A Novel Approach to Fuzzy Rough Set-Based Analysis of Information Systems

Alicja Mieszkowicz-Rolka and Leszek Rolka

Abstract This paper presents an approach to analysis of crisp and fuzzy information systems. It is based on comparison of elements of the universe to prototypes of condition and decision classes instead of using binary crisp indiscernibility or fuzzy similarity relations. We introduce several notions, such as dominating linguistic values, linguistic labels, characteristic elements, which lead to a new definition of fuzzy rough approximations. The presented method gives the same results as the original rough set theory of Pawlak, in the special case of crisp information systems. Furthermore, fuzzy information systems can be analyzed more efficiently than in the standard fuzzy rough set approach. Moreover, interpretation of results is quite natural and intuitive. Analysis of information systems will be illustrated with an extended example.

Keywords Information systems · Fuzzy sets · Rough sets · Fuzzy rough sets

1 Introduction

Fuzzy set theory, founded by Zadeh [1], is one of the most popular approaches used for modeling the human reasoning process. The characteristic feature and ability of a human operator to utilize vague and linguistic terms rather than numbers can be expressed with the help of fuzzy sets defined on a respective domain of interest.

Another way of dealing with uncertainty and imperfect knowledge was proposed by Pawlak [2] in the framework of the rough set theory. The crucial point of that approach consists of comparing elements of a universe of discourse (or rows of a decision table) by an indiscernibility relation with respect to selected condition and

A. Mieszkowicz-Rolka (✉) · L. Rolka
Department of Avionics and Control,
Rzeszów University of Technology, Rzeszów, Poland
e-mail: alicjamr@prz.edu.pl

L. Rolka
e-mail: leszekr@prz.edu.pl

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decision attributes. The consistency and redundancy of a given information system can then be evaluated by checking the dependencies between the obtained families of indiscernibility classes.

Since both theories consider distinct properties of the human reasoning process, an idea of combing them together into one approach seems to be quite obvious. The most important definition of fuzzy rough set was proposed by Dubois and Prade [3]. This concept was widely studied and developed by many authors, e.g. [4–7], [8].

In the present paper, we propose a modification of the basic assumptions and definitions of the fuzzy rough set model. The modified approach is in accordance with the original rough set theory of Pawlak, when the special case of crisp information system is considered. The crucial point of our proposal consists in changing the method of determination of fuzzy similarity classes. In the standard fuzzy rough set model, a binary fuzzy similarity relation is used for comparing all elements of the universe to each other. In our method, the fuzzy similarity, for all elements of the universe, is determined with respect to fuzzy prototypes (ideal elements), which can be seen as labels of linguistic values of particular condition and decision attributes. Furthermore, a new definition of fuzzy rough approximation is given.

Before we present the details of our approach, we need to recall basic notions and definitions of the crisp and fuzzy rough set theories.

2 Preliminaries

Although, both crisp and fuzzy information systems are taken into account in this paper, we only recall our definition of a fuzzy information system [4]: a crisp information system constitutes a special case.

Definition 1 A fuzzy information system ISF is the 4-tuple $S = \langle U, Q, \mathbb{V}, f \rangle$, where

U is a nonempty set, called the universe,

Q is a finite set of fuzzy attributes,

$\mathbb{V}$ is a set of fuzzy (linguistic) values of attributes, $\mathbb{V} = \bigcup_{q \in Q} \mathbb{V}_q$,
$\mathbb{V}_q$ is the set of linguistic values of an attribute $q \in Q$,

f is an information function, $f\colon U \times \mathbb{V} \to [0, 1]$,
$f(x, V) \in [0, 1]$ for every $V \in \mathbb{V}$ and every $x \in U$.

In practical applications, information systems are conveniently expressed in the form of a decision table, with the set of attributes Q composed of two disjoint sets: condition attributes C and decision attributes D. A column of the decision table contains values of a single condition or decision attribute for all elements of the universe U. Every row of the decision table contains a description (values of all attributes) of a single element of the universe U, which corresponds to a decision.

