What Are We Like ...

Snezana Lawrence

1 Introduction

At one of the conferences on *Mathematical Cultures* held at the London Mathematical Society offices in central London, in April 2013, one of the speakers asked the audience to raise their hand if they were a mathematician. Out of about fifty participants, three did so. In June of 2014, I repeated the experiment at the Festival of Mathematics, organized by the Institute of Mathematics and its Applications at the University of Manchester, where I addressed a community perhaps larger than seventy, and got about two positive responses.

If an 'outsider' were present, I would imagine their reaction would be that of a surprise and disbelief. They would probably ask what we were all doing then at a mathematical event such as those I described. Both were held within places that the general public would see as places where mathematicians work and meet. But if we were not mathematicians where were they? And why were they letting us run the show(s)? And who were we? Perhaps the latter is the easiest to answer: in both cases, the full set of people in some way dealt with mathematics in their profession and/or made a living from mathematics. There were, in both of those audiences, philosophers of mathematicians (the ones that 'live in the digital world' as they themselves described their existence) and mathematicians who considered themselves retired yet worked in mathematics in some capacity well beyond their retirement age. A few research mathematicians (those who raised their hands proudly) and a few students of mathematics who weren't sure what to do when asked the question (in question), were also present at both events. Note that the original problem was *not* 'identify

S. Lawrence (🖂)

Institute for Education, Bath Spa University, Bath, UK e-mail: snezana@mathsisgoodforyou.com

[©] Springer International Publishing Switzerland 2016

B. Larvor (ed.), *Mathematical Cultures*, Trends in the History of Science, DOI 10.1007/978-3-319-28582-5_7

yourself if you are working in a particular mathematical discipline'. It was broadly and generally put to the audience as simply 'mathematics'.

Then on the other hand, there are many descriptions of the public disbelieving that one is a mathematician if one, or any one of the following descriptions is applicable (separately, or in any combination): female, young, good looking, articulate, fashionable, able to hold an ordinary conversation, interested in architecture, or music, or dancing.¹ What instead should a mathematician be like? According to Rensaa's (2006) empirical evidence, a mathematician is usually imagined as: a man, middle aged, with not much hair, wearing glasses, dressed in an old-fashioned way, middle weighted, not completely unfit, antisocial, ordinary and a bore. Perhaps it doesn't help that apocryphal and funny descriptions of mathematicians are those which relate to extreme cases of introvert males.

An example of such extreme case of an antisocial, strangely behaved male mathematician is the reference given in the documentary film by Paul Erdös describing his colleague and friend Sidon (Simon Sidon, Hungarian mathematician, 1892–1941). As you read this description, consider whether this is an accurate representation of how many people would imagine a mathematician to be and behave. Let us see what Erdös said:

... In 1932 I met a Hungarian mathematician called Sidon who worked mostly in trigonometric series. And he was a very good mathematician, but he was a bit crazier than the average... mathematician... In fact he was a borderline schizophrenic. They tell about him that he usually talked this way to you (turning away from the audience), he turned towards the wall and talked. But when he talked about mathematics he talked sense. And he even made it into a Hungarian anecdote book, because once when in 1937, when Turán and I visited him – he (Sidon) also had a persecution complex – so he opened a door a crack and said 'Please come at another time and to another person'... 'Kérem, jöjjenek inkább máskor és máshoz!' It sounds better in Hungarian.²

Then, there are also medically related descriptions of mathematical ability as correlated to other physical traits, as Benbow, some time ago, linked the high mathematical ability with myopia, left-handedness, and even allergies (Benbow 1987). A number of other papers written on the link between mathematics, autism, Asperger's syndrome, and physical characteristics, all tell similar stories related to mathematicians' lack of empathy or understanding of the world.³ Some of the conclusions drawn about famous mathematicians, identified not only that they are curious, but that they may have had Asperger's Syndrome.⁴ Mathematicians, so it seems according to these researches, can be recognized by some biological or psychological traits they have in common.

¹Rensaa (2006).

²Video of Paul Erdös talking about Simon Sidon, Hungarian mathematician, (1892–1941), https:// www.youtube.com/watch?v=my0L2icGooU. Accessed July 13, 14.

³See in particular (Baron-Cohen 2002; Baron-Cohen et al. 2007).

⁴See for example James (2006).

We must note at this point, that although the history of psychology and psychiatry tells us also about empirical evidence being misinterpreted⁵ on occasions, that there seems to be a body of relevant and current documentary evidence that the mathematical talent is linked to autism. Baron-Cohen, the brother of the celebrated British comedian Sasha, describes autism as having two main features: impaired empathizing, concurrent with intact or even superior systemizing abilities (Baron-Cohen 2002, 2007).⁶ The empathizing-systematizing model of autism thus described would agree with the Erdös' description of one of his friends, or of Erdös by some of his friends in turn (Halasz et al. 2002).

