Chapter 3
Flow Processes

Hua Shao, Wenkui He, Milan Hokr, Payton W. Gardner, Herbert Kunz and Ales Balvin

3.1 Flow in Fracture/Matrix System

Milan Hokr, Hua Shao, Payton W. Gardner, Herbert Kunz, Ales Balvin

Flow velocity is a key parameter for the mass transport in fractured rock, which can be generally described by the groundwater flow equation (3.1.1). Lots of benchmark examples in different dimension have been performed for the plausibility of the code (Kolditz et al. 2015), but a coupled fracture and matrix system, numerically using an integrated 2D and 3D finite element mesh, was not considered. In such an integrated system, 2D finite elements may be used to simulate fracture and fracture network and 3D ones to rock matrix to increase the computational efficiency.

3.1.1 Theory

The transient saturated groundwater flow is described by:

$$S_0 \frac{\partial h}{\partial t} - \text{div}\,(K \text{ grad } h) = Q \tag{3.1.1}$$

H. Shao (✉) · H. Kunz
Federal Institute for Geosciences and Natural Resources, Hannover, Germany
e-mail: Hua.Shao@bgr.de

W. He (✉)
Helmholtz Centre for Environmental Research-UFZ, Leipzig, Germany
e-mail: wenkui.he@ufz.de

M. Hokr · A. Balvin
TUL, Technical University of Liberec, Liberec, Czech Republic

P.W. Gardner
SANDIA, Sandia National Laboratories, New Mexico, USA

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where $h(x, t)$ is the piezometric head, S_0 the specific storativity, q the fluid sink/source, and $\kappa_f(x, t)$ is the hydraulic conductivity tensor. The global boundary Γ consists of two parts $\Gamma = \Gamma_T + \Gamma_q$ (Γ_T Dirichlet's and Γ_q Cauchy's conditions). The average fluid velocity vector can be calculated using the generalised Darcy's law:

$$\mathbf{q} = K \text{ grad } h \qquad (3.1.2)$$

Analytical solutions for the partial differential equation (3.1.1) are only available for some simplified cases, e.g. steady state flow, radial pumping flow. A generalised solution for a fracture/matrix system is only possible with the help of numerical method.