In the case of a fuzzy information system, we define linguistic values for all condition and decision attributes [9]. Let us assume a finite universe U with

N elements: $U = \{x_1, x_2, \ldots, x_N\}$. An element x of the universe U will be described with fuzzy attributes, which consists of a subset of n condition attributes $C = \{c_1, c_2, \ldots, c_n\}$, and a subset of m decision attributes $D = \{d_1, d_2, \ldots, d_m\}$. Next, we assign to every fuzzy attribute a set of linguistic values: $\mathbb{C}_i = \{C_{i1}, C_{i2}, \ldots, C_{in_i}\}$, which is a family of linguistic values of the condition attribute c_i, and a set $\mathbb{D}_j = \{D_{j1}, D_{j2}, \ldots, D_{jm_j}\}$, which is a family of linguistic values of the decision attribute d_j, where n_i and m_j are the numbers of the linguistic values of the i-th condition and the j-th decision attribute, respectively, $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, m$.

In stage of building a decision table with fuzzy attributes, we need to determine the membership degrees for all elements $x \in U$, in all linguistic values of the condition attributes c_i and the decision attributes d_j, respectively, where $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, m$. This is done by fuzzification of the original values of particular attributes in their domains of interest. The obtained membership degrees for a single attribute and an element $x \in U$, can be expressed as a (fuzzy) value, which is a fuzzy set on the discrete domain of all linguistic values of that attribute [9].

The fuzzy value $\mathbb{C}_i(x)$ for any $x \in U$, and the condition attribute c_i, is a fuzzy set on the domain of the linguistic values of the attribute c_i

$$\mathbb{C}_i(x) = \{\mu_{C_{i1}}(x)/C_{i1}, \mu_{C_{i2}}(x)/C_{i2}, \ldots, \mu_{C_{in_i}}(x)/C_{in_i}\}. \tag{1}$$

The fuzzy value $\mathbb{D}_j(x)$ for any $x \in U$, and the decision attribute d_j, is a fuzzy set on the domain of the linguistic values of the attribute d_j

$$\mathbb{D}_j(x) = \{\mu_{D_{j1}}(x)/D_{j1}, \mu_{D_{j2}}(x)/D_{j2}, \ldots, \mu_{D_{jm_j}}(x)/D_{jm_j}\}. \tag{2}$$

For crisp condition and decision attributes, the sets (1) and (2) have only one single element with a membership degree equal to 1. For fuzzy attributes, several elements of the sets (1) and (2) can have a non-zero membership degree denoted with by a value in the interval [0, 1]. As we see, for crisp attributes, the cardinality of the sets (1) and (2) is always equal to 1. In order to be in accordance with this property, we should assume that for any $x \in U$, the fuzzy cardinality (power) for all linguistic values $\mathbb{C}_i(x)$ and $\mathbb{D}_j(x) (i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$ satisfies the requirements

$$\text{power}(\mathbb{C}_i(x)) = \sum_{k=1}^{n_i} \mu_{C_{ik}}(x) = 1, \quad \text{power}(\mathbb{D}_j(x)) = \sum_{k=1}^{m_j} \mu_{D_{jk}}(x) = 1. \tag{3}$$

The notion of fuzzy rough set was proposed by Dubois and Prade [3] and generalized by Radzikowska and Kerre [5]. For a given fuzzy set A and a fuzzy partition $\Phi = \{F_1, F_2, \ldots, F_n\}$ on the universe U, the membership functions of the lower and upper approximations of A by Φ are defined as follows

$$\mu_{\underline{\Phi}(A)}(F_i) = \inf_{x \in U} \mathrm{I}(\mu_{F_i}(x), \mu_A(x)), \tag{4}$$

$$\mu_{\bar{\Phi}(A)}(F_i) = \sup_{x \in U} \mathrm{T}(\mu_{F_i}(x), \mu_A(x)), \tag{5}$$

where T and I denote a T-norm operator and an implicator, respectively, $(i = 1, 2, \ldots, n)$. The pair of sets $(\underline{\Phi}A, \bar{\Phi}A)$ is called a fuzzy rough set.