Let us examine this a bit further. Firstly then, the medical, psychological, and anecdotal/apocryphal all tell us about the supposedly recognisable and easily identifiable common characteristics of mathematicians. Some of these are: physical frailty, mental illness, lack of empathy. Some of the characteristics give a glimpse of hope in terms of high (probably only related to academic) achievement: high systematizing and intellectual ability,... and here we run out of things to say. In fact, that too-the high academic ability and achievement-can be seen as something that is highly undesirable if the correlation is established between high academic achievement and physically unattractive demeanour. An exercise for a reader is suggested at this point, to draw a Venn diagram of popularly held beliefs about mathematicians just numbered above in one set and overlap with the set of characteristics that would be those referring to a popularly held image of a 'professor', and meditate upon it. You can take this thinking further if you spend some time watching the famous films such as *Beautiful Mind* or *Good Will Hunting*,⁷ and draw some more conclusions about the high systematizing and intellectual ability of mathematicians and their difficulties hence in finding lasting friendship, love and happiness. Correlation and causation are completely intermingled in such popular presentations of mathematicians; we are again left with images of inept human beings but who are good at mathematics.

However, these descriptions certainly form a picture that is rather surprisingly difficult to reconcile with the experience. Having spent the last twenty years in company of mathematicians from various continents and subgroups (academics, applied mathematicians, mathematics educators and mathematical authors), I tried

⁵This is particularly interesting in the actual interpretation of behaviours. See for example (Morris 2009).

⁶One of the famous quotes of Sasha Baron-Cohen is that from his interview with the Buzz Aldrin as Ali G: 'Are you upset that Michael Jackson gets all the credit for the moonwalk but you were the first geezer to actually do it?'; http://www.telegraph.co.uk/culture/sacha-baron-cohen/10440623/Sacha-Baron-Cohen-30-best-lines.html?frame=2730797. Accessed 25th July 2014. One cannot but wonder whether his knowledge of the empathizing-systemizing theory gave him an advantage in creating characters for his comedies.

⁷A Beautiful Mind, a 2001 film by Ron Howard, staring Russell Crowe, is based on the story of the life of John Nash, a Nobel Laureate in Economics for his work in Game Theory. *Good Will Hunting* is a 1997 film by Gus Van Sant, staring Matt Damon, telling a story of an unrecognized genius who works as a janitor at a university.

to recount occasions on which I would be able to recognize any of the above mentioned characteristics that mathematicians supposedly hold in great numbers. Perhaps the extent of my experience would be a noticeable absentmindedness of my colleagues. To this end, I can give an observation and a probable empirical evidence: watching a horde of mathematicians walking up the steps of a congressional hall such as that in Madrid where the ICM was held in 2006,⁸ I was surprised to see how many people can trip on the stairs at the same time on their way to the auditorium, but then I didn't measure the steps (maybe there was an architectural mistake in the design of the staircase in question?) or measure this against any other horde of professionals walking up the same steps in temperatures well above 30 °C, and under blinding sun.⁹

The descriptions and generalizations based on medical and anecdotal grounds may have some relevance in extreme medical cases but in everyday life, they only seem to give us difficulties in the perceptions of mathematics and mathematicians. The consequence of that, in turn, is that numerous difficulties in presenting what mathematics is and what mathematicians are like, will crop up in the context of education and in the context of educating mathematics teachers.

Finally, whilst this perception of a mathematician who is inept at relationships, lacks empathy, is myopic and allergic, may be doing something to the popular image and hence influence perceptions of young people faced with the everyday task of learning mathematics (and this may also be extended to their parents' views of mathematicians), it may also have tarnished the perception of those in my audience from the beginning of this chapter. That is at least what I have originally believed. But, examining their (my audience's behaviour) in the light of some feedback from the participants, we may be surprised to find that their original shyness to identify themselves as mathematicians wasn't actually due to this, it seems widely accepted negative public view of the mathematical animal. On the contrary, the disparity between portrayal of a mathematician as an inept human being and the view of a Mathematician by those of us who actually live and earn the living through mathematics, is huge.

So let us first then identify these two groups observing that mathematical animal. One is the general public, set P: a set very large (but finite) whose views range from the *observed* being slightly odd to a raving lunatic on the one hand, and the mathematicians, set M, relatively small (comparing to the other set) whose views range from the *observed* being perhaps a member of their set, to the *observed* being a kind of universal, in some sense specially perfect being, capable of grasping mathematics beyond their own capability.