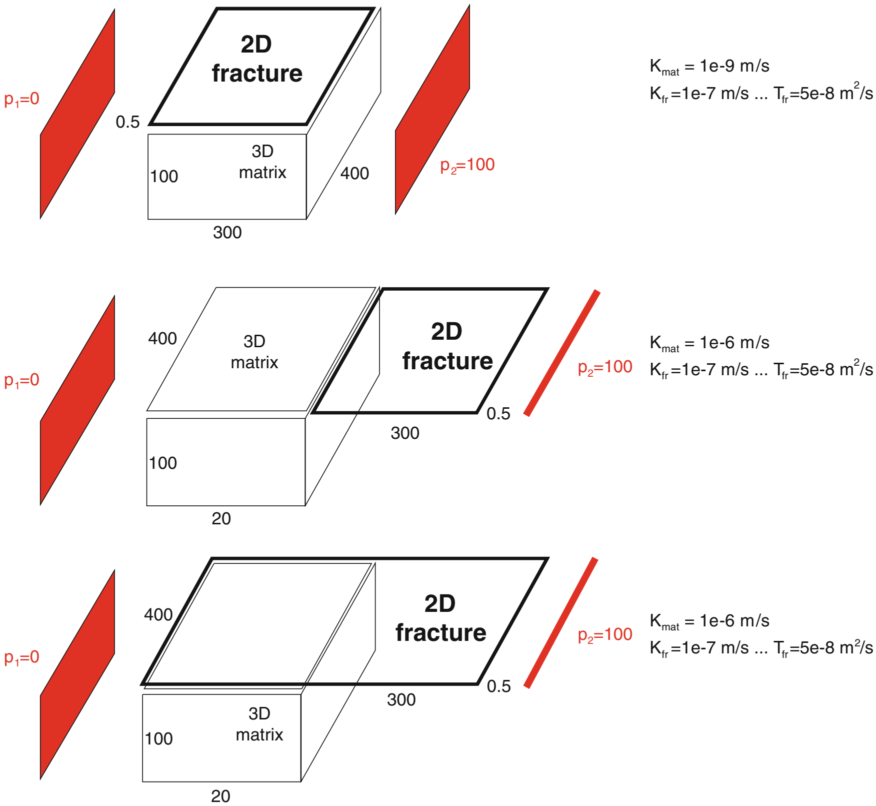


Fig. 3.1 Simplified benchmark cases

Table 3.1 Used parameters

Symbol	Quantity	Value
a	Fracture aperture	0.5 m
$T_{fracture}$	Transmissivity	5e-8 $m^2\ s^{-1}$
$K_{fracture}$	Fracture hydraulic conductivity	1e-7 m s^{-1}
K_{matrix}	Rock mass hydraulic conductivity	1e-9 m s^{-1}

3.1.2 *Problem Definition*

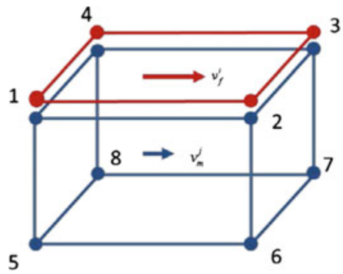
Based on a practical application, the Bedrichov tunnel case (CZ), four conceptual models have been developed within the international project DECOVALEX-2015. From that three cases (Fig. 3.1) have been simplified for the benchmark exercise.

Geometry of all three cases is comparable with she Bedrichov site. The dimension of the matrix block in the case (a) is 300 m × 400 m × 100 m (L×B×H) and in the cases (b) and (c) 20 m × 400 m × 100 m. The fracture has an aperture of 0.5 m and a hydraulic conductivity of 1e-7 m/s. The transmissivity of the fracture can be calculated to 5e-8 m^2/s (Table 3.1). The hydraulic conductivity of the rock mass is 1e-9 m/s in the first case (Fig. 3.1a) and 1e-6 m/s in the other two cases (Fig. 3.1b, c). The initial hydraulic head was set to 0. On the left side of the model the pressure was kept to 0.

3.1.3 *Analytical Solution*

There is no generalised analytical solution available for the groundwater flow (1) in a fracture/matrix system (Fig. 3.1). A comparison between codes is therefore necessary for a simplified case.

The first problem is constructed so that each dimension can be evaluated separately by an analytical solution (1D problem equivalent). Taking account of the



$$v_1 = (v_f^i + v_m^j)/2$$
$$v_2 = (v_f^i + v_m^j)/2$$
$$v_3 = (v_f^i + v_m^j)/2$$
$$v_4 = (v_f^i + v_m^j)/2$$

Fig. 3.2 Calculation of velocity of interface elements in a fracture/matrix system

Table 3.2 Results from case (a)

	P_2 = 100 m Q – fracture (m^3/s) (node/element)	Q – matrix (m^3/s)	Q2 = 2e-5 m^3/s P_2 – fracture (m)	P_2 – matrix (m) (maximal)
Analytical	6.67e-6	1.33e-5		
Flow123D	6.6667e-6	1.3333e-5	89.86	107.7
OGS	3.37e-6 / 6.67e-6	1.33e-5	90.7	109.2

case 1 (Fig. 3.1a), the flux through fracture zone and matrix zone can be calculated correspondingly according to the velocity:

$$T_{fracture} = T\frac{p_2 - p_1}{L}B = 5 \times 10^{-8}\frac{100 - 0}{300}400 = 6.67 \times 10^{-6}\,\mathrm{m}^3/\mathrm{s}$$

$$K_{matrix} = T\frac{p_2 - p_1}{L}B\,H = 1 \times 10^{-9}\frac{100 - 0}{300}400 \times 100 = 1.33 \times 10^{-5}\,\mathrm{m}^3/\mathrm{s}$$