Since we analyze a fuzzy information system, we use a fuzzy partition Φ_C, which is generated with respect to condition attributes C, and a fuzzy partition Φ_D, determined by taking into account decision attributes D, respectively. In order to investigate the properties of the information system, the partition Φ_C will be taken for approximation of fuzzy similarity classes from the partition Φ_D. The standard way of generating fuzzy similarity classes is based on comparing elements of the universe U. To this end, one can apply a symmetric binary T-transitive fuzzy similarity relation [10], which is expressed by means of the distance between the compared elements. If we want to compare any two elements x and y of the universe U with respect to the condition attributes c_i, $i = 1, 2, \ldots, n$, then the similarity between x and y can be expressed using a T-similarity relation [7] based on the Łukasiewicz T-norm

$$S_{c_i}(x, y) = 1 - \max_{k=1,n_i} \left| \mu_{C_{ik}}(x) - \mu_{C_{ik}}(y) \right|. \tag{6}$$

For determining the similarity $S_C(x, y)$, with respect to all condition attributes C, we aggregate the results obtained for particular attributes c_i, $i = 1, 2, \ldots, n$. This can be done by using the T-norm operator `min` as follows

$$S_C(x, y) = \min_{i=1,n} S_{c_i}(x, y) = \min_{i=1,n} (1 - \max_{k=1,n_i} \left| \mu_{C_{ik}}(x) - \mu_{C_{ik}}(y) \right|). \tag{7}$$

We determine in the same manner the similarity $S_D(x, y)$ for any two elements x and y of the universe U with respect to all decisions attributes d_j, $j = 1, 2, \ldots, m$.

We obtain two symmetric similarity matrices as the result of calculating the similarity for all pairs of elements of the universe U.

In the special case of a crisp information system, the similarity relations $S_C(x, y)$ and $S_D(x, y)$ assume the form of crisp binary indiscernibility (equivalence) relations R_C and R_D, obtained by taking into account the condition attributes C and the decision attributes D, respectively. This way, we get two crisp partitions of the universe U as families of disjoint indiscernibility classes. In consequence, the fuzzy rough approximations (4) and (5) become equivalent to the lower approximation $\underline{R}(A)$ and the upper approximation $\bar{R}(A)$ of a crisp set A, by an indiscernibility relation R, which were defined [1] as follows

$$\underline{R}(A) = \{x \in U\colon [x]_R \subseteq A\}, \tag{8}$$

$$\bar{R}(A) = \{x \in U\colon [x]_R \cap A \neq \emptyset\}, \tag{9}$$

where $[x]_R$ denotes an indiscernibility class that contains the element $x \in U$.

3 Similarity Classes Based on Linguistic Values of Attributes

The starting point of our approach is a different way of determination of fuzzy similarity classes. In contrast to the standard method recalled in previous section, we want to construct the similarity classes with respect to linguistic values of particular condition and decision attributes. This can be motivated by the fact that a human operator (expert) does not necessary compare every observed object (element) of a universe to each other. He or she performs rather a comparison of a new element to a limited group of selected prototypes. It seems quite natural to assume that those prototypes correspond to combinations of linguistic values of condition and decision attributes. Moreover, such prototypes can be ideals and may only exist in the mind of the human operator.

Now, we introduce all notions and definitions needed in the formal description of our approach. Basing on the definition of a fuzzy information system (Definition 1) given in previous section, we define a notion of dominating linguistic values.