⁸Convention Center Madrid, Paseo Castellana, 99.

⁹Or perhaps it wasn't the issue of mathematicians but the sandals they were wearing.

Both sets raise the status of a mathematician to a very high-status position in their different ways. The creative mathematician making a ground-breaking discovery or proving a conjecture posed centuries ago, such as Wiles¹⁰ or Perelman¹¹, is perhaps by members of the set M considered as The Mathematician, whilst we look back at ourselves as merely meddling and marvelling at the magic of such a master. The set P's views lack the insight into what it takes to achieve the task of proving the conjecture posed centuries ago, such as Wiles or Perelman were able to do, but on the other hand members of P are able to see all the obvious characteristics of Wiles and Perelman some of which do resemble the characteristics of mathematicians described above in this chapter. This is perhaps where the image of mathematics in education poses a problematic issue, so let us look at it a bit further.

2 Mathematics and School Mathematics

There are, therefore, two images of mathematics and mathematicians: held by those 'who know' what mathematics is like, as they engage with it in various ways constantly, and views held by those who are not a privy to such insight or information about what doing original mathematics is like. These two images are not necessarily precise and sharp—let us consider that they are very clouded by individual judgements and experiences. How do these images affect the school, classroom and classroom mathematics? Are mathematics teachers exactly like mathematicians and should they even they be so? In my recent surveys, a very small number of participants who were entering the teaching profession in the South West of England as secondary mathematics teachers had formal mathematics qualification or a mathematics degree. The numbers below show percentages safely below 50 % for entrants without a mathematics degree (but some or all of those may have completed some kind of *mathematics enhancement course*¹²).

Year	Group number	With a Mathematics degree	Without a Mathematics degree
2011/2012	29	5	24
2012/2013	31	11	20
2012/2013 (group 2)	28	6	22
2013/2014	25	3	22

¹⁰Andrew Wiles is a British mathematician (born 1953), currently working at the University of Oxford, who specializes in number theory, famously having solved Fermat's Last Theorem in 1994.

¹¹Grigori Perelman is a Russian mathematician (born 1966) who proved Poincaré conjecture (posed in 1904) in 2003 for which he was granted Fields Medal, the equivalent of Nobel Prize, which he refused to accept.

¹²At my university for example, the Mathematics Enhancement Course lasts for six months and covers mathematics up to university level.

This may suggest that great majority of secondary mathematics teachers have never been, for any prolonged period of time, working alongside practicing mathematicians or have indeed had experience of learning mathematics at a university level. In other words, less than half of mathematics teachers belong to the set M upon their entrance to the profession. It also means that mathematics teachers seem to have very small or negligible knowledge of what mathematicians are like as they had small or negligible experience of meeting with mathematicians. This inevitably extends to mathematics itself: the majority of mathematics teachers seem therefore to have a small or negligible experience of what doing (higher) mathematics is like.

I believe this poses two questions (at least). One is this: should the school mathematics resemble 'real' mathematics or mathematics 'proper' (this implies that we believe there are identifiable differences between the two)? And the second: if teachers are not mathematicians, can we expect them to be expert in initiating young people into mathematics 'proper' in any meaningful way?

The first questions is discussed and described well by Watson (2008) as a disparity between school and real mathematics and the inability of our educational system to mimic the 'real' mathematics.¹³ Whilst research mathematics relies on creative ability in mathematicians, and the cultural and social interaction over a protracted time among the groups of mathematicians, the school mathematics seems to disregard all three and insists instead on the mastery of the techniques, ability to transfer problem-solving skills from the textbook to the exam, and the performance under exam duress. One may argue that all these aspects of doing mathematics are also present in the work of mathematicians, but the emphasis is different (Watson 2008; Lawrence 2012; Burton 1999).

Like in any other culture related to learning and intellectual work of some kind—national, professional, organizational, the rules for joining may not be obvious, but they exist (Lawrence 2002). So let us look at some of the aspects of doing mathematics 'proper' that may translate into doing mathematics in a classroom, in the view that these rules may be made explicit to potential entrants to the mathematical field.

In mathematics 'proper', mathematicians cluster around different interests, universities, and research and interest groups.¹⁴ In school mathematics, the teachers' clustering is encouraged via subject associations (since the 19th century and more recently via online networks, such as 2006 born NCETM¹⁵ for example). The enculturation process is one that is perhaps not investigated enough and yet has a

¹³See also Lawrence (2012) who describes an experiment to introduce inquiry-led research model of learning of mathematics with children who opted to study mathematics beyond the curriculum and form research group with an overseas group of peers.