The total inflow/outflow in the system can be calculated as $Q2 = Q_{fracture} + Q_{matrix}$ = 2e-5 m^3/s. The calculated flow rate from both fracture and rock mass agree well with the analytical results (Table 3.2) in case of pressure is known. It is meaningful to check both types of boundary conditions for each case, e.g. given pressure to get flow and given flow to get pressure. Therefore, instead of pressure (p_2), Cauchy's condition (Q2) can be used as boundary condition, in particular the uniform flux is prescribed 2e-5 m^3/s per 40,200 m^2 area which is 4.97512e-10 m/s on both the fracture boundary (200 m^2) and the matrix boundary (40,000 m^2). As result the hydraulic head in the

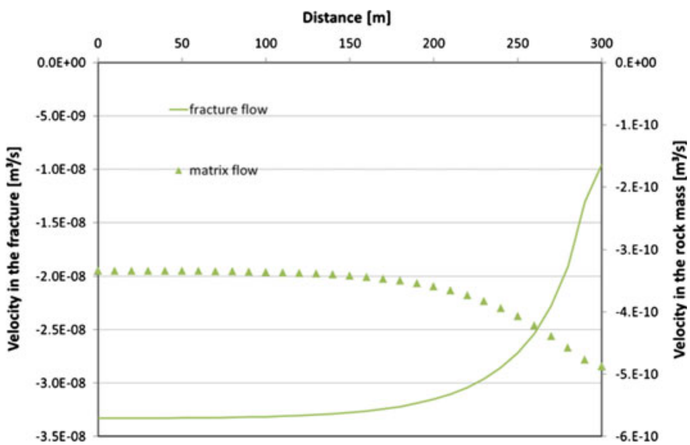


Fig. 3.3 Profile of velocity along fracture and matrix given by code OGS

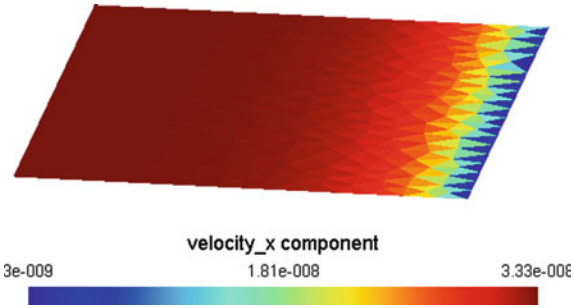


Fig. 3.4 Distribution of velocity Vx given by code Flow123

fracture (90.7 m) is lower than the maximal pressure in the matrix (109.2 m) because of high hydraulic conductivity of fracture (Fig. 3.3).

For the numerical method, the flow velocity is calculated within the element. The nodal output for velocity is a mean value from all neighbor elements (Fig. 3.2) and is therefore not suitable for the flux calculation especially in case of fracture/matrix system due to high difference on the hydraulic conductivities.

A spatial distribution of the velocity values in a 3D view (code Flow123D) (Fig. 3.4) are in correspondence with the graph (Fig. 3.3).

Flow123D results are in two variants for coarser (8 m) and finer mesh (2 m) at the 2D/3D contact edge. Practically, the case (b) is not exactly following the definition, as the mixed-hybrid formulation does not allow the 2D/3D communication through an edge (only through a side), so there is a 1 m wide overlap of the 2D domain above the 3D domain. The mesh here corresponds to the finer variant of the case (c) (Table 3.3).

It is worthwhile to have a benchmark exercise based on the application orientated cases. In most cases there are no analytical solutions available. Results comparison from different codes is therefore important to guarantee the quality in the development, implementation, and application of a code.