Definition 2 For a given fuzzy information system ISF, the set of dominating linguistic values of any element $x \in U$ and any fuzzy attribute $q \in Q$ is a subset $\widehat{\mathbb{V}}_q(x) \subseteq \mathbb{V}_q$ of the linguistic values of the attribute q, expressed as

$$\widehat{\mathbb{V}}_q(x) = \{\mathrm{V} \in \mathbb{V}_q\colon f(x, \mathrm{V}) \geq 0.5\}. \tag{10}$$

The above definition is written in a general form. In the following, we denote by $\widehat{\mathbb{C}}_i(x)$ the set of dominating linguistic values of $x \in U$ for a fuzzy condition attribute $c_i \in C$, and by $\widehat{\mathbb{D}}_j(x)$ the set of dominating linguistic values of $x \in U$ for a fuzzy decision attribute $d_j \in D$, respectively, where $i = 1, 2, \ldots, n$, and $j = 1, 2, \ldots, m$.

Let us take a closer look at the properties of sets of dominating linguistic values. We impose the requirements (3) on the linguistic values of every attribute. Hence, we find one or, in a rare case, two dominating linguistic values (when $f(x, \mathrm{V}) = 0.5$). Moreover, in a crisp information system only one dominating value is possible.

We want to find dominating combinations of linguistic values of any $x \in U$, with respect to a subset of attributes. Therefore, we introduce the notion of linguistic label.

Definition 3 The set of linguistic labels $\mathbb{E}^P(x)$ of any element $x \in U$ for a subset of fuzzy attributes $P \subseteq Q$ is the cartesian product of the sets of dominating linguistic values $\widehat{\mathbb{V}}_p$, for $p \in P$

$$\mathbb{E}^P(x) = \prod_{p \in P} \widehat{\mathbb{V}}_p(x) \tag{11}$$

We will denote by $\mathbb{E}^C(x)$ the set of linguistic labels of $x \in U$ with respect to the condition attributes C, and by $\mathbb{E}^D(x)$ the set of linguistic labels of $x \in U$ with respect to the decision attributes D.

Since, in most of the cases (always for a crisp information system), we have only one dominating linguistic value for every attribute $p \in P$, we only get one element in the cartesian product $\mathbb{E}^P(x)$.

Observe that several elements of the universe U can have the same linguistic labels. When we skip the argument x, a linguistic label $E^P \in \mathbb{E}^P$ can be used to denote a class of elements of the universe U which are similar with respect to a linguistic label E^P. Those elements of the universe U will be called the characteristic elements representing of a linguistic label $E^P \in \mathbb{E}^P$.

Definition 4 The set of characteristic elements X_{E^P} representing a linguistic label $E^P \in \mathbb{E}^P$, for a subset of fuzzy attributes $P \subseteq Q$, is a set of those elements $x \in U$ which possess the linguistic label $E^P \in \mathbb{E}^P$

$$X_{E^P} = \{x \in U : E^P \in \mathbb{E}^P(x)\}. \tag{12}$$

We can also interpret a decision table using the notions introduced in the framework of the crisp and fuzzy flow graph approach [9, 11, 12]. In such a case, a linguistic label $E^P \in \mathbb{E}^P$ denotes a unique path in the flow graph. The characteristic elements X_{E^P} of a linguistic label $E^P \in \mathbb{E}^P$ is a set of those elements $x \in U$ which flow (mainly) through the same path of the flow graph.

A single linguistic label $E^P \in \mathbb{E}^P$ is an ordered tuple of dominating linguistic values for all attributes $p \in P$: $E^P = (\widehat{V}_1, \widehat{V}_2, \ldots, \widehat{V}_{|P|})$, where $|P|$ denotes the cardinality of the set P. The resulting membership degree of an element $x \in U$ in the linguistic label $E^P \in \mathbb{E}^P$ can be determined by

$$\mu_{E^P}(x) = \min(\mu_{\widehat{V}_1}(x), \mu_{\widehat{V}_2}(x), \ldots, \mu_{\widehat{V}_{|P|}}(x)) \tag{13}$$

We should note that a higher membership degree in the linguistic label E^P can be obtained for its characteristic elements only.

