Multi-scale Analysis of Turbulent Rayleigh-Bénard Convection

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Abstract We report the results from a direct numerical simulation of turbulent Rayleigh-Bénard convection for Rayleigh number of 10^5 and Prandtl number of 0.7. The flow topology is characterized by the presence of coherent structures, the so-called thermal plumes, consisting of localized portions of fluid having a temperature contrast with the background. Two distinct events are identified close to the walls by using the wall-parallel divergence div_π of the velocity field: the impingement ($div_\pi > 0$) and the ejection of thermal plumes ($div_\pi < 0$). The impingement leads to the formation of larger velocity and temperature structures in the wall-parallel planes. Contrary to the classical picture of turbulence consisting of a direct transfer of energy from large toward smaller turbulent fluctuations, the impingement is conjectured to be probably responsible for a reverse transfer from small towards large scales in the near-wall region.

1 Introduction and Methodology

Thermally driven turbulence plays a major role in several natural and industrial processes, which range from atmospheric convection to cooling systems in nuclear power plant [1]. An idealized system for approaching all these cases is the Rayleigh-Bénard convection (RBC), which consists of a fluid layer heated from below and cooled from above in a vertically bounded domain. It is well known that the transport of thermal and kinetic energy across the fluid layer in turbulent RBC is carried out mainly by the so-called thermal plumes [3], which are commonly defined as localized portions of fluid having a temperature difference with the background [2]. Hot and cold plumes detach respectively from the bottom and the upper plate, moving toward the opposite wall driven by buoyancy forces. As nicely sketched by Kadanoff [4], thermal plumes carry out a self-sustained life cycle during which their morphology changes. In particular, the characteristic diameter of each structure enlarges as the distance from the impinging wall decreases due to the vertical constraint. The latter

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phenomenon, hereafter called impingement, is supposed to be related with a strong exchange of thermal and kinetic energy between turbulent scales, in particular, a transfer from small towards large scales. This conjecture can be verified by analyzing the budgets for the second-order structure functions, which are capable to capture both the local and the non-local phenomena of turbulence, i.e. the processes occurring simultaneously in physical space and in the space of turbulent scales, see Marati et al. [5] for a multi-scale analysis of a turbulent channel flow. An extensive analysis of the multi-scale energy budgets lies outside the purpose of the present work. Here, we want to investigate the life-cycle of thermal plumes in turbulent RBC in order to identify some fundamental events which could impress a clear footprint on the energy budgets. To that end, a data set coming from a direct numerical simulation (DNS) is employed. The Boussinesq equations are solved using a pseudospectral method which discretizes space with Fourier modes in the wall-parallel directions x and y, and with Chebychev polynomials in the wall-normal direction z. Time advancement is carried out with a fourth-order Runge–Kutta scheme for the nonlinear terms and a second-order accurate Crank-Nicholson scheme for the linear ones. The computational domain is a rectangular box of size $8 \times 8 \times 1$ along x, y, z respectively, where the Cartesian coordinate system is cell-centered, with the z-axis pointing in the direction opposite to that of gravity acceleration. The temperature of the lower and the upper walls are fixed at 0.5 and -0.5, respectively; no-slip boundary conditions are used on both horizontal plates, whereas periodic boundary conditions are imposed at the lateral sidewalls. The DNS is performed at $Pr = 0.7$ and $Ra = 1.7 \cdot 10^5$ using 128^2(horizontal) $\times$ 129 (vertical) modes and polynomials.

2 Analysis of the Flow Topology

According to the definition made in Sect. 1, thermal plumes are here displayed by isosurfaces of temperature. Further ways to extract these structures from the turbulent background consider thresholds of vertical velocity, thermal dissipation or vertical vorticity-temperature correlation. With respect to the latter, as an example, Zhou et al. [6] showed that the plumes are associated with a strong vertical vorticity component, hence the vorticity-temperature correlation is a suitable quantity for the identification. As can be seen in Fig. 1a, hot and cold plumes detach respectively from the lower and the upper wall, stretch across the domain and, finally, enlarge close to the opposite plate. Furthermore these structures have a sheet-like form at the beginning, whereas they take the appearance of a mushroom sufficiently away from the starting point. It seems quite evident from Fig. 1b that the sheet-like roots of thermal plumes create a fine network across the plates and that portions of fluid are emitted from the intersection spots. By considering the two-dimensional divergence of the velocity field in the xy-plane, it is possible to measure both the horizontal enlargement and narrowing of thermal plumes. It seems reasonable to apply this kind of analysis to the near-wall region, where both mushroom-like caps and sheet-like roots coexist, rather than the bulk, where the stalks have almost a constant section.

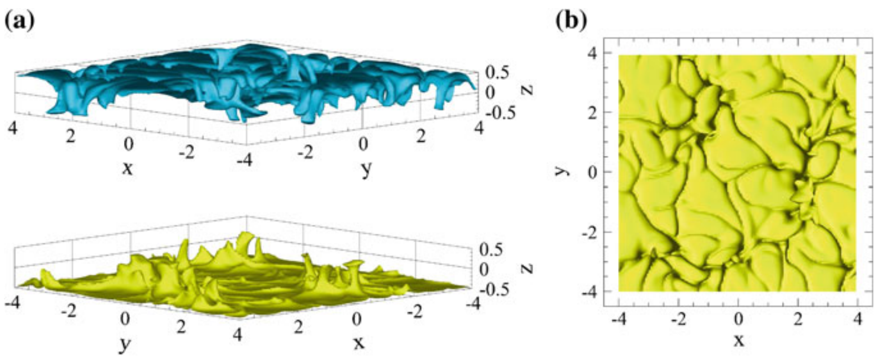

Fig. 1 **a** Isosurfaces of temperature ($\theta = 0.25$ *down* and $\theta = -0.25$ *up*) and **b** *top* view of the isosurface at $\theta = 0.25$