¹⁴See for example recent Michael Harris recent *Mathematics without Apologies: portrait of a problematic vocation*, Princeton University Press.

¹⁵National Centre for Excellence in the Teaching of Mathematics, founded by the government as one of the recommendations of the Smith (2004) report, whose primary aim was to promote networking between mathematics teachers and their exchange of information about mathematics pedagogy, including self-assessment mathematics knowledge online tools.

great impact on people wanting to become, and becoming mathematics teachers. This enculturation should consist of any or all of the following: the individual participatory appropriation, interpersonal or guided participation, and entering the community plane via an apprenticeship (Rogoff et al. 1995: 178). Likewise the enculturation process should then be transferred from teachers to their pupils in any number of ways, including the transference of the view of what doing mathematics should really be like (Masingila 2002).

In the mathematics classrooms instead we are changing the seating of pupils when it suits our behaviour management plan and put pupils into new groups in which they need to work collaboratively from 1 h (lesson) to the next, not knowing whether they will ever work in that same group again. This particular insistence on the collaborative, or group work within the mathematics classroom without giving pupils freedom to choose or form groups, is criticized by Popkewitz (2004: 3).

Now emerges also an image of mathematics as a field of activity in which transference from the mathematics 'proper' to mathematics teachers and further to mathematics pupils is problematic to specify and identify (Furinghetti 1993). There are many problems in trying to model collaboration that happens in a community of practitioners to that of a classroom. In a classroom, pupils all have to engage in activities whether they like the activities or indeed the topic that is being worked on or not. The research or industrial mathematicians have worked hard to achieve their independence from that necessity and engage in their communities of practice, albeit being aware of the constraints placed on them by their organizations and funding bodies (Rowlett 2011; Harris 2015).

Indeed, let us for a moment compare the view of mathematics classroom with a research mathematics laboratory (such as that described by Watson 2008). In the classroom the pace of learning is to be fast (all learning is to be achieved within prescribed 45 or 50 or 60 min), collaboration and group work are to be valued, discovery of mathematical facts to be shown and nurtured, and understanding to be assessed before the teacher 'moves on' with further explanations and questions. A mathematician instead seeks to find what works for them, in collaboration (grown over a prolonged period of time) or on their own, finding their own way through the dark chaos that Wiles described mathematics is, until things fall into place to "make the room illuminated".¹⁶ Our idealistic view that mathematics education will somehow resemble mathematics that is done by mathematicians momentarily seems to be unobtainable, but then we do remember that in other school subjects there is also mimicking of other disciplines. So if the school mathematics cannot be 'real' mathematics, is it possible to nurture the view of mathematics that is as close to 'real' mathematics that would enable children to strive to become 'real' rather than proto-mathematicians? And what allowances are there for teachers to develop this with the various groups of pupils in a mathematics classroom if they too are searching for an image of mathematics and themselves as mathematicians that they find ephemeral and fluid, if not unknown or unidentifiable?

¹⁶BBC documentary Horizon: Fermat's Last Theorem, first shown 15 Jan 1996.

Just as any other rhetorical question, so this one already has an answer. If we were to judge by the above described practices, there is no space or time for supporting the young to strive and be mathematicians: it is not because, but despite the practices of teaching for immediate and test-measured results that some of our pupils still want to pursue mathematics after their schooling.

On the other hand, to get all pupils to engage with lower-level mathematics and to behave as we imagine they should behave in a mathematics classroom is hence marred with the strategies of coercion. The 'successful' coercion strategies in the mathematics classroom go from motivation through grades, to motivation through the creation of an enjoyable classroom atmosphere, but the leading question-answer dialogue is all the same coercive as it often does 'not allow for contradiction, inhibits students from constructing analogies, explanations, and justifications' (Cobb et al. 1991: 451). This should not be surprising as the outcomes of research mathematics and the school mathematics are entirely different in their present form. Perhaps, if we imagined, as some dare to do,¹⁷ a school which allows for additional activities apart from those prescribed by the National Curriculum, and organised around the research interests of staff and students, some greater similarities may be developed between the two mathematics. The outputs would then include those that may not have necessarily been predicted, for example a collaboration with NASA (and to be cynical, how would Ofsted rate that?).¹⁸