Definition 5 For a finite universe U with N elements, the fuzzy similarity class of the elements of the universe U to the linguistic label E^P is a fuzzy set denoted by $\tilde{E}^P$ and defined as follows

$$\tilde{E}^P = \{\mu_{E^P}(x_1)/x_1, \mu_{E^P}(x_2)/x_2, \ldots, \mu_{E^P}(x_N)/x_N\}. \tag{14}$$

Definition 6 The set X_A of characteristic elements of a fuzzy set A is defined as

$$X_A = \{\mathrm{x} \in U\colon \mu_A(x) \geq 0.5\}. \tag{15}$$

Now, we are ready to define the lower and upper approximations of a fuzzy set A.

Definition 7 The lower approximation $\underline{\mathbb{E}^P}(\mathrm{A})$ of a fuzzy set A by the set of linguistic labels $\mathbb{E}^P$, with respect to a subset of fuzzy attributes $P \subseteq Q$, is expressed as

$$\underline{\mathbb{E}^P}(A) = \bigcup_{E^P \in \mathbb{E}^P} \tilde{E}^P\colon X_{E^P} \subseteq X_A \tag{16}$$

Definition 8 The upper approximation $\underline{\mathbb{E}^P}(\mathrm{A})$ of a fuzzy set A by the set of linguistic labels $\mathbb{E}^P$, with respect to a subset of fuzzy attributes $P \subseteq Q$, is expressed as

$$\overline{\mathbb{E}^P}(A) = \bigcup_{E^P \in \mathbb{E}^P} \tilde{E}^P : X_{E^P} \cap X_A \neq \emptyset \tag{17}$$

It can be proved that the approximations (16) and (17) are equivalent to the crisp approximations (8) and (9), in the special case of a crisp information system.

In order to evaluate the consistency of a fuzzy information system IFS, we require an adequate form of a widely used measure of approximation quality.

Definition 9 The approximation quality of the fuzzy similarity classes $\tilde{E}^D$ which are determined with respect to the decision attributes D, by the fuzzy similarity classes $\tilde{E}^C$ obtained with respect to the condition attributes C, is defined as

$$\gamma_C\left(\mathbb{E}^D\right) = \frac{\mathrm{power}\left(\mathrm{Pos}_C\left(\mathbb{E}^D\right)\right)}{\mathrm{card}\,\mathrm{U}} \tag{18}$$

$$\mathrm{Pos}_C(\mathbb{E}^D) = \bigcup_{E^D \in \mathbb{E}^D} \underline{\mathbb{E}^C}(\tilde{E}^D) \tag{19}$$

In the next section, we illustrate all presented notions by a computational example.

4 Example

Let us consider a decision table (Table 1) with three fuzzy condition attributes c_1, c_2, c_3, and one fuzzy decision attribute d_1. The condition attributes have two or three linguistic values: C_{11}, C_{12}, C_{21}, C_{22}, C_{31}, C_{32}, C_{33}, respectively. The decision attribute d can possess two linguistic values D_{11} and D_{12}. The membership functions of all linguistic values have triangular or trapezoidal shapes and they satisfy the requirement (3) of summing up to 1 for every element of the universe.

First, we want to determine the similarity between all elements of the universe for the condition attributes C, using the standard fuzzy similarity relation (6). We get a symmetric fuzzy similarity matrix given in Table 2.

In the same way, the similarity between all elements of the universe with respect to the decision attribute d is determined (Table 3).

We should notice that the rows of the obtained similarity matrices are unique. In consequence, we get six similarity classes generated with respect to the decision attribute that will be approximated by six similarity classes determined for the condition attributes (36 approximations).

Next, we perform the analysis the considered decision table basing on our approach presented in previous section. Table 4 contains the linguistic labels determined with respect to all condition attributes.