Table 3.3 Results from case (b) and (c)

	Case (b)	Case (b)	Case (c)	Case (c)
	$P_2 = 100\,\text{m}$	Q2 = 6.67e-5 m^3/e	$\text{P}_2 = 100\,\text{m}$	$\text{Q}_2 = 6.67\text{e-}5\,\text{m}^3/\text{s}$
	Q – outflow (m^3/s)	P2 – pressure (m)	Q – inflow (m^3/s)	P2 – pressure (m)
Flow123D	6.65312e-06	100.20364	6.62004e-06 (c)	100.70816 (c)
			6.64631e-05 (f)	100.30633 (f)
OGS	6.67e-6	100.03	6.66e-6	100.03

3.2 Water Table Experiment

Wenkui He

3.2.1 Description

This benchmark problem described here is based on the laboratory experiment of Vauclin et al. (1979), in which water infiltration in variably saturated sandy soil was conducted. The dimensions of the soil slab are 3 m long, 2 m high and 5 cm thick. The initial water table was 0.65 m above the bottom. A constant flux of 0.148 m/h was imposed over a region on the soil surface, which has a width of 0.5 m. There was no lateral flow on the left hand side of the domain. The pressure head of the right hand side of the domain was fixed by connecting with a constant head reservoir. The water table elevations on different positions of the domain were measured during the experiment which lasted 8 h.

3.2.2 Model Setup

The soil parameters applied for the two-dimensional model are given in Table 3.4, which are based on Vauclin et al. (1979) and Clement et al. (1994). The initial and boundary conditions are shown in Fig. 3.5. The entire model domain consists of 1200 uniform rectangular elements with a length of 0.10 m and a width of 0.05 m. A fixed time step size of 60 s is applied for the simulation.

Table 3.4 Soil properties

Symbol	Parameter	Value	Unit
ϕ	Porosity	0.30	–
k_s	Saturated hydraulic conductivity	8.40	md^{-1}
S_r	Residual water saturation	0.01	–
S_{max}	Maximum water saturation	1.0	–
α	Van Genuchten parameter	3.3	$1/m$
n	Van Genuchten parameter	4.1	–

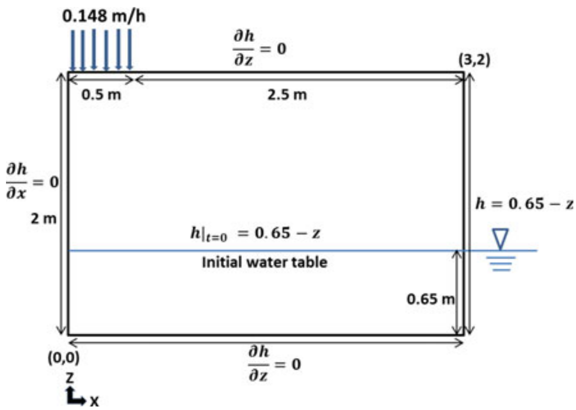


Fig. 3.5 Initial and boundary condition of the model (from He et al. 2015b). $h(x, z, t)$ is pressure head at coordinates x and z at time t

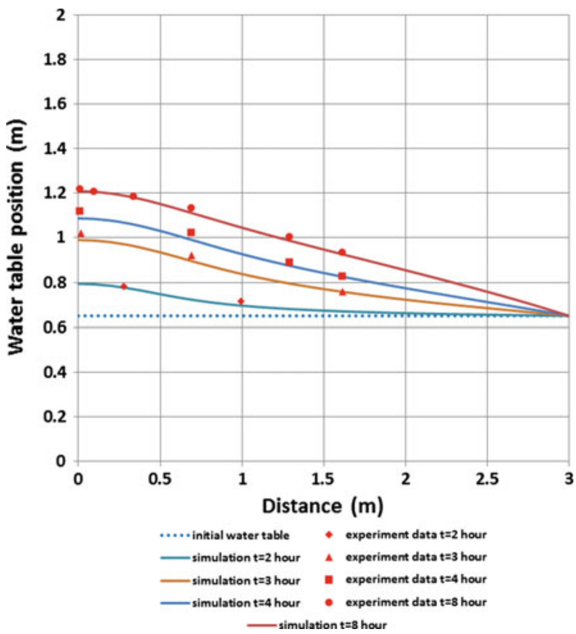


Fig. 3.6 A comparison of the water table positions simulated by using OGS with the experimental data (Vauclin et al. 1979)

3.2.3 Results

The simulated water table positions at different times are illustrated in Fig. 3.6 together with the experimental data of Vauclin et al. (1979). Generally, the simulated results can match to the experiment data quite well.