Mathematics, in the sense of Bakhtin (1981: 289), can be seen as a field of 'competing intellectual traditions whose relations form' mathematics as a discipline. But, unlike other subjects and their relative disciplines, school mathematics does not promote the view of mathematics as a field of cultural practice. We simply do not provide the frame of reference for culturally contextualized mathematical discovery mainly because of the lack of knowledge and training in the same (Lawrence and Ransom 2011). Do not think though that I am advocating a prescriptive 'cultural' training in mathematics education: culture is made from and through a dialogue, as conclusions from experimental studies on mathematicians' and mathematics educators' views of mathematics (Mura 1993, 1995) show us clearly. In other words, the field of dialogue about the nature of mathematics and the need we as species and societies have for mathematics is constantly being re-examined by mathematicians—this *is* culture, which is seen from the outside but from the inside (the circle of mathematicians) is just a way of being. Could that sense of *being*, or being immersed in mathematics be achieved in the mathematics classroom too? To attempt an answer to that, let us examine how mathematics can be portrayed within mathematics education.

¹⁷See for example the research culture developed and nurtured in a secondary school in Kent by allowing for teachers and pupils to spend allocated time each week on research projects around the research interests they have http://www.thelangton.org.uk/lucid/.

¹⁸See Lawrence (2012).

3 The Dichotomies of Mathematics

What do then mathematicians discuss that makes their environment rich in dialogue and 'mathematical culture'? And on the other hand, what is the prevailing view of mathematics in mathematics education? The generality of these questions makes them impossible to answer fully, and indeed, if any of these questions were put to us personally we would most probably attempt to answer them in a prolonged way that would itself generate a dialogue. But let us examine a few views that are most commonly put forward.

Most recently, the educational reform in general and the mathematics educational reform in the UK in particular, has brought to the surface many questions about what kind of mathematics we want in our educational system, and hence the questions on the dichotomies in mathematics itself. For mathematics to be regarded less of a bore and more relevant (Smith 2004), engaging (QCA 2009), and inspiring, a number of national enquiries and projects have been undertaken.¹⁹

Perhaps the most important image mathematics has projected into the field of mathematics education is its relation to the humanities and the arts, and the various dichotomies are thus being identified and discussed in this context. I list three occasions that may shed some light on the issue.

The first such occasion was general and looked at sciences, including mathematics, and set against the humanities. In his influential Rede lecture, Snow (1959), a chemist and an author, argued that the difference between humanities and sciences was harmful. He also developed an argument that the general knowledge and appreciation of sciences are as important as they are in the humanities yet not valued by western society in general:

A good many times I have been present at gatherings of people who, by the standards of the traditional culture, are thought highly educated and who have with considerable gusto been expressing their incredulity at the illiteracy of scientists. Once or twice I have been provoked and have asked the company how many of them could describe the Second Law of Thermodynamics. The response was cold: it was also negative. Yet I was asking something which is about the scientific equivalent of: *Have you read a work of Shakespeare's*?

Corollary of this is the view of a mathematician as someone dealing with the culturally obscure and difficult to grasp concepts and artefacts, in contrast to an artist or a writer. And then this theme has been taken further more recently, by Tim Gowers, a Cambridge based mathematician who was trying to exemplify the 'two cultures' in mathematics itself (Gowers 2002) by which he meant the differences of viewing mathematics in (to oversimplify) two distinct ways: a view first that we study problems in mathematics in order to understand it itself, and secondly that we study mathematics in order to be able to solve problems. Mathematics education

¹⁹See blog *Mathematics Reports* which lists more than 50 reports published in Great Britain on the state of mathematics education between beginning of 2011 and December 2013, http://mathsreports.wordpress.com. Accessed 15 July 2014.

seems firmly grounded in the latter view of mathematics and therefore the mathematician, as a consequence, is seen as an abstract problem solver.²⁰

The third dichotomy I wanted to describe was documented in Marcus du Sautoy's²¹ TED lecture on 6th Jan 2012 in Oxford. There he spoke about the dichotomy between arts and mathematics and how he, a creative and very productive mathematician, finds arts as crucial to his work as a mathematician, as he believes mathematics contributes to the work of great artists. A consequence of this view would be that a mathematician's ideas are sources of creativity and inspiration, something that is not obvious in mathematics education.

What seems to be a common theme running through these three famous statements is that, whilst we believe that generally in our culture we separate mathematics from the humanities and arts, mathematicians in their work behave in both 'artistic' and 'mathematical' ways but in the ways which are not obvious, easy for public to grasp, or appreciate.

There are many other distinctions between types of mathematics and opinions on its nature which make a difference to the image of what mathematics is for and therefore determine to a certain degree how mathematicians and mathematics teachers could and should behave (too numerous to mention here). The battles that are being fought outside of the mathematics classrooms between the various and ever multiplying camps of believers in the absolutism of the necessity of one of the approaches to mathematics as *the* one that should be adopted in mathematics education, are imposed on the innocent at the front line of mathematics education the teachers and pupils. On the one hand then, the mathematics as a school discipline is being shaped by the conflicting views of what mathematics should be and how it is taught, and on the other, the image of mathematician as an inept imaginary genius is being projected to the learners of the subject as we have shown above.