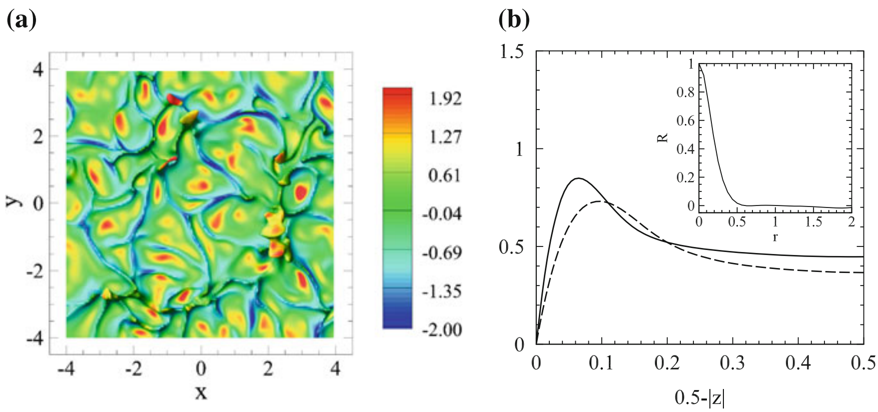

Fig. 2 **a** Top view of the isosurface at $\theta = 0.25$ coloured by div_π. **b** Main plot: $\langle div_\pi \rangle_+$ (*solid line*) and $-\langle div_\pi \rangle_-$ (*dashed line*) versus $z^* = 0.5 - |z|$. Inset: R versus r, evaluated at $z^* = 0.07$

Figure 2a shows an hot isosurface of temperature ($\theta = 0.25$) coloured with the horizontal divergence of the velocity field, $div_\pi = \partial u/\partial x + \partial v/\partial y$. Two distinct events can be identified in terms of the horizontal divergence: the emission ($div_\pi < 0$) and the impingement of thermal plumes ($div_\pi > 0$). As can be seen, concurrent regions of positive divergence are separated by thin filaments having a negative divergence, hence it is reasonable that sheet-like roots protrude from the thermal boundary layer mainly as a consequence of the mechanical action of impinging plumes. Furthermore, the intersection of different sheet-like roots leads to a large concentration of momentum, which is in turn responsible, together with the buoyancy forces, for the ejection of new structures.

Clearly, the impingement and the ejection are fundamental events in the self-sustained life cycle of thermal plumes. In order to locate the maximum of both phenomena, the contributions of impingement are separated from the ones of ejection. In the main plot of Fig. 2b the quantities $\langle div_\pi \rangle_+$ and $-\langle div_\pi \rangle_-$, where $\langle div_\pi \rangle_+ = \langle div_\pi \rangle$ for $div_\pi > 0$ and $\langle div_\pi \rangle_- = \langle div_\pi \rangle$ for $div_\pi < 0$, are displayed. These conditional statistics allow us to display the magnitude of both the impingement ($\langle div_\pi \rangle_+$) and the ejection ($\langle div_\pi \rangle_-$) as a function of the distance from the wall. As apparent, the impingement, $\langle div_\pi \rangle_+$, reaches its maximum at a distance from the wall $z^* = 0.5 - |z| = 0.07$, while the ejection, $-\langle div_\pi \rangle_-$, is peaked slightly further away from the wall, at $z^* = 0.10$, in accordance to the instantaneous field shown in Fig. 1a. Furthermore, the characteristic diameter of the impinging plume can be estimated by analyzing the two-points autocorrelation function in the wall-parallel plane at $z^* = 0.07$, where the impingement dominates the ejection. The two-points autocorrelation function is $R = \langle div_\pi(\boldsymbol{\pi}, z) div_\pi(\boldsymbol{\pi} + \mathbf{r}, z)\rangle \,/\, \left\langle div_\pi^2(\boldsymbol{\pi}, z)\right\rangle$, where $\boldsymbol{\pi} = (x, y)$ and $\mathbf{r} = (r_x, r_y)$ are respectively the position and the separation vector in the wall-parallel planes. Taking into account the statistical homogeneity and isotropy in the $x - y$ planes, the dependence of R on both the position vector $\boldsymbol{\pi}$ and the direction of $\mathbf{r}$ vanishes, hence $R = R(r, z)$, where $r = |\mathbf{r}|$. The inset of Fig. 2b displays R as a function of r evaluated at $z^* = 0.07$. As can be seen, R drops to zero at $r = 0.5$, hence this separation can be considered as the characteristic diameter of the impinging plumes.

3 Final Remarks

In conclusion, the rich dynamics observed in the near-wall region can be separated into impingement and emission events by using the planar divergence of the velocity field. From this topological analysis emerges that these events are related to each other and control the life cycle of turbulence. Since the impingement is stronger than the emission, as can be seen in Fig. 2b, a reverse transfer of thermal and kinetic energy from small towards large fluctuations is conjectured to take place in the near-wall region. However, given the multi-scale nature of these phenomena, a scale-by-scale budget is needed (and planned) in order to quantify and possibly verify these conjectures on the cascade processes occurring near the wall of RBC.

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