We obtain the fuzzy similarity classes to the linguistic labels for the condition attributes C:

$$\tilde{E}_1^C = \{0.80/x_1, 0.10/x_2, 0.70/x_3, 0.00/x_4, 0.10/x_5, 0.15/x_6\},$$
$$\tilde{E}_2^C = \{0.00/x_1, 0.85/x_2, 0.00/x_3, 0.00/x_4, 0.25/x_5, 0.80/x_6\},$$
$$\tilde{E}_3^C = \{0.20/x_1, 0.00/x_2, 0.00/x_3, 0.65/x_4, 0.00/x_5, 0.00/x_6\},$$
$$\tilde{E}_4^C = \{0.00/x_1, 0.10/x_2, 0.00/x_3, 0.00/x_4, 0.75/x_5, 0.15/x_6\},$$

the fuzzy similarity classes to the linguistic labels for the decision attribute d:

$$\tilde{E}_1^D = \{1.00/x_1, 0.10/x_2, 0.90/x_3, 0.00/x_4, 0.70/x_5, 0.20/x_6\},$$
$$\tilde{E}_2^D = \{0.00/x_1, 0.90/x_2, 0.10/x_3, 1.00/x_4, 0.30/x_5, 0.80/x_6\}.$$

Table 1 Decision table with fuzzy attributes

	c_1		c_2		c_3			d_1	
	C_{11}	C_{12}	C_{21}	C_{22}	C_{31}	C_{32}	C_{33}	D_{11}	D_{12}
x_1	**0.80**	0.20	**0.90**	0.10	0.20	**0.80**	0.00	**1.00**	0.00
x_2	0.10	**0.90**	0.15	**0.85**	0.00	0.10	**0.90**	0.10	**0.90**
x_3	**0.70**	0.30	**0.80**	0.20	0.00	**1.00**	0.00	**0.90**	0.10
x_4	0.00	**1.00**	**0.70**	0.30	**0.65**	0.35	0.00	0.00	**1.00**
x_5	**0.75**	0.25	0.10	**0.90**	0.00	0.20	**0.80**	**0.70**	0.30
x_6	0.15	**0.85**	0.20	**0.80**	0.00	0.15	**0.85**	0.20	**0.80**

Table 2 Fuzzy similarity matrix with respect to condition attributes

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1.00	0.10	0.80	0.20	0.20	0.15
x_2	0.10	1.00	0.10	0.10	0.35	0.95
x_3	0.80	0.10	1.00	0.30	0.20	0.15
x_4	0.20	0.10	0.30	1.00	0.20	0.15
x_5	0.20	0.35	0.20	0.20	1.00	0.40
x_6	0.15	0.95	0.15	0.15	0.40	1.00

Table 3 Fuzzy similarity matrix with respect to decision attribute

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	1.00	0.10	0.90	0.00	0.70	0.20
x_2	0.10	1.00	0.20	0.90	0.40	0.90
x_3	0.90	0.20	1.00	0.10	0.80	0.30
x_4	0.00	0.90	0.10	1.00	0.30	0.80
x_5	0.70	0.40	0.80	0.30	1.00	0.50
x_6	0.20	0.90	0.30	0.80	0.50	1.00

Table 4 Linguistic labels with respect to condition attributes

	E_1^C	E_2^C	E_3^C	E_4^C
	(C_{11}, C_{21}, C_{32})	(C_{12}, C_{22}, C_{33})	(C_{12}, C_{21}, C_{31})	(C_{11}, C_{22}, C_{33})
x_1	**0.80**	0.00	0.20	0.00
x_2	0.10	**0.85**	0.00	0.10
x_3	**0.70**	0.00	0.00	0.00
x_4	0.00	0.00	**0.65**	0.00
x_5	0.10	0.25	0.00	**0.75**
x_6	0.15	**0.80**	0.00	0.15

Now, we approximate the fuzzy similarity classes $\tilde{E}_1^D$, and $\tilde{E}_2^D$, by the fuzzy similarity classes $\tilde{E}_1^C, \tilde{E}_2^C, \tilde{E}_3^C$, and $\tilde{E}_4^C$. Observe that we need to calculate at most 8 approximations. According to formulae (17) and (18), we get the lower approximations

$$\underline{\mathbb{E}}^C(\tilde{E}_1^D) = \tilde{E}_1^C \cup \tilde{E}_4^C, \quad \underline{\mathbb{E}}^C(\tilde{E}_2^D) = \tilde{E}_2^C \cup \tilde{E}_3^C,$$

and the upper approximations

$$\overline{\mathbb{E}}^C(\tilde{E}_1^D) = \tilde{E}_1^C \cup \tilde{E}_4^C, \quad \overline{\mathbb{E}}^C(\tilde{E}_2^D) = \tilde{E}_2^C \cup \tilde{E}_3^C.$$