How can then mathematics education answer these problems to provide meaningful and productive outcomes? And should it attempt to do so? Thompson (1984) suggests that not only teachers' views of mathematics vary to such a degree that they are sometimes not possible to reconcile with the views of mathematics shaped by the programmes of study (such as national curricula), but goes on to suggest that it is difficult or near-impossible to change the teachers' conceptions of mathematics once they are formed. Furthermore, she noticed the same trend that I have witnessed from talking to mathematics teachers which is that the bridge that needs to be built by the 'reformers' and academic communities between the views of mathematics and mathematics education they hold, and the views of teachers, are projected and

 $^{^{20}}$ An issue that is perhaps interesting to investigate here would be the influence of the *How to Solve it*, a book published at the start of the Cold War in 1945 by George Polya in the USA (Polya was originally from the Eastern Block).

²¹The Simonyi Professor for the Public Understanding of Science and a Professor of Mathematics at the University of Oxford, see https://www.youtube.com/watch?v=2v3IWGiThKA. Accessed 15th July 2014.

designed on one premise: that it is teachers who need to change for mathematics education to change or succeed, or be more successful.

Perhaps, then, it would be prudent to ask teachers to tell us whether they are prepared to change their views, what their views of mathematics are, and therefore how can mathematics education respond to preparing the children to participate in it. Because 'their change is not our business; how when and if they change is surely their concern alone...' (Pimm 1993: 31).

4 The Multiple Identities

So what should we be like? And who is this 'we'? Mathematicians, mathematical engineers, mathematics computer scientists, applied mathematicians, research mathematicians, philosophers of mathematics, mathematics teachers and mathematics teacher educators—all have in common mathematics, but the mathematics they do is very different in each case. (Let us go back to our set M and consider all these groups to be its subsets.)

The multiplicity of views, the mathematicians' own and the disparity between mathematics education and mathematics 'proper' seems to be something that is of crucial importance for a teacher in training, and those supporting them in that task. An experiment that dealt particularly with this aspect of the training of mathematics teachers came to the conclusion that there are four stages in teachers' beliefs about what mathematics is and what they therefore should be like. These were listed as belonging to four major stages in the building of identity of a mathematics teacher:

- 1. Dualism—any proposition or act must be right or wrong
- 2. Multiplicity—a plurality of view-points exist, but no internal structure or external relationships exist
- 3. Relativism-a plurality of viewpoints exist, and context is very important
- 4. Commitment—one personally commits to a mode of action and belief. (Bush et al. 1990:43).

This agrees well with the studies mentioned above that conclude that the teachers' beliefs about mathematics are difficult or impossible to change once they have formed, and explains the phenomenon further. It may then, not be so much about the views but rather about selecting a set of beliefs that one commits to propagating through one's work in mathematics education. This 'set of beliefs' however does not relate to beliefs in the right or wrong sense of the word: it can refer to beliefs of one's own ability and orientation. A certain sense of inevitability sets if this is an accepted view. The teacher training programmes encourage teachers to form their own identities and position themselves in relation to mathematics.

What about the mathematics learners, the pupils of those teachers that will soon have their own classes to take care of? What do we do in schools that is similar to allowing pupils to position themselves in relation to mathematics and mathematics learning? How do we account for students' beliefs about mathematics and their place in mathematics education—how do they position themselves during that process? Ignaci et al. (2006) in fact defined such *positioning* as 'mathematics self-concept': it is a self-contained personal positioning that includes personal beliefs and judgments about mathematics, mathematical ability and the experience of doing mathematics. It turns out that this "mathematics self-concept" is the third most important one in the students' actual results in mathematics. The first two were the actual performance on tests and the difficulties in learning mathematics; leaving behind things such as family support, teacher and learning support to name but a few (Ngirande 2014).

A wild thought to leave with you, which takes us a little back in the chapter: as teachers themselves are not mathematicians (usually come from non-mathematical backgrounds) could this orientation of their own, and of their pupils, somehow be interlinked and should their exploration of mathematics and their positioning against, or within it be nurtured? Teachers learning along their pupils (some of whom will eventually hopefully become better mathematicians 'proper' than their teachers) may be another field to explore in this context.

