table 1, for $K = 7, 11, 13$. In the experiment, K^2 -d universal eigenface was used for each K , and in the PUS approach K -d subspace was used in each PUS.

For the full projection, a subspace partition, which consists of K subspaces, was used when concurrency = K . When concurrency = $4K$, 4 orthogonal subspace partitions were used. When concurrency = $(K + 1)K$, $K + 1$ orthogonal subspace partitions were used. Note that when K is a prime number, there are exactly $K + 1$ orthogonal subspace partitions. For the parallel partial projections, 6 partial projections were used along with a full projection. Therefore, concurrency is magnified 7 times by the parallel partial projections.

Table 1 shows that the PUS method provided much better results than the NN method in both the full and parallel partial projections. In the PUS method, recognition rates were improved when the concurrency increases. The table also shows that results of full projection were worse than those of the parallel partial

Table 1. Face-recognition rates (%) for K^2 and K^3 registrations

method	concurrency	K^2 registration			K^3 registration		
		K=7	K=11	K=13	K=7	K=11	K=13
NN(FP)	1	72.45	78.46	79.07	62.92	64.74	64.96
PUS(FP)	K	92.86	98.50	99.11	84.49	95.46	95.24
	4K	90.94	98.30	99.37	83.71	95.49	96.57
	(K+1)K	90.69	98.45	99.52	83.33	95.59	96.67
NN(PPP)	7	91.07	92.30	90.01	89.50	87.04	84.22
PUS(PPP)	7*K	99.49	99.95	99.96	98.92	99.77	99.73
	7*4K	99.62	100	100	99.22	99.79	99.78
	7*(K+1)K	99.62	100	100	99.23	99.81	99.80

projections for both the NN and PUS methods. In K^3 registration, combination of PUS and parallel partial projections gave the best recognition rate for each K , but a perfect recognition could not be obtained even when the concurrency was $7(K+1)K$. However, it is noted that recognition rates reached 99.8% for $K = 11, 13$ at the maximum concurrency. The PUS method could accomplish almost perfect recognition for 1331 and 2197 faces. In K^2 registration, the PUS method accomplished a perfect recognition for 121 and 169 faces.

Because the dimensionality of the universal eigenface was set to K^2 , recognition rate got worse in NN(FP), PUS(FP) and PUS(PPP), while K decreased. However, in NN(PPP), recognition rates of two partial projections lowered as K increased, and affected the final recognition rates.

4.6 Fundamental Experiments of Preliminary Elimination

As fundamental experiments of preliminary elimination, face selection rates were compared between the nearest neighbor (NN) method and the parallel underdetermined system (PUS). In both methods, only the parallel partial projections (PPP) was used because of the face recognition results in Sect. 4.5.

Table 2 shows K - and K^2 -face selection rates for K^3 faces. The result shows that PUS method could perfectly select K candidates from K^3 candidates for $K = 11, 13$ with any concurrency, and K^2 candidates for $K = 7$. Therefore, the PUS method is effective for preliminary selection of K^2 candidates from K^3 faces.

The table also shows that rank- K^2 selection rates of the NN method could not reach 100%, and the simple NN method was less scalable to the number of registered faces than the PUS method.

4.7 Challenge to Scalability of Face Recognition

From experimental results shown above, the parallel underdetermined systems could effectively work for preliminary elimination. Table 2 shows that

Table 2. Rank- K and K^2 selection rates (%) from K^3 persons

method	concurrency	Rank- K			Rank- K^2		
		K=7	K=11	K=13	K=7	K=11	K=13
NN(PPP)	7	96.47	94.89	93.26	99.03	98.50	98.02
PUS(PPP)	7*K	99.98	100	100	100	100	100
	7*4K	99.98	100	100	100	100	100
	7*(K+1)K	99.98	100	100	100	100	100

Table 3. Final recognition rate and rank-13 selection rates(%) for 2,197 faces

Preliminary	Final	Recognition	Rank-13
-	PUS(7*13)	99.73	100
-	PUS(7*14*13)	99.80	100
PUS(7*13)	PUS(7*13)	99.85	100
PUS(7*13)	PUS(7*14*13)	99.87	99.997

169-candidate selection was perfectly done from 2,197 faces using 169d universal eigenface. Furthermore, Table 1 shows that face recognition from 169 was also perfectly done using the same eigenface. Therefore, if 169 candidates selected by the preliminary elimination has similar properties to the 169 faces used in Table 1, combination of the two methods can accomplish a scalable face recognition. Otherwise, the two methods may be inconsistent.

In order to confirm if the combination works or not, the following experiment was tried for 2,197 faces. In the experiment, PUS was used for final recognition, and the final recognition rate and rank-13 selection rates were compared between the cases with/without preliminary elimination.

Experimental result as shown in Table 3 indicates that the combination of preliminary elimination and final recognition works well. The final recognition rates of 99.85 % and 99.87 % were better than those of the cases without using the preliminary elimination. In our current implementation, the processing time of the combination of preliminary elimination and final recognition using PUS (PPP, 2,197 persons, concurrency = $7 * 14 * 13$) was about 0.8 seconds/image on Intel Corei7-5820K 3.30 GHz without any GPU.

5 Conclusions

This paper reported a challenge to scalability of face recognition using the universal eigenface and the parallel underdetermined systems of linear equations. Based on the characteristics of the underdetermined linear systems, this paper indicated that effective preliminary elimination is possible with little loss by the parallel underdetermined systems. From these experimental results, this paper proposed a preliminary elimination followed by a small-scale face recognition.

In order to confirm the effectiveness of the method, a scalable database was constructed by an extension database of CMU Multi-PIE. Our final experiments show that the proposed method worked well on 2,197 faces with 99.87% correct face recognition. Comparison of the proposed method and the state-of-the-art methods is in future work. We hope the proposed method will effectively work in wide variety of computer vision and pattern recognition problems.

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