Since, the lower approximations are equal to the upper approximations, we obtain the following certain decision rules:

$$C_{11}C_{21}C_{32} \rightarrow D_{11}, \quad C_{11}C_{22}C_{33} \rightarrow D_{11},$$
$$C_{12}C_{22}C_{33} \rightarrow D_{12} \quad C_{12}C_{21}C_{31} \rightarrow D_{12}.$$

Finally, we calculate the approximation quality $\gamma_C(\mathbb{E}^D)$ of the fuzzy similarity classes $\tilde{E}^D$ by the fuzzy similarity classes $\tilde{E}^C$. The operator `max` was applied for determination of the sum of fuzzy sets. We get $\gamma_C(\mathbb{E}^D) = 4.55/6 = 0.76$.

Let us now omit the fuzzy condition attribute c1. We repeat the above steps for a reduced information system with the set of condition attributes denoted by C' (Table 5).

We obtain the lower approximations

$$\underline{\mathbb{E}}^{C'}(\tilde{E}_1^D) = \tilde{E}_1^{C'}, \quad \underline{\mathbb{E}}^{C'}(\tilde{E}_2^D) = \tilde{E}_3^{C'},$$

and the upper approximations

$$\overline{\mathbb{E}}^{C'}(\tilde{E}_1^D) = \tilde{E}_1^{C'} \cup \tilde{E}_2^{C'}, \quad \overline{\mathbb{E}}^{C'}(\tilde{E}_2^D) = \tilde{E}_2^{C'} \cup \tilde{E}_3^{C'}.$$

Because the lower approximations are not equal to the upper approximations, the reduced information system is not consistent. We have now two certain decision rules

$$C_{21}C_{32} \rightarrow D_{11}, \quad C_{21}C_{31} \rightarrow D_{12},$$

and two uncertain decision rules

$$C_{22}C_{33} \rightarrow D_{11}, \quad C_{22}C_{33} \rightarrow D_{12}.$$

The approximation quality $\gamma_{C'}\left(\mathbb{E}^D\right) = 2.60/6 = 0.43$. Since the quality of approximation has significantly decreased, the attribute c_1 cannot be removed from the information system. After analyzing the influence of every fuzzy condition attribute, we find that the attributes c_2, c_3 could be removed from the fuzzy information system.

Summarizing, the proposed method is suitable for investigating the properties of fuzzy information systems. We are able to determine the dependencies between groups of attributes, evaluate the approximation quality, find which attributes can be removed, and finally, obtain a set of decision rules. The presented method requires less computation, in comparison to methods which base on a fuzzy similarity relation.

Table 5 Linguistic labels with respect to condition attributes (omitted attribute c_1)

	$E_1^{C'}$ (C_{21}, C_{32})	$E_2^{C'}$ (C_{22}, C_{33})	$E_3^{C'}$ (C_{21}, C_{31})
x_1	**0.80**	0.00	0.00
x_2	0.10	**0.85**	0.00
x_3	**0.80**	0.00	0.00
x_4	0.35	0.00	**0.65**
x_5	0.10	**0.80**	0.00
x_6	0.15	**0.80**	0.00

5 Conclusions

We propose a new way of determination of fuzzy similarity classes with respect to linguistic values of particular condition and decision attributes. This is a simpler and more effective method than the standard approach, in which a fuzzy similarity needs to be computed. The introduced notions can be further developed and applied in various fuzzy rough sets models. Especially, an extension of the variable precision fuzzy rough set model is possible. Furthermore, the mathematical properties of the modified fuzzy rough approximations should be investigated in future research.

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