5 Conclusion

Towards the end of this chapter, one may exclaim, "Is there any hope for mathematics education then?" and equally "Is there any hope that the image of mathematicians will change to be one of authenticity and diversity rather than caricature?" Do not despair, these are not new questions, and although they have not been resolved, they offer opportunities for further investigation rather than depression. These problems have existed since at least Plato: look at his example such as the dialogue between Socrates and Meno (Plato c. 402BC) and reflect upon the many examples of publications of textbooks during the French Revolution such as Monge's (1798)²² book in which he made a proclamation of the importance of mathematics to national prestige giving birth to an image of a *revolutionary* mathematician (Alexander 2011), or some centuries earlier the proclamation Dee made on the importance of mathematics in developing and nurturing 'beautiful' minds (Dee 1570). Then equally, consider that mathematics can be seen as a corruptor of youth in times of social turmoil or revolution²³.

²²Monge, in the first edition of his *Geometrié Descriptive* published in an III exclaimed: In order to raise the French nation from the position of dependence on foreign industry, in which it has continued to the present time, it is necessary in the first place to direct national education towards an acquaintance with matters which demand exactness, a study which hitherto has been totally neglected; and to accustom the hands of our artificers to the handling of tools of all kinds, which serve to give precision to workmanship, and for estimating its different degrees of excellence. Then the consumer, appreciating exactness, will be able to insist upon it in the various types of workmanship and to fix its proper price; and our craftsmen, accustomed to it from an early age, will be capable of attaining it (Monge 1798: ix).

²³See Lawrence (2008), noting example warning by Patriarch Grigorios V that was issued in 1819 against mathematicians and mathematical studies: 'cubes and triangles, logarithms and symbolic calculus... bring apathy... jeopardizing our irreproachable faith'.

History shows us that the images of mathematics and mathematicians can in this way be set against each other to purposes that may be entirely opposing and desirable to entirely opposing sections of the society—even more so as societies and their definitions and interests change (Lim 1999). This makes the situation a new teacher faces very complex indeed: he or she will in every case have to reconcile the image of mathematics with the image of a mathematician in such a multiplicity of possibilities in the most natural and authentic way it feels to them. In this respect any opportunity that is given to teachers to explore the nature of mathematics by working on mathematics content would mean an opportunity to explore what it is they are doing and how it affects them, so that they can, in time, begin to articulate that and communicate it to, or with, their students. But to resemble 'real mathematical culture' this must also be given social context for discussions to develop between colleagues, and between pupils too. In other words, we need to develop mechanisms for including teachers and pupils into the set M.

I will now ask the reader to close their eyes and imagine various mathematicians according to the multiple descriptions of mathematics and mathematicians she/he has read about in this chapter. Then think of recent images of mathematicians you may have come across, and consider whether that makes a positive change to the "What are we like?" question.

An authentic engagement with the dialogues about mathematics would end the stereotyping of mathematicians (Devlin 2001), but the opportunities for this need to exist. 'Mathematical positioning' could then take diverse manifestations with different students of mathematics, be they young, old, or even maths teachers. And then, perhaps, at the conferences as described at the beginning of this chapter, all of us who make living out of mathematics, and do some mathematics for living, attend mathematical conferences and have business cards with a word 'mathematics' embossed on them, could possibly be able to identify ourselves as purely, simply, mathematicians—no super-, or sub- but just mathematicians.

References

- Alexander, A. (2011). Heroes, Martyrs, and the rise of modern mathematics. Cambridge Massachusetts US and London UK: Harvard University Press.
- Bakhtin, M. M. (1981). *The dialogic imagination: Four essays*. University of Texas Press, Austin and London.
- Baron-Cohen, S. (2002). The extreme male brain theory of autism. *Trends in Cognitive Science*, 6, 248–254.
- Baron-Cohen, S., Wheelwright, S., Burtenshaw, A., & Hobson, E. (2007). Mathematical talent is linked to autism. *Human Nature*, 18, 125–131.
- Benbow, C. B. (1987). Possible biological correlates of precocious mathematical reasoning ability. *Trends in the Neurosciences*, 10, 17–20.
- Burton, L. (1999). The practices of mathematicians: What do they tell us about coming to know mathematics? In *Educational studies in mathematics*, (Vol. 37, No. 2, 1998–1999, pp. 121–143). Berlin: Springer.
- Bush, W. S., Lamb, C. E., & Alsina, I. (1990). Gaining certification to teach secondary mathematics: A study of three teachers from other disciplines. *Focus on Learning Problems in Mathematics*, 12(1), 41–60.

- Cobb, P., Wood, T., & Yackel, E. (1991). A constructivist approach to second grade mathematics. In E. von Glaserfield (Ed.), *Radical constructivism in mathematics education* (pp. 157–176). Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Dee, J. (1570). Mathematical preface (Euclid translated by Billingsley, Trans.). London.
- Devlin, K. (2001). As others see us, MAA Online http://202.38.126.65/mirror/www.maa.org/ devlin/devlin_2_01.html
- Furinghetti, F. (1993). Images of mathematics outside the community of mathematicians: Evidence and explanations. For the Learning of Mathematics, 12(2), 33–38.
- Gowers, T. (2002). Two cultures of mathematics. In: V. I. Arnold, M. Atiyah, P. D. Lax & B Mazur (Eds.), *Mathematics: Frontiers and perspectives*.
- Halasz, G., Lovasz, L., Simonovits, M., Sos, V. eds. (2002). *Paul Erdös and His Mathematics*. Bolyai Society Mathematical Studies). Berlin: Springer.
- Harris, M. (2015). *Mathematics without apologies: Portrait of a problematic vocation*. Princeton University Press. Princeton, US.
- Ignacio, N. G., Nieto, L. J. B., & Barona, E. G. (2006). The affective domain in mathematics learning. *International Electronic Journal of Mathematics Education*, 1(1), 16–32.
- James, I. (2006). Asperger's syndrome and high achievement: Some very remarkable people. London and Philadelphia: Jessica Kingsley Publishers.
- Lawrence, S. (2002). *Geometry of architecture and freemasonry in 19th century England*. PhD Thesis submitted at the Open University England.
- Lawrence, S. (2008). A balkan trilogy: Mathematics in the balkans before world war I. In E. Robson & J. Steddall (Eds.), *The oxford handbook of the history of mathematics*. Oxford University Press, Oxford.
- Lawrence S. & Ransom, P. (2011). How much meaning can we construct around geometric constructions? In *Proceedings of the 7th european congress on research in mathematics education*, Poland.
- Lawrence, S. (2012). Enquiry led learning and the history of mathematics. In Thomas de Vittori (Ed.), *Innovative methods for science Education: History of science, ICT and Inquiry based science teaching.* Berlin: Frank & Time.
- Lim, C. S. (1999). The public images of Mathematics. Unpublished doctoral thesis, Universiti of Exeter, United Kingdom.
- Masingila, J. O. (2002). Examining students' perceptions of their everyday Mathematics practice. Journal for Research in Mathematics Education, Monograph, 11: 30–39. NCTM.
- Monge, G. (1798). Géometrie Descriptive. Leçons donnée aux Écoles Normales, L'an 3 de la République. Baudouin, Paris.
- Morris, E. K. (2009). A case study in the Mi representation of applied behaviour analysis in autism: The Gernsbacher lectures. *Behavior Analysis*, 32(1), 205–240.
- Mura, R. (1993). Images of Mathematics Held by University Teachers of Mathematical Sciences. *Educational Studies in Mathematics*, 25(4), 375–385.
- Mura, R. (1995). Images of Mathematics Held by University Teachers of Mathematics Education. *Educational Studies in Mathematics*, 28(4), 385–399.
- Ngirande, H., & Mutodi, P. (2014). The influence of students' perceptions on Mathematics performance. A case of a selected high school in South Africa. *Mediterranean Journal of Social Sciences*, 5:3, 431–446. Rome.
- Pimm, D. (1993). From should to could: Reflections on possibilities of mathematics teacher education. For the Learning of Mathematics, 13(2), 27–32.
- Popkewitz, T. S. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. In *American educational research journal*, (Vol. 41, Spring 2004, pp. 3–34). Routledge, New York & Abingdon, Oxon.
- Rensaa, R. J. (2006). The image of a Mathematician, *Philosophy of Mathematics Education Journal*, 19. Retrieved from http://www.people.ex.ac.uk/PErnest/

- Rogoff, B., Baker-Sennett, J., Lacasa, P., & Goldsmith, D. (1995). Development through participation in sociocultural activity. In *Cultural practices as contexts for development* (Vol. 67, pp. 45–65). Springer.
- Rowlett, P. (2011). The unplanned impact of mathematics. Nature, 475, 166-169 (14 July 2011).
- Smith, A. (2004). Inquiry into post-14 mathematics education, www.tda.gov.uk/upload/resources/ pdf/m/mathsinquiry_finalreport.pdf.

Snow, C. P. (1959). The two cultures. The Rede Lecture: University of Cambridge.

- Thompson, A. G. (1984). Teachers' beliefs and conception: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.
- Watson, A. (2008). School mathematics as a special kind of Mathematics. For the Learning of Mathematics, 28(3), 3–8.